Broken Chiral Symmetry and the Transformation Properties of Asymptotic Fields

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The chiral transformation of asymptotic fields in the presence of symmetry breaking is discussed in a model of interaction between a quantized Dirac field and an external electromagnetic field. We conclude from this model that the asymptotic fields will generally transform nonlocally and nonlinearly. Only if the symmetry breaking is small, or if the interaction is sufficiently weak, will the asymptotic fields transform locally and linearly as a first approximation.

I. INTRODUCTION

QUITE often one would like to know the transformation properties of asymptotic fields in the presence of symmetry breaking. For example, if we knew the transformation properties of the asymptotic pion field $\boldsymbol{\pi}_{\text{in}}$ under chiral $SU(2)\otimes SU(2)$, we would be able to determine the $\pi\rightarrow\mu+\nu$ decay amplitude by taking the vacuum expectation value of $i[X_i, \pi_{j,\text{in}}]$,[1] where X_i $(i=1, 2, 3)$ are generators of the chiral transformation.

Unfortunately, there is not yet known any *a priori* simple way to determine the transformation properties of asymptotic fields except in the case of exact symmetry, when the generators are constants of motion. In this case quantities like $[X_i, \pi_{j,\text{in}}]$ satisfy the same free-field equations as $\pi_{i,\text{in}}$, generally suggesting linear and local transformation laws for asymptotic fields.

In order to get some insight as to how the asymptotic fields transform in the presence of the symmetry breaking, we discuss in Sec. II a simple model of a quantized Dirac field interacting with an external electromagnetic field. This system will not be invariant under the linear and local chiral transformation $\psi\rightarrow\exp(i\epsilon\gamma_5)\psi$, as long as the Dirac particle has a finite mass. ψ is the Heisenberg field. Solving the Heisenberg equation of motion, i.e., expressing ψ in terms of the asymptotic field ψ_{in}, we see that ψ_{in} will transform nonlocally and, if A_μ is quantized, nonlinearly.

II. MODEL

In our model the Heisenberg spinor field $\psi(x)$ satisfies the following equation:

$$(-i\gamma_\mu\partial^\mu+m)\psi(x)=gA_\mu(x)\gamma^\mu\psi(x), \tag{1}$$

where $A_\mu(x)$ is an external field.

We concentrate on the chiral transformation whose generator is

$$X=-\int d^3x\,\bar{\psi}(x)\beta\gamma_5\psi(x). \tag{2}$$

The field $\psi(x)$ satisfies the linear transformation law

$$[X,\psi(x)]=\gamma_5\psi(x), \tag{3}$$

and it is readily seen that only the mass term in (1) breaks the symmetry. The symmetry-breaking part of the Hamiltonian H_{SB} in our case is given by a mass term

$$H_{\text{SB}}=m\int d^3x\,\bar{\psi}(x)\psi(x),$$

and its "transformation" is given as

$$i\dot{X}=[X,H_{\text{SB}}]=2m\int\bar{\psi}(x)\gamma_5\psi(x)d^3x. \tag{4}$$

In what follows we need the solutions of (1) in terms of in-fields. From (1) we can write the corresponding integral equations:

$$\psi(x)=\psi_{\text{in}}(x)-g\int S_R(x-y)\boldsymbol{A}(y)\psi(y)d^4y,$$
$$\bar{\psi}(x)=\bar{\psi}_{\text{in}}(x)-g\int\bar{\psi}(y)\boldsymbol{A}(y)S_A(y-x)d^4y, \tag{5a}$$

where S_R and S_A are retarded and advanced Green's functions, respectively.

Because of the linearity of (5a) in $\psi(x)$ and $\bar{\psi}(x)$, respectively, we can write its solutions in the form

$$\psi(x)=\psi_{\text{in}}(x)-\int d^4y\,M_R(x,y)\psi_{\text{in}}(y),$$
$$\bar{\psi}(x)=\bar{\psi}_{\text{in}}(x)-\int d^4y\,\bar{\psi}_{\text{in}}(y)M_A(y,x). \tag{5b}$$

M_R and M_A satisfy the following integral equations[2]:

$$M_R(x,y)=gS_R(x-y)\boldsymbol{A}(y)-g\int d^4z\,S_R(x-z)\boldsymbol{A}(z)M_R(z,y),$$
$$M_A(x,y)=g\boldsymbol{A}(x)S_A(x-y)-g\int d^4z\,M_A(x,z)\boldsymbol{A}(z)S_A(z-y). \tag{6}$$

By means of (5b) and (6), X and $i\dot{X}$ from (2) and

[1] It can be shown that if one imposes strictly linear chiral transformations for the quartet of asymptotic fields $\pi_{i,\text{in}}$, σ_{in}, in the form $i[X_i,\pi_{j,\text{in}}]=\sigma_{\text{in}}\delta_{ij}$, $i[X_i,\sigma_{\text{in}}]=-\pi_{i,\text{in}}$, the pion would not decay. See T. Muta and H. Umezawa, University of Wisconsin-Milwaukee Report No. UWM-4867-69-4 (unpublished).

[2] It is assumed that the external field $A_\mu(x)$ vanishes in space-time infinites. Salam and Matthews studied the scattering on an external field in momentum space. They also discussed the condition on an external field in order to apply the Fredholm method in this problem [A. Salam and P. T. Matthews, Phys. Rev. **90**, 610 (1953)]. See also R. P. Feynman, *ibid.* **75**, 486 (1949); **75**, 1736 (1949); M. Neuman, *ibid.* **83**, 1258 (1952); **85**, 129 (1952); J. Schwinger, *ibid.* **93**, 615 (1954); J. Šoln, Nuovo Cimento **18**, 914 (1960); **32**, 1301 (1964).

(4) are expressed in terms of ψ_{in} and $\bar{\psi}_{\text{in}}$ as follows[3]:

$$X(t)=-\int d^3x\,\bar{\psi}_{\text{in}}(x)\beta\gamma_5\psi_{\text{in}}(x)$$
$$+\int d^3xd^4y\,\bar{\psi}_{\text{in}}(x)\beta\gamma_5M_R(x,y)\psi_{\text{in}}(y)$$
$$+\int d^3xd^4z\,\bar{\psi}_{\text{in}}(z)M_A(z,x)\beta\gamma_5\psi_{\text{in}}(x)$$
$$-\int d^3xd^4zd^4y\,\bar{\psi}_{\text{in}}(z)M_A(z,x)\beta\gamma_5M_R(x,y)\psi_{\text{in}}(y),\quad(7)$$

$$i\dot{X}(t)=2m\int d^3x\,\bar{\psi}_{\text{in}}(x)\gamma_5\psi_{\text{in}}(x)$$
$$-2m\int d^3xd^4y\,\bar{\psi}_{\text{in}}(x)\gamma_5M_R(x,y)\psi_{\text{in}}(y)$$
$$-2m\int d^3xd^4z\,\bar{\psi}_{\text{in}}(z)M_A(z,x)\gamma_5\psi_{\text{in}}(x)$$
$$+2m\int d^3xd^4zd^4y\,\bar{\psi}_{\text{in}}(z)M_A(z,x)\gamma_5M_R(x,y)\psi_{\text{in}}(y).\quad(8)$$

Having the Heisenberg fields expressed in terms of respective asymptotic in fields, we may easily see what kind of transformation $\psi_{\text{in}}(x)$ satisfies with X as a generator. Equation (7), together with the anticommutation relations for ψ_{in}, gives us

$$[X(t),\psi_{\text{in}}(\mathbf{x},t)]=\gamma_5\psi_{\text{in}}(x)-\int d^4y\,\gamma_5M_R(x,y)\psi_{\text{in}}(y)$$
$$+i\int d^3x'd^4z\,S(x-z)M_A(z,x')\beta\gamma_5$$
$$\times[\psi_{\text{in}}(x')-\int d^4y\,M_R(x',y)\psi_{\text{in}}(y)],\qquad t=t_{x'}.\quad(9)$$

From (9) we see that in our model, ψ_{in} transforms nonlocally. Generally, ψ_{in} will transform *nonlocally* and *nonlinearly*, which can be seen if one treats the electromagnetic field as a dynamical variable and repeats the derivation of Eq. (9), say, up to the second order in g.[4] The important thing we learn from (9), however, is that the nonlocal and generally also nonlinear term which appears in the transformation of asymptotic fields is "small" if the interaction is weak, regardless of how strong the symmetry breaking is. Note that in our model, $M_{R,A}\to0$ as $g\to0$.

We expect that the asymptotic field will transform approximately linearly if the parameter of the symmetry breaking is small. In our model, this would correspond to small fermion mass m. This expectation is usually justified by the fact that $\dot{X}$ is of the order of the symmetry breaking, thus small if the symmetry-breaking parameter is small [see (8)]. It does not appear to be simple to rewrite (9) in such a way that the mass m appears explicitly on the right-hand side of (9). However, we can measure the degree of the nonlinearity and the nonlocality of the transformation with the quantity $D_x[X(t),\psi_{\text{in}}(x,t)]$, where the operator D_x is defined as

$$D_x=\beta(\mathbf{p}-m),\qquad p\!\!\!/=\gamma_\mu p^\mu=-i\gamma^\mu(\partial/\partial x^\mu).$$

This is clear, since $D_x\gamma_5\psi_{\text{in}}(x)=0$.

In our model we can compute the quantity $D_x[X(t),\psi_{\text{in}}(x)]$ from (9) using the differential equations for M_R and M_A,

$$(-i\gamma^\mu(\partial/\partial x^\mu)+m-g\mathbf{A}(x))M_R(x,y)=-g\mathbf{A}(x)\,\delta^{(4)}(x-y)$$

and

$$M_A(y,x)(i\gamma^\mu(\partial/\partial x^\mu)+m-g\mathbf{A}(x))=-g\mathbf{A}(x)\delta^{(4)}(y-x),$$

which can easily be derived from (6).

The result is

$$D_x[X(t),\psi_{\text{in}}(\mathbf{x},t)]$$
$$=2im\int d^3x'd^4z\,\beta[\beta,\,S(x-z)M_A(z,x')]\gamma_5$$
$$\times(\psi_{\text{in}}(x')-\int d^4y\,M_R(x',y)\psi_{\text{in}}(y)),\qquad t_{x'}=t.\quad(10)$$

First of all, as $g\to0$, $D_x[X(t),\psi_{\text{in}}(\mathbf{x},t)]\to0$, confirming what we already found: $[X(t),\psi_{\text{in}}(\mathbf{x},t)]\to\gamma_5\psi_{\text{in}}(\mathbf{x},t)$. However, if the mass m becomes smaller, we shall also have that $D_x[X(t),\psi_{\text{in}}(\mathbf{x},t)]\to0$, meaning that $[X(t),\psi_{\text{in}}(\mathbf{x},t)]$ approaches $\gamma_5\psi_{\text{in}}(\mathbf{x},t)$. This result is clear, since in the $m\to0$ limit, $X(t)$ is a constant of the motion and we can evaluate $[X(t),\psi_{\text{in}}(\mathbf{x},t)]$ as $t\to-\infty$.

III. CONCLUSION AND DISCUSSION

From the relations (9) and (10) we read off some interesting properties. First of all, we see that if the dynamics is absent (which is achieved by putting $g=0$), ψ_{in} will transform locally and linearly. In other words, if we demand a local linear transformation law for the asymptotic field ψ_{in} in the presence at the symmetry breaking, we shall have to ignore the dynamics; i.e., we shall be dealing with a free system. We believe that this property which we extracted from our model is generally true, e.g., in a broken chiral $SU(2)\otimes SU(2)$ or $SU(3)\otimes SU(3)$. From our model, we also learn that the degree of the nonlinearity and the nonlocality in the transformation of asymptotic fields is small if the parameter of the symmetry breaking is small; in our model this parameter is related to the fermion mass m [see Eq. (10) and the discussion thereafter]. It is plausible to expect that this is generally true even in cases where the symmetry breaking would enter into the interaction part of the Lagrangian.

A natural question that now arises is: How much sense does it make to classify the elementary particles according to linear representations of $SU(3)$ or $SU(3)\otimes SU(3)$, when we know that these groups are broken in nature, and since we expect, as illustrated by

[3] The products like $\psi_{\text{in}}^\dagger(z)\cdots\psi_{\text{in}}(y)$ should be imagined as normal products, $:\psi_{\text{in}}^\dagger(z)\cdots\psi_{\text{in}}(y):$. See, for example, S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, Evanston, Ill., 1961).

[4] When the electromagnetic field is treated as a dynamical variable, besides Eq. (1) for the spinor field $\psi(x)$, we shall have the corresponding differential equation for $A_\mu(x)$. Now, however, both equations will have to contain some extra terms in order to take into account the mass and coupling constant (charge) renormalizations. See G. Källén, Helv. Phys. Acta **25**, 853 (1952); **26**, 755 (1953).

our model, the physical asymptotic fields to belong to the nonlinear, nonlocal representations?

For $SU(3)$, one of the answers which is suggested by Eq. (10) is well known[5]: As long as the symmetry breaking is small, the classification of the elementary particles according to linear representations of $SU(3)$ is good in the first approximation even when the particles interact strongly. This is reflected in the fact that the strongly interacting particles can be grouped into $SU(3)$ multiplets with members having nearly the same mass. For chiral $SU(3)\otimes SU(3)$, however, the symmetry breaking does not appear necessarily to be small, for if it were, the baryon masses, for example, would have to be nearly vanishing.[6] Thus we expect, as suggested by Eqs. (9) and (10), that the asymptotic fields of strongly interacting particles belong to the nonlinear, nonlocal representations of chiral $SU(3)\otimes SU(3)$.

The relations (9) and (10) allow us to discuss another interesting possibility. Even when the symmetry is broken strongly (m being large in our model), the asymptotic fields will transform linearly in the first approximation if the particles interact weakly. This possibility is certainly appealing for the theory of weak interactions and should be further explored.

[5] M. Gell-Mann, Caltech Report No. CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[6] A very nice discussion of a problem of transformation properties of elementary particles in the presence of symmetry breaking on the $SU(3)\otimes SU(3)$ level was given by S. L. Glashow, in *Proceedings of the Seventh Internationale Universitätswochen für Kernphysik, 1968, Schladming, Austria* (Springer-Verlag, Vienna, 1968).

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Method for Testing Toller ω Dependence in the Multi-Regge Model

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Given data that are plausibly described by the multi-Regge model, it is shown that a distribution in a certain angle related to a triple subenergy (which in turn is related to ω) is *flat* if (1) the amplitude is independent of that subenergy and (2) a certain selection of data is made. The test can be applied to up to four internal vertices, thus allowing a *complete* analysis of ω dependence in reactions with six, seven, or eight particles in the final state.

I. INTRODUCTION

SINCE its inception a few years ago, the multi-Regge model has enjoyed much success in fitting experimental data.[1] A significant feature in these applications of the model has been the essential neglect of the internal Reggeon-Reggeon-particle vertex function. Theoretically,[2] these are expected to depend on the two momentum transfers involved and on a Toller angle ω. Recent theoretical considerations[3–5] have shown that in the applications to date, neglect of dependence on ω is justified only because of the highly peripheral nature of the reactions. That is, the smallness of the momentum transfers for the bulk of the events effectively uncouples the amplitude from dependence on ω. The form of the dependence on the momentum transfers is not known, however.

Certainly, in developing models of multi-Regge behavior, or multiperipheral models in general, it is desirable to have stringent experimental tests of the dependence of the amplitudes on such variables. In the case of three-particle final states, such a test for ω dependence has already been proposed[6] but it has the drawback of requiring data over a range of incident energies. It is the purpose of the present paper to develop a test of ω dependence that is particularly suited to four-, five-, and six-particle final states and has some use in reactions with higher-multiplicity final states. (It is not applicable to three-particle final states, however.) Furthermore, only data at a fixed incident energy are needed, although a certain selection of events must be made.

In point of fact, the test to be explained does not aim specifically at ω dependence. Rather, it tests for dependence of the amplitude on the *triple* subenergy (the square of the sum of three final-state momenta)

[1] For a review see O. Czyzewski, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1969), p. 367.

[2] N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. Letters **19**, 614 (1967); Phys. Rev. **163**, 1572 (1967).

[3] J. M. Kosterlitz, Nucl. Phys. **B9**, 273 (1969).

[4] Chung-I Tan and Jiunn-Ming Wang, Phys. Rev. **185**, 1899 (1969).

[5] When one of the Reggeons is the pion, the approximate vanishing of the pion's trajectory further aids the decoupling of ω. See R. A. Morrow, Nuovo Cimento **61A**, 215 (1969).

[6] R. A. Morrow, Phys. Rev. **176**, 2147 (1968).