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Fluctuations at the Threshold of Classical Cosmology

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The "threshold epoch" of classical cosmology is first discussed from a classical point of view. It is shown that if the density fluctuations at this epoch are proportional to N^{-n} (where N is the number of particles in a disturbance), then n must exactly, or very closely, equal the value $\frac{2}{3}$ in order that such fluctuations develop into protogalaxies. This result is independent of the value of the threshold epoch, and of the equation of state for the very early universe. Arguments of a speculative nature are then presented which indicate that the threshold epoch occurs at the Planck density. It is also proposed that the initial conditions of galaxy formation are metric fluctuations at the threshold epoch of classical cosmology. The evidence in favor of this theory is (i) the density fluctuation obeys the $n=\frac{2}{3}$ rule, and (ii) the spin fluctuation gives an eventual angular momentum of the order $N^{4/3}\hbar$, in reasonable agrement with that of the Galaxy.

I. INTRODUCTION

Of the various problems encountered in the study of galaxy formation, the most perplexing is the nature and origin of the initial conditions. It is argued elsewhere[1] that the initial inhomogeneities which eventually develop into galaxies cannot arise spontaneously in an expanding universe, and must therefore already exist in primitive form when the universe begins to expand. In this discussion it is proposed that the initial inhomogeneities are fluctuations in the metric at the threshold of classical cosmology.[2]

An attractive possibility is that the initial conditions of galaxy formation consist of density variations in an otherwise uniform universe.[3–12] These variations are expressed generally by the relation $\mu_0=\text{const}\times L_0^{-n/3}$, where $\mu=\delta\rho/\rho$, $\delta\rho$ is a density perturbation of wavelength L, n is a constant, and a zero subscript denotes the value at an initial epoch t_0. Hence,

$$\mu_0=\beta_E N^{-n}, \tag{1}$$

where $N\propto L_0^3$ is the number of particles in the perturbed region, and β_E is a constant. (Thus in the case of thermal fluctuations $n=\frac{1}{2}$, $\beta_E\sim 1$.) From linear theory, μ grows with time in the early Friedmann models according to

$$\mu=a\mu_0(t/t_0)^m, \tag{2}$$

for $t\gg t_0$, where a and m are constants, and therefore, from (1) and (2),

$$\mu=a\beta_E N^{-n}(t/t_0)^m. \tag{3}$$

In order that μ attain sufficient values to lay the foundations of a structured universe, it is usual, for a given m, to postulate appropriate values of either μ_0 and t_0 in (2), or n and t_0 in (3). In this discussion, we adopt an alternative approach; instead of postulating initial conditions that appear appropriate, we set t_0 equal to the threshold epoch t^* of classical cosomology, and show quite generally that density fluctuations must obey an $n=\frac{2}{3}$ law in order to evolve into galaxies.

In the following we discuss first the threshold epoch, and show that n is closely or exactly equal to $\frac{2}{3}$, independently of the actual value of t^*. It is then argued heuristically that t^* is the Planck time and shown that metric fluctuations at this epoch conform to the $n=\frac{2}{3}$ law. In addition, it is shown that the spin component of the fluctuation gives an angular momentum in reasonable agreement with the observed value for the Galaxy.

II. THRESHOLD EPOCH

It is assumed that the early universe is represented by the Friedmann models in which the pressure is provisionally $p=\frac{1}{3}\rho c^2$. The curvature term is negligible in the early stages from the dynamic point of view and therefore

$$(dR/dt)^2=C/R^2, \qquad C=8\pi G\rho R^4/3, \tag{4}$$

where $R(t)$ is the scaling variable and C is a constant. It follows that

$$\rho t^2=3/32\pi G, \tag{5}$$

where time is measured from the singularity $\rho=\infty$. Let mc^2 be the mean particle energy, and λ_p a mean interparticle distance defined by the density $\rho=3m/4\pi\lambda_p^3$. A typical particle gravitational length is $\lambda_g=2Gm/c^2$; and hence, from (5), $t=(\lambda_p^3/4c^2\lambda_g)^{1/2}$. The Hubble distance (or "particle horizon") is $\lambda_H=cR/\dot{R}=2ct$, and therefore an equivalent way of writing (5) is

$$\lambda_H/\lambda_p=(\lambda_p/\lambda_g)^{1/2}. \tag{6}$$

[1] E. R. Harrison, Monthly Notices Roy. Astron. Soc. **141**, 397 (1968).
[2] E. R. Harrison, Nature **215**, 151 (1967).
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As we go back in time, the mean particle energy increases, and since $\lambda_H \propto t$, $\lambda_p \propto (mt^2)^{1/3}$, it follows that $\lambda_H/\lambda_p \propto (t/m)^{1/3}$ and therefore that the ratio λ_H/λ_p diminishes.

The continuous fluid models are acceptable representations of the universe when the Hubble distance is large compared with the mean interparticle distance, or when $\lambda_H \gg \lambda_p \gg \lambda_g$, from (6). But as t goes to zero, we enter an impossibly bizarre era of $\lambda_H < \lambda_p < \lambda_g$ in which the continuous fluid representations break down[2] (for example, particles recede from each other at speeds greater than c and are smaller than their gravitational length). Our models of an expanding universe apply, therefore, only when $t > t^*$, where t^* is a "threshold epoch" defined by $2ct^* = \lambda^*$, and $\lambda_H = \lambda_p = \lambda_g \equiv \lambda^*$.

The $n = \frac{2}{3}$ law discussed below is independent of the actual value of t^*; we shall need, however, the following result: At the threshold epoch, the number of particles in an initial region of radius L^* is $N = (N^*/\lambda^*)^3$, and hence, from (5),

$$8\pi G\rho^* L^{*3} = 3c^2 N^{2/3}. \tag{7}$$

III. $n = \frac{2}{3}$ LAW

We consider the effect of density fluctuations and ignore rotation. At time t^* a perturbed region of $N \gg 1$ is large compared with the Hubble distance, and (because the spatial derivatives are small) it can be treated as a section of a Friedmann model.[7,8,12] Thus, in a perturbed region,

$$ds^2 = d\tau^2 - S^2(\Gamma^{-2}dr^2 + r^2 d\Omega^2), \tag{8a}$$

$$\Gamma^2 = 1 - \kappa r^2, \tag{8b}$$

where $S(\tau)$ is a scaling variable, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. In place of (4), we now have

$$(dS/d\tau)^2 = (C + \delta C) S^{-2} - \kappa. \tag{9}$$

The initial perturbations in "kinetic," "potential," and total "energy" per unit mass are $\delta(dS/d\tau)_0^2$, $\delta(C/S^2)_0$, and $-\kappa$, respectively, and we suppose the perturbations originate at time τ_0. For convenience, it is assumed that $S_0 = R_0$, $(dS/d\tau)_0 = (dR/dt)_0$, and, therefore, from (1) and (9),

$$S_0^2\kappa/C = \delta C/C = \mu_0 = \beta_E N^{-n}. \tag{10}$$

Thus (9) becomes

$$(dS/d\tau)^2 = C[(1+\mu_0)S^{-2} - \mu_0 R_0^{-2}], \tag{11}$$

and density and total energy perturbations are equivalent.

To relate t and τ, we use[13]

$$\frac{\partial\phi}{\partial r} = -\frac{\partial p/\partial r}{p + \rho c^2} = -\frac{\partial \rho/\partial r}{4\rho},$$

where $d\tau = e^{\phi}dt$. If ρ_s is the density in the perturbed region, then $\rho_s S^4 = \rho_0(1+\mu_0)R_0^4$, $\rho R^4 = \rho_0 R_0^4$, and

$$\rho/\rho_s = S^4/(1+\mu_0)R^4, \tag{12}$$

and, therefore,

$$\phi = \int_{\rho_s}^{\rho} d\rho/4\rho = \ln[S/(1+\mu_0)^{1/4}R].$$

The relation between the proper intervals of time dt in the unperturbed region and $d\tau$ in the perturbed region is thus

$$R(1+\mu_0)^{1/4}d\tau = S dt. \tag{13}$$

Equations (4), (11), and (13) give

$$dS/dR = (1+\mu_0)^{1/4}(1 - S^2/S^{\dagger 2})^{1/2}, \tag{14}$$

where $S^\dagger = (1+\mu_0^{-1})^{1/2}R_0$ is the maximum value of S.

For $t \gg t_0$ we can neglect the small quantities t_0, R_0, and with $\mu_0 \ll 1$ it follows that

$$S = S^\dagger \sin\psi, \qquad \psi = (\mu_0 t/t_0)^{1/2}, \tag{15}$$

and maximum expansion occurs at $t^\dagger = (\frac{1}{2}\pi)^2 t_0/\mu_0$. Alternatively, from (11),

$$S = S^\dagger[1 - (1 - \tau/\tau^\dagger)^2]^{1/2}, \tag{16}$$

and the time to attain $S^\dagger$ is $\tau^\dagger = (3/8\pi G\rho^\dagger)^{1/2} = 2t_0/\mu_0$. Hence t and τ are related by

$$\tau/\tau^\dagger = 1 - \cos\psi. \tag{17}$$

The contrast density $\mu = (\rho_s - \rho)/\rho$ is

$$\mu = (\psi/\sin\psi)^4 - 1, \tag{18}$$

and at maximum expansion the density is $\rho_s^\dagger = \mu_0^2\rho_0$ and is $(\frac{1}{2}\pi)^4$ times the unperturbed density. Finally, let τ_H be the epoch when the Hubble distance equals the radius of the perturbed region, that is, $2\tau_H = S_H r_b$, where b denotes the boundary. From (15) and (17), it follows that

$$2\pi G\rho_0\mu_0 R_0^2 r_b^2 = 3\tan^2(\tfrac{1}{2}\psi_H). \tag{19}$$

At the threshold epoch the radius is $L^* = R^* c r_b$, and since $\rho \propto 1/t^2$, $R \propto t^{1/2}$, we obtain, from (7) and (19),

$$\mu_0 N^{2/3} t^*/t_0 = 4\tan^2(\tfrac{1}{2}\psi_H), \tag{20}$$

where $\psi_H = (\mu_0 t_H/t_0)^{1/2} = \frac{1}{2}\pi(t_H/t^\dagger)^{1/2}$. The treatment of perturbed regions as sections of Friedmann models is a reasonable approximation while $\tau \leq \tau_H$ or $\psi \leq \psi_H$; thereafter, the approximation breaks down and boundary conditions become important.

From these results we show first that n in (1) has a lower limit. The perturbed region is part of the universe and not a separate closed Friedmann model if $\Gamma^2 > 0$ in (8a). From (7), (8b), and (10), the boundary value Γ_b is given by

$$1 - \Gamma_b^2 = 8\pi G\rho_0\mu_0 R_0^2 r_b^2/3 = \beta_E N^{-n+2/3} t^*/t_0. \tag{21}$$

[13] L. D. Landau and E. M. Lifshitz, *Classical Theory of Fields* (Pergamon, Oxford, 1962), p. 332.

Thus when $\Gamma_b^2<0$, or when

$$n<\tfrac{2}{3}+\frac{\ln(\beta_E t^*/t_0)}{\ln N}, \qquad (22)$$

the perturbations are too strong and form closed metrics.

We now show that n has also an upper limit. It is easily seen that the t_H epoch occurs in the radiation era for all galactic masses of $10^3M_\odot<M<10^{18}M_\odot$. Also, when μ_H is small, μ ceases to grow and oscillates[3] for $t>t_H$ and, furthermore, is subject to damping[14] during the remainder of the radiation era. But at the end of the radiation era, μ must have a value of at least $\mu_m\sim10^{-2}$ for galaxy formation.[15] Thus, when $\mu_H<\mu_m$, the perturbations are in fact too weak to form galaxies, and under these conditions (i.e., $\psi_H\ll1$), we obtain from (18) and (20)

$$\mu_H=2\mu_0t_H/3t_0=2\beta_EN^{-n}t_H/3t_0, \qquad (23)$$

$$t_H=t^*N^{2/3}. \qquad (24)$$

{More exactly, from a first-order treatment of (14),

$$\mu=\mu_0[1+\tfrac{2}{3}(t/t_0-t_0^{1/2}/t^{1/2})],$$

and for $t_H\gg t_0$ we obtain (23).} From (23) and (24), we have

$$\mu_H=2\beta_EN^{-n+2/3}t^*/3t_0. \qquad (25)$$

Hence all values of n given by

$$n>\tfrac{2}{3}+\frac{\ln(2\beta_Et^*/3\mu_mt_0)}{\ln N} \qquad (26)$$

are excluded on the grounds that $\mu_H<\mu_m$.

The perturbations are too strong for n given by (22) and are too weak for n given by (26), and therefore

$$0<(n-\tfrac{2}{3})\ln N+\ln(t_0/\beta_Et^*)<\ln(2/3\mu_m) \qquad (27)$$

determines the possible values of n. The assumptions underlying the derivation of this result are: (a) The density perturbations are of the form $\mu_0=\beta_EN^{-n}$ and commence at time $t_0\geq t^*$; (b) the perturbations can be treated as sections of Friedmann models for $t\leq t_H$; and (c) the contrast density μ_H cannot be less than μ_m.

In the derivation of (27) we have assumed $p=\alpha\rho c^2$, with $\alpha=\frac{1}{3}$. However, a more general treatment with arbitrary α, involving somewhat cumbersome equations, does not alter the main conclusion. Equation (7) is unchanged, and in place of (22) we find

$$n<\tfrac{2}{3}+\frac{\ln[\beta_E(t^*/t_0)^q]}{\ln N}, \qquad (22')$$

where $q=(6\alpha+2)/(3\alpha+2)$; and, in place of (26), we find

$$n>\frac{\alpha+1}{3\alpha+1}+\ln\left(\frac{6\alpha+6}{15\alpha+7}\frac{\beta_Et^*}{\mu_mt_0}\right)\Big/\ln N. \qquad (26')$$

The main modification is therefore in (26′); and since $\alpha\leq\frac{1}{3}$, it implies that values of $n>w+O(1/\ln N)$, where $w\geq\frac{2}{3}$ are excluded on the grounds that $\mu_H<\mu_m$. However, throughout the radiation era α is very closely equal to $\frac{1}{3}$ and it is only prior to the radiation era (in the "hadron era"), when $T\gtrsim m_\pi c^2/k$ (where T is temperature) or $t<10^{-4}$ sec, that there is uncertainty in the value of α. Because $t_H\gg10^{-4}$ sec, and $\alpha=\frac{1}{3}$ throughout most of the lifetime of the perturbation, (23) and (24) are found to be essentially unchanged and therefore condition (26) is still true. Hence (22′) and (26) give, for arbitrary α in the hadron era,

$$0<(n-\tfrac{2}{3})\ln N+\ln[(1/\beta_E)(t_0/t^*)^q]<\ln[(2/3\mu_m)(t_0/t^*)^{q-1}]. \qquad (27')$$

It has been argued[1] that perturbations which evolve into galaxies are part of the universe from the beginning of its expansion and cannot arise spontaneously at some later epoch. The earliest epoch in classical cosmology is t^*; hence we have $t_0=t^*$, and (27) and (27′) both give

$$0<(n-\tfrac{2}{3})\ln N-\ln\beta_E<\ln(2/3\mu_m). \qquad (28)$$

It is evident that when N is large, n is close to $\frac{2}{3}$ for a wide range of β_E. If N_n is the present number of nucleons in a galaxy, the original number of particles up to the time of photon decoupling at the end of the radiation era is[16] $N\sim N_n\eta^{-1}$, where $\eta=b(\hbar c/kT)^3\sim10^{-9}$, $b\sim10^{-6}$ cm^{-3} is the present average nucleon density, and $T\simeq3$°K is the background radiation temperature. Thus we have $N\sim10^{77}$ for a galaxy of mass $10^{11}M_\odot$. If, for example, $\beta_E=10^{-1}$, $2/3\mu_m=10^2$, then from (28) we find $0.65<n<0.68$.

Our conclusion is that n has a value either close to or exactly equal to $\frac{2}{3}$. When $n=\frac{2}{3}$ exactly, then (28) gives $1>\beta_E\gtrsim10^{-2}$; also we find

$$\Gamma^2=1-\beta_Er^2/r_b^2, \qquad (29)$$

and, at maximum expansion, the radius is β_E^{-1} times the Schwarzschild radius.

It must be emphasized that the $n=\frac{2}{3}$ rule does not assert that there is a spectrum of wavelengths for which $n=\frac{2}{3}$ for all values of N. The rule merely states that those fluctuations which develop into galaxies (for which N has a limited range of values) must obey $n=\frac{2}{3}$. It is conceivable, for example, that the clustering of galaxies is due to longer-wavelength perturbations of $n>\frac{2}{3}$.

IV. FLUCTUATIONS AT THRESHOLD EPOCH

In this section some speculative comments are offered on the possible nature of the fluctuations at the threshold epoch. The question of whether or not these comments are correct does not affect the validity of the $n=\frac{2}{3}$ rule.

Quantum fluctuations of the metric discussed by Wheeler[17] apparently provide an additional reason for

[14] J. Silk, Astrophys. J. **151**, 569 (1968).

[15] See also C. W. Misner, Rev. Mod. Phys. **29**, 497 (1957).

[16] E. R. Harrison, Phys. Rev. **167**, 1170 (1968).

[17] J. A. Wheeler, Ann. Phys. (N.Y.) **2**, 604 (1957); *Geometrodynamics* (Academic, New York, 1962), pp. 76–77; in *Relativity, Groups, and Topology*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964).

the failure of the classical cosmologies at the threshold epoch. We follow, with slight modification, Wheeler's argument,[15] which is based on the Feynman propagator

$$\langle f_2\sigma_2/f_1\sigma_1\rangle = \sum_H \exp(iS_H), \tag{30}$$

where f_1 and f_2 are field configurations on hypersurface boundaries σ_1 and σ_2, and the summation is over all possible fields, classical or otherwise, with appropriate normalization. The dimensionless action is

$$S_H = (c^3/16\pi G\hbar)\int R_\mu{}^\mu(\sqrt{-g})\,d^4x,$$

and variations due to field fluctuations are

$$\delta S_H = (8\pi\lambda_P{}^2)^{-1}\int\delta(R_\mu{}^\mu\sqrt{-g})\,d^4x. \tag{31}$$

In this equation, $\lambda_P=\hbar/m_Pc=2Gm_P/c^2$ is the Planck length. Let δg represent a fluctuation in any of the metric coefficients $g_{\mu\nu}$; then $\delta(R_\mu{}^\mu\sqrt{-g})$ contains terms of $k^2\delta g$ and $(k\delta g)^2$, where $k=2\pi/L$ for wavelength L. From (31) and $d^4x\sim L^4$, it follows that

$$\delta S_H = \tfrac{1}{2}\pi(L/\lambda_P)^2\times\text{terms of order } \delta g \text{ and } (\delta g)^2. \tag{32}$$

The significant contributions in (30) are $\delta S_H=\frac{1}{2}\pi$ and, according to (32), they form a spectrum of fluctuations of

$$\delta g\sim(\lambda_P/L)^2, \qquad L\gg\lambda_P \tag{33}$$

with $\delta g\sim 1$ at wavelength $L=\lambda_P$.

A variety of arguments, commencing with (33), lead to the results expressed below in (36) and (37). A particularly simple approach[2] entails the plausible hypothesis that $\lambda_P\sim\lambda^*=\hbar/m^*c$ at the threshold epoch; hence,

$$t^*=(G\hbar/2c^5)^{1/2}=(3/32\pi G\rho^*)^{1/2}\simeq 4\times10^{-44}\text{ sec}, \tag{34}$$

and $\rho^*=3m^*/4\pi\lambda^{*3}$ is the Planck density

$$\rho^*=3c^5/16\pi G^2\hbar\simeq 3\times10^{92}\text{g cm}^{-3}. \tag{35}$$

According to this picture, particle masses of $m^*c^2=(\hbar c^5/2G)^{1/2}\sim10^{19}$ GeV, for which $Gm^{*2}/\hbar c\sim1$, are the basic constituents created by interactions with the metric at the Planck energy density. One advantage of this hypothesis is that the short-wavelength fluctuations of $\delta g\sim1$ explain why classical cosmology has a threshold epoch. When the age of the universe is large compared with t^*, the classical models of nested hypersurfaces orthogonal to cosmic time are little affected by the quantum states of the metric. At the threshold epoch, occurring at the Planck energy density, the microcurvature resonances of wavelength λ^* smear together all hypersurfaces of $t<t^*$, and the conventional concepts of a space-time manifold break down. Thus again we have reason for believing that classical cosmology holds only when $t>t^*$.

Since the Planck length and λ^* are of the same order of magnitude, (33) now becomes $\delta g\sim N^{-2/3}$, for $N\gg1$. At time $t\gtrsim t^*$, we have, from the Einstein field equation,

$$\delta g\sim\lambda^{*2}\delta R_\mu{}^\mu\sim t^{*2}G\delta\rho^*\sim(\delta\rho/\rho)^*,$$

and, therefore,

$$\mu^*=\beta_E N^{-2/3} \tag{36}$$

is the energy fluctuation, and β_E is an undetermined constant. Also, if ω is an angular velocity, then similarly the spin fluctuation is given by[18]

$$\delta g\sim\lambda^{*2}\delta R_\mu{}^\mu\sim\omega^{*2}t^{*2}\sim\omega^{*2}/G\rho^*,$$

and it follows that

$$\chi^*=\beta_S N^{-2/3}, \tag{37}$$

where $\chi=3\omega^2/8\pi G\rho$ and β_S is a second undetermined constant. The angular momentum is $J\sim\rho^*\omega^*L^{*5}$, or

$$J=A\beta_S{}^{1/2}N^{4/3}\hbar, \tag{38}$$

and A is a constant of order unity.

It should be stressed that (36) and (37) have not been derived by formal analysis and depend on, among other things, the validity of (32) at the threshold epoch. An interesting consequence of (36) and (37) is that all fluctuations are resonant states at the Planck energy density.[19] The basic resonances have mass m^* and spin $s\sim1$; these are grouped to form massive resonances[20] of $M=Nm^*$, $s\sim N^{4/3}$, and wavelength $L\sim s\hbar/Mc\sim GM\mu^*/c^2$.

V. COMMENTS

We have shown that initial density variations in the universe must obey exactly or almost exactly an $n=\frac{2}{3}$ law if they are to develop into galaxies. This result is independent of the actual value of the threshold epoch, and of the precise form of the equation of state in the hadron era.

It is suggested that the universe is initially structured with quantized fluctuations of the metric, or massive resonances, and among other things this structure contains the initial conditions of galaxy formation. In favor of this undoubtedly speculative theory is its conformity with the $n=\frac{2}{3}$ law. It also predicts acceptable values for the angular momenta of galaxies. For if photons decouple from a rotating region at the

[18] See, for example, J. L. Synge, *Relativity: General Theory* (North-Holland, Amsterdam, 1960), pp. 331–337; R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw Hill, New York, 1965), pp. 367–377.

[19] Whether or not these states are associated with baryon inhomogeneity (Ref. 16) is a matter for speculation.

[20] One might indeed argue that initially the universe itself is a resonance, of wavelength $L_u{}^*$ and $\beta_E=1$, embracing all states of $L^*<L_u{}^*$, $\beta_E<1$. We notice that if $\lambda=\hbar/mc$, where m is the nucleon mass, then $L_u{}^*/\lambda\sim N_u{}^{1/3}m/m^*\sim\eta^{-1}$, which may be no more than a numerical coincidence. The present angular velocity of the universe can be calculated from (37) and is given by $\omega_ut\sim10^{-6}\beta_S{}^{1/2}$. From the anisotropy of the microwave background radiation, observed by R. R. Partridge and D. T. Wilkinson [Phys. Rev. Letters **18**, 577 (1967)], it is estimated by S. Hawking [Monthly Notices Roy. Astron. Soc. **142**, 129 (1969)] that $\omega_ut\sim10^{-3}z^{-1}$ in a closed universe, where z is the red shift since the radiation was last scattered. Since $8<z<10^3$, depending on the thermal history of the intergalactic medium, it is by no means certain that the observed anisotropy is due to rotation of the universe.

end of the radiation era then J, given by (38), is approximately the present angular momentum of a galaxy, with allowance made for viscous damping in the fireball and subsequent tidal interactions. It is shown[21] that viscous damping is not important for masses of $M > 10^{11} M_{\odot}$. Moreover, it is argued that tidal interactions are generally not of great importance in the determination of galactic spin. For a galaxy of mass $M = 10^{11} M_{\odot}$, we find

$$J \sim 10^{75} A \beta_S^{1/2} \text{g cm}^2 \text{ sec}^{-1}.$$

The estimated value[22] for the Galaxy is 1.4×10^{74}g cm^2 sec^{-1} and this is in reasonable agreement for β_S of order unity.[23]

In Sec. III rotation is ignored and, in effect, we consider the case of β_S equal or close to zero. Rotation will cause anisotropy in the expansion, and lacking a formal theory of rotating Friedmann models, we can regard $\chi = 3\omega^2/8\pi G\rho$ as a measure of the departure from uniform expansion. Since ωS and ρS^4 are constants for a pressure $p = \frac{1}{3}\rho c^2$, it follows $\chi = \chi^* S^2/R^{*2}$. If $\beta_E = 0$, then

$$\chi = \chi^* t/t^* = \beta_S t/t_H, \tag{39}$$

from (24) and (37), and therefore $\chi_H = \beta_S$. When β_E is not zero,

$$\chi = (\chi^*/\mu^*) \sin^2\psi = (\beta_S/\beta_E) \sin^2\psi, \tag{40}$$

from (15), (36), and (37), and $\chi^\dagger = \beta_S/\beta_E$. Elsewhere[23] it is argued that $\beta_E < \beta_S/\beta_E < 1$ and $\beta_S > \beta_E$ lead to distinctly different configurations, which later develop into elliptical and spiral galaxies, respectively.

The possibility that galaxies originate in the early universe from thermal fluctuations of $n = \frac{1}{2}$, $\beta_E \sim 1$ has been considered by many authors.[24] With these values of n and β_E in (27), we obtain

$$1 < t_0/N^{1/6} t^* < 2/3\mu_m. \tag{41}$$

In the case of a typical galaxy, with $2/3\mu_m = 10^2$, it follows that $10^{13} < t_0/t^* < 10^{15}$, as in Peeble's model.[25] Thermal fluctuations must therefore originate in an era beginning at $t \sim 10^{-30}$ sec and ending at $t \sim 10^{-28}$ sec. Those fluctuations existing prior to this era are too strong and form closed metrics, whereas those developing after this era are too weak to form galaxies. Until we discover reasons for believing in a special era in which thermal fluctuations suddenly emerge, this theory remains in a rather unattractive state. There is also some room for doubt concerning the reality of statistical fluctuations of wavelength $\lambda \gg ct$ in the early universe. In the case of metric fluctuations this objection is circumvented by the argument that they originate prior to classical cosmology.

A significant result of the present discussion is that the structure of protogalaxies is determined by the energy and spin fluctuation parameters β_E and β_S. According to the usual view, protogalaxies are slight inhomogeneities of $\mu \lesssim 10^{-2}$ in the fireball. Instead, we now propose that the fireball, and particularly the radiation era, is populated with relatively dense embryonic galaxies that possess spin. This presents a picture in some respects similar to the lagging-core theory,[26] and has much in common with an Ambartsumian-Weizsacker type of cosmogony. The energy fluctuations specified by β_E lay the foundations of an Ambartsumian[27] cosmogony in which galaxies develop from dense embryos, and the spin fluctuations specified by β_S endow the primordial cosmic medium with turbulence-like properties as in the Weizsäcker[28] cosmogony.

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[22] Estimated by P. J. E. Peebles [Astrophys. J. **155**, 393 (1969)] from the models of K. Innanen [Astrophys. J. **143**, 150 (1966)].

[23] E. R. Harrison, Monthly Notices Roy. Astron. Soc. **148**, 119 (1970).

[24] In Hagedorn's theory [R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965); **6**, 311 (1968); Nuovo Cimento **52A**, 1336 (1967); R. Hagedorn and J. Ranft, Nuovo Cimento Suppl. **6**, 311 (1968)], the statistical fluctuations are of the form $\delta\rho/\rho \sim (\lambda_\pi/\lambda)^{3/2}$, where $\lambda_\pi = \hbar/m_\pi c$ and $N \propto \lambda^3$, in the limit of high density. This fluctuation expression applies only when the density is less than $\rho \sim M/\lambda_\pi^3 \sim m_\pi^2 c^4/G\hbar^2 \sim 10^{54}$g cm^{-3}, where M is the mass in a nucleon volume and $GM/c^2 \sim \lambda_\pi$. At this density, the Hubble distance equals the pion wavelength, and $t \sim 10^{-23}$ sec. At higher densities and earlier epochs the low-mass end of the hadron spectrum has wavelengths greater than the Hubble distance. This suggests that the low-mass states are depopulated, and Hagedorn's equation of state and fluctuation expression require modification. At the Planck density the only surviving particles are presumably those of mass $m^* \sim (\hbar c/G)^{1/2}$. Nevertheless, if correlation can exist over distances of λ_π even at the threshold epoch, then we have the interesting result that the mass in a nucleon volume is $M \sim \rho^* \lambda_\pi^3$, and the eventual material mass is $M_g \sim \eta M$, or $M_g \sim \eta m_\pi (\hbar c/G m_\pi^2)^2 \sim 10^{11} M$. The order-of-magnitude value of η is $(G m_\pi^2/\hbar c)^{1/4}$, and therefore $M_g \sim m_\pi (\hbar c/G m_\pi^2)^{7/4}$, as compared with $M_{\odot} \sim m_n (\hbar c/G m_n^2)^{3/2}$.

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