Regge behavior can also be demonstrated by standard methods.[2,3,7]

The formula for F_N is a simple generalization of the one for F_5. The external momenta are first fixed in some order $p_1, p_2, \ldots, p_N$. The function F_N for this particular order is given by

$$F_N = \int_0^1 \prod_{i=2}^{N-2} dx_{1i} [J(x_{12}, x_{13}, \ldots, x_{1N-2})]^{-1} \prod x_{ij}^{-\alpha_{ij}-1} \tag{12}$$

up to an over-all normalization. The product over $x_{ij}^{-\alpha_{ij}-1}$ is over all possible "multiperipheral" channels allowed by this order. The constraints on x_{ij} are obtained by defining $u_{ij} = \Omega(x_{ij})$ and imposing the N-point beta-function equations on u_{ij}. The expression for J is also evident: If $\tilde{J}(u_{12}, u_{13}, \ldots, u_{1N-2})^{-1}$ is the invariant measure factor for beta functions,[2]

$$J(u_{12}, u_{13}, \ldots, u_{1N-2})^{-1} = \prod_{i=2}^{N-2} [1-\omega'(x_{1i})] / \tilde{J}(u_{12}, u_{13}, \ldots, u_{1N-2}). \tag{13}$$

The resulting F_N is dual and factorizable.

Recently, some work has been done on the factorization properties of the beta functions at the poles $\alpha_{ij} = n$.[8] A paper by D. K. Sinclair develops a similar formalism for these F_N for a general ω.[9]

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[7] The foregoing assertions on the nature of F_5 are not completely precise. For example, (δ) does not guarantee that F_5 does not behave like a pole plus a logarithm when $\alpha_{12} \to 0$. Such difficulties can be avoided if Ω and Ω^{-1} are holomorphic in some domain which contains the interval [0, 1]. That there are infinitely many such Ω is clear from (2) and (11).

[8] K. Bardakci and S. Mandelstam, Phys. Rev. **184**, 1640 (1969); S. Fubini and G. Veneziano, MIT report, 1969 (unpublished).

[9] D. K. Sinclair, Stoney Brook report, 1970 (unpublished).

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Time-Symmetric Interactions and Quantum Electrodynamics*

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The derivation of an action form from a time-symmetric formalism for quantum electrodynamics is given. A constraint added to a Fokker-type functional leads directly to an expression given recently by Schwinger; however, no supplementary renormalization condition is required here.

I. INTRODUCTION

THE problem of a satisfactory basis for quantum electrodynamics has led to attempts to reinterpret procedures used (via axiomatic field theory, for instance) or to attempts to find a new physical basis which will at least duplicate the successful quantitative predictions of the original theory. Schwinger[1] has recently formulated his phenomenological source theory with the purpose of reestablishing old electrodynamic results, and also hopefully of going beyond these; he has also applied the theory to the calculation of electromagnetic masses[2] and to gravitation.[3]

In regard to electrodynamics there is some formal resemblance between the action form as written by Schwinger[4] and its c-number formulation and the classical action of Wheeler and Feynman.[5] The latter, preceded by the work of Fokker[6] and others, leads to a formalism which is equivalent to the Maxwell theory, but lacks the classical divergences inherent in field theory. As has been emphasized by Havas,[7] such particle theories have not been fully exploited; in a similar vein, Dresden[8] has conjectured that the Wheeler-Feynman formulation may be extendible to the quantum domain.

The idea put forward here represents a particle view of electrodynamics (in principle it should be extendible to other types of interactions). It shares in common with

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[1] J. S. Schwinger, Phys. Rev. **165**, 1714 (1968); **167**, 1546(E) (1968). See also J. S. Schwinger, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967*, edited by C. R. Hagen, G. Guralnik, and V. S. Mathur (Interscience, New York, 1967), p. 128.

[2] J. S. Schwinger, Phys. Rev. **167**, 1432 (1968).

[3] J. S. Schwinger, Phys. Rev. **173**, 1264 (1968).

[4] J. S. Schwinger, Phys. Rev. **173**, 1536 (1968). See particularly p. 1542.

[5] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945). See also A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (Macmillan, New York, 1964), p. 213.

[6] A. D. Fokker, Z. Physik **58**, 386 (1929).

[7] P. Havas, in Argonne National Laboratory Summer Lectures on Theoretical Physics, 1958, ANL-5982, p. 124 (unpublished).

[8] M. Dresden, in Ref. 7, p. 101.

previous particle theories[5–7] a time-symmetric basis, from which radiative effects are obtained by imposing additional constaints. In common with the Schwinger[1–4] and the Wheeler and Feynman formalisms, the acts of emission and absorption are treated together. The systems that are considered, however, are assumed to be closed, in the sense that all interactions are taken into account, in principle. This does not preclude applying the theory to situations which are not closed or only approximately so; one does not lose the prerogative of examining a particularly interesting but open physical subspace of the entire system.

II. ACTION WITH CONSTRAINT

We examine the variation of

$$W=\int L\,ds, \tag{1}$$

where

$$L=L_0+\tfrac{1}{2}\sum_{a,b;\,a\neq b}(e_ae_b/c)\int \dot{x}_a{}^{\mu}(s)\dot{x}_{b\mu}(s')\delta(\Delta s^2{}_{ab})ds', \tag{2}$$

with

$$\Delta s^2{}_{ab}\equiv(x_{a\nu}-x_{b\nu})(x_a{}^{\nu}-x_b{}^{\nu}) \tag{3}$$

(the metric is chosen to be $-+++$), L_0 is the free-particle Lagrangian, and the particle trajectories are treated symmetrically (integration over all proper time). The variation of this W, viewed as a classical quantity, is sufficient to give ordinary classical electrodynamics with radiation damping, if "absorber" conditions are applied.[5,7]

The action form given by (1) will lead to quantum-electrodynamic results, when used (for instance) in conjunction with the path-integral formalism of Feynman,[9] *provided that an additional constraint is imposed*. To see what this appropriate constraint is, we first seek it out in a quasiclassical domain.

The constraint was used in a recent paper by Buneman.[10] Following earlier work of Landau and Peierls,[11] Buneman obtained a relativistic invariant, written in terms of the space-time variables, which characterizes the "photon number" in emission from what he calls "noncausally connected" charged particles. That is, the system is open and the motion of the charges is prescribed, regardless of mutual action (although the invariant itself contains terms identifiable as "mutual interaction" and "self-interaction").

We postulate that the invariant is true for all interactions involving a closed system of charged particles which mutually interact and self-interact. The "photon count" is thus viewed as an interaction number—a number which describes the complete world history of the particles in terms of the number of times they have interacted with themselves and with others. Since we assume that we wait a sufficiently long time for radiation to be absorbed, every photon is "virtual" in the usual quantum-electrodynamic sense. The distinction between "interaction" and "photon" is intended to be more than semantical, however, especially if we think of possible extension to situations where an intermediary particle may not be in evidence (for instance, in weak interactions or gravitation). In addition, one may wish to retain a philosophic distinction between the two concepts.

Adding the integral constraint to the action, with λ as an invariant multiplier, we get the form

$$\bar{W}=W+\lambda W', \tag{4}$$

where

$$\begin{aligned}W'&=\int L'\,ds\\ &=\tfrac{1}{4}\sum_{a,b}(e_ae_b/c)\int\int \dot{x}_a{}^{\mu}(s)\dot{x}_{b\mu}(s')\\ &\qquad\times[D_-(\Delta s^2{}_{ab})-D_+(\Delta s^2{}_{ab})]\,ds\,ds'.\end{aligned} \tag{5}$$

The invariant defined above is the interaction number multiplied by $i\hbar$, where the number is integral. Thus quantization is apparent in this demand, in contrast to the situation in Ref. 10 where one views it as a classical nonintegral multiple, in general. In Eq. (5) we use the notation (note that this Schwinger convention differs from the more common D_F and $D_F{}^*$)

$$D_{\pm}(\Delta s^2)\equiv\delta(\Delta s^2)\mp(i/\pi)P(1/\Delta s^2). \tag{6}$$

Taking the variation of (4) and using (1), it follows that

$$\frac{d}{ds}\left(\frac{\partial L}{\partial u_a{}^{\mu}}\right)-\frac{\partial L}{\partial x_a{}^{\mu}}+\lambda\left[\frac{d}{ds}\left(\frac{\partial L'}{\partial u_a{}^{\mu}}\right)-\frac{\partial L'}{\partial x_a{}^{\mu}}\right]=0. \tag{7}$$

Upon multiplying (7) by $u_a{}^{\mu}\equiv\dot{x}_a{}^{\mu}$ and integrating, we get

$$\sum_a\{[L_a-u_a{}^{\mu}(\partial L_a/\partial u_a{}^{\mu})]+\lambda[L_a'-u_a{}^{\mu}(\partial L_a'/\partial u_a{}^{\mu})]\}=0, \tag{8}$$

where $L\equiv\sum_a L_a$ and $L'\equiv\sum_a L_a'$ [defined from (5)]. Since L_a and L_a' are homogeneous in first order in $u_a{}^{\mu}$, the integrated condition does not determine λ. This implies that the weaker condition (7) cannot be solved for λ, but that one must have

$$\frac{d}{ds}\left(\frac{\partial L}{\partial u_a{}^{\mu}}\right)-\frac{\partial L}{\partial x_a{}^{\mu}}=\frac{d}{ds}\left(\frac{\partial L'}{\partial u_a{}^{\mu}}\right)-\frac{\partial L'}{\partial x_a{}^{\mu}}=0. \tag{9}$$

The vanishing in (9) of the differential expression involving L' is similar to the Wheeler-Feynman absorber condition

$$\sum_b(F_{b\mu\nu}{}^{\mathrm{ret}}-F_{b\mu\nu}{}^{\mathrm{adv}})=0, \tag{10}$$

[9] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965). See especially Sec. 9-4. The resultant action form may also be used in Schwinger's source theory in the restricted sense that one would be dealing with an open subspace of the original space.

[10] O. Buneman, Phys. Rev. **157**, 1167 (1967). In this paper Buneman suggested a possible connection with the Wheeler-Feynman approach. A factor of 1/2 follows for a closed system since two particles "share" an interaction, in contrast to the situation concerned only with emission.

[11] L. Landau and R. Peierls, Z. Physik **62**, 188 (1930).

where

$$F_{b\mu\nu}{}^{\mathrm{ret}}=e_b\int\left(\dot{x}_b{}^\nu(s')\,\frac{\partial D^{\mathrm{ret}}}{\partial x_a{}^\mu}-\dot{x}_b{}^\mu(s')\,\frac{\partial D^{\mathrm{ret}}}{\partial x_a{}^\nu}\right)ds' \tag{11a}$$

and

$$F_{b\mu\nu}{}^{\mathrm{adv}}=e_b\int\left(\dot{x}_b{}^\nu(s')\,\frac{\partial D^{\mathrm{adv}}}{\partial x_a{}^\mu}-\dot{x}_b{}^\mu(s')\,\frac{\partial D^{\mathrm{adv}}}{\partial x_a{}^\nu}\right)ds'. \tag{11b}$$

However, from (9) we get the functions D_+ and D_- (D_F and $D_F{}^*$) in place of the usual D^{ret} and D^{adv}. These functions differ from D^{ret} and D^{adv} by a solution (the same solution in each case) of the homogeneous equation.

Thus it is interesting that a condition similar to (10) follows directly in this time-symmetric argument for the closed system. Note, however, that since the system is closed, there is no possibility of consistently invoking conditions on the "field" solutions outside of the system, in contrast to what is done in the Wheeler-Feynman treatment.

What is the value of λ? This is dictated by physical considerations. If we wish purely retarded solutions with radiation damping for the individual particles, then $\lambda=-1$. Assuming this is the case, then from (4),

$$\bar{W}=\tfrac{1}{2}\sum_{a\neq b}(e_ae_b/c)\int\int u_a{}^\mu(s)\,u_{b\mu}(s')\,D_+(\Delta s^2{}_{ab})\;ds\,ds' +\tfrac{1}{2}i\sum_a(e_a{}^2/c)\int\int u_a{}^\mu(s)\,u_{a\mu}(s') \times\mathrm{Im}D_+(\Delta s_a{}^2)\;ds\,ds', \tag{12}$$

which is the action obtained recently by Schwinger,[4] and from which one obtains some of the usual results of quantum electrodynamics, including radiative corrections. In contrast to the argument of Schwinger, however, it is unnecessary to say that since the physical mass is already accounted for in L_0 in (2), and since the real part of the self-interaction term corresponds to a mass renormalization, the real part should be omitted.

The argument considered here is assumed to be capable of generalization to strong-interaction processes involving the creating and annihilation of massive particles, although the symmetry of the emission and absorption process would require inverse processes also to take place.

Another point to be considered is that in the weak-coupling case, one has a possible demonstration of a classical quantum-electrodynamic limiting process. We would observe that this limit would appear to be crucial for the extrapolation to the microscopic domain.

We have based the plausibility of the argument here largely on the derivation of the appropriate action, without the use of any supplementary renormalization requirement; the rationale was also motivated by a desire to follow a path somewhat analogous to the classical Wheeler-Feynman treatment. Lastly, one should not overlook the overriding reason for such an attempt: the absence of renormalization schemes.

One final comment: Since the only system that can be rigorously closed[12] would seem to be our universe, the finite value of the invariant given in (5) possibly suggests a connection involving the fine-structure constant and the macrocosm.

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[12] For a lucid discussion of the question of separability and open and closed systems, see P. Havas, in *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science* (North-Holland, Amsterdam, 1965), p. 347.