Baryon Trajectories without MacDowell Partners*

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It is shown that baryon trajectories can be constructed which are in agreement with the observed π-N resonance spectrum as well as backward scattering data and yet have *no* MacDowell partners. This is accomplished by including appropriate physical branch points in the trajectory function.

BARYON Regge trajectories are expected to be analytic functions of W (energy) rather than $s = W^2$. Experimental evidence, however, favors a trajectory function much like

$$\alpha(W)=a+bW^2. \tag{1}$$

In view of the MacDowell symmetry,

$$T_J^+(W)=T_J^-(-W),$$

the parametrization (1) implies that every baryon has a partner of nearly the same mass, but opposite parity, unless the residue function vanishes at the appropriate values of $W < 0$. With the exception of the nucleon $\frac{5}{2}^+$-$\frac{5}{2}^-$, these MacDowell partners have not been seen. It is completely apparent that the partners of the $N^*(1238)$ and the nucleon are absent.

The absence of the low-mass partners can be produced by the introduction of arbitrary zeros into the residue function,[1] but as these are purely phenomenological and required in increasing numbers as one goes to higher energy, an alternate solution based on a modification of the parabolic trajectory is worth considering.[2]

It is well known that (1) cannot really be quite right, as it is real above the π-N threshold and hence violates unitarity in that region. Furthermore, the trajectory function is expected to have branch points at both thresholds ($W=W_1$ and $W=-W_1$) and perhaps even at $s = W^2 = 0$.[3] These modifications, while experimentally rather small for $W>W_1$, where α of the form given in (1) fits the resonance masses that reoccur on the trajectory, may be considerably larger in both the negative W and W^2 regions.

We propose here a simple model that produces large distortions in the $-W$ region possibly so large as to do away with MacDowell partners altogether, and is still approximately consistent with form (1) for W above threshold. We take as model for the nucleon trajectory the form given in Eq. (2), which is again certainly not completely right, although it contains imaginary parts required by unitarity in the physical regions, ignored in (1).

$$\alpha_N^{(W)}=a+bW^2+cW(W+W_1)^{3/2}+idW(W-W_1)^{3/2}. \tag{2}$$

Note that above W_1 (π-N threshold), the fourth term is pure imaginary, and below $-W_1$, the third term is pure imaginary and the fourth term is real.

A rough adjustment of these parameters to fit the known behavior of α results in the following set of parameters:

$$a=-0.45, \quad b=+0.45, \quad c=d=0.21.$$

The trajectories produced by these parameters are shown in Figs. 1(a) and 1(b) as functions of W and W^2, respectively. For comparison we have also included a parabolic and polynomial fit by Barger and Cline.[1]

The following features are worth noting: (a) In the positive and experimentally available s region, the trajectory is virtually indistinguishable from the usual linear trajectory with the canonical unit slope.[4] (b) There are no MacDowell partners and the trajectory falls in the negative W region. (c) On the negative s (imaginary W) axis, the trajectory has both a real and imaginary part; the real part is given by

$$\text{Re}\alpha(W)=-0.45-0.45\,|W^2|.$$

The slope is much smaller than the slope usually quoted,[1] but as is seen from Fig. 1(b), the real part

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[1] V. Barger and D. Cline, Phys. Rev. Letters **21**, 392 (1968); **21**, 1132(E) (1968).

[2] Another attractive possibility has been suggested by R. Carlitz and M. Kislinger, Phys. Rev. Letters **24**, 186 (1970); here the trajectory remains parabolic, but retreats to another sheet of the J-Plane for negative W.

[3] J. S. Ball and F. Zachariasen, Phys. Rev. Letters **23**, 346 (1969).

[4] The first recurrence, i.e., $N_{5/2}^+$ is seen to have a mass $M\approx1.7$ BeV and a width $\Gamma\approx120$ MeV.

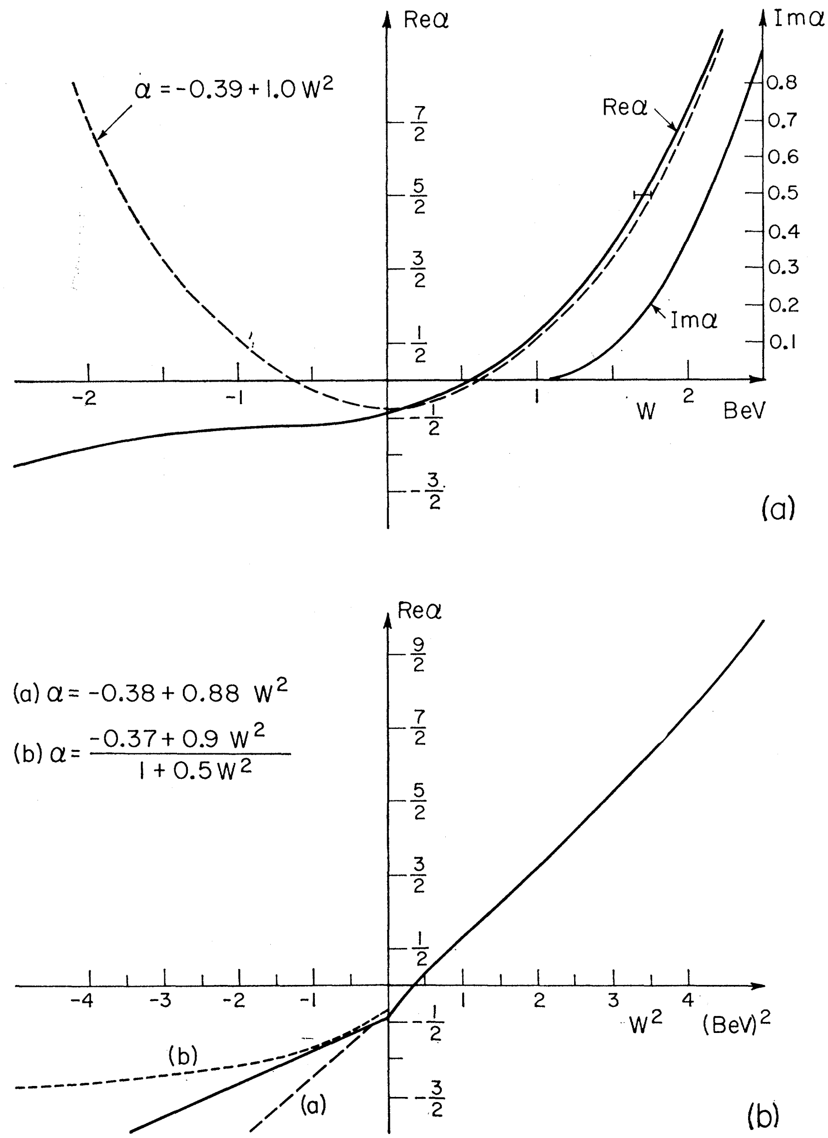

FIG. 1. Nucleon N_α trajectory α as a function of W[Fig. 1(a)] and of $W^2=s$ [Fig. 1(b)]. The solid line is the present model. The dashed lines, for proposes of comparison, are a parabolic fit to the Chew-Frautschi plot [Fig. 1(a)], and a parabolic and ratio-of-polynomials fit (Ref. 1) to π-p backward scattering [Fig. 1(b)].

falls somewhere between the two fits of Barger and Cline. No attempt has been made here to optimize the parameters in (2) taking both real and imaginary α into account for fitting the data.[5] This model serves as an illustration that more realistic trajectories can appear to give a quasilinear trajectory for positive W and yet produce no MacDowell partners.[6]

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[5] While the threshold condition for $W=W_1$ is correct (in the $m_\pi\approx 0$ approximation), i.e., $\mathrm{Im}\alpha(W)\sim(W-W_1)^{|\alpha(W_1)+1|}$, the condition at $W=-W_1$, namely, $\mathrm{Im}\alpha(W)\sim(W+W_1)^{|\alpha(-W_1)|}$ is not satisfied. However, the leading term may be small and the next term in Imα would properly go as the third term in (2).

[6] A fit using dispersion relations, but only a positive W cut, was made by B. R. Desai, D. Gregorich, and R. Ramachandran, Phys. Rev. Letters **18**, 565 (1967).