Group-Geometrical Formulation of an $SU(3)\times SU(3)$ Effective Lagrangian

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An $SU(3)\times SU(3)$ effective Lagrangian, generated by a massive vector field, has been formulated with the aid of the group-geometrical method employed in our earlier papers. This formalism is manifestly invariant under nonlinear transformations, and permits an exact and compact treatment of the nonlinearities. Our $SU(3)\times SU(3)$ treatment incorporates only those minimal departures from $SU(2)\times SU(2)$ that are required to secure a reasonable model, and the new Lagrangian contains no free parameters not already present in the $SU(2)\times SU(2)$ model. These parameters are determined by fitting the linearized theory to the pion-nucleon system. Results for other members of the octets, such as the Callan-Treiman formula, are then in satisfactory agreement with experiment or with the current algebra. It is hoped that the formalism will permit the treatment of the nonlinearities to be extended beyond the discussion of the tree diagrams.

I. INTRODUCTION

THE generation of effective Lagrangians by gauge fields has been extensively studied.[1,2] According to the fundamental physical assumption in this approach, there is a local gauge group and it is therefore not possible to compare currents at distinct space-time points without introducing a displacement field. In the Yang-Mills[3] version this displacement field is a vector field which permits currents to be related at different points by parallel transfer. The displacement field may therefore be regarded as a localized displacement operator that corresponds to the momentum operator in the Poincaré algebra. Similarly, in the theory to be described,[4] the pseudoscalar field may be regarded as a localization of the γ^5 reflection operation. Furthermore, the pseudoscalar and vector fields are not independent, but are parts of a larger complex which may transform into each other under changes of gauge. In this respect the theory to be described resembles $SU(6)$, which combines the vector and pseudoscalar particles into the regular representation.[5]

The existence of rest mass is foreign to these ideas and in our approach forces us to introduce two privileged gauges. In the first gauge the vector mass is specified, while in the second the fermion mass is given. The gauge transformation connecting these privileged gauges in turn defines the pseudoscalar field. Therefore the local gauge is specified by the local pseudoscalar field. The resulting theory is no longer locally gauge invariant, but it is still invariant under constant gauge transformations.

These ideas may be formally implemented by choosing the components of the pseudoscalar octet to be the local parameters of the gauge group. Then the eight pseudoscalar variables provide a coordinate system on the local group space.[6,7] The arbitrariness in the choice of such a coordinate system corresponds to the possibility of point transformations on the pseudoscalar fields. By making use of the group geometry, such a theory may be simply formulated in a manner which is manifestly invariant under nonlinear transformations of the pseudoscalar fields.

We have repeated for $SU(3)$ our earlier work on $SU(2)$.[1] Accordingly, the Lagrangian model which is presented below incorporates only those minimal departures from the $SU(2)$ theory that are necessary to

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[1] R. Finkelstein and L. Staunton, Physica (to be published), hereafter called A. Notice the difference in notation between A and the present paper: There vector gauge is denoted by °; here it is denoted by an overbar.
[2] A. Salam and J. C. Ward, Phys. Rev. **136**, 763 (1964); J. Schwinger, Phys. Letters **24B**, 473 (1967); S. Weinberg, *ibid.* **18**, 188 (1967); J. A. Cronin, Phys. Rev. **161**, 1483 (1967); H. S. Mani, Y. Tomozawa, and Y. P. Yao, Phys. Rev. Letters **18**, 1084 (1967); W. A. Bardeen, L. S. Brown, B. W. Lee, and H. T. Nieh, *ibid.* **18**, 1170 (1967); J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); L. S. Brown, *ibid.* **163**, 1802 (1967); P. Chang and F. Gürsey, *ibid.* **164**, 1752 (1967); **169**, 1397(E) (1968); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters **19**, 1085 (1967); B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968); W. A. Bardeen and B. W. Lee, *ibid.* **177**, 2389 (1969); S. Weinberg, *ibid.* **177**, 2C05 (1969).
[3] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954). More general displacement fields have been discussed in R. Finkelstein and W. Ramsay [Ann. Phys. (N.Y.) **21**, 408 (1963)] and earlier papers referenced there.
[4] R. Finkelstein and L. Staunton, Ann. Phys. (N.Y.) **54**, 97 (1969); and also A.
[5] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964).
[6] R. Finkelstein, Physica **44**, 260 (1969). This paper is based on a method which had been used elsewhere: R. Finkelstein and D. Levy, J. Math. Phys. **8**, 2147 (1967); and earlier papers referenced there.
[7] C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2247 (1969); K. Meetz (unpublished report); C. J. Isham, Nuovo Cimento **59A**, 356 (1969); D. V. Volkov, report, 1968 (unpublished). All of these papers parametrize the group space with the components of the meson field.

assure a reasonable physical system, namely, $SU(3)$ symmetry breaking in the vector and fermion mass terms. The parameters of the theory are evaluated in the $SU(2)$ subspace consisting of the pion, nucleon, and ρ meson. The new $SU(3)$ relations which include the Callan-Treiman relation,[8] for example, are then found to be in satisfactory agreement either with experiments or with current algebra (when these are different). In these new relations the couplings of the pseudoscalars depend in a fundamental way on the masses of the associated vector mesons.

Just as in our earlier work, the agreement with experiment depends on only the linearized form of the theory. However, we believe that the main interest of the present work is the group-geometrical formulation of the nonlinear theory. Although it has been shown possible to obtain the linear results with a less elaborate formalism,[9] the possibility of making further progress along these lines may depend upon the preservation of the exact nonlinear structure in a tractable form.

II. $SU(2)$ MODEL

A local gauge group implies a vector displacement field, say, P_μ, and a corresponding curvature or tensor field:

$$P_{\mu\nu}=\nabla_\mu P_\nu-\nabla_\nu P_\mu, \tag{2.1}$$

$$\nabla_\mu=\partial_\mu+P_\mu. \tag{2.2}$$

Under a local gauge transformation,

$$P_\mu=U\bar{P}_\mu U^{-1}+U\partial_\mu U^{-1}, \tag{2.3}$$

$$P_{\mu\nu}=U\bar{P}_{\mu\nu}U^{-1}, \tag{2.4}$$

where U is unitary and the fields are anti-Hermitian. We make the parity decomposition

$$P_\mu=V_\mu+\gamma^5 A_\mu \tag{2.5}$$

and the chiral decomposition

$$U=U(l)a(l)+U(r)a(r) \tag{2.6}$$

in terms of the projection operators

$$a(l)=\tfrac{1}{2}(1+\gamma^5),\qquad a(r)=\tfrac{1}{2}(1-\gamma^5), \tag{2.7}$$

and where $U(l)$ and $U(r)$ are unitary and independent of γ^5.

We assume that U is chiral, namely,

$$U(l)=U(r)^{-1}=U(r)^\dagger. \tag{2.8}$$

It follows that V_μ and A_μ undergo different gauge transformations.

We consider the minimal coupling of a fermion field ψ to P_μ through the Lagrangian

$$-L=-\tfrac{1}{4}\,\mathrm{Tr}P_{\mu\nu}P^{\mu\nu}+\tfrac{1}{2}M^2\,\mathrm{Tr}\bar{P}_\mu\bar{P}^\mu -i\bar{\psi}^0\gamma^\mu(\partial_\mu+\bar{P}_\mu)\psi^0+\bar{\psi}^0M^0\psi^0. \tag{2.9}$$

The first term is invariant under local gauge transformations, as is the third term if we assume that ψ obeys the usual chiral transformation law

$$\psi=U\psi^0,\qquad \bar{\psi}=\bar{\psi}^0U. \tag{2.10}$$

However, the second term is not similarly invariant and must be given in a special gauge (up to a constant gauge transformation). Therefore, the vector mass term defines a special gauge which has been denoted by $(\bar{P}_\mu,\psi^0)$. Now we express the last two terms in a second special gauge (P_μ,N), where N is so chosen that the fermion bilinear becomes a mass term. Then the total Lagrangian becomes

$$-L=-\tfrac{1}{4}\,\mathrm{Tr}P_{\mu\nu}P^{\mu\nu}+\tfrac{1}{2}M^2\,\mathrm{Tr}\bar{P}_\mu\bar{P}^\mu -i\bar{N}\gamma^\mu(\partial_\mu+P_\mu)N+m\bar{N}N, \tag{2.11}$$

where U is so chosen that

$$M^0=mU^2. \tag{2.12}$$

We have now introduced two special gauges which are defined by the vector and nucleon mass terms, and which may be called the vector and nucleon gauges.[1] The gauge transformation connecting these two special gauges we associate with the pseudoscalar field. These special gauges are, however, defined only up to a constant gauge transformation, and the Lagrangian remains invariant under these constant gauge transformations. Furthermore, we may distinguish dynamically between V_μ and A_μ without losing any of the foregoing symmetry by adding the terms

$$\kappa M^2\,\mathrm{Tr}A_\mu A^\mu-i\epsilon\bar{N}\gamma^\mu\gamma^5A_\mu N, \tag{2.13}$$

where A_μ is the axial vector in the nucleon gauge and κ and ϵ are free parameters. Similar terms in V_μ would not remain invariant under the full group. Some further justification of these terms has been given in A.

The total Lagrangian is then

$$-L=-\tfrac{1}{4}\,\mathrm{Tr}P_{\mu\nu}P^{\mu\nu}-i\bar{N}\gamma^\mu(\partial_\mu+P_\mu)N+m\bar{N}N +\tfrac{1}{2}M^2\,\mathrm{Tr}\bar{P}_\mu\bar{P}^\mu+\kappa M^2\,\mathrm{Tr}A_\mu A^\mu-i\epsilon\bar{N}\gamma^\mu\gamma^5A_\mu N. \tag{2.14}$$

The first three terms would describe a massless vector which is minimally coupled to a massive fermion just as in electrodynamics.[1] The total Lagrangian, which is expressed partly in the vector and partly in the fermion gauge, describes a massive vector interacting with a massive fermion field. This Lagrangian does not explicitly contain pseudoscalar fields; on the other hand, since it is expressed in a mixed gauge (partly vector and partly nucleon), pseudoscalar fields are implied.

[8] C. G. Callan and S. T. Treiman, Phys. Rev. Letters **16**, 153 (1966).

[9] See, for instance, B. W. Lee, Phys. Rev. **170**, 1359 (1968); Phys. Rev. Letters **20**, 617 (1968); and other papers in Ref. 2.

Alternatively, one may write (2.14) entirely in the nucleon gauge:

$$-L=-\tfrac{1}{4}\,\mathrm{Tr}P_{\mu\nu}P^{\mu\nu}-\tfrac{1}{2}M^2\,\mathrm{Tr}(\nabla_\mu U)^\dagger(\nabla^\mu U)$$
$$-i\bar{N}\gamma^\mu\nabla_\mu N+m\bar{N}N+\kappa M^2\,\mathrm{Tr}A_\mu A^\mu-i\epsilon\bar{N}\gamma^\mu\gamma^5A_\mu N, \quad (2.15)$$

where a formal kinetic energy replaces the vector mass term. It was shown in A that this kinetic energy term does indeed describe pseudoscalar mesons. If the last two terms in (2.14) and (2.15) are dropped, it is still possible to describe (a) massive vectors without pseudoscalars according to the so-truncated (2.14), or (b) massless vectors minimally coupled to pseudoscalars according to the similarly truncated (2.15). To describe massive vectors and pseudoscalars together, however, at least the κ term is needed; and, furthermore, this term is independently required as soon as the fermions are assumed to be massive. Then, as shown in A, all the usual low-energy results also follow, namely, the Feynman–Gell-Mann currents,[10] the Goldberger-Treiman[11] and the Adler-Weisberger results,[12] the Weinberg formula for the mass of the axial-vector meson,[13] and the KSRF relation.[14]

In A, the gauge group was assumed to be $SU(2)$ and N was identified with the two-component nucleon, but the discussion there was largely independent of this assumption. There is consequently no formal obstacle to extending the previous discussion to $SU(3)$ by simply replacing the two-rowed N by a three-rowed quark basis or an eight-rowed baryon basis. Although we shall finally specialize to the octet basis to make physical application, it will be convenient to begin our discussion by regarding (2.14) or (2.15) as a $SU(3)\times SU(3)$ effective Lagrangian without specializing to either the fundamental (quark) or regular (baryon) basis. In interpreting (2.14), one should substitute 3×3 matrices for the bosons everywhere if the spinor belongs to the quark basis. On the other hand, if the spinor belongs to the octet basis, then the boson matrices are 3×3 in the trace terms but 8×8 in the spinor terms.

Of course, $SU(3)$ is also distinguished from $SU(2)$ by large mass splittings of the regular representation and by the existence of a D/F ratio. It will be necessary to take these facts into account to give a realistic discussion, but the necessary modifications of (2.15) will be postponed until Sec. VI.

[10] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[11] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).
[12] S. L. Adler, Phys. Rev. **140**, B736 (1965); W. I. Weisberger, *ibid.* **143**, 1302 (1966).
[13] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).
[14] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

III. INVARIANCE GROUP

We shall now discuss the invariance group of the Lagrangians just introduced. Although most of the discussion in this section and in Sec. IV holds for a general unitary group, we are interested only in $SU(3)$.

It has been assumed that the fermion gauge is connected with the vector gauge by a chiral transformation according to (2.8). Then the local pseudoscalar fields correspond to a point in the group space of either $U(l)$ or $U(r)$, since these are not independent. For definiteness, we shall choose $U(l)$: The coordinates of a point in the group space of $U(l)$ are then the independent pseudoscalar fields. In the $SU(3)$ case, there are eight such fields.

The invariance group of the theory consists of all transformations which leave the Lagrangian invariant and preserve the chirality of U. These are the left and right invariance groups of (2.9) and have been formulated as follows by Gürsey[15]:

$$\text{(I)}\quad \psi^{0\prime}=e^{i\alpha a(l)}\psi^0,\qquad \bar{\psi}^{0\prime}=\bar{\psi}^0e^{-i\alpha a(r)},$$
$$\bar{P}_\mu{}'=e^{i\alpha a(l)}\bar{P}_\mu e^{-i\alpha a(l)},\qquad U^{2\prime}=e^{i\alpha a(r)}U^2e^{-i\alpha a(l)}. \quad (3.1)$$

$$\text{(II)}\quad \psi^{0\prime}=e^{i\alpha a(r)}\psi^0,\qquad \bar{\psi}^{0\prime}=\bar{\psi}^0e^{-i\alpha a(l)},$$
$$\bar{P}_\mu{}'=e^{i\alpha a(r)}\bar{P}_\mu e^{-i\alpha a(r)},\qquad U^{2\prime}=e^{i\alpha a(l)}U^2e^{-i\alpha a(r)}. \quad (3.2)$$

The α in these equations is a generator of the symmetry group, which is here $SU(3)$. The transformations (I) and (II) belong to commuting groups which separately preserve the invariance of the Lagrangian as well as the chirality of U^2 (and therefore of U). The invariance groups are defined on U^2 because of (2.12).

Let Ω be any tensor of mixed parity:

$$\Omega=\Omega(l)a(l)+\Omega(r)a(r). \quad (3.3)$$

Then

$$e^{i\alpha a(s)}\Omega e^{-i\alpha a(s)}=[e^{i\alpha}\Omega(s)e^{-i\alpha}]a(s)+\Omega(t)a(t), \quad (3.4)$$
$$e^{i\alpha a(s)}\Omega e^{-i\alpha a(t)}=[e^{i\alpha}\Omega(s)]a(s)+[\Omega(t)e^{-i\alpha}]a(t), \quad (3.5)$$

where $s=l,r$; $t=l,r$; $s\neq t$.

Now let $\Omega=U^2$. Then U and Ω are both chiral, or

$$\Omega(l)=\Omega(r)^\dagger. \quad (3.6)$$

Then by (3.1) and (3.5),

$$\Omega(l)'=\Omega(l)e^{-i\alpha}, \quad (3.7a)$$
$$\Omega(r)'=e^{i\alpha}\Omega(r). \quad (3.7b)$$

By (3.6) and (3.7),

$$\Omega(l)'=[e^{i\alpha}\Omega(l)^\dagger]^\dagger$$
$$=[\Omega(r)']^\dagger. \quad (3.8)$$

[15] F. Gürsey, Nuovo Cimento **16**, 1254 (1960); Ann. Phys. (N.Y.) **12**, 91 (1961); P. Chang and F. Gürsey, Phys. Rev. **164**, 1752 (1967).

Therefore, Ω' is also chiral. Similarly, under (3.2)

$$\Omega(l)'=e^{i\alpha}\Omega(l), \quad (3.9a)$$

$$\Omega(r)'=\Omega(r)e^{-i\alpha}. \quad (3.9b)$$

The corresponding transformations of P_μ follow from (3.4).

The effect of (3.1) and (3.2) on the space of $\Omega(l)$ alone may be expressed as follows:

$$\Omega(l)'=\Omega(l)e^{-i\alpha} \quad \text{for (I)} \quad (3.10a)$$

$$=e^{i\alpha}\Omega(l) \quad \text{for (II)}. \quad (3.10b)$$

Therefore, (I) and (II) induce group multiplication on the right and left. Then the invariance group of the theory consists of all motions of the group space into itself; it is thus the product of the right and left parameter groups, or $G\times G$ if the gauge group is G.

The infinitesimal form of (I) in the vector gauge is

$$\delta\psi^0=ia(l)\delta\alpha\psi^0, \qquad \delta\bar{\psi}^0=-i\bar{\psi}^0\delta\alpha a(r), \quad (3.11)$$

$$\delta\bar{P}_\mu=ia(l)(\delta\alpha,\bar{P}_\mu)_-, \qquad \delta\Omega(l)=-i\Omega(l)\delta\alpha.$$

(II) is obtained by interchanging l and r. The $\delta\alpha$ appearing in these equations is position independent. Therefore the invariance operations expressed in the vector gauge are independent of the pseudoscalar field but do depend on the pseudoscalar element γ^5.

To express the same transformations in the fermion gauge, we need the relations

$$B=U\psi^0, \quad (3.12)$$

$$P_\mu=U\bar{P}_\mu U^{-1}+U\partial_\mu U^{-1}. \quad (3.13)$$

One then finds

$$\delta B=\delta\mathcal{H}B, \quad (3.14)$$

$$\delta P_\mu=(\delta\mathcal{H},P_\mu)_- -\partial_\mu(\delta\mathcal{H}), \quad (3.15)$$

where, for instance,

$$\delta\mathcal{H}^{(\mathrm{I})}=ia(l)U\delta\alpha^{\mathrm{I}}U^{-1}+\delta UU^{-1}. \quad (3.16)$$

Again (II) is obtained by replacing l by r.

It will be shown in Sec. V that $\delta\mathcal{H}$ is independent of γ^5. On the other hand, $\delta\mathcal{H}$ does depend on the pseudoscalar field and therefore on position.

One may say that the symmetry operators are realized linearly in the vector gauge and nonlinearly in the fermion gauge. The situation may also be described by saying that γ^5 linearizes the symmetry transformations.

The fact that $\delta\mathcal{H}$ is, in fact, independent of γ^5 has the following important consequences. By (3.15) one finds

$$\delta V_\mu=(\delta\mathcal{H},V_\mu)-\partial_\mu(\delta\mathcal{H}), \quad (3.17)$$

$$\delta A_\mu=(\delta\mathcal{H},A_\mu). \quad (3.18)$$

Therefore the terms (2.13) are invariant, while corresponding terms depending on V_μ would not be invariant. It follows that the complete Lagrangian (2.14) is invariant under the chiral group (I)$\times$(II).

Further discussion is facilitated by the introduction of group-geometrical methods.

IV. GROUP GEOMETRY

Define

$$U_k(\pm,\alpha)=\pm i[\partial U^{\pm1}(\alpha)/\partial\phi^k]U^{\mp1}(\alpha),$$
$$\alpha=l,r;\quad k=1,\ldots,8 \quad (4.1)$$

where U belongs to a fundamental representation. Then n-uples of (real) parallel vector fields $u_{Ak}(\pm,\alpha)$ on the group space may be introduced as follows:

$$R_0U_k(\pm,\alpha)=cu_{Ak}(\pm,\alpha)\lambda_A \quad (4.2a)$$

$$=2cu_{Ak}(\pm,\alpha)\mathbf{F}_A, \quad (4.2b)$$

and the relation between left and right fields is

$$U_k(\pm,r)=-U_k(\mp,l), \quad (4.3a)$$

$$u_{Ak}(\pm,r)=-u_{Ak}(\mp,l). \quad (4.3b)$$

Here R_0 is the radius of curvature of the group space and c is a normalization constant. The $\mathbf{F}_A$ are generators of a fundamental representation. The λ_A satisfy the normalization condition

$$\mathrm{Tr}\lambda_A\lambda_B=2\delta_{AB} \quad (4.4)$$

and the commutation relations[16]

$$(\lambda_A,\lambda_B)_-=2if_{ABC}\lambda_C, \quad (4.5a)$$

$$\{\lambda_A,\lambda_B\}_+=\tfrac{4}{3}\delta_{AB}1+2d_{ABC}\lambda_C. \quad (4.5b)$$

The metric of the group space is then[17]

$$\gamma_{kl}=u_{Ak}(\pm,\alpha)u_{Al}(\pm,\alpha) \quad (4.6a)$$

$$=(R_0^2/2c^2)\,\mathrm{Tr}U_k(\pm,\alpha)U_l(\pm,\alpha). \quad (4.6b)$$

We define the "rotation" matrix

$$R_{AB}(\alpha)=u_{Ak}(+,\alpha)u^k{}_B(-,\alpha), \quad (4.7)$$

where $u^k{}_B(-,\alpha)$ is obtained from $u_{Bk}(-,\alpha)$ by raising the index in the usual way with the aid of the metric tensor of the group space. Then

$$R_{AB}(\alpha)\lambda_B=U^{-1}(\alpha)\lambda_AU(\alpha), \quad (4.8a)$$

$$R_{BA}(\alpha)\lambda_B=U(\alpha)\lambda_AU^{-1}(\alpha). \quad (4.8b)$$

Here $R(\alpha)$ is the element of the regular (adjoint) representation corresponding to $U(\alpha)$.

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[16] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).

[17] Throughout this paper, symbols which refer to left or right, such as α in this case, are not summed over when repeated.

By differentiating (4.8), one obtains

$$(R_0/c)[\partial R_{AB}(\alpha)/\partial\phi^k]=2f_{BCD}R_{AC}(\alpha)u_{Dk}(-,\alpha) \quad (4.9a)$$

$$=2f_{ADC}R_{CB}(\alpha)u_{Dk}(+,\alpha). \quad (4.9b)$$

Now we define the analog of (4.1) in the regular representation

$$R_k(\pm,\alpha)=\pm i[\partial R^{\pm1}(\alpha)/\partial\phi^k]R^{\mp1}(\alpha). \quad (4.10)$$

Then, using (4.9), we obtain

$$(R_0/c)[R_k(\pm,\alpha)]_{AB}=2if_{ADB}u_{Dk}(\pm,\alpha) \quad (4.11)$$

or

$$R_0R_k(\pm,\alpha)=2cu_{Ak}(\pm,\alpha)F_A, \quad (4.12)$$

where F_A is a generator of the regular representation.

Equations (4.2) and (4.12) express the same relation in the fundamental and regular representations, respectively. In $SU(3)$, (4.2) is a 3×3 matrix relation, while (4.12) is an 8×8 matrix relation. The coefficients $u_{Ak}(\pm,\alpha)$ are, of course, the same for (4.2) and (4.12).

Let us also note that $\mathbf{F}_A$ and F_A are differently normalized. Thus,

$$\mathrm{Tr}\mathbf{F}_A\mathbf{F}_B=\tfrac{1}{2}\delta_{AB}, \quad (4.13a)$$

while

$$\mathrm{Tr}F_AF_B=3\delta_{AB} \quad (4.13b)$$

in order that the commutation relations be the same:

$$(\mathbf{F}_A,\mathbf{F}_B)=if_{ABC}\mathbf{F}_C, \quad (4.14a)$$

$$(F_A,F_B)=if_{ABC}F_C. \quad (4.14b)$$

Therefore,

$$u_{Ak}(\pm,\alpha)=(R_0/c)\ \mathrm{Tr}U_k(\pm,\alpha)\mathbf{F}_A \quad (4.15a)$$

$$=(R_0/6c)\ \mathrm{Tr}R_k(\pm,\alpha)F_A. \quad (4.15b)$$

Let us next calculate the coefficients $u_{Ak}(\pm,\alpha)$. Because of the relationship between the right and left n-uples (4.3), we shall work only with the left parallel fields and, therefore, define the short notation

$$u_{Ak}(\pm)=u_{Ak}(\pm,l). \quad (4.16)$$

In a fundamental representation, one may write

$$U(l)=\tfrac{1}{3}\chi+i(c/R_0)\sum_A h_A\lambda_A, \quad (4.17)$$

where

$$\chi=\mathrm{Tr}U(l). \quad (4.18)$$

Making use of (4.5) and the reality of the $u_{Ak}(\pm)$, we find

$$\tfrac{1}{2}[u_{Ak}(+)+u_{Ak}(-)]$$
$$=\tfrac{1}{3}\ \mathrm{Re}[(\partial\chi/\partial\phi^k)h_A^*-\chi^*(\partial h_A/\partial\phi^k)]$$
$$-(c/R_0)d_{ABC}\ \mathrm{Im}[(\partial h_B/\partial\phi^k)h_C^*], \quad (4.19a)$$

$$\tfrac{1}{2}[u_{Ak}(+)-u_{Ak}(-)]$$
$$=-(c/R_0)f_{ABC}\ \mathrm{Re}[(\partial h_B/\partial\phi^k)h_C^*]. \quad (4.19b)$$

In Appendix A these vectors are given in a particular coordinate system.

Near the identity $\frac{1}{3}\chi\rightarrow1$, and the h_A themselves may be chosen as a coordinate system, since they become real. Then, to lowest order,

$$h_A=\phi_A, \quad (4.20)$$

$$\tfrac{1}{2}[u_{Ak}(+)+u_{Ak}(-)]=-\delta_{Ak}, \quad (4.21a)$$

$$\tfrac{1}{2}[u_{Ak}(+)-u_{Ak}(-)]=-(c/R_0)f_{AkB}\phi_B, \quad (4.21b)$$

$$\gamma_{kl}=\delta_{kl}, \quad (4.21c)$$

$$R_{AB}(l)=\delta_{AB}+(2c/R_0)f_{ABC}\phi_C. \quad (4.21d)$$

V. GEOMETRY OF INVARIANCE GROUP

The left and right invariance groups induce, according to (3.7) and (3.9), the transformations

$$\delta\Omega(l)=-i\Omega(l)\delta\alpha^{\mathrm{I}}, \quad (5.1a)$$

$$\delta\Omega(r)=-i\Omega(r)\delta\alpha^{\mathrm{II}}. \quad (5.1b)$$

An equivalent form of (5.1b) is

$$\delta\Omega(l)=i\delta\alpha^{\mathrm{II}}\Omega(l), \quad (5.1c)$$

as pointed out in (3.10).

These equations may be expressed in geometrical form via the introduction of the coordinates ϕ^k and the matrix vector fields

$$\Omega_k(\pm,\alpha)=\pm i[\partial\Omega(\alpha)^{\pm1}/\partial\phi^k]\Omega(\alpha)^{\mp1},\qquad \alpha=l,r. \quad (5.2)$$

These fields are related to Ω in the same way that U_k and R_k are related to U and R, respectively. Here again we have the left-right relation

$$\Omega_k(r,\pm)=-\Omega_k(l,\mp). \quad (5.3)$$

Without specifying the representation, let us introduce F_A satisfying

$$\mathrm{Tr}F_AF_B=N\delta_{AB}, \quad (5.4)$$

where $N=\frac{1}{2}$ for the fundamental representation and $N=3$ for the regular representation.

Let Ω be given in either the fundamental (quark) or regular (octet) representation. Now define $\omega_{Ak}(\pm,\alpha)$ and Γ_{kl} by the following equations:

$$R_0\Omega_k(\pm,\alpha)=(1/\sqrt{N})\omega_{Ak}(\pm,\alpha)F_A, \quad (5.5)$$

$$\Gamma_{kl}=\omega_{Ak}(\pm,\alpha)\omega_{Al}(\pm,\alpha) \quad (5.6)$$

and, therefore,

$$=R_0^2\ \mathrm{Tr}\Omega_k(\pm,\alpha)\Omega_l(\pm,\alpha). \quad (5.7)$$

According to (4.2), (4.12), and (5.5), the expansions of U_k, R_k, and Ω_k are normalized in the same way if

$$2c=1/\sqrt{N}. \quad (5.8)$$

If Ω is chosen in the regular representation, $c=\sqrt{3}/6$. In the quark representation, $c=1/\sqrt{2}$. It may then be shown that

$$\omega_{Ak}(+,\alpha)=R_{AB}(\alpha)[u_{Bk}(+,\alpha)+u_{Bk}(-,\alpha)], \quad (5.9a)$$

$$\omega_{Ak}(-,\alpha)=R_{BA}(\alpha)[u_{Bk}(+,\alpha)+u_{Bk}(-,\alpha)]. \quad (5.9b)$$

We shall also introduce Δ^{kl} according to

$$\Delta^{kl}\Gamma_{lm}=\delta^k{}_m. \quad (5.10)$$

Then

$$\Delta^{kl}\omega_{Ak}(\pm,\alpha)\omega_{Bl}(\pm,\alpha)=\delta_{AB}, \quad (5.11)$$

$$\Delta^{kl}\omega_{Al}(\pm,\alpha)\omega_{Am}(\pm,\alpha)=\delta^k{}_m. \quad (5.12)$$

One may now express (5.1a) as

$$(\delta^{\rm I}\phi^k)[\partial\Omega(l)/\partial\phi^k]=-i\Omega(l)\delta\alpha^{\rm I}. \quad (5.13)$$

Introducing Γ_{km} by (5.7), one finds

$$\Gamma_{km}(\delta^{\rm I}\phi^k)=R_0^2\,{\rm Tr}\Omega_m(-,l)\delta\alpha^{\rm I}. \quad (5.14)$$

Now put

$$\delta\alpha^{\rm I}=F_A\delta\alpha_A{}^{\rm I}. \quad (5.15)$$

Then

$$\Gamma_{kn}(\delta\phi^k/\delta\alpha_A{}^{\rm I})=R_0^2\,{\rm Tr}\Omega_n(-,l)F_A$$
$$=(R_0/2c)\omega_{An}(-,l),$$
$$\delta\phi^k/\delta\alpha_A{}^{\rm I}=(R_0/2c)\Delta^{kn}\omega_{An}(-,l). \quad (5.16a)$$

Similarly,

$$\delta\phi^k/\delta\alpha_A{}^{\rm II}=(R_0/2c)\Delta^{kn}\omega_{An}(-,r). \quad (5.16b)$$

Equations (5.16) are equivalent to (5.1).

We may now complete the geometric transcription of $\delta\mathcal{H}$ which appears in (3.16), namely,

$$\delta\mathcal{H}^{\rm I}=ia(l)U\delta\alpha^{\rm I}U^{-1}+\delta U\,U^{-1}. \quad (5.17)$$

This equation is true in every representation (U does not refer solely to the fundamental representation here). By (5.15)

$$a(l)U\delta\alpha^{\rm I}U^{-1}=a(l)U(l)\delta\alpha^{\rm I}U^{-1}(l)$$
$$=a(l)U(l)F_AU^{-1}(l)\delta\alpha_A{}^{\rm I},$$

or

$$a(l)U\delta\alpha^{\rm I}U^{-1}=a(l)R_{BA}(l)F_B\delta\alpha_A{}^{\rm I}. \quad (5.18)$$

One also finds

$$i\delta U\,U^{-1}=[U_k(+,l)a(l)+U_k(+,r)a(r)]$$
$$\times(\delta\phi^k/\delta\alpha_A{}^{\rm I})\delta\alpha_A{}^{\rm I}. \quad (5.19)$$

By (5.17), (5.18), and (5.19) it then follows that

$$\delta\mathcal{H}^{\rm I}=iH_{AB}{}^{\rm I}F_B\delta\alpha_A{}^{\rm I}, \quad (5.20)$$

where

$$H_{AB}{}^{\rm I}=a(l)R_{BA}(l)-(2c/R_0)$$
$$\times[u_{Bk}(+,l)a(l)+u_{Bk}(+,r)a(r)]\delta\phi^k/\delta\alpha_A{}^{\rm I}. \quad (5.21a)$$

Similarly,

$$H_{AB}{}^{\rm II}=a(r)R_{BA}(r)-(2c/R_0)$$
$$\times[u_{Bk}(+,l)a(l)+u_{Bk}(+,r)a(r)]\delta\phi^k/\delta\alpha_A{}^{\rm II}, \quad (5.21b)$$

with $\delta^k/\delta\alpha_A$ given by (5.16). These equations may also be written in terms of l vectors only by (4.3). Therefore,

$$H_{AB}{}^{\rm I}=a(l)R_{BA}(l)-[u_{Bk}(+,l)a(l)$$
$$-u_{Bk}(-,l)a(r)]\Delta^{kn}\omega_{An}(-,l). \quad (5.22)$$

The γ^5 part is

$$H_{AB}{}^{\rm I(5)}=\tfrac{1}{2}R_{BA}(l)$$
$$-\tfrac{1}{2}[u_{Bk}(+,l)+u_{Bk}(-,l)]\Delta^{kn}\omega_{An}(-,l). \quad (5.23)$$

But, by (5.9) and (5.11),

$$H_{AB}{}^{\rm I(5)}=\tfrac{1}{2}R_{BA}(l)-\tfrac{1}{2}R_{BC}(l)\Delta^{kn}\omega_{Ck}(-,l)\omega_{An}(-,l)$$
$$=0. \quad (5.24)$$

Therefore, $H_{AB}{}^{\rm I}$ is independent of γ^5, and one has simply

$$H_{AB}{}^{\rm I}=\tfrac{1}{2}R_{BA}(l)-\tfrac{1}{2}[u_{Bk}(+,l)+u_{Bk}(+,r)]\Delta^{kn}\omega_{An}(-,l) \quad (5.25a)$$

and

$$H_{AB}{}^{\rm II}=\tfrac{1}{2}R_{BA}(r)$$
$$-\tfrac{1}{2}[u_{Bk}(+,l)+u_{Bk}(+,r)]\Delta^{kn}\omega_{An}(-,r). \quad (5.25b)$$

The following relations may also be established:

$$H_{AB}{}^{\rm I}+H_{AB}{}^{\rm II}=\delta_{AB}, \quad (5.26)$$

$$H_{AB}{}^{\rm I}-H_{AB}{}^{\rm II}=\Delta^{mk}[u_{Am}(+,l)+u_{Am}(-,l)]$$
$$\times[u_{Bk}(+,l)-u_{Bk}(-,l)]. \quad (5.27)$$

Therefore also

$$\delta\mathcal{H}^{\rm (I,II)}=\tfrac{1}{2}i[\delta_{AB}\pm\Delta^{kn}(u_{Ak}(+,l)+u_{Ak}(-,l))$$
$$\times(u_{Bn}(+,l)-u_{Bn}(-,l))]F_B\delta\alpha_A{}^{\rm (I,II)}. \quad (5.28)$$

In the weak-field limit, by (4.21), (5.6), and (5.9),

$$\Gamma_{kl}\to4\delta_{kl}. \quad (5.29)$$

Therefore, by (4.21),

$$\delta\mathcal{H}^{\rm (I,II)}\to\tfrac{1}{2}i[\delta_{AB}\mp(c/R_0)f_{ABC}\phi^C]F_B\delta\alpha_A{}^{\rm (I,II)}, \quad (5.30)$$

where $c=1/\sqrt{2}$ and $c=1/2\sqrt{3}$ for the fundamental and regular representations, respectively, as the fermions are chosen to be quarks or baryons.

VI. EFFECTIVE LAGRANGIAN

We shall study the following Lagrangian:

$$-L=-\tfrac{1}{4}\,{\rm Tr}P_{\mu\nu}P^{\mu\nu}+\tfrac{1}{2}\,{\rm Tr}\bar{P}_\mu\bar{P}^\mu Q+\kappa\,{\rm Tr}A_\mu A^\mu\tilde{Q}(+)$$
$$-i\bar{B}\gamma^\mu\nabla_\mu B+\bar{B}\mathfrak{M}B-i\bar{B}\gamma^\mu\gamma^5A_\mu{}^dB. \quad (6.1)$$

Here

$$\nabla_\mu=\partial_\mu+P_\mu{}^f, \quad (6.2a)$$

where

$$P_\mu{}^f=P_{\mu B}F_B=(V_{\mu B}+\gamma^5A_{\mu B})F_B \quad (6.2b)$$

and

$$A_\mu{}^d=A_{\mu B}{}^dD_B. \quad (6.3)$$

Equation (6.1) reduces to (2.14) if one puts $B=N$, $Q=\tilde{Q}(+)=M^2$, $\mathfrak{M}=m1$, and $A_{\mu B}{}^d=\epsilon A_{\mu B}$. On the other hand, (6.1) has more structure than (2.14) because of these differences and, in particular, because of the existence of D as well as F couplings. One consequence is that the Lagrangian (2.14) is exactly invariant under the chiral group $G\times G$ but (6.1) is not, because of the mass operators given below.

In A the structure (2.14) was studied under the assumption that the symmetry group is $SU(2)$. This assumption implies that the electromagnetic mass differences are neglected. If we now attempted to study (6.1) in an analogous fashion, it would be necessary to neglect octet (or quark) mass differences. Since such an assumption would not be sufficiently realistic, the mass operators Q, $\tilde{Q}(+)$, and $\mathfrak{M}$ have been introduced.

Like (2.14), Eq. (6.1) is written in a mixed gauge. The kinetic energy terms are of course gauge invariant. However, the second term is expressed in the vector gauge, while all other terms are written in the fermion gauge.

The boson fields, as well as their mass operators, Q and $\tilde{Q}(+)$, appear as 3×3 matrices in the trace terms. However, if B is an eight-rowed basis, then ∇_μ, $\mathfrak{M}$, and $A_\mu{}^d$ in the last term are 8×8 matrices. A_μ in the κ term is, of course, expressed in its 3×3 form. (The superscripts f and d are needed only in the regular representation.)

We assume the mass operators

$$Q=M^2 1+\mu^2\lambda_8, \tag{6.4}$$

$$\mathfrak{M}=m1+m_- F_8+m_+ D_8 \tag{6.5}$$

in order to obtain the usual octet mass formulas. Here λ_8 is 3×3, while F_8 and D_8 are 8×8 if B is an eight-rowed basis, or the appropriate 3×3 versions if the quark model is under consideration.

In (6.4) the parameters M^2 and μ^2 are fixed by the masses of the ρ and K^*. The eighth mass then lies between the masses of the ϕ and the ω; however, the fact that ϕ and ω do not satisfy the octet mass formula is not important for the applications discussed in this paper.

The mass operator $\tilde{Q}(+)$ and the associated $\tilde{Q}(-)$ are defined as follows:

$$\tilde{Q}(\pm)=\tfrac{1}{2}[\tilde{Q}(l)\pm\tilde{Q}(r)], \tag{6.6a}$$

where

$$\tilde{Q}(\alpha)=U(\alpha)QU(\alpha)^{-1}, \qquad \alpha=l,r. \tag{6.6b}$$

While Q might appear to be the first choice for the κ term, it turns out that $\tilde{Q}(+)$ leads to a simpler nonlinear structure as well as to the same results as Q in lowest order (see Sec. VII). At this point let us also note the relations

$$\tilde{Q}(\alpha)=M^2 1+\mu^2 R_{B8}(\alpha)\lambda_B, \tag{6.7}$$

$$\tilde{Q}(+)=M^2 1+\tfrac{1}{2}\mu^2[R_{B8}(l)+R_{B8}(r)]\lambda_B, \tag{6.8a}$$

$$\tilde{Q}(-)=\tfrac{1}{2}\mu^2[R_{B8}(l)-R_{B8}(r)]\lambda_B. \tag{6.8b}$$

The boson matrices are three dimensional as just expressed. If the eight-dimensional basis were adopted, these matrices would be pure D.

We have assumed in (6.2) that P_μ appearing in ∇_μ is pure F, although it would be possible for a D part to enter here. The reason for this choice depends on the fact that a change in gauge adds to the connection P_μ an inhomogeneous term which is pure F. Therefore a P_μ which is pure F will remain so under a gauge transformation. For this reason it appears natural to adopt (6.2). This is the exact analog of our procedure for $SU(2)$.

It is finally necessary to decide the F or D character of the ϵ term. It is not possible to settle this question within the nucleon subspace because F_k and D_k ($k=1, 2, 3,$) both behave like $\tfrac{1}{2}\tau_k$ in this subspace.

The ϵ term is required to renormalize g_A/g_V and therefore should contain a D part. It appears possible and simplest to assume that this term, like the κ term, is exclusively D, and we have therefore adopted (6.3). However, we also assume

$$A_{\mu B}{}^d=\epsilon A_{\mu B}. \tag{6.9}$$

According to this assumption, the D components are not dynamically independent of the F components and therefore are not associated with independent particles.

Notice that $g_A/g_V=-1$ corresponds to $\epsilon=1$. It will be shown in Sec. X that the observed value of g_A/g_V corresponds to $\epsilon\cong\tfrac{3}{2}$.

Finally, just as in the $SU(2)$ case, the ϵ term does not break the symmetry of the Lagrangian.

VII. ANALYSIS OF VECTOR LAGRANGIAN

The analysis of (6.1) follows almost exactly the $SU(2)$ analysis of (2.14). Introduce the following Hermitian, dimensional fields:

$$P_\mu=i(g/\sqrt{2})(v_\mu+a_\mu\gamma^5) \tag{7.1a}$$

$$=ig(v_{\mu A}+a_{\mu A}\gamma^5)\mathbf{F}_A, \tag{7.1b}$$

where

$$v_\mu=\sqrt{2}v_{\mu A}\mathbf{F}_A, \tag{7.2a}$$

$$a_\mu=\sqrt{2}a_{\mu A}\mathbf{F}_A, \tag{7.2b}$$

so that

$$\mathrm{Tr}v_\mu v^\mu=v_{\mu A}v^\mu{}_B\delta_{AB}, \tag{7.3a}$$

$$\mathrm{Tr}a_\mu a^\mu=a_{\mu A}a^\mu{}_B\delta_{AB}. \tag{7.3b}$$

We then rescale the Lagrangian so that

$$\begin{aligned}L=&-\tfrac{1}{4}\,\mathrm{Tr}(v_{\mu\lambda}v^{\mu\lambda}+a_{\mu\lambda}a^{\mu\lambda})+\tfrac{1}{2}\,\mathrm{Tr}(\bar{v}_\mu\bar{v}^\mu+\bar{a}_\mu\bar{a}^\mu)Q\\&+\kappa\,\mathrm{Tr}a_\mu a^\mu\tilde{Q}(+)+i\bar{B}\gamma^\mu(\partial_\mu+igv_{\mu A}F_A\\&+iga_{\mu A}F_A\gamma^5+i\epsilon g a_{\mu A}D_A\gamma^5)B-\bar{B}\mathfrak{M}B,\end{aligned} \tag{7.4a}$$

where we have introduced the decomposition

$$(\sqrt{2}/ig)P_{\mu\lambda}=v_{\mu\lambda}+a_{\mu\lambda}\gamma^5. \tag{7.4b}$$

We now focus on the partial Lagrangian

$$\tilde{L}=\tfrac{1}{4}\,\mathrm{Tr}[\bar{P}_\mu(l)\bar{P}^\mu(l)+\bar{P}_\mu(r)\bar{P}^\mu(r)]Q+\kappa\,\mathrm{Tr}a_\mu a^\mu\tilde{Q}(+), \tag{7.5}$$

where, for convenience, we define the Hermitian fields

$$P_\mu(l)=v_\mu+a_\mu, \tag{7.6a}$$

$$P_\mu(r)=v_\mu-a_\mu. \tag{7.6b}$$

Then, in order to express (7.4) entirely in one gauge, we make the substitution

$$\bar{P}_\mu(\alpha)=U^{-1}(\alpha)P_\mu(\alpha)U(\alpha)+\Delta_\mu(-,\alpha), \tag{7.7}$$

where

$$\Delta_\mu(-,\alpha)=(\sqrt{2}/ig)U^{-1}(\alpha)\partial_\mu U(\alpha). \tag{7.8}$$

One then finds

$$\tilde{L}=L_v+L_a+L_\Delta+L_-. \tag{7.9}$$

Here,

$$2L_v=\mathrm{Tr}[v_\mu v^\mu+\tfrac{1}{2}\{v_\mu,\Delta^\mu(l)+\Delta^\mu(r)\}_+]\tilde{Q}(+), \tag{7.10}$$

$$2L_a=\mathrm{Tr}[(1+2\kappa)a_\mu a^\mu+\tfrac{1}{2}\{a_\mu,\Delta^\mu(l)-\Delta^\mu(r)\}_+]\tilde{Q}(+), \tag{7.11}$$

$$2L_\Delta=\tfrac{1}{2}\mathrm{Tr}[\Delta_\mu(l)\Delta^\mu(l)\tilde{Q}(l)+\Delta_\mu(r)\Delta^\mu(r)\tilde{Q}(r)], \tag{7.12}$$

$$2L_-=\mathrm{Tr}[\{a_\mu,v^\mu\}_++\tfrac{1}{2}\{v_\mu,\Delta^\mu(l)-\Delta^\mu(r)\}_+ +\tfrac{1}{2}\{a_\mu,\Delta^\mu(l)+\Delta^\mu(r)\}_+]\tilde{Q}(-), \tag{7.13}$$

where

$$\Delta_\mu(\alpha)=\Delta_\mu(+,\alpha)=U(\alpha)\Delta_\mu(-,\alpha)U^{-1}(\alpha) \tag{7.14}$$

and $\tilde{Q}(\pm)$ has been defined in (6.6).

The simplification of L_a in (7.11) effected by the introduction of $\tilde{Q}(+)$ in (6.1) is now apparent. Just as in the $SU(2)$ case, L_a contains a term linear in a_μ which must be eliminated by the following displacement[18]:

$$a_\mu=\hat{a}_\mu+\Delta_\mu, \tag{7.15}$$

where

$$\Delta_\mu=\tfrac{1}{2}[1/(1+2\kappa)][\Delta_\mu(r)-\Delta_\mu(l)]. \tag{7.16}$$

Then the partial Lagrangian $\tilde{L}$ may be rewritten in terms of the physical fields v_μ and $\hat{a}_\mu$ as follows:

$$\tilde{L}=\tfrac{1}{2}\mathrm{Tr}[v_\mu v^\mu+(1+2\kappa)\hat{a}_\mu\hat{a}^\mu]\tilde{Q}(+)+L_\phi+L_{V\phi\phi}+\hat{L}_-, \tag{7.17}$$

where

$$8(1+2\kappa)L_\phi=\mathrm{Tr}[(1+4\kappa)(\Delta_\mu(r)\Delta^\mu(r)+\Delta_\mu(l)\Delta^\mu(l)) +\{\Delta_\mu(r),\Delta^\mu(l)\}_+]\tilde{Q}(+), \tag{7.18}$$

$$4L_{V\phi\phi}=\mathrm{Tr}[\{v_\mu,\Delta^\mu(l)+\Delta^\mu(r)\}_+\tilde{Q}(+)+(2\kappa/(1+2\kappa)) \times\{v_\mu,\Delta^\mu(l)-\Delta^\mu(r)\}_+\tilde{Q}(-)], \tag{7.19}$$

$$2\hat{L}_-=\mathrm{Tr}[\{\hat{a}_\mu,v^\mu\}_++\tfrac{1}{2}\{\hat{a}_\mu,\Delta^\mu(l)+\Delta^\mu(r)\}_+ +(\kappa/(1+2\kappa))(\Delta_\mu(l)\Delta^\mu(l)-\Delta_\mu(r)\Delta^\mu(r))]\tilde{Q}(-). \tag{7.20}$$

All boson matrices appearing in $\tilde{L}$ are 3×3. Therefore, to introduce components of the displacement Δ_μ, we make use of (4.2). The reduction of $\tilde{L}$ may then be carried out in a straightforward way. We find for L_ϕ, the kinetic energy of the pseudoscalar field,

$$L_\phi=[c^2\tilde{Q}_{AB}(+)/2g^2R_0^2(1+2\kappa)] \times(\partial_\mu\phi^k)(\partial^\mu\phi^m)[(1+4\kappa)(u_{Ak}(+)u_{Bm}(+) +u_{Ak}(-)u_{Bm}(-))-2u_{Ak}(-)u_{Bm}(+)], \tag{7.21}$$

where we have expressed L_ϕ entirely in the l space with the aid of (4.3) and where we have also introduced the notation

$$\tilde{Q}_{AB}(\pm)=\tfrac{1}{4}\mathrm{Tr}\{\lambda_A,\lambda_B\}_+\tilde{Q}(\pm), \tag{7.22}$$

$$u_{Ak}(\pm)=u_{Ak}(\pm,l). \tag{7.23}$$

The interaction energy between the vector and pseudoscalar fields is

$$L_{V\phi\phi}=(-c/gR_0)v_{\mu A}\partial^\mu\phi^k\{[u_{Bk}(+)-u_{Bk}(-)]\tilde{Q}_{AB}(+) +[2\kappa/(1+2\kappa)][u_{Bk}(+)+u_{Bk}(-)]\tilde{Q}_{AB}(-)\}. \tag{7.24}$$

We next compute the weak-field limits. Then to lowest order, from (4.21) and (6.8),

$$\tilde{Q}(+)=M^2 1+u^2\lambda_8=Q, \tag{7.25a}$$

$$\tilde{Q}(-)=-2\mu^2(c/R_0)f_{8BC}\phi^C\lambda_B \tag{7.25b}$$

and[19]

$$\tilde{Q}_{AB}(+)=Q_{(A)}\delta_{AB}, \tag{7.26a}$$

$$\tilde{Q}_{AB}(-)=-2\mu^2(c/R_0)d_{ABC}f_{8CD}\phi^D, \tag{7.26b}$$

where

$$Q_A=M^2+\mu^2 d_{(A)A8}. \tag{7.27}$$

Then in this limit, according to (4.21), and using the properties of the f and d tensors of $SU(3)$,

$$L_\phi\to[4\kappa/(1+2\kappa)](c^2/g^2R_0^2)Q_A\partial_\mu\phi^A\partial^\mu\phi^A, \tag{7.28}$$

$$L_{V\phi\phi}\to(2c^2/gR_0^2)v_{\mu A}(\partial^\mu\phi^B)\phi^C \times\{f_{ABC}Q_A-[4\kappa/(1+2\kappa)]\mu^2(d_{8AD}f_{DBC}+d_{8BD}f_{DAC})\}. \tag{7.29}$$

Now we normalize the pseudoscalar fields so that

$$Q_{(A)}^{1/2}\phi^A=\eta\Phi^A, \tag{7.30}$$

where Φ^A are the physical pseudoscalar fields and η is a renormalization constant which is the same for all of the pseudoscalars. Then

$$L_\Phi\to[4\kappa/(1+2\kappa)](c^2\eta^2/g^2R_0^2)\partial_\mu\Phi^A\partial^\mu\Phi^A \tag{7.31}$$

and

$$L_{V\Phi\Phi}\to[2c^2\eta^2/gR_0^2(Q_BQ_C)^{1/2}]v_{\mu A}(\partial^\mu\Phi_B)\Phi_C \times\{f_{ABC}Q_A-[4\kappa/(1+2\kappa)]\mu^2(d_{8AD}f_{DBC}+d_{8BD}f_{DAC})\}. \tag{7.32}$$

It is next necessary to formulate "vector universality."[20] This may be done by going back to the degenerate multiplets in which the eight vectors, as well

[18] See, for example, J. Wess and B. Zumino, Ref. 2.

[19] The parenthesis is introduced to prevent a summation. Compare (7.26) with (7.28).

[20] J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).

as the eight pseudoscalars, are not distinguishable. Then $\mu=0$ and $Q_A=M^2$, where M is the mass of the degenerate multiplet. The corresponding interaction is

$$L_{V\Phi\Phi}\to(2c^2\eta^2/gR_0^2)f_{ABC}v_{\mu A}\partial^\mu\Phi_B\Phi_C. \qquad (7.33)$$

Vector universality then becomes

$$g_{V\Phi\Phi}=g_{VBB}=g \qquad (7.34)$$

or

$$2c^2\eta^2/gR_0^2=g. \qquad (7.35)$$

Of course, we must also require that the kinetic energies of the pseudoscalar fields approach the correct weak-field limits; that is,

$$[4\kappa/(1+2\kappa)](c^2\eta^2/g^2R_0^2)=\tfrac{1}{2}. \qquad (7.36)$$

From (7.35) and (7.36),

$$\kappa=\tfrac{1}{2}. \qquad (7.37)$$

Then (7.32) becomes

$$L_{V\Phi\Phi}\to gv_{\mu A}(\partial^\mu\Phi_B)\Phi_C[f_{ABC}(Q_B/Q_C)^{1/2}], \qquad (7.38)$$

and, from (7.16), the displacement of the axial vector is

$$\Delta_\mu=(1/2\sqrt{2}g)[U_k(+,l)+U_k(-,l)]\partial_\mu\phi^k \qquad (7.39a)$$

$$=(c/\sqrt{2}gR_0)[u_{Ak}(+)+u_{Ak}(-)]\partial_\mu\phi^k\mathbf{F}_A. \qquad (7.39b)$$

In the limit

$$\Delta_\mu\to-(\sqrt{2}c/gR_0)\mathbf{F}_A\partial_\mu\phi_A \qquad (7.40a)$$

$$=-Q_A^{-1/2}\mathbf{F}_A\partial_\mu\Phi_A. \qquad (7.40b)$$

The Lagrangian (7.4) then becomes, in a mixed notation,

$$L=-\tfrac{1}{4}\,\mathrm{Tr}(v_{\mu\nu}v^{\mu\nu}+a_{\mu\nu}a^{\mu\nu})+\tfrac{1}{2}M_A{}^2v_{\mu A}v^\mu{}_A$$
$$+\tfrac{1}{2}(2M_A{}^2)\hat{a}_{\mu A}\hat{a}^\mu{}_A+\tfrac{1}{2}(\partial_\mu\Phi_A)(\partial^\mu\Phi_A)$$
$$+gv_{\mu A}(\partial^\mu\Phi_B)\Phi_C f_{ABC}(M_B/M_C)+\hat{L}_-+L_B, \qquad (7.41)$$

where the pseudoscalar terms are shown only in lowest order, L_- is not shown explicitly, and L_B is the part of L which depends on the baryon field if B is assigned to the octet. Here, $M_A{}^2=Q_A$ is the mass of the vector particle A. Of course, owing to the form of Q_A, $M_1=M_2=M_3=$ mass of the ρ meson, and $M_4=M_5=M_6=M_7$ mass of the K^* meson. Interpreted as an effective Lagrangian, (7.41) implies an entire series of Weinberg relations[13] between the masses of the axial vectors and those of the associated vectors:

$$M_{aA}=\sqrt{2}M_{vA}=\sqrt{2}Q_A{}^{1/2}, \qquad A=1,\ldots,8. \qquad (7.42)$$

These relations are a direct consequence of the hypothesis of vector universality (7.34). Notice also that the physical pseudoscalar fields are normalized to the masses of the associated vector mesons, from (7.30) with $M_A{}^2=Q_A$:

$$M_{(A)}\phi^A=\eta\Phi^A. \qquad (7.43)$$

VIII. BARYON COUPLINGS

If, as we now assume, B belongs to the baryon basis, then the baryon part of (7.41) is, exclusive of the mass term,

$$L_B=i\bar{B}\gamma^\mu[\partial_\mu+igv_{\mu A}F_A+ig\hat{a}_{\mu A}(F_A+\epsilon D_A)\gamma^5]B$$
$$-g\bar{B}\gamma^\mu\gamma^5(F_A+\epsilon D_A)B\Delta_{\mu A}, \qquad (8.1)$$

where

$$\Delta_{\mu A}=(c/2gR_0)[u_{Ak}(+)+u_{Ak}(-)]\partial_\mu\phi^k \qquad (8.2)$$

and F_A and D_A are the usual 8×8 matrices. By choosing the appropriate three-dimensional matrices, Eq. (8.1) can also be interpreted as a quark Lagrangian.

According to (8.1) there are direct pseudoscalar-baryon couplings in addition to fundamental vector-baryon couplings. These result from the displacement of the axial-vector field (7.15) and in the lowest order give rise to the gradient couplings

$$(c\eta/R_0M_A)[\bar{B}\gamma^\mu\gamma^5(F_A+\epsilon D_A)B]\partial_\mu\Phi_A \qquad (8.3)$$

according to (8.2) and (7.40). Here $(F_A+\epsilon D_A)/M_{(A)}$ measures the p-wave coupling of the pseudoscalar A in terms of the mass M_A of the associated vector.

On the other hand, there are no direct s-wave couplings. These are mediated by the vector mesons through the following lowest-order partial Lagrangian[20]:

$$-g\bar{B}\gamma^\mu v_{\mu A}F_AB+gv_{\mu A}(\partial^\mu\Phi_B)\Phi_C f_{ABC}(M_B/M_C). \qquad (8.4)$$

The resulting direct s-wave pseudoscalar-baryon couplings are

$$(g^2/M_A{}^2)(\bar{B}\gamma^\mu F_AB)f_{ABC}(\partial_\mu\Phi_B)\Phi_C(M_B/M_C). \qquad (8.5)$$

We may also rewrite the p-wave term (8.3) making use of (7.35):

$$(g/\sqrt{2}M_A)\bar{B}\gamma^\mu\gamma^5(F_A+\epsilon D_A)B\partial_\mu\Phi_A. \qquad (8.6)$$

The ratios of s- and p-wave terms for all the members of the pseudoscalar octet are then determined by the structure of the group (F and D matrices), by the masses of the various associated vector mesons, and by the parameter ϵ.

Also, from (8.6) above, the ratios of the various pseudoscalar-baryon coupling constants to $g_{\pi NN}$, the pion-nucleon coupling constant, depend not only on ϵ, the D/F ratio (which will be fixed in Sec. X), but also on the masses of the vector mesons.

IX. PARTIAL CURRENTS

The contribution of the fermions and of the pseudoscalars to the right chiral current is

$$J_A{}^{\mathrm{II}}=i\bar{B}\gamma_\mu(\delta B/\delta\alpha_A{}^{\mathrm{II}})+[\partial L/\partial(\partial^\mu\Phi^k)](\delta\Phi^k/\delta\alpha_A{}^{\mathrm{II}}), \qquad (9.1)$$

where the constant parameters $\alpha_A{}^{\mathrm{II}}$ belong to the right chiral group.

According to (3.14), (5.16), and (5.20),

$$J_A{}^{\mathrm{II}}=-\bar{B}\gamma_\mu H_{AB}{}^{\mathrm{II}}F_BB+(R_0M_k/2c\eta)$$
$$\times[\partial L/\partial(\partial^\mu\Phi^k)]\Delta^{kn}\omega_{An}(-,r). \qquad (9.2)$$

The preceding expression is exact and already depends on the particular form of the kinetic energy of the fermions. The dependence of L on the pseudoscalar velocity is more complicated. However, in this section we shall keep only the leading terms, according to (7.41) and (8.3), as follows:

$$\partial L/\partial(\partial^\mu\Phi^B)\rightarrow\partial_\mu\Phi_B+(c\eta/R_0M_{(B)})\bar{B}\gamma_\mu\gamma^5\,(F_B+\epsilon D_B)B. \tag{9.3}$$

We shall also replace the exact variations by the following expressions:

$$\delta\Phi^B/\delta\alpha_A{}^{\text{II}}\rightarrow(M_{(B)}R_0/4c\eta)[\delta_{AB}+(2c\eta/R_0M_{(C)})f_{ABC}\Phi_C], \tag{9.4}$$

according to (4.21), (5.16), and (5.29), and

$$\delta B/\delta\alpha_A{}^{\text{II}}\rightarrow\tfrac{1}{2}iF_AB, \tag{9.5}$$

according to (3.14) and (5.30).

Then the current in this approximation is

$$J_{A\mu}{}^{\text{II}}=-\tfrac{1}{2}\bar{B}\gamma_\mu F_AB+\tfrac{1}{4}\bar{B}\gamma_\mu\gamma^5(F_A+\epsilon D_A)B$$
$$+(M_{(A)}R_0/4c\eta)\partial_\mu\Phi_A-(M_C/2M_B)f_{ABC}\Phi_B(\partial_\mu\Phi_C). \tag{9.6}$$

By construction the preceding currents characterize the symmetry of the strong interactions. According to $SU(2)$ theory, however, the weak current is itself proportional to $J_{A\mu}{}^{\text{II}}$. Then, in going on to $SU(3)$, one must introduce the $\Delta S=\Delta Q$ selection rule as well as the Cabibbo angle.[21] We shall introduce these features in just the usual way, and shall write the matrix element

$$\langle\,|\,J_{\mu A}{}^h(\text{weak})\,|\,\rangle=F_\theta(\Delta S,\Delta Q)\langle\,|\,2J_{\mu A}{}^{\text{II}}\,|\,\rangle, \tag{9.7}$$

where $F_\theta(\Delta S,\Delta Q)$ contains the selection rules and the Cabibbo angle θ. (We have also assumed that θ_A and θ_V are equal.)

X. APPLICATIONS

The main consequences of the preceding work may be investigated by relating the strong pseudoscalar-baryon Lagrangian to the renormalized weak currents. By (9.6) the relevant expressions are, to lowest order,

$$2J_{\mu A}{}^{\text{II}}=-j_{\mu A}+k_{\mu A}+a^{-1}(M_{(A)}/M)\partial_\mu\Phi_A$$
$$-(M_C/M_B)f_{ABC}\Phi_B\partial_\mu\Phi_C \tag{10.1}$$

and

$$L_{BB\Phi}=a(M/M_A)k^\mu{}_A\partial_\mu\Phi_A+\tfrac{1}{2}a^2j^\mu{}_A\bar{f}_{ABC}\partial_\mu\Phi_B\Phi_C. \tag{10.2}$$

Here $j_{\mu A}$ and $k_{\mu A}$ are the vector and axial-vector currents associated with the fermions, namely,

$$j_{\mu A}=\bar{B}\gamma_\mu F_AB, \tag{10.3}$$

$$k_{\mu A}=\tfrac{1}{2}\bar{B}\gamma_\mu\gamma_5(F_A+\epsilon D_A)B. \tag{10.4}$$

We have introduced the mass M of the degenerate octet and the gradient coupling constant a. By (7.35),

$$a\equiv2c\eta/R_0M=\sqrt{2}g/M. \tag{10.5}$$

Finally the $\bar{f}_{ABC}$ are structure constants modified by the mass spectrum in the following way:

$$\bar{f}_{ABC}=(M^2M_{(B)}/M^2{}_{(A)}M_{(C)})f_{ABC}. \tag{10.6}$$

The vector current is pure F, while the axial-vector current is mostly D since ϵ will be shown to be nearly $\frac{3}{2}$. Therefore, according to (10.2), the s-wave interaction is pure F, while the p-wave interaction is mixed with $D/F\cong\frac{3}{2}$.

These results also differ from the $SU(2)$ results because they are modified by the mass spectrum. If the mass splitting is ignored, then $M/M_{(A)}\rightarrow1$ and $\bar{f}_{ABC}\rightarrow f_{ABC}$, and (10.1) and (10.2) reduce to the form which holds for $SU(2)$, although the f_{ABC} of course refer to $SU(3)$.

The free parameters of the $SU(2)$ theory are the mass M and coupling constant g of the ρ meson, and the somewhat mysterious ϵ. The corresponding parameters of the $SU(3)$ theory are the masses of the vector mesons (or M and μ) and again the universal coupling constant g as well as ϵ, which is now the D/F ratio. These parameters may be fixed by either the weak or the strong processes.

Let us next consider (10.1) and (10.2) in the $SU(2)$ pion-nucleon subspace. Then, since D_A and F_A both reduce to $\frac{1}{2}\tau_A$ ($A=1,2,3$), we have

$$J_{\mu A}(\text{weak})\rightarrow\cos\theta\,[-\tfrac{1}{2}\bar{N}\gamma_\mu\tau_AN+\tfrac{1}{2}(1+\epsilon)\tfrac{1}{2}\bar{N}\gamma_\mu\gamma^5\tau_AN$$
$$+(M_\rho/\sqrt{2}g)\partial_\mu\pi_A-\epsilon_{ABC}\pi_B\partial_\mu\pi_C], \tag{10.7}$$

and, evaluating the f tensor in (10.6) also in this subspace, we obtain

$$L_{BB\Phi}\rightarrow L_{\pi NN}=[g/(\sqrt{2}M_\rho)]\tfrac{1}{2}(1+\epsilon)(\bar{N}\gamma^\mu\gamma^5\tau_AN)\partial_\mu\pi_A$$
$$+[g/(\sqrt{2}M_\rho)]^2(\bar{N}\gamma^\mu\tau_AN)\epsilon_{ABC}(\partial_\mu\pi_B)\pi_C. \tag{10.8}$$

These are just the $SU(2)$ expressions and therefore the various $SU(2)$ results still hold. In particular,[22] from (10.7)

$$\tfrac{1}{2}(1+\epsilon)=-g_A/g_V\cong1.231. \tag{10.9}$$

Therefore,

$$\epsilon=1.462\cong\tfrac{3}{2}. \tag{10.10}$$

Also, from (10.8), we obtain the KSRF relation[14]

$$g_{\pi NN}/2M_N=(g/\sqrt{2}M_\rho)(-g_A/g_V). \tag{10.11}$$

Once the parameters of the theory have been fixed in this way by the pion-nucleon system, one may go on to calculate the interesting quantities for the full octets. The effective pseudoscalar-baryon coupling constants may be read off from the axial-vector couplings in (10.2), and are presented in Table I. Also, the ratios $-g_A/g_V$ for the various baryon leptonic weak decays may be read off from (10.1), and are sum-

[21] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[22] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

TABLE I. Pseudoscalar baryon coupling constants, $g_{PBB'}^2/4\pi = (\text{factor})^2 \times [(M_B+M_{B'})^2/(2M_N)^2] g_{\pi NN}^2/4\pi$.

Type	Factor[a]	Value	Comparison values
πNN	1	14.5 (input)	14.5
$\pi\Xi\Xi$	$(1-\epsilon)/(1+\epsilon)$	1.004	1.82[b]
$\pi\Lambda\Sigma$	$2\epsilon/\sqrt{3}(1+\epsilon)$	10.288	10.2[b]
$\pi\Sigma\Sigma$	$2/(1+\epsilon)$	15.420	15.9[b]
$KN\Lambda$	$-[(3+\epsilon)/\sqrt{3}(1+\epsilon)](M_\rho/M_{K^*})$	14.031	16.0 ± 2.5[c]
$K\Xi\Lambda$	$[(3-\epsilon)/\sqrt{3}(1+\epsilon)](M_\rho/M_{K^*})$	2.337	
$KN\Sigma$	$-[(1-\epsilon)/(1+\epsilon)](M_\rho/M_{K^*})$	0.486	0.3 ± 0.5[c]
$K\Xi\Sigma$	$-1(M_\rho/M_{K^*})$	19.121	
ηNN	$[(3-\epsilon)/\sqrt{3}(1+\epsilon)](M_\rho/M_{\omega 8})$	1.274	
$\eta\Xi\Xi$	$-[(3+\epsilon)/\sqrt{3}(1+\epsilon)](M_\rho/M_{\omega 8})$	21.086	
$\eta\Lambda\Lambda$	$-[2\epsilon/\sqrt{3}(1+\epsilon)](M_\rho/M_{\omega 8})$	6.490	
$\eta\Sigma\Sigma$	$[2\epsilon/\sqrt{3}(1+\epsilon)](M_\rho/M_{\omega 8})$	7.418	

[a] We have used the values $\epsilon=1.462$, $M_\rho=765$ MeV, $M_{\omega_8}=931$ MeV.
[b] S. Matsuda, S. Oneda, and P. Desai, Phys. Rev. **178**, 2129 (1969).
[c] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967).

marized in Table II. Notice that the pseudoscalar coupling constants in Table I depend on the masses of the associated vector mesons in a significant way.

From the gradient (chiral) term in the current, one recovers the Goldberger-Treiman relation for the pion[11]

$$f_\pi = (M_N/g_{\pi NN})(-g_A/g_V)\cos\theta \qquad (10.12)$$

and the corresponding relation for the kaon. It follows from these relations and more directly from the gradient term of (10.1) that

$$f_K/f_\pi = (M_{K^*}/M_\rho)\tan\theta \cong 1.165\tan\theta. \qquad (10.13\text{a})$$

This result agrees with the spectral function relation[23]

$$f_K/f_\pi = 1.16\tan\theta \qquad (10.13\text{b})$$

and is close to the current-algebra result.[24]

From the final (isotopic) term in the current, one may determine the rates of the reactions

$$\pi^+ \to \pi^0 + e^+ + \nu, \qquad (10.14\text{a})$$

$$K^+ \to \pi^0 + e^+ + \nu. \qquad (10.14\text{b})$$

The relevant partial currents are

$$J_\mu(1-i2) = i\cos\theta\,[\sqrt{2}(\pi^0\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^0) + \cdots], \quad (10.15\text{a})$$

$$J_\mu(4-i5) = i\sin\theta\,\{(1/\sqrt{2})[(M_{K^*}/M_\rho)\pi^0\partial_\mu K^+ - (M_\rho/M_{K^*})K^+\partial_\mu\pi^0] + \cdots\} \quad (10.15\text{b})$$

The corresponding matrix elements are

$$\langle\pi^0|J_\mu(1-i2)|\pi^+\rangle = (p_{\pi^+}+p_{\pi^0})_\mu(f_+)_\pi \times \exp[-i(p_{\pi^+}-p_{\pi^0})x] \quad (10.16\text{a})$$

and

$$\langle\pi^0|J_\mu(4-i5)|K^+\rangle = [(p_{K^+}+p_{\pi^0})_\mu(f_+)_K + (p_{K^+}-p_{\pi^0})_\mu(f_-)_K]\exp[-i(p_{K^+}-p_{\pi^0})x], \quad (10.16\text{b})$$

where

$$(f_+)_\pi = \sqrt{2}\cos\theta, \qquad (10.17)$$

$$(f_+)_K = \tfrac{1}{4}\sqrt{2}\sin\theta\,(M_{K^*}/M_\rho + M_\rho/M_{K^*}), \quad (10.18)$$

$$(f_-)_K = \tfrac{1}{4}\sqrt{2}\sin\theta\,(M_{K^*}/M_\rho - M_\rho/M_{K^*}). \quad (10.19)$$

By adding (10.18) and (10.19) and substituting (10.13) one finds the Callan-Treiman relation[8]

$$(f_+)_K + (f_-)_K = (1/\sqrt{2})\sin\theta\,(M_{K^*}/M_\rho) = (1/\sqrt{2})\cos\theta\, f_K/f_\pi. \qquad (10.20)$$

The agreement up to this point with experiment is satisfactory.

On the other hand, while the vector coupling in (10.2) is known to yield reasonable agreement for the pion-nucleon s-wave scattering lengths,[25] the kaon-nucleon s-wave scattering lengths calculated from the same term yield values close to the usual current-algebra results[26] which are known to disagree with experimental values.[27] The model needs to be modified at this point to put the full nonet of vectors into proper correspondence with the pseudoscalar multiplet.[28]

XI. REMARKS

The underlying physical picture is a version of $SU(3)\times SU(3)$ characterized by Goldstone bosons rather than by the doubling of the usual $SU(3)$

[23] S. L. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).
[24] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).
[25] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); **18**, 188 (1967).
[26] Y. Tomazawa, Nuovo Cimento **46**, 803 (1967); A. P. Balachandran, G. M. Gundzik, and F. Nicodemi, *ibid.* **44A**, 1257 (1966); P. Roy, Phys. Rev. **162**, 1644 (1967).
[27] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962); V. J. Stenger *et al.*, Phys. Rev. **134**, B1111 (1964).
[28] See, for instance, H. Sugawara and F. Von Hippel, Phys. Rev. **145**, 1331 (1966); J. Schechter, Y. Ueda, and G. Venturi, *ibid.* **177**, 2311 (1969).

multiplets.[29,30] The chiral $SU(3)$ symmetry is broken in such a way that the next approximation is not $SU(3)$ but chiral $SU(2)$, as can be seen by noting that all currents are exactly conserved in the nucleon-pion subspace. Therefore, while no pseudoscalar masses appear in our Lagrangian, the theory behaves like a partially conserved axial-vector current (PCAC) model in which $m_\pi=0$ but $m_K\neq 0$.

Our model corresponds to case (1) of Dashen and Weinstein,[29] namely, exact conservation of axial-vector current, $m_\pi=0$, and an exact Goldberger-Treiman relation. It resembles the model of Gell-Mann, Oakes, and Renner[30] in that chiral $SU(3)$ is broken down to chiral $SU(2)$ by the mass-splitting operator.

Although these physical assumptions are not new, their implementation and, in particular, the introduction of the pseudoscalar fields are treated rather differently. The fundamental formal objects are now the local gauge group and the associated displacement field. Since there are two mass terms, however, one is forced to introduce two privileged gauges (baryon and vector). Although the introduction of privileged gauges implies that the Lagrangian is no longer invariant under local gauge transformations, the local gauge group does not lose its significance—the local parameters of this group (describing the transformation connecting the baryon and vector gauges) are just the various components of the pseudoscalar field and, in fact, the nonlinearity of the Lagrangian is expressed entirely in terms of geometrical invariants on the local group space.

The formalism unifies the vector and pseudoscalar fields into parts of a single complex which mix under gauge transformations. In particular, one and the same term appears either as a vector mass term (in the vector gauge) or as a minimally coupled pseudoscalar kinetic energy term (in the baryon gauge). The dual interpretation of this particular key term implies the relations [(7.30) and (7.36)]

$$\eta\Phi_a=M_a\phi_a,\qquad 4\kappa=\tfrac{1}{2}(1+2\kappa)(g^2R_0^2/c^2\eta^2).$$

The first relation states that every pseudoscalar field is renormalized by the mass of the corresponding vector field and has the consequence that vector masses appear in our equations in a characteristic way.

The second relation may be fixed by imposing universality as follows (7.35):

$$gR_0=\sqrt{2}c\eta.$$

In this form universality relates g ($=g_{VBB}$) to the radius (R_0) of curvature of the local group space, which in turn fixes the scale of the pseudoscalar couplings.

All of the usual relations now follow, namely,

TABLE II. Baryon leptonic vector–axial-vector ratios for octet decays.

Decay	Ratio	Formula	Value	Expt. (Ref. 22)
$n\rightarrow p$	g_A/g_V	$-\frac{1}{2}(1+\epsilon)$	-1.231[a]	-1.231 ± 0.010
$\Lambda\rightarrow p$	g_A/g_V	$-\frac{1}{6}(3+\epsilon)$	-0.744	$-0.97_{-0.22}^{+0.14}$
$\Sigma^+\rightarrow\Lambda$	g_V/g_A	0	0	
$\Sigma^-\rightarrow\Lambda$	g_V/g_A	0	0	0.3 ± 0.3
$\Sigma^-\rightarrow n$	g_A/g_V	$\frac{1}{2}(\epsilon-1)$	$+0.231$	$+0.28\pm0.16$
$\Xi^0\rightarrow\Sigma^+$	g_A/g_V	$-\frac{1}{2}(1+\epsilon)$	-1.231	
$\Xi^-\rightarrow\Sigma^0$	g_A/g_V	$-\frac{1}{2}(1+\epsilon)$	-1.231	
$\Xi^-\rightarrow\Lambda$	g_A/g_V	$-\frac{1}{6}(3-\epsilon)$	-0.256	

[a] Input gives $\epsilon=1.462$.

$M_A=\sqrt{2}M$, KSRF, the p- and s-wave scattering lengths of pseudoscalars against baryons, and also the weak currents, implying, for example, generalized Adler-Weisberger, Goldberger-Treiman, and Callan-Treiman relations. The final effective Lagrangian depends on two parameters: the vector coupling constant $g^2/4\pi\cong 3$ and $\epsilon=1.46$, which fixes D/F.

The usual $SU(3)\times SU(3)$ current algebra is satisfied in lowest order. We plan to discuss the field algebra elsewhere.

Compared to other effective Lagrangians the present formulation is, in our opinion, conceptually simple and its structure is relatively strongly determined.

APPENDIX A: $u_{Ak}(\pm)$ AND γ_{kl} IN A PARTICULAR COORDINATE SYSTEM

For completeness, we exhibit below the parallel fields $u_{Ak}(\pm)$ and the metric tensor γ_{kl} in one particular coordinate system.

The problem of exhibiting a parametrization suitable for calculating the $u_{Ak}(\pm)$ in a closed form is considerably more difficult for $SU(3)$ than for $SU(2)$, and has but recently been solved by Macfarlane, Sudbery, and Weisz.[31] In our notation, their result is

$$U(l)=(Z^2Z^*)^{-1/3}\times[1-\tfrac{1}{3}X^2-\tfrac{1}{12}iY+i(c/R_0)\phi_A\lambda_A-\tfrac{1}{2}N_A\lambda_A],\qquad\text{(A1)}$$

where

$$X^2=(c^2/4R_0^2)\phi_A\phi_A,\qquad\text{(A2)}$$

$$Y=(c^3/R_0^3)d_{ABC}\phi_A\phi_B\phi_C,\qquad\text{(A3)}$$

$$N_A=(c^2/R_0^2)d_{ABC}\phi_B\phi_C,\qquad\text{(A4)}$$

$$Z=1+X^2+\tfrac{1}{12}iY.\qquad\text{(A5)}$$

Then with

$$\pm i[\partial U^{\pm1}(l)/\partial\phi^k]U^{\mp1}(l)=(c/R_0)u_{Ak}(\pm)\lambda_A,\qquad\text{(A6)}$$

[29] R. Dashen and M. Weinstein, Phys. Rev. **183**, 1261 (1969).
[30] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[31] A. J. Macfarlane, A. Sudbery, and P. Weisz, Cambridge University Report No. DAMTP 69/16 (unpublished).

we have

$$u_{Ak}(\pm)=-(Z^*Z)^{-1}[(1+\tfrac{1}{3}X^2)\delta_{Ak}+\tfrac{1}{2}(c^2/R_0^2)\phi_k\phi_A \pm(c/R_0)f_{AkB}\phi_B-d_{AkB}N_B+\tfrac{1}{8}N_AN_k-\tfrac{1}{12}(c/R_0)d_{AkB}\phi_BY \pm\tfrac{1}{2}(c/R_0)f_{ABC}d_{kBD}\phi_DN_C], \quad \text{(A7)}$$

where

$$Z^*Z=1+2X^2+X^4+(1/144)Y^2 \quad \text{(A8)}$$

and

$$\gamma_{kl}=(Z^*Z)^{-2}\{(Z^*Z)\ (1+\tfrac{2}{3}X^2)\delta_{kl}-\tfrac{1}{3}(c/R_0)^2 \times[\tfrac{1}{3}X^2-(1/144)Y^2]\phi_k\phi_l-\tfrac{1}{2}[1+2X^2-(4/9)X^4 +(1/144)Y^2]d_{klA}N_A+\tfrac{1}{12}(1+6X^2+X^4)N_kN_l -\tfrac{1}{3}(c/R_0)d_{klA}\phi_AX^2Y-(1/72)(c/R_0) \times(1+X^2)Y(\phi_kN_l+\phi_lN_k)\}. \quad \text{(A9)}$$

APPENDIX B: COMPLETE CURRENTS

First, from (7.4a) we have

$$v_{\mu\nu}=\partial_\mu v_\nu-\partial_\nu v_\mu+(ig/\sqrt{2})(v_\mu,v_\nu)_--(ig/\sqrt{2})(a_\nu,a_\mu)_-, \quad \text{(B1a)}$$

$$a_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu+(ig/\sqrt{2})(v_\mu,a_\nu)_--(ig/\sqrt{2})(v_\nu,a_\mu)_-. \quad \text{(B1b)}$$

Then, because of (7.15) and (7.39), (B1) becomes

$$v_{\mu\nu}=\partial_\mu v_\nu-\partial_\nu v_\mu+(ig/\sqrt{2})(v_\mu,v_\nu)_--(ig/\sqrt{2})(\hat{a}_\nu,\hat{a}_\mu)_-+\tfrac{1}{4}i(\hat{a}_\mu,[U_k(+)+U_k(-)])_-\partial_\nu\phi^k -\tfrac{1}{4}i(\hat{a}_\nu,[U_k(+)+U_k(-)])_-\partial_\mu\phi^k+(i/8\sqrt{2}g) \times([U_k(+)+U_k(-)],[U_m(+)+U_m(-)])_-\partial_\mu\phi^k\partial_\nu\phi^m, \quad \text{(B2a)}$$

$$a_{\mu\nu}=\partial_\mu\hat{a}_\nu-\partial_\nu\hat{a}_\mu+(ig/\sqrt{2})(v_\mu,\hat{a}_\nu)_--(ig/\sqrt{2})(v_\nu,\hat{a}_\mu)_- +\tfrac{1}{4}i(v_\mu,[U_k(+)+U_k(-)])_-\partial_\nu\phi^k-\tfrac{1}{4}i(v_\nu,[U_k(+)+U_k(-)])_-\partial_\mu\phi^k -(i/2\sqrt{2}g)[(U_k(+),U_m(+))_--(U_k(-),U_m(-))_-]\partial_\mu\phi^k\partial_\nu\phi^m. \quad \text{(B2b)}$$

The currents depend on only the velocity-dependent parts of the Lagrangian, namely,

$$\hat{L}=-\tfrac{1}{4}\operatorname{Tr}(v_{\mu\nu}v^{\mu\nu}+a_{\mu\nu}a^{\mu\nu})+(c^2/g^2R_0^2)\tilde{Q}_{BC}(+)(\xi_{Bk}\xi_{Cm}+2\tau_{Bk}\tau_{Cm})\partial_\mu\phi^k\partial^\mu\phi^m +(2c^2/g^2R_0^2)\tilde{Q}_{BC}(-)(\xi_{Bk}\tau_{Cm})\partial_\mu\phi^k\partial^\mu\phi^m-(2c/gR_0)v_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(+)+\tfrac{1}{2}\xi_{Ck}\tilde{Q}_{BC}(-)]\partial^\mu\phi^k -(2c/gR_0)\hat{a}_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(-)]\partial^\mu\phi^k+i\bar{B}\gamma_\mu\partial^\mu B-(c/R_0)\bar{B}\gamma_\mu\gamma^5(F_B+\epsilon D_B)B\xi_{Bk}\partial^\mu\phi^k, \quad \text{(B3)}$$

where we have written $\hat{L}$ in a more convenient form with the definitions

$$\xi_{Ak}=\tfrac{1}{2}[u_{Ak}(+,l)+u_{Ak}(-,l)], \quad \text{(B4a)}$$

$$\tau_{Ak}=\tfrac{1}{2}[u_{Ak}(+,l)-u_{Ak}(-,l)]. \quad \text{(B4b)}$$

The vectors ξ and τ determine the chiral and isotopic generators in U space and appear quadratically in the above expressions that hold in the U^2 space.[6]

The full right current obtained in the usual way is, then,

$$J_{\mu A}{}^{\text{II}}=-\operatorname{Tr}\left[v_{\mu\nu}\left(\frac{\delta v^\nu}{\delta\alpha_A{}^{\text{II}}}-\tfrac{1}{4}i(\hat{a}^\nu,[U_k(+)+U_k(-)])_-\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}} +\frac{i}{8\sqrt{2}g}([U_k(+)+U_k(-)],[U_m(+)+U_m(-)])_-\partial^\nu\phi^m\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}}\right)\right] -\operatorname{Tr}\left[a_{\mu\nu}\left(\frac{\delta\hat{a}^\nu}{\delta\alpha_A{}^{\text{II}}}-\tfrac{1}{4}i(v^\nu,[U_k(+)+U_k(-)])_-\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}} -\frac{i}{2\sqrt{2}g}[(U_k(+),U_m(+))_--(U_k(-),U_m(-))_-]\partial^\nu\phi^m\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}}\right)\right]+\frac{2c^2}{g^2R_0^2} \times\{[\tilde{Q}_{BC}(+)(\xi_{Bn}\xi_{Cm}+2\tau_{Bk}\tau_{Cm})+\tilde{Q}_{BC}(-)(\xi_{Bk}\tau_{Cm}+\tau_{Bk}\xi_{Cm})]\partial_\mu\phi^m -(gR_0/c)v_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(+)+\tfrac{1}{2}\xi_{Ck}\tilde{Q}_{BC}(-)]-(gR_0/c)\hat{a}_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(-)]\} \times\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}}+i\bar{B}\gamma_\mu\frac{\delta B}{\delta\alpha_A{}^{\text{II}}}-\frac{c}{R_0}\bar{B}\gamma_\mu\gamma^5(F_B+\epsilon D_B)B\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\text{II}}}. \quad \text{(B5)}$$

Now, however, from (7.15) and (7.39),

$$\delta\hat{a}_\mu=\delta a_\mu+(i/2\sqrt{2}g)[(U_k(+),U_m(+))_- -(U_k(-),U_m(-))_-]\partial_\mu\phi^m\partial\phi^k - (1/2\sqrt{2}g)\partial_\mu\{[U_k(+)+U_k(-)]\delta\phi^k\}, \quad \text{(B6)}$$

and (B5) becomes

$$J_{\mu A}{}^{\rm II}=-\operatorname{Tr}\left[v_{\mu\nu}\frac{\delta v^\nu}{\delta\alpha_A{}^{\rm II}}+\tfrac{1}{4}i(a^\nu,v_{\mu\nu})\frac{4c}{R_0}\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}\mathbf{F}_B\right] - \operatorname{Tr}\left[a_{\mu\nu}\frac{\delta a^\nu}{\delta\alpha_A{}^{\rm II}}+\tfrac{1}{4}i(v^\nu,a_{\mu\nu})\frac{4c}{R_0}\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}\mathbf{F}_B - \frac{1}{2\sqrt{2}g}a_{\mu\nu}\partial^\nu\left(\frac{4c}{R_0}\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}\right)\mathbf{F}_B\right]+\frac{2c^2}{g^2R_0{}^2}$$
$$\times\{[\tilde{Q}_{BC}(+)(\xi_{Bk}\xi_{Cm}+2\tau_{Bk}\tau_{Cm})+\tilde{Q}_{BC}(-)(\xi_{Bk}\tau_{Cm}+\tau_{Bk}\xi_{Cm})]\partial_\mu\phi^m - (gR_0/c)v_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(+)+\tfrac{1}{2}\xi_{Ck}\tilde{Q}_{BC}(-)]-(gR_0/c)\hat{a}_{\mu B}[\tau_{Ck}\tilde{Q}_{BC}(-)]\}$$
$$\times\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}+i\bar{B}\gamma_\mu\frac{\delta B}{\delta\alpha_A{}^{\rm II}}-\frac{c}{R_0}\bar{B}\gamma_\mu\gamma^5(F_B+\epsilon D_B)B\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}, \quad \text{(B7)}$$

where, from (3.14), (3.17), (3.18), (5.16), and (5.20),

$$\frac{\delta B}{\delta\alpha_A{}^{\rm II}}=iH_{AB}{}^{\rm II}\mathbf{F}_BB, \quad \text{(B8a)}$$

$$\frac{\delta v^\nu}{\delta\alpha_A{}^{\rm II}}=i(H_{AB}{}^{\rm II}\mathbf{F}_B,v^\nu)_- -\frac{\sqrt{2}}{g}\partial^\nu(H_{AB}{}^{\rm II}\mathbf{F}_B), \quad \text{(B8b)}$$

$$\frac{\delta a^\nu}{\delta\alpha_A{}^{\rm II}}=i(H_{AB}{}^{\rm II}\mathbf{F}_B,a^\nu)_-, \quad \text{(B8c)}$$

and

$$\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}=\frac{R_0}{2c}\Delta^{kn}\omega_{An}(-,r). \quad \text{(B8d)}$$

Finally, from the analog of (5.23), and from (5.25), we have

$$\xi_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}=-\frac{R_0}{4c}R_{AB}, \quad \text{(B9a)}$$

$$\tau_{Bk}\frac{\delta\phi^k}{\delta\alpha_A{}^{\rm II}}=-\frac{R_0}{4c}[2H_{AB}{}^{\rm II}-R_{AB}], \quad \text{(B9b)}$$

where

$$R_{AB}=R_{AB}(l)=R_{BA}(r). \quad \text{(B10)}$$

Then the current is

$$J_{\mu A}{}^{\rm II}=-i\operatorname{Tr}\{[(v^\nu,v_{\mu\nu})_-H_{AB}{}^{\rm II}-\tfrac{1}{4}(a^\nu,v_{\mu\nu})R_{AB}+(i\sqrt{2}/g)v_{\mu\nu}\partial^\nu H_{AB}{}^{\rm II}]\mathbf{F}_B\}$$
$$-i\operatorname{Tr}\{[(a^\nu,a_{\mu\nu})H_{AB}{}^{\rm II}-\tfrac{1}{4}(v^\nu,a_{\mu\nu})R_{AB}-(i/2\sqrt{2}g)a_{\mu\nu}\partial^\nu R_{AB}]\mathbf{F}_B\}$$
$$-(c/2g^2R_0)\{\tilde{Q}_{BC}(+)[R_{AB}\xi_{Cm}+2(2H_{AB}{}^{\rm II}-R_{AB})\tau_{Cm}] +\tilde{Q}_{BC}(-)[R_{AB}\tau_{Cm}+(2H_{AB}{}^{\rm II}-R_{AB})\xi_{Cm}]\}\partial_\mu\phi^m$$
$$+(1/2g)\{v_{\mu B}[(2H_{AC}{}^{\rm II}-R_{AC})\tilde{Q}_{BC}(+)+\tfrac{1}{2}R_{AC}\tilde{Q}_{BC}(-)]+\hat{a}_{\mu B}(2H_{AC}{}^{\rm II}-R_{AC})\tilde{Q}_{BC}(-)\}$$
$$-\bar{B}\gamma_\mu H_{AB}{}^{\rm II}F_BB+\tfrac{1}{4}\bar{B}\gamma_\mu\gamma^5(F_B+\epsilon D_B)R_{AB}B. \quad \text{(B11)}$$