# Theory of $\Omega^-$ Decay*

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The $\Omega^-$ nonleptonic decays are investigated in the framework of a chiral Lagrangian model. In this way, ambiguities of extension of meson momenta to zero which are present in the current-algebra approach are made explicit. We find that these decays may possess a rather complicated structure and a discussion of approximation schemes is given which may help to explain the present experimental data. The approximation of the $\Xi 2\pi$ decay mode by $\Xi^*\pi$ is also discussed.

## I. INTRODUCTION

In this paper we shall investigate the nonleptonic decay modes of the $\Omega^-$ particle in a phenomenological chiral Lagrangian model. A similar model[1,2] was previously considered for the nonleptonic decays of the ordinary octet hyperons. It was found that in this way the "current-algebra" results[3] could be obtained very simply, thus eliminating the usual delicate arguments that are needed when the pion four-momentum is extrapolated to zero. Furthermore, the phenomenological Lagrangian gives terms which may make sizable contributions but which vanish in the zero-pion-momentum limit. This may help to clear up the discrepancy between the current-algebra $p$-wave prediction and experimental results.

Two treatments[4,5] of the $\Omega^-$ decay in the current-algebra scheme have already appeared. However, the newer experimental data[6] seem to disagree with these predictions. This situation gives a motivation for studying the $\Omega^-$ decays in more detail. We also investigate additional (rare) nonleptonic decay modes.

We remark that the $\Omega^-$ nonleptonic decays are not simply isolated curiosities but involve interactions which also appear in ordinary hyperon decays. This point, coupled with the fact that the energy is somewhat higher than in the hyperon case, suggests that these processes may be among the most useful probes of the structure of the hadronic weak interactions.

The following nonleptonic modes are allowed by energy conservation:

$$\Omega_-^-:\quad \Omega^- \to \Xi^0\pi^- \quad (217)\,, \tag{1a}$$

$$\Omega_0^-:\quad \Omega^- \to \Xi^-\pi^0 \quad (216)\,, \tag{1b}$$

$$\Omega_K^-:\quad \Omega^- \to \Lambda^0 K^- \quad (61)\,, \tag{1c}$$

$$\Omega^*{}_-^-:\quad \Omega^- \to \Xi^{*0}\pi^- \quad (3.5\pm2.1)\,, \tag{2a}$$

$$\Omega^*{}_0^-:\quad \Omega^- \to \Xi^{*-}\pi^0 \quad (3.3\pm2.9)\,, \tag{2b}$$

$$\Omega_{-0}^-:\quad \Omega^- \to \Xi^0\pi^-\pi^0 \quad (82)\,, \tag{3a}$$

$$\Omega_{00}^-:\quad \Omega^- \to \Xi^0\pi^0\pi^0 \quad (81)\,, \tag{3b}$$

$$\Omega_{+-}^-:\quad \Omega^- \to \Xi^-\pi^+\pi^- \quad (72)\,. \tag{3c}$$

In the above, the $Q$ value in MeV for each decay has been indicated in parentheses. It is clear that the decays (2) are really special cases of (3). However, in the present method of calculation they may be distinguished (see Fig. 1), and the question of how well (2) approximates to (3) can be discussed. It may also be possible to discriminate between (2) and (3) experimentally when it is remembered that the $Q$ values of (2) are quite small and the width[6] of $\Xi^*$ is only about 7.3 MeV. A predominance of very-low-energy $\pi^0$ or $\pi^-$ mesons in the $\Omega^-$ rest frame would probably mean that (2) holds.

For the most common decay modes (1), the general matrix element may be written as

$$\bar{u}(p')(A+B\gamma_5)k_\lambda u_\lambda(p)\,, \tag{4}$$

where $u_\lambda(p)$ is the $\Omega^-$ wave function, $\bar{u}(p')$ the $\Xi$ or $\Lambda$ wave function, and $k_\lambda$ the meson four-momentum. $A$ is a real quantity corresponding to the parity-conserving "$p$-wave" transition, while $B$ is a real quantity corresponding to parity-violating "$d$-wave" transition. The formula for the decay width in terms of $A$ and $B$ is given in Appendix A. If $\Delta I=\frac{1}{2}$ transitions are dominant for the $\Xi\pi$ mode, we would have the amplitude rule

$$\Omega_0^- = (\sqrt{\tfrac{1}{2}})\Omega_-^-\,. \tag{5}$$

The very scanty present experimental results[6] support this rule.

For the rare modes (2), we have the general amplitude

$$\bar{u}_\mu(p')[C_1\delta_{\mu\nu}+C_2\gamma_5\delta_{\mu\nu}+C_3p_\mu p_\nu'+C_4\gamma_5 p_\mu p_\nu']u_\nu(p)\,. \tag{6}$$

The decay rate is also given in Appendix A, and the $\Delta I=\frac{1}{2}$ rule reads

$$\Omega^*{}_0^- = -(\sqrt{\tfrac{1}{2}})\Omega^*{}_-^-\,. \tag{7}$$

Finally, decays into $\Xi$, $\pi^a(k_1)$, and $\pi^b(k_2)$ can be

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[1] J. Schechter, Phys. Rev. **174**, 1829 (1968).
[2] B. W. Lee, Phys. Rev. **170**, 1359 (1968).
[3] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997 (1965); M. Suzuki, *ibid.* **15**, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); S. Badier and C. Bouchiat, Phys. Letters **20**, 529 (1966); L. S. Brown and C. Sommerfield, Phys. Rev. Letters **16**, 751 (1966).
[4] Y. Hara, Phys. Rev. **150**, 1175 (1966). See also R. Rajaraman, Phys. Letters **22**, 102 (1966).
[5] X. Y. Pham and R. Zaoui, Phys. Rev. **167**, 1319 (1968).
[6] See Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

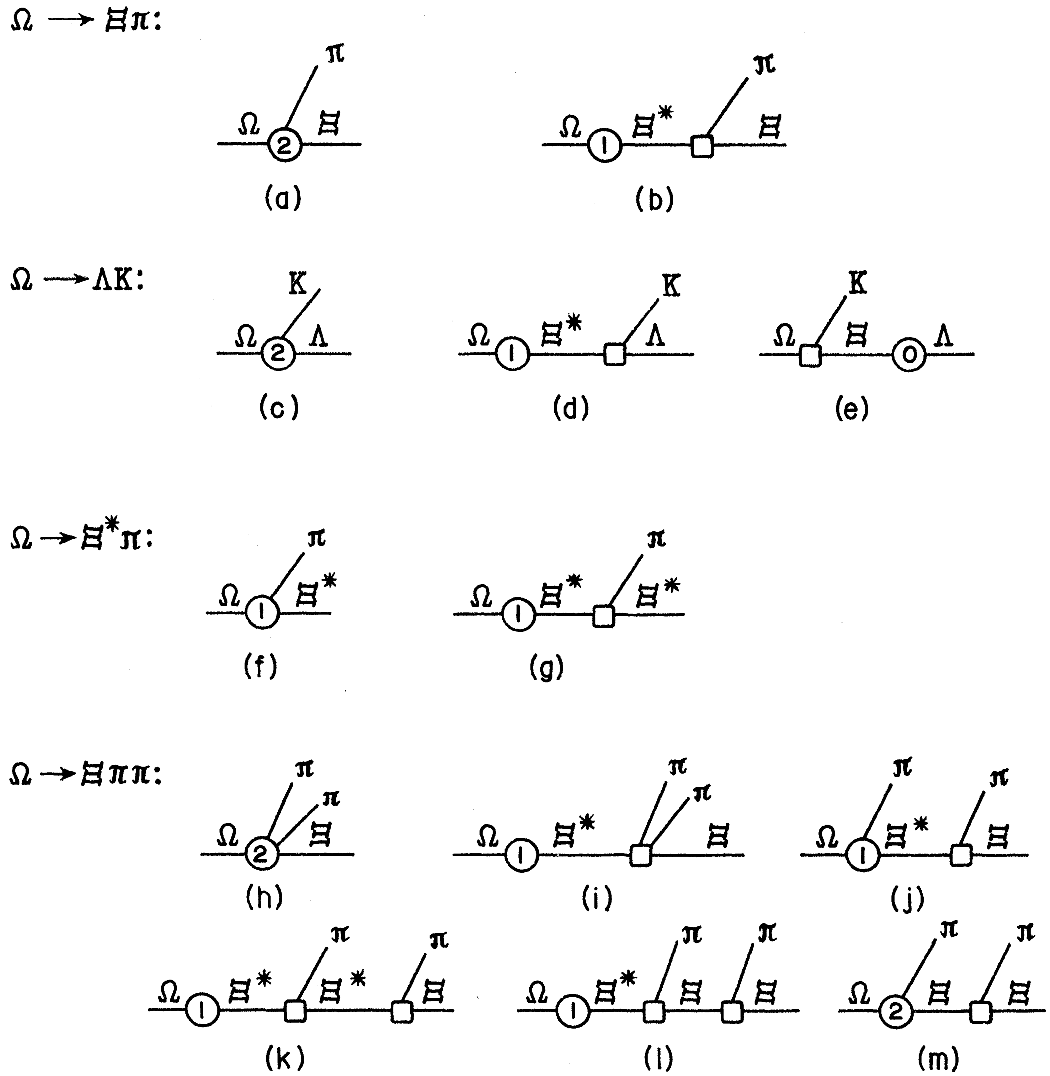


FIG. 1. Diagrams for nonleptonic $\Omega^-$ decay. The squares indicate strong vertices, while the circles indicate weak vertices of various types.

represented by the general matrix element

$$\begin{aligned} M_{ab}(k_1,k_2) &= \bar{u}(p')[\{F_1+F_2\gamma_5+iF_3(\gamma\cdot k_1)+iF_4(\gamma\cdot k_1)\gamma_5\}(k_1)_\mu \\ &\quad +\{G_1+G_2\gamma_5+iG_3(\gamma\cdot k_1) \\ &\quad +iG_4(\gamma\cdot k_1)\gamma_5\}(k_2)_\mu]u_\mu(p)\,. \end{aligned} \tag{8}$$

In this case the $\Delta I=\frac{1}{2}$ predictions take the form

$$M_{00}(k_1,k_2)=\tfrac{1}{2}[M_{+-}(k_1,k_2)+M_{+-}(k_2,k_1)]\,, \tag{9a}$$

$$M_{0-}(k_1,k_2)=-(\sqrt{\tfrac{1}{2}})[M_{+-}(k_1,k_2)-M_{+-}(k_2,k_1)]\,. \tag{9b}$$

These may be combined to give

$$M_{+-}-M_{00}+(\sqrt{\tfrac{1}{2}})M_{0-}=0\,. \tag{10}$$

The decay rate corresponding to Eq. (8) is also given in Appendix A.

## II. CALCULATION OF DECAY AMPLITUDES

We will construct an effective Lagrangian out of octet baryons, decuplet baryons, and octet pseudoscalar mesons. The octet baryons will be taken to transform as [(8,1), (1,8)] under the chiral $SU(3)\times SU(3)$ group so that in a representation of the Dirac matrices where $\gamma_5$ is diagonal, we have

$$N_b{}^a=\begin{pmatrix} L_b{}^a \\ R_{b'}{}^{a'} \end{pmatrix},\quad \bar{N}_b{}^a=(\bar{R}_{b'}{}^{a'},\bar{L}_b{}^a)\,, \tag{11}$$

where unprimed indices refer to the "left-handed" $SU(3)$ and primed indices to the "right-handed" $SU(3)$. The decuplet baryons then can be described[7] by the

[7] J. Schechter and Y. Ueda, Phys. Rev. 177, 2300 (1969).

[(1,10), (10,1)] representation

$$D_{\mu abc}=\begin{pmatrix} L_{\mu a'b'c'} \\ R_{\mu abc}\end{pmatrix},\quad \bar{D}_\mu{}^{abc}=(\bar{R}_\mu{}^{abc},\bar{L}_\mu{}^{a'b'c'}),\quad (12)$$

where our normalization is such that $\Omega^-{}_\mu=(\sqrt{\tfrac{1}{6}})D_{\mu 333}$. The pseudoscalar mesons[8] will be taken to transform nonlinearly but the combinations

$$M_a{}^{b'}=\delta_a{}^b+2if^2\phi_a{}^b-2f^2(\phi^2)_a{}^b+\cdots,\quad (13a)$$

$$M_{a'}{}^b=\delta_a{}^b-2if^2\phi_a{}^b-2f^2(\phi^2)_a{}^b+\cdots,\quad (13b)$$

satisfying

$$M_a{}^{b'}M_{b'}{}^c=\delta_a{}^{c'},\quad (14a)$$

$$M_{a'}{}^bM_b{}^{c'}=\delta_{a'}{}^{c'},\quad (14b)$$

transform as (3,3*) and (3*,3). The quantity $f$ is identical[8] to the pion decay constant and is numerically close to $1.0\mu^{-1}$ ($\mu$ is the pion mass). Although, as written, the baryons transform linearly under chiral $SU(3)\times SU(3)$, it is possible to perform an equivalence transformation to obtain new baryon fields which transform nonlinearly but which yield the same matrix elements. Which procedure is adopted is evidently a matter of taste.

The characteristic feature of a chiral-invariant interaction (or an interaction with simple chiral transformation properties) is that, because of the presence of $M_a{}^{b'}$, vertices with any number of pions coming out are present and are all related to each other in powers of $f$. For example, the nucleon vertex with no pions (mass term) is related to the nucleon vertex with one pion (Yukawa-type term), etc. This is the essential content of the Goldberger-Treiman relation. A similar situation exists for the weak vertices as previously noted, so that the relation between the $s$-wave and $p$-wave hyperon decays is the weak analog of the Goldberger-Treiman relation. For the case of $\Omega^-$ decay, all possible ("tree"-type) diagrams involving at least two baryons at the weak vertex are shown in Fig. 1. In order to compute these, it is necessary to have at hand both the strong (square) vertices and the different types of weak (round) vertices.

We need the strong vertices which involve at least one decuplet particle. The usual strong vertices involving two baryon-octet lines are given, for example, in Eq. (7) of Ref. 1. The vertices which involve two decuplet lines come from the decuplet mass term in the Lagrangian, which may be written as

$$\begin{aligned}-\tfrac{1}{6}m(N^*)[&\bar{L}_\mu{}^{a'b'c'}R_{\mu def}M_{a'}{}^dM_{b'}{}^eM_{c'}{}^f\\ &+\bar{R}_\mu{}^{abc}L_{\mu d'e'f'}M_a{}^{d'}M_b{}^{e'}M_c{}^{f'}]\\ &-\tfrac{1}{2}[m(Y^*)-m(N^*)]\bar{D}_\mu{}^{3bc}D_{\mu 3bc}\\ =-\tfrac{1}{6}&m(N^*)\bar{D}_\mu{}^{abc}D_{\mu abc}\\ &-ifm(N^*)\bar{D}_\mu{}^{abc}\gamma_5D_{\mu dbc}\phi_a{}^d+\cdots\\ &-\tfrac{1}{2}[m(Y^*)-m(N^*)]\bar{D}_\mu{}^{3bc}D_{\mu 3bc}.\end{aligned}\quad (15)$$

In (15) the last term breaks the $SU(3)$ as well as the chiral symmetry in order to give each member of the decuplet its proper mass. It is evident that we can arrange things so that the "degenerate" mass multiplying the chiral-invariant term in (15) is something other than $m(N^*)$. However, based on an analogy to the nucleon case,[9] the present choice seems reasonable. The $SU(3)$-breaking term can be written (using the $M$'s) with different chiral transformation properties but we shall not discuss this point here.

The strong vertices involving one decuplet line may result from the following chiral-invariant part of the Lagrangian:

$$\begin{aligned}\tfrac{1}{2}ih[&\epsilon^{ebd}\bar{L}_d{}^cR_{\mu abc}M_{f'}{}^a\partial_\mu M_e{}^{f'}\\ &-\epsilon^{e'b'd'}\bar{R}_{d'}{}^{c'}L_{\mu a'b'c'}M_f{}^{a'}\partial_\mu M_{e'}{}^f]+\text{H.c.}\\ =-&hf\epsilon^{ebd}\bar{N}_d{}^cD_{\mu abc}\partial_\mu\phi_e{}^a\\ &-ihf^2\epsilon^{ebd}\bar{N}_d{}^c\gamma_5D_{\mu abc}\phi_f{}^a\overleftrightarrow{\partial_\mu}\phi_e{}^f+\cdots.\end{aligned}\quad (16)$$

The unknown parameter $h$ in (16) may be determined from the widths of any of the decuplet particles, assuming symmetry breaking to be unimportant. From the $N^*$ width, we find $|h|\simeq0.88$, while from the $\Xi^*$ width, we find $|h|\simeq0.53$. The range gives us an idea of the $SU(3)$ breaking for decuplet decays if a decay interaction without form factors is used. Since in the present context we will be interested in $\Xi^*$ couplings, $|h|\simeq0.53$ seems more reasonable.

The construction of the weak vertices is somewhat more complicated and involves a certain number of assumptions:

(i) The $\Delta I=\tfrac{1}{2}$ rule will be assumed for simplicity, since this is generally roughly true for other nonleptonic decays and seems presently[10] to be all right for $\Omega^-\to\Xi\pi$.

(ii) The chiral transformation property of the effective weak interaction will be taken to be $T_3{}^2+T_2{}^3$, where all indices are unprimed. This would correspond to a fundamental "quark-type" current-current interaction with octet dominance (or $\Delta I=\tfrac{1}{2}$ rule) assumed. We remark that it is not necessarily clear that the induced effective interaction need have the same transformation properties as the fundamental one (since symmetry breaking may be rather important).

(iii) Weak vertices not involving a baryon will not be considered in the present paper (neither will strong vertices involving mesons other than the pseudoscalar octet).

(iv) The weak vertex of each type will be constructed with the minimum number of derivatives necessary. This assumption is made to reduce the number of arbitrary parameters appearing in the theory.

Figure 1(e) shows that the weak octet-octet spurion of ordinary hyperon decays is needed. This will be taken

[8] See J. A. Cronin, Phys. Rev. **161**, 1483 (1967).

[9] J. Schechter, Y. Ueda, and G. Venturi, Phys. Rev. **177**, 2311 (1969).

[10] Reference 6 gives $\Gamma(\Omega_0^-)/\Gamma(\Omega_-^-)=\tfrac{3}{8}$ for 11 events, while the prediction of Eq. (5) is $\tfrac{1}{2}$.

as the weak interaction of type 0:

$$
\begin{aligned}
H_w{}^{(0)} &= (\delta+\phi)[\bar{L}_a{}^2 M_{b'}{}^a R_{c'}{}^{b'} M_3{}^{c'} + \bar{R}_{b'}{}^{c'} M_{c'}{}^3 L_2{}^a M_a{}^{b'}] \\
&\qquad + (\delta-\phi)[\bar{L}_3{}^a M_a{}^{b'} R_{b'}{}^{c'} M_{c'}{}^2 + \bar{R}_{c'}{}^{b'} M_2{}^{c'} L_a{}^3 M_{b'}{}^a] + (2\leftrightarrow 3) \\
&= (\delta+\phi)[\bar{N}_a{}^2 N_3{}^a + 2if\bar{N}_a{}^2\gamma_5 N_3{}^b\phi_b{}^a + if\bar{N}_a{}^3(1-\gamma_5)N_c{}^a\phi_2{}^c] \\
&\qquad + (\delta-\phi)[\bar{N}_3{}^a N_a{}^2 - 2if\bar{N}_3{}^a\gamma_5 N_b{}^2\phi_a{}^b - if\bar{N}_3{}^a(1-\gamma_5)N_a{}^c\phi_c{}^2] + \text{H.c.} + \cdots. \qquad (17)
\end{aligned}
$$

Fitting the $s$-wave hyperon decays gives $\phi \simeq -3\delta \simeq 3\times 10^{-7}\mu$, while fitting the $p$-wave decays[1] (and adjusting the $s$ waves with a $K^*$-$\pi$ pole diagram) gives $\phi \simeq -\frac{4}{3}\delta \simeq 6\times 10^{-7}\mu$.

The type-1 weak interaction involving two decuplet lines may be written as

$$
\begin{aligned}
H_w{}^{(1)} &= g_1[\bar{L}_\mu{}^{a'b'c'} R_{\mu 3ef} M_{a'}{}^2 M_{b'}{}^e M_{c'}{}^f + \bar{R}_\mu{}^{2bc} L_{\mu d'e'f'} M_3{}^{d'} M_b{}^{e'} M_c{}^{f'} + (2\leftrightarrow 3)] \\
&= g_1[\bar{D}_\mu{}^{2bc} D_{\mu 3bc} + 4if\bar{D}_\mu{}^{2bc}\gamma_5 D_{\mu 3ec}\phi_b{}^e - if\bar{D}_\mu{}^{abc}(1-\gamma_5)D_{\mu 3bc}\phi_a{}^2 \\
&\qquad + if\bar{D}_\mu{}^{2bc}(1+\gamma_5)D_{\mu dbc}\phi_3{}^d + (2\leftrightarrow 3) + \cdots], \qquad (18)
\end{aligned}
$$

where $g_1$ is *a priori* unknown but may be estimated on the basis of certain models.

We define a type-2 weak vertex as one involving a decuplet line and a baryon-octet line. The most general form containing only one derivative is

$$
\begin{aligned}
H_w{}^{(2)} &= i(a_1\epsilon^{2bc}\bar{L}_3{}^d R_{\mu bde} S_{\mu c}{}^e + a_2\epsilon^{a2c}\bar{L}_a{}^d R_{\mu 3de} S_{\mu c}{}^e + a_3\epsilon^{ab2}\bar{L}_a{}^d R_{\mu bde} S_{\mu 3}{}^e + a_4\epsilon^{abc}\bar{L}_a{}^2 R_{\mu b3e} S_{\mu c}{}^e + a_5\epsilon^{abc}\bar{L}_a{}^d R_{\mu bd3} S_{\mu c}{}^2) \\
&\quad + i\epsilon^{a'b'c'}[d_1\bar{L}_d{}^2 R_{\mu 3fg} M_{a'}{}^d \partial_\mu M_{b'}{}^f M_{c'}{}^g + d_2\bar{L}_3{}^e R_{\mu efg} M_{a'}{}^2 \partial_\mu M_{b'}{}^f M_{c'}{}^g \\
&\quad + \bar{L}_d{}^e R_{\mu ef3}(d_3\partial_\mu M_{a'}{}^d M_{b'}{}^2 M_{c'}{}^f + d_4 M_{a'}{}^d \partial_\mu M_{b'}{}^2 M_{c'}{}^f + d_5 M_{a'}{}^d M_{b'}{}^2 \partial_\mu M_{c'}{}^f)] \\
&\quad + i\epsilon^{a2c}\bar{R}_{d'}{}^{e'} L_{\mu e'f'g'}[e_1 M_3{}^{d'} \partial_\mu M_c{}^{f'} M_a{}^{g'} + e_2 \partial_\mu M_a{}^{d'} M_3{}^{f'} M_c{}^{g'} \\
&\qquad + e_3 M_a{}^{d'} \partial_\mu M_3{}^{f'} M_c{}^{g'} + e_4 M_a{}^{d'} M_3{}^{f'} \partial_\mu M_c{}^{g'}] + \text{H.c.}, \qquad (19)
\end{aligned}
$$

where $S_{\mu b}{}^a = M_{c'}{}^a \overleftrightarrow{\partial}_\mu M_b{}^{c'} = 2M_{c'}{}^a \partial_\mu M_b{}^{c'}$ and the $a_i$'s, $d_i$'s, and $e_i$'s are real arbitrary constants. Although there appear to be a horrible number of arbitrary parameters in (19), the situation is actually not so bad when we expand it to give the terms which are interesting for $\Omega^-$ decay. These may be written as

$$
\begin{aligned}
H_w{}^{(2)} &= f[(\surd\tfrac{1}{2})\bar{\Xi}^+(\rho_1+\rho_2\gamma_5)\Omega_\mu{}^-\partial_\mu\pi^0 + \bar{\Xi}^0(\rho_1+\rho_2\gamma_5)\Omega_\mu{}^-\partial_\mu\pi^+ + (2/\surd 6)\bar{\Lambda}(\rho_3+\rho_4\gamma_5)\Omega_\mu{}^-\partial_\mu K^+] \\
&\quad - 2if^2\{\bar{\Xi}^+(\rho_5+\rho_6\gamma_5)\Omega_\mu{}^-\pi^+\partial_\mu\pi^- + \bar{\Xi}^+(\rho_7+\rho_8\gamma_5)\Omega_\mu{}^-\pi^-\partial_\mu\pi^+ + \tfrac{1}{2}\bar{\Xi}^+[(\rho_5+\rho_7)+(\rho_6+\rho_8)\gamma_5]\Omega_\mu{}^-\pi^0\partial_\mu\pi^0 \\
&\qquad - (\surd\tfrac{1}{2})\bar{\Xi}^0(\rho_5-\rho_7)(1-\gamma_5)\Omega_\mu{}^-\pi^+\overleftrightarrow{\partial}_\mu\pi^0\} + \text{H.c.} + \cdots. \qquad (20)
\end{aligned}
$$

Here we have only the constants[11] $\rho_1 \cdots \rho_8$. The trilinear vertex involving a $K$ meson is unrelated to the others. The other vertices are related by the $\Delta I = \frac{1}{2}$ rule, which are displayed in (20), as well as the equations

$$\rho_6 = -\rho_1 - \rho_2 - \rho_5, \qquad (21a)$$

$$\rho_7 = \tfrac{1}{2}\rho_1 - \tfrac{1}{2}\rho_2 + \rho_5, \qquad (21b)$$

$$\rho_8 = -\tfrac{3}{2}\rho_1 - \tfrac{1}{2}\rho_2 - \rho_5. \qquad (21c)$$

Equations (21) represent additional information over the $\Delta I = \frac{1}{2}$ rule that is given to us by chiral symmetry. It means that of the six possible constants appearing at the trilinear and quadrilinear ($\Omega\Xi$) type-2 vertices, only three are independent.

In writing (19), we did not include terms where the derivative acted on the octet-baryon fields; these can be transformed by partial integration to terms of the type given and also terms where the derivative acts on the decuplet field. Since we are only interested here in diagrams with a physical decuplet particle, the latter terms would vanish when account is taken of the spin-$\frac{3}{2}$ field subsidiary condition $[p_\mu u_\mu(p) = 0]$. Thus in our case a type-2 vertex with no mesons present makes no contribution, and this is the reason that no such diagrams have been included in Fig. 1. However, a vertex of this type could contribute when the spin-$\frac{3}{2}$ particle is virtual. Such diagrams may arise in computing ordinary hyperon decay. Unfortunately, as we have just seen, chiral symmetry does not uniquely relate these vertices to quantities appearing in $\Omega^-$ decay. In fact, the quantity

$$\epsilon^{2ab}\partial_\mu \bar{L}_a{}^c R_{\mu bc3} + \text{H.c.},$$

which does not involve any meson, has the correct chiral transformation property and could contribute only to hyperon decays.

Now using the strong and weak interactions that we have given, it is a straightforward matter to compute the amplitudes corresponding to Fig. 1. In diagrams (j) and (k) it is possible for the $\Xi^*$ propagator to develop a pole in the physical region. To avoid this, we adopt the usual procedure of adding a small imaginary part pro-

[11] The relations between the independent $\rho_i$'s and the parameters of (19) are

$$\rho_1 = (\surd\tfrac{1}{6})(-2a_5+d_3-d_4-e_2),$$
$$\rho_2 = (\surd\tfrac{1}{6})(2a_5-d_3+d_4-e_2),$$
$$\rho_3 = (\surd\tfrac{1}{6})(-2a_1+2a_2+a_4-\tfrac{1}{2}d_1+d_2-d_3+d_5-e_1+e_2-e_4),$$
$$\rho_4 = (\surd\tfrac{1}{6})(2a_1-2a_2-a_4+\tfrac{1}{2}d_1-d_2+d_3-d_5-e_1+e_2-e_4),$$

and

$$\rho_5 = (\surd\tfrac{1}{6})(a_5+d_4+e_2).$$

portional to the $\Xi^*$ width in the denominator. It is seen that diagrams (j) and (k) are essentially the same as (f) and (g).

The results for the amplitudes are given in Appendix B. These provide the framework for the different types of approximation schemes and plausible estimates which can be made.

## III. DISCUSSION OF TWO-BODY DECAY MODES

The only decays of $\Omega^-$ observed so far[6,12] are the two-body modes listed in (1): $\Omega_-^-$, $\Omega_0^-$, and $\Omega_K^-$. The experimental results show that there is no apparent violation of the $\Delta I=\frac{1}{2}$ relation between $\Omega_-^-$ and $\Omega_0^-$. Furthermore, the $\Omega_K^-$ mode seems to be slightly more frequent than the $\Omega_-^-$ and $\Omega_0^-$ modes put together. However, it may be more difficult to observe a $\Xi\pi$ decay than a $\Lambda K$ decay for experimental reasons.[12] Therefore, we should probably reserve final judgment on the data until more events are seen.

Let us first consider a theoretical model containing only type-0 and type-1 weak vertices. This corresponds to the previously considered[4,5] current-algebra-type models. The relevant diagrams are (b), (d), and (e) of Fig. 1. From Appendices A and B we may then completely calculate these decays in terms of the type-1 weak parameter $g_1$ [see Eq. (18)] and the type-0 weak-parameter combination $(3\phi-\delta)$. Since we have experimental findings on both the $\Lambda K$ and $\Xi\pi$ rates, we may then determine these parameters.

It is convenient to make use of the curves of Fig. 2 to present our results. There, the quantity $3\phi-\delta$ is plotted against the branching ratio

$$R=\Gamma(\Omega\to\Lambda K)/\Gamma(\Omega\to\Xi\pi)=R(g_1,(3\phi-\delta))$$

for the experimental $\Omega$ lifetime $\tau$, which is evidently well approximated by

$$1/\tau=\Gamma(\Omega\to\Lambda K)+\Gamma(\Omega\to\Xi\pi)=1/\tau(g_1,(3\phi-\delta)).$$

The motivation for presenting the results in this form is that $R$ seems to be more unreliable[12] than $\tau$, so that it is worthwhile to see how the predictions would change if $R$ changes. The value of $3\phi-\delta$ can be determined from ordinary hyperon decays, as noted after Eq. (17). In this method of presentation, the unknown type-1 weak-coupling parameter $g_1$ has been eliminated for convenience but, of course, is known.

Two cases can be distinguished depending on the relative signs of $g_1$ and $3\phi-\delta$. The same sign would correspond to destructive interference between dia-

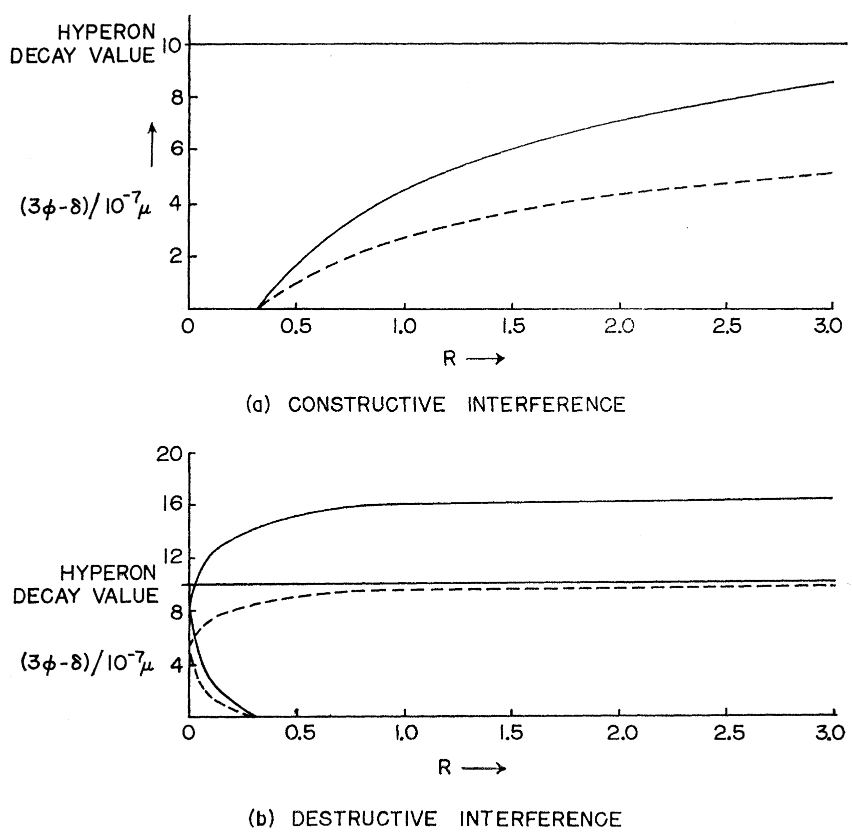


FIG. 2. Type-0 parameter combination $3\phi-\delta$ versus $R$. In each case the dashed curve corresponds to $|h|=0.88$, while the continuous curve corresponds to $|h|=0.53$. Note that in (a) no solution exists for $R<0.32$ and in (b) the solution is two-valued for $R<0.32$.

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[12] R. Speth *et al.*, Phys. Letters **29B**, 252 (1969).

grams 1(d) and 1(e), while the opposite sign would correspond to constructive interference. These cases are shown in Figs. 2(a) and 2(b), respectively. The main curves in Figs. 2 are computed with the value of $|h|$ [see (16)] computed from the $\Xi^*$ width, while the dashed curves were computed with the less plausible value of $|h|$ found from the $N^*$ width.

For orientation, we note that $R$ seems to be,[6,12] with large uncertainty, a little greater than 1. We see from Fig. 2 that the case of constructive interference is ruled out. Furthermore, the case of destructive interference does not lead to a solution with $R>1$ if the most reasonable value of $h$ is used and if $3\phi-\delta$ is obtained by fitting $s$-wave hyperon decays. Even if $3\phi-\delta$ is obtained by fitting $p$-wave hyperon decays, there is no solution. If the less reasonable value of $h$ is used, there exist solutions for $R>1$. This corresponds to $g_1<0.4\times10^{-7}$, which means that the weak decuplet spurion would be rather smaller than the weak octet spurion. Hara's previous solution[4] had a larger $g_1$ but led to very small $R$, which now seems to be ruled out experimentally.

Thus we cannot conclude that a very reasonable solution exists if only type-0 and type-1 weak vertices are included in our model. Of course, a more careful study of symmetry breaking in Eq. (16) is very desirable in this context. Our tentative conclusion is that additional weak interactions, possibly of type-2, are required. Unfortunately reference to Eq. (20) shows that there are too many arbitrary parameters. In the future, when the experimental results on the three-body modes and on the asymmetry parameters become available, we may expect to determine these, however.

A type of diagram which is often considered useful for nonleptonic decays is the meson-pole diagram. This is illustrated for $\Omega\to\Xi\pi$ in Fig. 3. However, by strangeness conservation at the strong vertex, we see that this diagram cannot contribute to $\Omega\to\Lambda K$. Thus a pure meson-pole model would predict $R=0$, in violent disagreement with experiment. Interference effects with the pole diagram may be important though.

Finally, we note that exactly the same weak vertex is found in Eqs. (1b) and (1d) and only the $\Delta I=\frac{1}{2}$ part can contribute to a $\Xi^*$-$\Omega$ transition. Thus the two-body decay modes by themselves give no information on disentangling any possible $\Delta I=\frac{3}{2}$ component which may be present in type-1 weak interaction.

## IV. DISCUSSION OF RARE DECAY MODES

The energetically allowed modes $\Xi^*\pi$ and $\Xi\pi\pi$ have not yet been seen, but since in our model they involve the same parameters as the $\Xi\pi$ and $\Lambda K$ modes, it is expected that they can yield much useful information in the future.

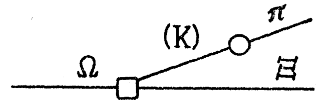


FIG. 3. Meson-pole diagrams for $\Omega\to\Xi\pi$. $(K)$ represents any $S=-1$ meson.

First let us consider the $\Xi\pi\pi$ modes. These correspond to diagrams (h)–(m) of Fig. 1 and the amplitudes are given in Appendix B. The decay rate can be calculated using formula (A3) of Appendix A. Actual details of the method of evaluation are given in Appendix C. Again, since we presently have no information on the type-2 vertices, let us assume for simplicity that they are absent (although we have seen in the previous section that this is doubtful). Then we are to evaluate only diagrams (i)–(l) of Fig. 1. It turns out that diagrams (i) and (l) are less important than (j) and (k). The reason is that the $\Xi^*$ propagator in (j) and (k) passes through resonance in the physical region. We note that these two graphs are essentially the same as (f) and (g), so that the approximation of the $\Xi\pi\pi$ mode by $\Xi^*\pi$ is not too bad. If the type-2 interaction turns out to be large, we cannot, of course, draw this conclusion.

For a rough estimate of the branching ratio,

$$\begin{aligned} R_1 &= \Gamma(\Omega\to\Xi\pi\pi)/\Gamma(\Omega\to\Lambda K) \\ &\simeq \Gamma(\Omega\to\Xi^*\pi)/\Gamma(\Omega\to\Lambda K), \end{aligned}$$

we may use diagrams (f) and (g) with the value of $g_1$ determined by fitting $R$ and $\tau$ (disregarding whether or not $3\phi-\delta$ is the value which fits hyperon decay). Then we find

$$\begin{aligned} g_1 &= 0.6\times10^{-7}, \\ 3\phi-\delta &= 16.2\times10^{-7}, \\ R_1 &\simeq 1/20. \end{aligned}$$

If $R_1$ should turn out to be much less than this, it might be interpreted to mean that the type-1 weak interaction is very small compared to the type-2, since diagrams (f) and (g) show that only type-1 vertices contribute in the $\Xi^*\pi$ approximation.

Finally, we may point out that, since in the case of the $\Xi\pi\pi$ mode the $\pi^+$-$\pi^0$, $\Xi^0$-$\Xi^-$, and $\Xi^{*0}$-$\Xi^{*-}$ mass differences are not completely negligible fractions of the $Q$ value of the decay, there may be apparent violations of the $\Delta I=\frac{1}{2}$ predictions, Eqs. (9).

Further discussion will be given elsewhere.

## APPENDIX A

The decay rate for reactions (1) is found from (4) to be

$$\Gamma(\Omega\to B\pi)=\frac{E'^2-m'^2}{12\pi m(\Omega)}[A^2(E'+m')+B^2(E'-m')], \tag{A1}$$

where $E'$ and $m'$ are the energy and mass of the final baryon, respectively, in the $\Omega^-$ rest frame.

The decay rate for reactions (2) is found from (6) to be

$$\Gamma(\Omega\to\Xi^*\pi)=\frac{E'^2-m'^2{}^{1/2}}{4\pi m(\Omega)}\Bigg\{C_1^2(E'+m')\left[1+\frac{2}{9}\frac{E'-m'}{m'}\frac{E'+2m'}{m'}\right]$$
$$+C_2^2(E'-m')\left[1+\frac{2}{9}\frac{E'+m'}{m'}\frac{E'-2m'}{m'}\right]+\frac{2}{9}\frac{m(\Omega)^2}{m'^2}(E'^2-m'^2)^2[C_3^2(E'+m')+C_4^2(E'-m')]$$
$$+\frac{2}{9}\frac{m(\Omega)}{m'}(E'^2-m'^2)\left[C_1C_3(E'+m')\frac{2E'+m'}{m'}+C_2C_4(E'-m')\frac{2E'-m'}{m'}\right]\Bigg\}\,, \qquad \text{(A2)}$$

where again $E'$ and $m'$ refer to the final baryon in the $\Omega^-$ rest frame.

The decay rates for the three-body decays in Eqs. (3) can be written in terms of $M_{ab}$ in Eq. (8) as

$$\Gamma(\Omega(p)\to\Xi(p'),\pi^a(k_1),\pi^b(k_2))=(1-\tfrac{1}{2}\delta_{ab})\frac{2m'}{(2\pi)^5}\int d^4p'd^4k_1d^4k_2\delta(p'^2+m'^2)\delta(k_1^2+\mu_1^2)$$
$$\times\delta(k_2^2+\mu_2^2)\theta(p_0')\theta(k_{10})\theta(k_{20})\delta^4(p-p'-k_1-k_2)\tfrac{1}{4}\sum_{\text{spins}}|M_{ab}(k_1,k_2)|^2\,, \qquad \text{(A3)}$$

where

$$\sum_{\text{spins}}|M_{ab}(k_1,k_2)|^2=\frac{2}{3m'm}\Bigg[\left(\frac{(k_1\cdot p)^2}{m^2}-\mu_1^2\right)\{F_1^2(m'm-p'\cdot p)-F_2^2(m'm+p'\cdot p)$$
$$+F_3^2[\mu_1^2m'm+\mu_1^2(p'\cdot p)+2(k_1\cdot p)(k_1\cdot p')]-F_4^2[\mu_1^2m'm-\mu_1^2(p'\cdot p)-2(k_1\cdot p)(k_1\cdot p')]$$
$$+2F_1F_3[m(k_1\cdot p')+m'(k_1\cdot p)]-2F_2F_4[m(k_1\cdot p')-m'(k_1\cdot p)]\}$$
$$+\left(\frac{(k_2\cdot p)^2}{m^2}-\mu_2^2\right)\{F_i\to G_i\}+\left(\frac{(k_1\cdot p)(k_2\cdot p)}{m^2}+k_1\cdot k_2\right)\{2F_1G_1(m'm-p'\cdot p)-2F_2G_2(m'm+p'\cdot p)$$
$$+2F_3G_3[\mu_1^2m'm+\mu_1^2(p'\cdot p)+2(k_1\cdot p)(k_1\cdot p')]-2F_4G_4[\mu_1^2m'm-\mu_1^2(p'\cdot p)-2(k_1\cdot p)(k_1\cdot p')]$$
$$+F_1G_3[m(k_1\cdot p')+m'(k_1\cdot p)]-F_2G_4[m(k_1\cdot p')-m'(k_1\cdot p)]$$
$$+F_3G_1[3m(k_1\cdot p')+m'(k_1\cdot p)]-F_4G_2[3m(k_1\cdot p')-m'(k_1\cdot p)]\}$$
$$-(F_1G_3-F_3G_1-F_2G_4+F_4G_2)(1/m)\{\mu_1^2m^2(k_2\cdot p')+\mu_1^2(k_2\cdot p)(p'\cdot p)-(k_1\cdot p)^2(k_2\cdot p')+(k_1\cdot p)(k_1\cdot k_2)(p'\cdot p)\}\Bigg]\,,$$

where $m$, $m'$, $\mu_1$, and $\mu_2$ are the masses of $\Omega^-$, the final baryon, and the mesons, respectively.

## APPENDIX B

Using the interactions given in Sec. II, we compute the following matrix elements corresponding to the diagrams in Fig. 1:

diagram (a): $$M_a(\Omega^-\to\Xi^0\pi^-)=-f\bar{u}(p')(\rho_1+\rho_2\gamma_5)k_\mu u_\mu(p)\,,$$

diagram (b): $$M_b(\Omega^-\to\Xi^0\pi^-)=\frac{-2(\sqrt{6})hg_1f}{m(\Omega)-m(\Xi^*)}\bar{u}(p')k_\mu u_\mu(p)\,,$$

diagram (c): $$M_c(\Omega^-\to\Lambda K^-)=-(2f/\sqrt{6})\bar{u}(p')(\rho_3+\rho_4\gamma_5)k_\mu u_\mu(p)\,,$$

diagram (d): $$M_d(\Omega^-\to\Lambda K^-)=-\frac{6hg_1f}{m(\Omega)-m(\Xi^*)}\bar{u}(p')k_\mu u_\mu(p)\,,$$

diagram (e): $$M_e(\Omega^-\to\Lambda K^-)=\frac{hf(3\phi-\delta)}{m(\Xi^0)-m(\Lambda)}\bar{u}(p')k_\mu u_\mu(p)\,.$$

The total matrix element $\Omega_-^-=M_a+M_b$ is to be identified with (4). $\Omega_0^-$ can then be found from (5). Similarly, $\Omega_k^-=M_c+M_d+M_e$.

For the $\Xi^{*0}\pi^-$ decay mode we have $\Omega^*{}_-^-=M_f+M_g$, where

diagram (f): $$M_f(\Omega^-\to\Xi^{*0}\pi^-)=-2\sqrt{3}g_1f\bar{u}_\mu(p')(1-\gamma_5)u_\mu(p)\,,$$

diagram (g): $$M_g(\Omega^-\to\Xi^{*0}\pi^-)=\frac{4\sqrt{3}m(N^*)}{m(\Omega)-m(\Xi^*)}g_1f\bar{u}_\mu(p')\gamma_5u_\mu(p)\,.$$

This corresponds to (6), and $\Omega^*{}_0^-$ is given by the $\Delta I=\frac{1}{2}$ rule (7).

Finally, the amplitude $M_{+-}(k_1,k_2)$ is $M_h+M_i+M_j+M_k+M_l+M_m$, where

diagram (h): $$M_h(\Omega^-\to\Xi^-,\pi^+(k_1),\pi^-(k_2))$$
$$=2if^2\bar{u}(p')[(\rho_5+\rho_6\gamma_5)k_{1\mu}+(\rho_7+\rho_8\gamma_5)k_{2\mu}]u_\mu(p)\,,$$

diagram (i): $$M_i=\frac{2(\sqrt{6})ig_1hf^2}{m(\Omega)-m(\Xi^*)}\bar{u}(p')(k_1-k_2)_\mu\gamma_5u_\mu(p)\,,$$

diagram (j): $$M_j=-\frac{2(\sqrt{6})ig_1hf^2}{(p-k_2)^2+m(\Xi^*)^2+im(\Xi^*)\Gamma(\Xi^*)}\bar{u}(p')\{(b_1+i\gamma\cdot k_1)k_{1\mu}+(b_2+ib_3\gamma\cdot k_1)k_{2\mu}\}(1-\gamma_5)u_\mu(p)\,,$$

diagram (k): $$M_k=\frac{4(\sqrt{6})ig_1hf^2}{(p-k_2)^2+m(\Xi^*)^2+im(\Xi^*)\Gamma(\Xi^*)}\frac{m(N^*)}{m(\Omega)-m(\Xi^*)}$$
$$\times\bar{u}(p')[(b_1+i\gamma\cdot k_1)k_{1\mu}+(b_2+ib_3\gamma\cdot k_1)k_{2\mu}]\gamma_5u_\mu(p)$$

diagram (l): $$M_l=\frac{4(\sqrt{6})ig_1hf^2}{(p-k_2)^2+m(\Xi)^2}\frac{1}{m(\Omega)-m(\Xi^*)}\bar{u}(p')(b_4+ib_5\gamma\cdot k_1)\gamma_5k_{2\mu}u_\mu(p)\,,$$

diagram (m): $$M_m=\frac{2if^2}{(p-k_2)^2+m(\Xi^*)^2}\bar{u}(p')(b_4+ib_5\gamma\cdot k_1)(\rho_1\gamma_5+\rho_2)k_{2\mu}u_\mu(p)\,.$$

In the last four equations we used the abbreviations

$$b_1=-m(\Xi)-m(\Xi^*)\,,$$
$$b_2=[1/3m(\Xi^*)^2]\{2[m(\Xi)+m(\Xi^*)]k_1\cdot(p-k_2)-\mu_1{}^2m(\Xi^*)\}\,,$$
$$b_3=[1/3m(\Xi^*)^2]\{m(\Xi^*)[m(\Xi)+m(\Xi^*)]-2k_1\cdot(p-k_2)\}\,,$$
$$b_4=2\beta(\mu_1{}^2-2k_1\cdot p')\,,$$
$$b_5=(1+4\beta)m(\Xi)\,,$$

where $\beta\simeq-0.33$ as defined in Ref. 1. The $\Delta I=\frac{1}{2}$ relations (9) enable us to find $M_{00}(k_1,k_2)$ and $M_{0-}(k_1,k_2)$ from $M_{+-}(k_1,k_2)$ given above.

## APPENDIX C

The amplitudes for diagrams (j) and (k) contain a factor

$$\frac{h}{(p-k_2)^2+m(\Xi^*)^2+im(\Xi^*)\Gamma(\Xi^*)}\,,$$

so that $|M_{ab}(k_1,k_2)|^2$ contains a factor

$$\frac{h^2}{[(p-k_2)^2+m(\Xi^*)^2]^2+m(\Xi^*)^2\Gamma^2}\propto\frac{\Gamma}{[(p-k_2)^2+m(\Xi^*)^2]^2+m(\Xi^*)^2\Gamma^2}\,.$$

Since the $\Xi^*$ width is fairly small, we make the zero-width approximation. Then,

$$\frac{\Gamma}{[(p-k_2)^2+m(\Xi^*)^2]^2+m(\Xi^*)^2\Gamma^2} \to \frac{\pi}{m(\Xi^*)}\delta[(p-k_2)^2+m(\Xi^*)^2].$$

We define $\tilde{M}_{ab}(k_1,k_2)$ by

$$|M_{ab}(k_1,k_2)|^2=\delta[(p-k_2)^2+m(\Xi^*)^2]|\tilde{M}_{ab}(k_1,k_2)|^2.$$

The decay rate for $\Omega(p)\to\Xi(p')$, $\pi^a(k_1)$, and $\pi^b(k_2)$ is given by

$$\Gamma(\Omega\to\Xi\pi^a\pi^b)=(1-\tfrac{1}{2}\delta_{ab})\frac{2m'}{(2\pi)^5}\int d^4p'd^4k_1d^4k_2\delta(p'^2+m'^2)\theta(p_0')\delta(k_1^2+\mu_1^2)\theta(k_{10})$$
$$\times\delta(k_2^2+\mu_2^2)\theta(k_{20})\delta[(p-k_2)^2+m(\Xi^*)^2]\delta^4(p-p'-k_1-k_2)\tfrac{1}{4}\sum_{\text{spins}}|\tilde{M}_{ab}(k_1,k_2)|^2.$$

We introduce new variables

$$K=k_1+p',\quad Q=k_1-p',$$

with the properties

$$d^4p'd^4k_1=\tfrac{1}{16}d^4Kd^4Q$$

and

$$\delta(p'^2+m'^2)\delta(k_1^2+\mu_1^2)=4\delta(K^2+Q^2+2\rho_1^2)\delta(K\cdot Q+\rho_2^2),$$

where

$$\rho_1^2=\mu_1^2+m'^2\quad\text{and}\quad\rho_2^2=\mu_1^2-m'^2.$$

In terms of these new variables,

$$\Gamma(\Omega\to\Xi\pi^a(k_1)\pi^b(k_2))=(1-\tfrac{1}{2}\delta_{ab})\frac{2m'}{(2\pi)^5}\int d^4k_2d^4Kd^4Q\,\delta(k_2^2+\mu_2^2)\delta(K^2+Q^2+2\rho_1^2)\delta(K\cdot Q+\rho_2^2)$$
$$\times\delta(K^2+m(\Xi^*)^2)\delta^4(p-k_2-K)\sum_{\text{spins}}|\tilde{M}_{ab}(k_1,k_2)|^2$$
$$=(1-\tfrac{1}{2}\delta_{ab})\frac{2m'}{(2\pi)^5}\int d^4k_2d^4K\,\delta(k_2^2+\mu_2^2)\delta(K^2+m(\Xi^*)^2)\delta^4(p-k_2-K)I(p,k_2,K,\text{masses}),\qquad\text{(C1)}$$

where

$$I=\int d^4Q\delta(K^2+Q^2+2\rho_1^2)\delta(K\cdot Q+\rho_2^2)\sum_{\text{spins}}|\tilde{M}_{ab}(k_1,k_2)|^2.$$

$I$ is a Lorentz-invariant quantity and can be evaluated straightforwardly[13] by choosing the frame where $K=(0,iK_0)$. The evaluation of the decay rate (C1) then becomes equivalent to the calculation of the decay rate for the two-body final state, and this can be easily carried out.

[13] J. D. Jackson, in *Elementary Particle Physics and Field Theory, 1962 Brandeis Lectures* (W. A. Benjamin, Inc., New York, 1963), Vol. 1, p. 392.