where L is the exact value to which L_n is an approximation. In Figs. 4 and 5, L_1, L_2, and L_3 are plotted for $\theta_R=5$, 10 keV and $0\leq\theta_e\leq 100$ keV. The results of Warham's and Corman's calculations are also plotted.

Convergence of the series in (7) is satisfactory over the range $0\leq\theta_e\leq 100$ keV and $0\leq\theta_R\leq 10$ keV. When θ_R is as large as 15 keV the convergence of the series is not so strong, since the $\ll$ signs in (8) can no longer be taken so strictly. At $\theta_e=0$ and $\theta_e=100$ keV, $|L_3-L_2|$ is more than half $|L_2-L_1|$.

This analysis indicates, using Eq. (9), that Eq. (7) gives dE/dt to within about 3% of the exact value for $0\leq\theta_e\leq 92$ keV and for $0\leq\theta_R\leq 7.2$ keV. The limit on θ_R can be raised to 10 keV if we desire only about 10% accuracy.

IV. CONCLUSIONS

The rate of energy transfer from an isotropic Maxwell-Boltzmann electron gas with temperature $T_e=\theta_e/k$ to an isotropic Planckian photon gas with temperature $T_R=\theta_R/k$ can be accurately calculated using the series expansion in Eq. (7) when $\theta_e \lesssim 100$ keV and $\theta_R \lesssim 10$ keV. The size of the relativistic corrections incorporated in the higher-order terms in (7) indicates that the nonrelativistic rate cannot be used when $\theta_e>20$ keV or when $\theta_R>2.5$ keV.

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Relativistic Compton Energy Exchange*

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A relativistically accurate table of values is calculated for the Compton energy exchange rate between photons in a Planckian distribution at temperature T_R varying from 5 to 50 keV and electrons in a nondegenerate Maxwell-Boltzmann distribution at temperatures T_e varying from 0 to 100 keV.

INTRODUCTION

THE Compton energy exchange between electrons and photons (both direct and inverse) has been discussed in recent literature.[1–4] Such calculational methods have involved series expansions in v/c and $h\nu/mc^2$ or numerical integration procedures.

In this treatment Monte Carlo techniques are applied in lieu of series expansions or multiple integrations to obtain the Compton energy exchange rate. At any given temperature as much accuracy as desired can be achieved simply by using a large enough sample of photon "bundles", each bundle representing a large number of true photons of a given energy K. For these calculations a sampling size of 100 000 photon bundles was selected, which required approximately 10-sec computing time on the CDC 7600 computer, for each dE/dt value.

I. METHOD OF CALCULATION

The photons were selected from a Planckian distribution at temperature T_R. Each photon of energy K collided with an electron of velocity v approaching the point of collision at an angle whose cosine is γ_1 (see Fig. 1). The electron velocity is selected from a relativistic Maxwell-Boltzmann distribution $\exp(-\epsilon/kT_e)P^2dP$, where the electron kinetic energy

$$\epsilon=mc^2/(1-\beta^2)^{1/2}-mc^2, \qquad \beta=v/c,$$

and P is the electron momentum. The normalized distribution used for selecting β was a third-order expansion[5]

$$f(\eta)d\eta=(P_1f_1+P_2f_2+P_3f_3)d\eta,$$

where

$$\beta\equiv v/c=(1-[1+(kT_e/mc^2)\eta^2]^{-2})^{1/2},$$

and

$$f_1(\eta)=(4/\sqrt{\pi})\eta^2\exp(-\eta^2),$$
$$f_2=(8/3\sqrt{\pi})\eta^4\exp(-\eta^2),$$
$$f_3=(16/15\sqrt{\pi})\eta^6\exp(-\eta^2),$$
$$D=1+(15/8)(kT_e/mc^2)+(105/128)[(kT_e)^2/m^2c^4],$$
$$P_1=D^{-1}, \qquad P_2=(15/8)(kT_e/mc^2)D^{-1},$$
$$P_3=(105/128)[(kT_e)^2/m^2c^4]D^{-1},$$

each f_i being separately normalized to unity. Note that f_1 is the nonrelativistic velocity distribution, f_2 and f_3 being relativistic corrections.

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[1] R. Weymann, Phys. Fluids **8**, 2112 (1965).
[2] F. C. Jones, Phys. Rev. **167**, 1159 (1968).
[3] A. Warham, Atomic Weapons Research Establishment Report No. 003/68 (unpublished).
[4] P. Woodward, preceding paper, Phys. Rev. D **1**, 2731 (1970).

[5] A. M. Winslow, UCRL Report No. UCIR-141, 1967 (unpublished).

TABLE I. $n_e^{-1}dE/dt$ (keV per shake).[a]

T_R (keV) \ T_e (keV)	0	10	20	30	40	50	60	70	80	90	100
5	-3.54×10^2	3.67×10^2	1.13×10^3	1.94×10^3	2.80×10^3	3.71×10^3	4.68×10^3	5.71×10^3	6.78×10^3	7.87×10^3	8.96×10^3
	(-3.51×10^2)	(3.62×10^2)				(3.76×10^3)					(9.13×10^3)
10	-9.47×10^3	0	9.86×10^3	2.04×10^4	3.07×10^4	4.28×10^4	5.39×10^4	6.68×10^4	7.94×10^4	9.28×10^4	1.06×10^5
	(-9.70×10^3)					(4.44×10^4)					(1.12×10^5)
15	-6.19×10^4	-2.08×10^4	2.14×10^4	6.44×10^4	1.13×10^5	1.56×10^5	2.09×10^5	2.58×10^5	3.10×10^5	3.62×10^5	4.19×10^5
20	-2.29×10^5	-1.16×10^5	0	1.18×10^5	2.41×10^5	3.76×10^5	5.05×10^5	6.38×10^5	7.73×10^5	9.10×10^5	1.07×10^6
25	-6.24×10^5	-3.75×10^5	-1.32×10^5	1.23×10^5	3.77×10^5	6.60×10^5	9.25×10^5	1.23×10^6	1.53×10^6	1.81×10^6	2.13×10^6
	(-6.76×10^5)	(-4.10×10^5)				(7.38×10^5)					(2.33×10^6)
30	-1.40×10^6	-9.54×10^5	-4.74×10^5	0	5.14×10^5	1.01×10^6	1.49×10^6	2.03×10^6	2.54×10^6	3.03×10^6	3.62×10^6
35	-2.78×10^6	-2.01×10^6	-1.21×10^6	-4.08×10^5	3.98×10^5	1.25×10^6	2.09×10^6	2.98×10^6	3.80×10^6	4.66×10^6	5.59×10^6
40	-5.01×10^6	-3.74×10^6	-2.53×10^6	-1.34×10^6	0	1.29×10^6	2.60×10^6	3.94×10^6	5.21×10^6	6.74×10^6	8.07×10^6
45	-8.38×10^6	-6.51×10^6	-4.66×10^6	-2.90×10^6	-9.37×10^5	8.26×10^5	2.90×10^6	4.92×10^6	6.73×10^6	8.91×10^6	1.08×10^7
50	-1.32×10^7	-1.06×10^7	-8.06×10^6	-5.47×10^6	-2.68×10^6	0	2.64×10^6	5.53×10^6	8.47×10^6	1.10×10^7	1.40×10^7
	(-1.48×10^7)	(-1.18×10^7)									(1.58×10^7)

[a] In some cases the corresponding values which were obtainable from Warham (Ref. 3) are printed in parentheses just below those of the author.

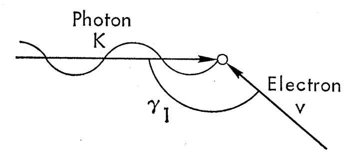

FIG. 1. Before collision.

A Lorentz transformation was effected to the rest system (*) of the electron according to the following relations:

$$K^*=[(1-\beta\gamma_1)/(1-\beta^2)^{1/2}]K, \quad \gamma_1^*=(\gamma_1-\beta)/(1-\beta\gamma_1).$$

The photon scattering angle γ_2^* was selected by a point rejection scheme from the Klein-Nishina distribution[6]:

$$P(\gamma_2^*)=\tfrac{1}{2}(1+\Delta)^{-2}[1+\gamma_2^*+\Delta^2/(1+\Delta)],$$

where $\Delta=(1-\gamma_2^*)K^*/mc^2$. The azimuthal angle was selected randomly. The energy K'^* after collision was then calculated according to[7]

$$K'^*=\frac{K^*}{1+(1-\gamma_2^*)K^*/mc^2}.$$

With γ_2^* and K'^*-K^*, another Lorentz transformation was effected back to the laboratory system:

$$\gamma_4^*=\gamma_1^*\gamma_2^*+[(1-\gamma_1^{*2})(1-\gamma_2^{*2})]^{1/2}\gamma_3^*,$$
$$K'=[(1+\beta\gamma_4^*)/(1-\beta^2)^{1/2}]K'^*,$$

where γ_4 is the cosine of the angle between the electron before collision and the photon after collision (see Fig. 2) and γ_3^* is the cosine of a random angle (azimuthal angle of photon K').

The Compton rate of energy exchange can be written

$$dE/dt=(c/4\pi)\int d^3P\int d^3K\int d\Omega\, n_e(P)n_p(K)(K'-K)\times(d\sigma/d\Omega)[1+\tfrac{1}{2}(hc)^3n_p(K')],$$

where $n_e(P)$ is the density of electrons of momentum P,

$$n_p(K)=[2/(hc)^3][\exp(K/kT_R)-1]^{-1}$$

is the density of photons of energy K, and $d\sigma/d\Omega$ is the differential Klein-Nishina cross section in the laboratory system.

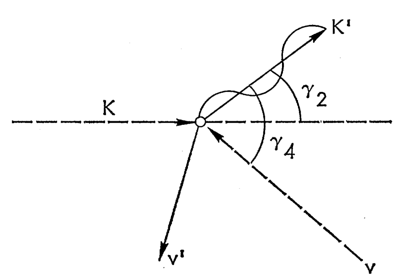

FIG. 2. After collision.

[6] W. Heitler, *Quantum Theory of Radiation*, 3rd ed. (Oxford U. P., New York, 1954), p. 219.

[7] W. Heitler, *Quantum Theory of Radiation*, 3rd ed. (Oxford U. P., New York, 1954), p. 211.

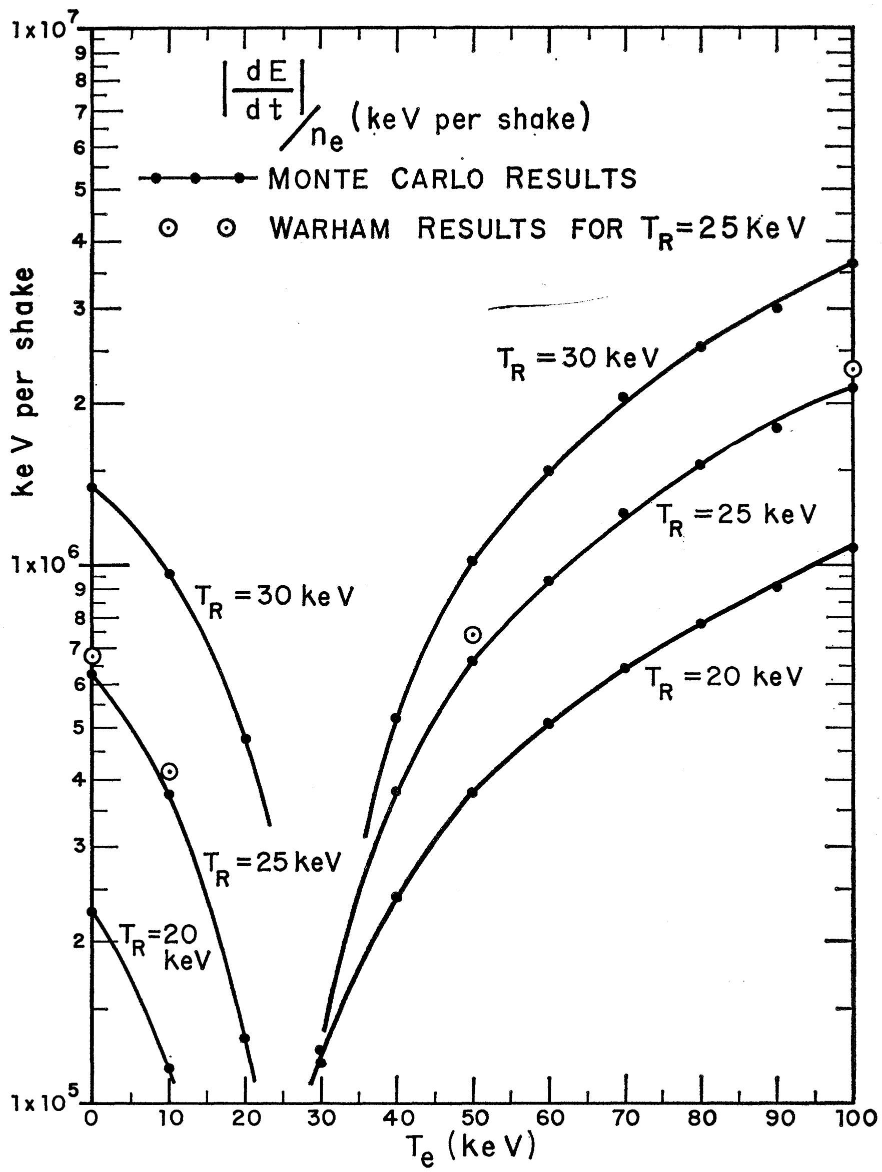

FIG. 3. $n_e^{-1}|dE/dt|$ in keV per shake for $kT_R=20$, 25, and 30 keV. Solid lines are Monte Carlo results and circles are those of Warham.

Numerical integrations were simulated by Monte Carlo samplings over the range of variables of integration. For integrating over K, the energy of all the photon bundles was summed and compared with the total energy of a Planckian radiation field. When the total energy was depleted the sampling was ceased. For integrating over electron velocities, a velocity sampling was taken from the normalized distribution above. The electron density n_e becomes a coefficient of the integral. The angular integration $d\Omega$ was effected by selecting the γ_1's uniformly over the domain -1 to $+1$. The $\sigma\Delta K$ product was calculated in the laboratory system for each event ($\Delta K=K'-K$). Then $\sigma\Delta K$ was summed over all scattering events. σ was calculated[5] from the expression $(1-\beta\gamma_1)d\sigma^*/d\Omega^*(K^*, K'^*)$, where σ^* is the Compton cross section in the electron rest system[6]:

$$d\sigma^*/d\Omega^*=\tfrac{1}{2}r_0^2(K'^{*2}/K^{*2})(K^*/K'^*+K'^*/K^*-1+\gamma_2^{*2}),$$

where $r_0=e^2/mc^2$. The factor $[1+\frac{1}{2}(hc)^3n_p(K')]$ was included because photons are bosons, and $n_p(K')$ is the density of photons in the final state K'. For a Planckian distribution the factor becomes simply $\{1+[\exp(K'/kT_R)-1]^{-1}\}$. The present discussion is confined to the case where the electron density is sufficiently low that degeneracy effects are unimportant $[\frac{1}{2}h^3n_e(P')\ll 1]$.

II. RESULTS

Table I contains $(dE/dt)/n_e$ to the first three significant digits. dE/dt is the energy exchange rate in keV

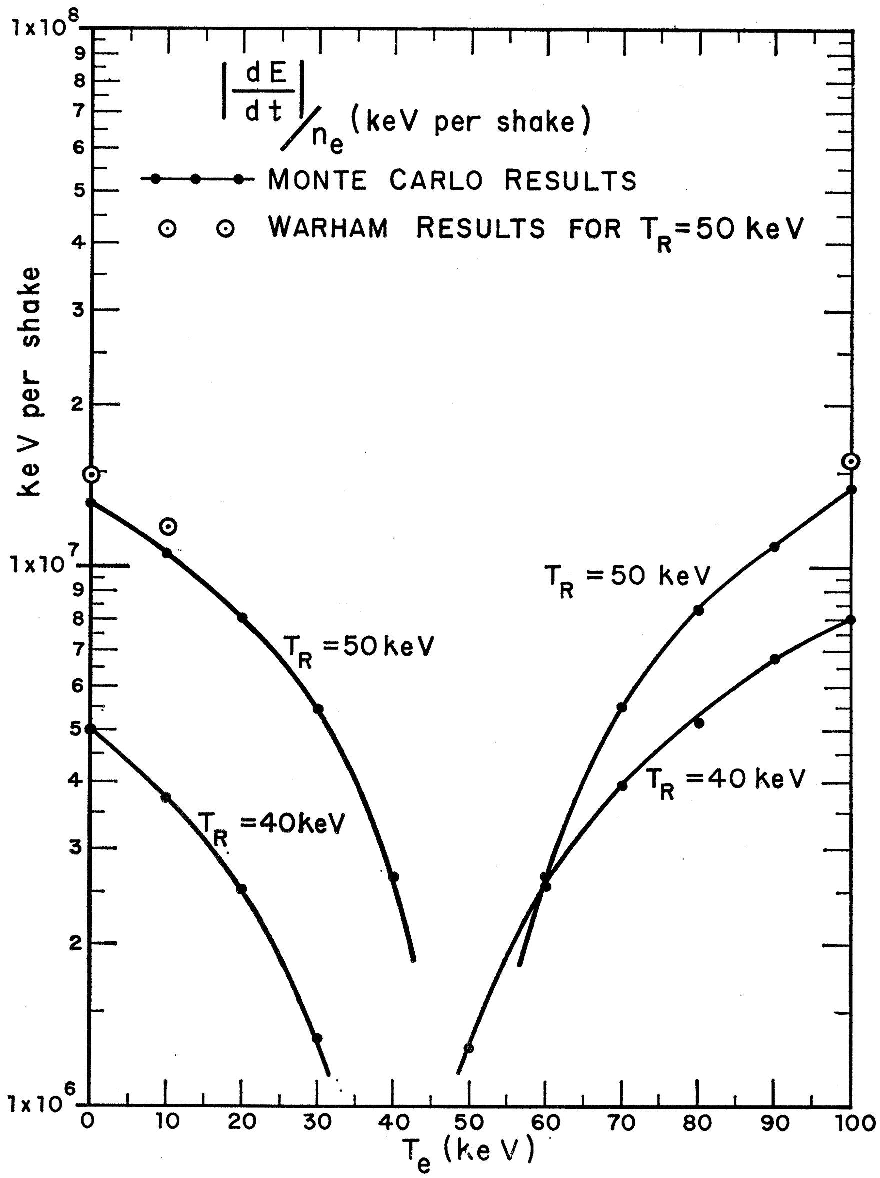

FIG. 4. $n_e^{-1}\,|\,dE/dt\,|$ for $kT_R=40$ and 50 keV.

per shake per cm³ *from electrons to photons*.[8] A few of the values have the corresponding values which were obtainable from Warham[3] printed in parentheses below those of the author.

Such values have been plotted in Figs. 3 and 4 for kT_R= 20, 25, 30, 40, and 50 keV.

III. DISCUSSION

In all the Monte Carlo results presented above it must be emphasized that there are possible statistical errors on the order of about 2%, on account of the finite size of the photon sample used. Such errors are inherent in any such Monte Carlo treatment. When the exchange rates are small the percentage error may be quite appreciable.

The agreement with Warham is generally good; the differences are about 1% at low T_e and T_R, but about 12% for high T_e and T_R.

See Woodward[4] for comparisons with nonrelativistic approximations and the deviations therefrom. It was found[4] that the nonrelativistic expression is accurate only for $kT_R \leq 2.5$ keV and $kT_e \leq 20$ keV.

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[8] A shake equals 10^{-8} sec; see, for example, J. W. Bond, Jr., K. M. Watson, and J. A. Welch, Jr., *Atomic Theory of Gas Dynamics* (Addison-Wesley, Reading, Mass., 1965), p. 502.