Generalizations of W Spin*

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The generalization of W spin to orbital extensions of the quark model is discussed. We point out the Lorentz-invariant properties and the predictive power of the assumptions, and discuss a possible manifestation of the breaking of the rest (and collinear) symmetry.

In a recent paper, Lipkin[1] pointed out difficulties that arise in spin treatments in the quark model. As expected, the results that stem from W-spin[2] considerations do not contradict general properties that are due to Lorentz invariance. The W spin can be defined in terms of the quark fields; however, it can also be abstracted and put into a general algebraic form.[3] In the present comments, we point out that this can be achieved also for orbital extensions of the problem and analyze some possible consequences and implications.

The $SU(6)_S$ generators are defined in terms of eightfold extensions of a spinlike quantity, usually identified as the quark spin $\mathbf{S}$. All that one has to require is that $\mathbf{S}$ forms an $SU(2)$ and behaves like 1^{++} under J^{PC}. To it we add now another $SU(2)$ axial-vector operator with opposite charge conjugation $\mathbf{R}$ behaving like 1^{+-}. $\mathbf{S}$ and $\mathbf{R}$ constitute together an $SO(4)$ that can be identified with the $[SU(2)\times SU(2)]_\beta$ defined by the quark field generators $\int d^3x{:}\psi^\dagger\boldsymbol{\sigma}\psi{:}$ and $\int d^3x{:}\psi^\dagger\beta\boldsymbol{\sigma}\psi{:}$. We note that if we want the complete Dirac $SU(4)$ algebra, we have to add the vector operators $\mathbf{V}=\int d^3x{:}\psi^\dagger\gamma_5\boldsymbol{\sigma}\psi{:}$ and $\mathbf{N}=\int d^3x{:}\psi^\dagger i\beta\gamma_5\boldsymbol{\sigma}\psi{:}$, both behaving like 1^{--} under J^{PC}. These were also sometimes used in a non-Hermitian form to generate an $SU(2,2)$; however, the J^{PC} properties remain unchanged. Note that none of the generators has the quantum numbers of a Lorentz boost, 1^{-+}, and therefore cannot represent it in any $SU(2,2)$ version. This point was often overlooked in the early days of "relativization" of $SU(6)$.

$[SU(2)\times SU(2)]_\beta$ can serve as a rest symmetry for classification of particles. That is so since, in the quark field model, $\mathbf{S}$ and $\mathbf{R}$ commute with the Hamiltonian. Similarly, in order to find candidates for a collinear symmetry, we look for the $[SU(2)\times SU(2)]_\beta$ generators that commute with Λ_z, the boost along the z direction. These operators form the W spin,

$$W_{x,y}=R_{x,y},\qquad W_z=S_z. \tag{1}$$

The extension to $[SU(6)\times SU(6)]_\beta\to SU(6)_W$ is straightforward.

The Lorentz group is generated by $\mathbf{J}$ (1^{++}) and $\mathbf{\Lambda}$ (1^{-+}). We already saw that $\mathbf{\Lambda}$ cannot have a part that belongs to the quark Dirac algebra. $\mathbf{J}$, however, does have a part there. Following Gell-Mann,[4] we now define

$$\mathbf{J}=\mathbf{S}+\mathbf{L}, \tag{2}$$

thus denoting by $\mathbf{L}$ that part of $\mathbf{J}$ not included in the $SU(6)_S$ generators. Since $\mathbf{S}$ is assumed to behave like a vector under rotations, so does $\mathbf{L}$. Moreover, one finds

$$[L_i,L_j]=i\epsilon_{ijk}L_k,\qquad [\mathbf{S},\mathbf{L}]=0. \tag{3}$$

Thus we arrive at a rest symmetry of

$$[SU(6)\times SU(6)]_\beta\times O(3)_L.$$

Since $[L_z,\Lambda_z]=0$, we can define the collinear symmetry $SU(6)_W\times O(2)_{L_z}$, where $\mathbf{W}$ remains as defined above in Eq. (1).

This was proposed several years ago by Freund *et al.*[5] and by Lipkin.[6] Freund *et al.* used a Lagrangian approach to implement this symmetry while Lipkin obtained it as a result of quark model calculations. It can be, of course, most simply used in its algebraic abstract form to reproduce the results of these authors. As an example of the simplicity of the calculations and the power of its prediction, we show in Table I the predicted couplings of all **35** mesons with arbitrary L to a $B\bar{B}$ pair (B is an octet within **56**, $L=0$). Since these results are true for all particles, they will hold for the Regge trajectory to which these particles belong. Thus we predict that the leading trajectory at $t=0$ ($^3L_{L+1}$ coupling of $J_z=0$) will couple to the $B\bar{B}$ pair with pure F coupling. This coincides with the prediction of Cabibbo, Horwitz,

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TABLE I. W-spin predictions for **8** meson (**35**, L) couplings to $B\bar{B}$ ($B\subset$**56**, $L=0$). All couplings can be multiplied by the same arbitrary function of L.

	$(L_z=0), J_z=0, W=0$	$(L_z=0), J_z=1, W=1$
${}^3L_{L+1}$	$[(L+1)/(2L+1)]^{1/2}F$	$[(L+2)/(4L+2)]^{1/2}(\frac{2}{3}F+D)$
${}^3L_L$	0	$-(\sqrt{\frac{1}{2}})(\frac{2}{3}F+D)(1-\delta_{L,0})$
${}^3L_{L-1}$	$-[L/(2L+1)]^{1/2}F$	$[(L-1)/(4L+2)]^{1/2}(\frac{2}{3}F+D)$
	$(L_z=0), J_z=0, W=1$	$(L_z=0), J_z=1$, W nonexistent
${}^1L_L$	$-(\frac{2}{3}F+D)$	0

and Ne'eman[7] and was discussed quite extensively in the literature. Similarly, one finds predictions for baryonic decays. Thus all B^* octets that belong to a **56** with arbitrary L decay into BP with $D/F=\frac{3}{2}$. These predictions can help to identify the classification of resonances.

Trouble arises in the mesonic decays. If one identifies the A_1 with a ${}^3P_1$, and the B with ${}^1P_1$, one finds that A_1 should decay transversely into $\rho\pi$, and B longitudinally into $\omega\pi$. This result was pointed out by Lipkin.[6] Experimentally, the A_1 question is not finally settled, but the B looks definitely transversal and not longitudinal.[8] It is very simple to see how the selection rule for $B\to\omega\pi$ comes about. A ${}^1P_1$ state has $J_z=L_z$, and since both ω and π have $L=0$, it turns out that L_z conservation implies that the decay can proceed only from a $J_z=0$ state. Therefore, a possible explanation of the paradox is that B is not a pure ${}^1P_1$ state. This would seem attractive if other similar states are discovered nearby. It might also mean that an $L=0$ $qq\bar{q}\bar{q}$ classification is better than the $L=1$ $q\bar{q}$. However, the most natural way out of this dilemma is to attribute it to a genuine symmetry-breaking effect. Gell-Mann[4] showed that a natural choice for the symmetry-breaking Hamiltonian is a **35** with $\mathbf{L}=1$. The existence of such a breaking is, of course, most strongly felt in violations of selection rules. This will obviously affect the above-mentioned decay modes.

Indeed, we note that the situation here is different from the usual decays of **56** or **35** with $L=0$. Usually, the prominent p-wave couplings (e.g., $\Delta\to N\pi$ and $\rho\to\pi\pi$) are forbidden by the rest symmetry

$$[SU(6)\times SU(6)]_\beta\times O(3)_L$$

even to first order in the symmetry breaking. They are, however, allowed by the collinear $SU(6)_W\times O(2)_{L_z}$. Here we deal with couplings that are allowed to first order in the symmetry breaking even at the level of the rest symmetry. Therefore, we should not be astonished to find such a big effect in the $B\omega\pi$ system.

The generalization of the decay symmetry to extended quark models is straightforward. By adding internal degrees of freedom, the discussed structure of the decay symmetry remains untouched.[9] The usual L-excitation scheme adopted in quark-model assignments involves at least an orbital $SL(2,C)$ spectrum-generating algebra with generators $\mathbf{L}$ and $\mathbf{Q}$ (another 1^{--} set). Additional collinear symmetries may exist in $\mathbf{Q}$, depending upon the explicit structure[10] of these operators.

In summary, let us point out that the decay symmetry based on $\mathbf{W}$ and L_z does not lead to contradictions with Lorentz invariance. This can be directly checked in the free-quark field theory. It is the natural extension of the W-spin symmetry defined for $L=0$ and can be formulated in an abstract way independent of any particular model. This leads then to predictions for collinear processes and can be simply generalized for more complicated models. Its usefulness will become clear as the structures of the various decay modes become experimentally available.

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