# Selection Rule in the Quark Model and the Real Part of the Forward Scattering Amplitude*

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The selection rule derived from vector-meson decay is further investigated. It is found that the selection rule can be formulated consistently in the framework of field theory, if we work with a quark model with three degrees of freedom. Applying this theory to evaluate the real part of the high-energy forward scattering amplitudes, it is found that the fit with experiment is quite satisfactory.

## I. INTRODUCTION

After the discovery of the $\phi(1^-)$ meson it was soon recognized that the physical states of $\phi$ and $\omega$ are not in the pure irreducible representations of $SU(3)$. However, the recognition of $\phi$-$\omega$ mixing[1] was not enough to explain the fact that the $\phi$ meson decays mainly into a $K\bar{K}$ pair but not into $\rho\pi$, and this led Okubo[2] to introduce the following argument to get rid of this difficulty.

Consider that the vector mesons form a nonet and write them in a $3\times3$ matrix defined by $\mathbf{V}_9=\mathbf{V}_8+\frac{1}{3}\sqrt{3}\omega_0\mathbf{1}$. Then the $\phi$ meson, being the $V_{33}$ component of $\mathbf{V}_9$, can only decay into a $K\bar{K}$ pair if one type of interaction, $\mathrm{Tr}(\mathbf{V}_9\mathbf{P}_8\mathbf{P}_8)$, out of the possible $SU(3)$-invariant interactions, is dominant and those such as $\mathrm{Tr}(\mathbf{V}_9)\,\mathrm{Tr}(\mathbf{P}_8\mathbf{P}_8)$ and $\mathrm{Tr}(\mathbf{V}_9)\,\mathrm{Tr}(\mathbf{V}_9\mathbf{P}_8)$ do not exist.

Glashow and Socolow[3] applied this principle to the decay of the $2^+$ meson and successfully explained the fact that the $f'$ meson only decays into a $K\bar{K}$ pair. It was Iizuka[4] who expressed this principle in terms of the quark coupling, by requiring that Fig. 1(a) should be dominant while Fig. 1(b) is suppressed.

Assuming that the meson decay takes place through the decay of one of the quarks or antiquarks in the meson, Figs. 1(a) and 1(b) are the same as Figs. 1(a′) and 1(b′). In Figs 1(a′) and 1(b′), respectively, it is understood that $\bar{\delta}$-$\gamma$ and $\bar{\delta}$-$\delta$ form one of the decay-produced mesons. The selection rule which forbids Fig. 1(b) while allowing Fig. 1(a) is hard to understand field theoretically, since the antiquark in the final state hardly knows which of the two quarks was in the initial state. This is clearly seen in the special case of $\gamma=\delta$.

We would like to point out in this paper that this kind of selection rule can be easily formulated in the framework of field theory if we work with parafermion quarks[5] or three-triplet quarks.[6]

A quark field in these models is decomposed into $t_i^{\alpha}$, where the upper suffix denotes the $\mathcal{P}$, $\mathcal{N}$, or $\lambda$ and the lower suffix represents a Green's[5] component for the parafermion quark or specifies a triplet for the three-triplet model. The meson and baryon states are expressed in these theories[5] as

$$\text{meson:}\quad |[\alpha,\bar{\beta}]\rangle=\sum_{i=1}^{3}a_i^{*\alpha}(k)b_i^{*\beta}(k')\,|\,0\rangle, \tag{1}$$

$$\text{baryon:}\quad |\{\alpha,\{\beta,\gamma\}\}\rangle=\sum_{i,j,k=1}^{3}a_i^{*\alpha}(k)a_j^{*\beta}(k')\times a_k^{*\gamma}(k'')\,|\,0\rangle f_{ijk}{}^{\alpha\beta\gamma}, \tag{2}$$

where we have omitted the properly weighted integrations over $k$, $k'$, and $k''$ for the sake of simplicity. The $f_{ijk}$ in Eq. (2) takes a nonzero value only when $i\neq j\neq k$.

Now let us supply the lower subscripts for quarks participating the process of Figs. 1(a′) and 1(b′). Since by Eq. (1) the decay-produced boson has to be summed for quark-antiquark pairs with the same lower subscripts, Figs. 1(a′) and 1(b′) should be expressed as Figs. 2(a) and 2(b), respectively. We can now distinguish Fig. 2(a) from Fig. 2(b) because each corresponds to different processes in terms of the decomposed quark components. It is, then, not difficult to build up a Hamiltonian model which leads to the desired selection rule.

In Sec. III, such a Hamiltonian is actually derived in the parafermion quark model. The equivalence of this model to the three-triplet quark model is proved in the Appendix. In Sec. II we discuss a possible consequence of the selection rule. It is shown that high-energy behavior of the real part of the forward scattering amplitudes can be well understood with this selection rule.

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[1] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

[2] S. Okubo, Phys. Letters **5**, 165 (1963).

[3] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[4] J. Iizuka, K. Okada, and O. Shito, Progr. Theoret. Phys. (Kyoto) **35**, 1061 (1966).

[5] H. S. Green, Phys. Rev. **90**, 270 (1953); O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[6] Y. Nambu, in *Proceedings of Second Coral Gables Conference on Symmetry Principles at High Energy, University of Miami, 1965* (Freeman, San Francisco, 1965).

## II. REAL PART OF FORWARD SCATTERING AMPLITUDES

Because of the crossing relation we have two types of scattering diagrams (Fig. 3) which are obtainable from Fig. 2(a). The characteristics of these two diagrams are that (a) there exist only exchange scattering in terms of lower subscripts for quark-quark scattering, and (b) there exist $\gamma$-quark and $\delta$-antiquark scattering only when $\gamma=\delta$. We would like to apply these restrictions to high-energy forward scattering where no complication[7] of the quark distribution in the baryons and mesons comes in. For this purpose we also assume that (c) the forward scattering amplitude (FSA) is given by a sum over the scattering amplitudes between constituent quarks.

A remark should be added, however, before applying the selection rule to the FSA, namely, that the restrictions (a) and (b) cannot be imposed on the theory in general since the imaginary part of the FSA of the nonexchanged process $\gamma_i\delta_j\to\gamma_i\delta_j$ has to exist, owing to the unitarity of the $S$ matrix when there exist elastic and inelastic processes of the exchanged type $\gamma_i\delta_j\to\gamma_j\delta_i$. In other words, the restrictions (a) and (b) are masked in the imaginary part of the FSA and the result derived from only the assumption (c) was already discussed by Lipkin and Scheck.[8] For the real part of the FSA, on the other hand, the restrictions may survive.

Since only $\gamma=\delta$ processes in both (a) $\gamma_i\delta_j\to\gamma_j\delta_i$ and (b) $\gamma_i\bar{\gamma}_j\to\delta_i\bar{\delta}_j$ can contribute to the elastic scattering,[9]

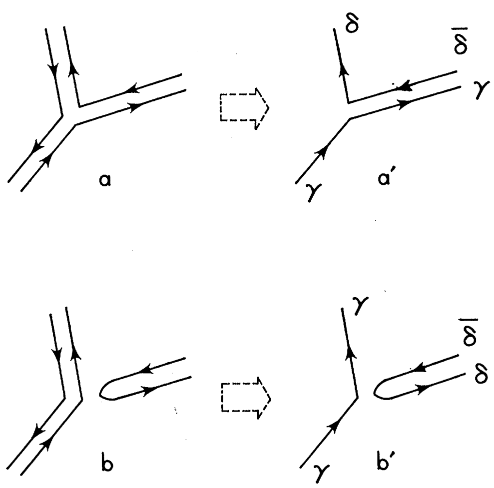


FIG. 1. Diagrams for a meson decay. In (a) each decay-produced meson contains one of the quarks of the parent meson. In (b) the created quark-antiquark pair forms one of the daughter mesons. One of the quark lines is omitted from (a′) and (b′) according to the assumption of the additivity of the quark model.

[7] H. J. Lipkin, Phys. Rev. **183**, 1221 (1969).
[8] H. J. Lipkin and H. Scheck, Phys. Rev. Letters **18**, 347 (1967).
[9] In some elastic processes, such as $pn\to pn$ and $p\Lambda\to p\Lambda$, the $\gamma=\delta$ scattering of $\gamma_i\delta_j\to\gamma_j\delta_i$ can also contribute.

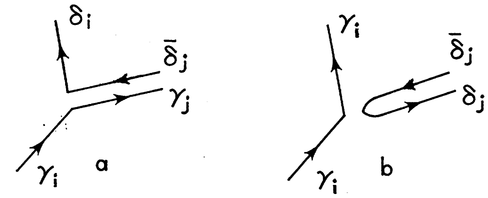


FIG. 2. Same as Figs. 1(a′) and (b′) but here the participating quarks have lower indices. In (a) $\sum_j\gamma_j\bar{\delta}_j$ and in (b) $\sum_j\delta_j\bar{\delta}_j$ form the mesons.

the real part of the FSA can be expressed with the aid of two amplitudes $A$ and $\tilde{A}$ defined by

$$\begin{aligned}\mathrm{Re}T(\gamma\gamma\to\gamma\gamma)&=A,\\ \mathrm{Re}T(\gamma\bar{\gamma}\to\gamma\bar{\gamma})&=\tilde{A},\end{aligned}\tag{3}$$

where we have assumed $SU(3)$ invariance and spin independence of the amplitude at high energy. The real parts of the FSA for processes in which we are interested are explicitly expressed as

$$\begin{aligned}&\mathrm{Re}(\pi^+p)=2A+\tilde{A}, &&\mathrm{Re}(K^-p)=2\tilde{A},\\ &\mathrm{Re}(\pi^-p)=2\tilde{A}+A, &&\mathrm{Re}(pp)=5A,\\ &\mathrm{Re}(K^+p)=2A, &&\mathrm{Re}(p\bar{p})=5\tilde{A}.\end{aligned}\tag{4}$$

We wish to note here that the rearrangement model proposed by Rubinstein and Stern[10] corresponds to our model with $\tilde{A}=0$ except for $\mathrm{Re}(pp)$. The $\mathrm{Re}(pp)$ is given in the rearrangement model spin dependently by

$$\begin{aligned}\mathrm{Re}(pp)_{J=1}&=(31/9)2A,\\ \mathrm{Re}(pp)_{J=0}&=(2/9)2A.\end{aligned}$$

The ratio of the real part to the imaginary part of the FSA is given from the optical theorem by

$$\alpha=\mathrm{Re}T/2MP_L\sigma_{\mathrm{tot}},\tag{5}$$

where $M$ is the nucleon mass of the target and $P_L$ is the momentum of the incoming particle in the laboratory system. The values of $\alpha$ calculated from Eq. (4), using experimental values of $\alpha(\pi^{\pm}p)$ as inputs,[11] are given in Table I. There we also show the theoretical values calculated in the rearrangement model ($\alpha_R$). For $\alpha_R(pp)$ we took the spin-averaged value

$$[\tfrac{3}{4}\,\mathrm{Re}(pp)_{J=1}+\tfrac{1}{4}\,\mathrm{Re}(pp)_{J=0}]/2M_{LL}\sigma_{\mathrm{tot}}.$$

It may be seen from Table I that the rearrangement model is clearly in disagreement with experiment.

We may also consider a model which has no restriction like (a) and (b). This is the kind of model con-

[10] H. R. Rubinstein and H. Stern, Phys. Letters **21**, 447 (1966); M. Elitzur and H. R. Rubinstein, Phys. Rev. Letters **18**, 417 (1967).
[11] K. J. Foley *et al.*, Phys. Rev. Letters **18**, 193 (1967); **18**, 330 (1967); **18**, 857 (1967); W. De Baere *et al.*, Nuovo Cimento **45A**, 885 (1966).

TABLE I. Ratios ($\alpha$) of the real part to the imaginary part of the FSA at $P_L=8$, 14, and 20 GeV/$c$. For experimental values of $\sigma_{\text{tot}}$ and $\alpha_{\text{expt}}$ see Ref. 11. All $\alpha_{\text{expt}}$ contain systematic errors of $\pm0.02$ besides errors given in the Table. $\alpha_{\text{expt}}(K^-p)$ is consistent with 0. The dispersion calculation gives $(\alpha p\bar{p})\sim-0.06$ at 10 GeV/$c$.

| | $P_L$ (GeV/$c$) | Experimental $\sigma_{\text{tot}}$ (mb) | Our values $\alpha$ | Rearrangement model $\alpha_R$ | Experimental $\alpha_{\text{expt}}$ |
|---|---|---|---|---|---|
| $\pi^-p$ | 8 | 27.6 | input | input | $-0.15\pm0.02$ |
| | 14 | 25.9 | | | $-0.13\pm0.02$ |
| | 20 | 25.1 | | | $-0.12\pm0.02$ |
| $\pi^+p$ | 8 | 25.5 | input | $-0.32$ | $-0.23\pm0.02$ |
| | 14 | 24.2 | | $-0.28$ | $-0.19\pm0.02$ |
| | 20 | 23.7 | | $-0.25$ | $-0.14\pm0.02$ |
| $K^+p$ | 8 | 17.3 | $-0.30$ | $-0.47$ | $\lvert\alpha\rvert\sim0.31\pm0.21$ at 3 GeV/$c$ |
| | 14 | 17.4 | $-0.22$ | $-0.38$ | $\sim0.45\pm0.14$ at 5 GeV/$c$ |
| | 20 | 17.2 | $-0.16$ | $-0.35$ | |
| $K^-p$ | 8 | 23.6 | $-0.06$ | 0 | $\lvert\alpha\rvert<0.2$ |
| | 14 | 21.5 | $-0.07$ | | |
| | 20 | 21.0 | $-0.08$ | | |
| $pp$ | 8 | 40.3 | $-0.32$ | $-0.72$ | $-0.33\pm0.01$ |
| | 14 | 39.4 | $-0.25$ | $-0.62$ | $-0.27\pm0.01$ |
| | 20 | 39.0 | $-0.18$ | $-0.58$ | $-0.20\pm0.01$ |
| $p\bar{p}$ | 8 | 56.4 | $-0.06$ | 0 | $-0.006\pm0.03\pm0.06$ at 10 GeV/$c$ |
| | 14 | 50.7 | $-0.07$ | | |
| | 20 | 50.3 | $-0.08$ | | |

sidered by Lipkin and Scheck,[8] but unlike the imaginary part of the FSA, the real part is by no means positive definite. Therefore we have to introduce six parameters defined by

$$\begin{aligned}
&\mathrm{Re}(\mathcal{P}\mathcal{P})=\mathrm{Re}(\mathcal{P}\mathcal{N})=P,\\
&\mathrm{Re}(\mathcal{P}\bar{\mathcal{N}})=\tilde{P}',\\
&\mathrm{Re}(\mathcal{P}\bar{\mathcal{P}})=\mathrm{Re}(\mathcal{N}\bar{\mathcal{N}})=\tilde{P},\\
&\mathrm{Re}(\mathcal{P}\lambda)=\mathrm{Re}(\mathcal{N}\lambda)=P'-S,\\
&\mathrm{Re}(\mathcal{P}\bar{\lambda})=\mathrm{Re}(\lambda\bar{\mathcal{P}})=\mathrm{Re}(\mathcal{N}\bar{\lambda})=\mathrm{Re}(\bar{\mathcal{N}}\lambda)=\tilde{P}'-\tilde{S}.
\end{aligned}\tag{6}$$

We have thus, instead of Eq. (4),

$$\begin{aligned}
&\mathrm{Re}(\pi^+p)=3P+2\tilde{P}'+\tilde{P},\\
&\mathrm{Re}(\pi^-p)=3P+\tilde{P}'+2\tilde{P},\\
&\mathrm{Re}(K^+p)=3P+3\tilde{P}'-3\tilde{S},\\
&\mathrm{Re}(K^-p)=2\tilde{P}+3P+\tilde{P}'-3S,\\
&\mathrm{Re}(pp)=9P,\\
&\mathrm{Re}(\bar{p}S)=5\tilde{P}+4\tilde{P}'.
\end{aligned}\tag{4'}$$

Reducing $P$, $\tilde{P}$, and $\tilde{P}'$ from the experiments of $\alpha(\pi^\pm p)$ and $\alpha(pp)$, then at 8 GeV/c we get $P/2MP_L\sim-1.47$ mb, $\tilde{P}'/2MP_L\sim-1.1$ mb, and $\tilde{P}/2MP_L\sim0.73$ mb. These values indicate that $\mathrm{Re}(\mathcal{P}\mathcal{P})=\mathrm{Re}(\mathcal{P}\mathcal{N})$ and $\mathrm{Re}(\mathcal{P}\bar{\mathcal{N}})$ are of almost the same magnitude, whereas $\mathrm{Re}(\mathcal{P}\bar{\mathcal{P}})$ has the opposite sign. This seems hard to understand, because for the real part of the FSA only the Born term is supposed to contribute, and there is no reason for a difference in sign between $\mathrm{Re}(\mathcal{P}\bar{\mathcal{N}})$ and $\mathrm{Re}(\mathcal{P}\bar{\mathcal{P}})$ at high energy, which would be in contrast with the result $\mathrm{Re}(\mathcal{P}\mathcal{P})=\mathrm{Re}(\mathcal{P}\mathcal{N})$ that follows from the assumption that there is no selection rule in this model.

Besides this problem, if we proceed to evaluate $\alpha(K^\pm p)$, we have to assume again the value of the parameter $S$ in Eq. (4′) and at the $SU(3)$-invariant limit, that is, in the case $S=0$, we get $\alpha(K^+p)$ and $\alpha(K^-p)$ at 8 GeV/$c$ as $-0.45$ and $-0.18$, respectively. In order to get values as small as our values of $\alpha(K^\pm p)$, we have to take $S/P=S/P'=70\%$, which means very large $SU(3)$ breaking. In this regard it is highly desirable to have accurate measurements of $\alpha(K^\pm p)$ at high energy.

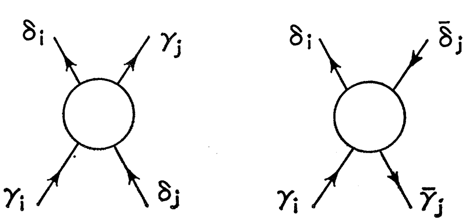


FIG. 3. Diagrams for quark-quark and quark-antiquark scattering obtained by the crossing relation from Fig. 2(a).

## III. FORMULATION OF SELECTION RULE

We would like to construct in this section a Hamiltonian which leads to the selection rules (a) and (b). For definiteness we shall assume that the $t_i^\alpha$'s are Green's components and satisfy Green's commutation relations, that is, fields with the same lower indices anticommute, and those with different lower indices commute. The coefficient in the baryon state in Eq. (2), $f_{ijk}{}^{\alpha\beta\gamma}$, is in this case completely symmetric among $\alpha$, $\beta$, $\gamma$ and $i$, $j$, $k$, respectively.

The baryon and meson operators commute with each other, and operators in a baryon operator also commute with each other, because of Eqs. (1) and (2). Therefore, the order of field operators in a baryon operator may be chosen arbitrarily, and we may write the scattering amplitude between a meson and a baryon as

$$\langle[\gamma',\bar{\epsilon}']\xi'\eta'\zeta' \mid -i\int H dt \mid [\gamma,\bar{\epsilon}]\xi\eta\zeta\rangle. \tag{7}$$

Since we also assume by (c) that only one of the quarks in each particle participates in the scattering and the others go through without interaction, Eq. (7) can be reduced to a sum over possible combinations of three noninteracting quarks or antiquarks. For example, a matrix element corresponding to a process, say, $\bar{\epsilon}\to\bar{\epsilon}'$, $\xi\to\xi'$, $\zeta\to\zeta'$ is given by

$$\sum_{l,m,n}\sum_i \langle a_m{}^{\eta'}a_i{}^{\gamma'} \mid -i\int H dt \mid a_i{}^{*\gamma}a_m{}^{*\eta}\rangle f_{lmn}{}^{\xi\eta\zeta}(f_{lmn}{}^{\xi\eta'\zeta})^*, \tag{8}$$

where use has been made of the property of $f_{lmn}$ that $f_{lmn}\neq 0$ only if $l\neq m\neq n$ and thus $m$ has to be fixed when $l$ and $n$ are fixed.

We shall study two types of Hamiltonian:

$$H_1=G\int\sum_{lm}\sum_{\alpha\beta}:(\bar{t}_l{}^\alpha\Gamma t_l{}^\alpha)(\bar{t}_m{}^\beta\Gamma t_m{}^\beta):d^3x, \tag{9}$$

$$H_2=G\int\sum_{lm}\sum_{\alpha\beta}:(\bar{t}_l{}^\alpha\Gamma t_m{}^\alpha)(\bar{t}_m{}^\beta\Gamma t_l{}^\beta):d^3x. \tag{10}$$

A typical matrix element under the sum over $l$, $m$, $n$ in Eq. (8) is calculated, except for the common proportionality factor $-(2\pi)^4V^{-2}iG\delta^4(k_\gamma+k_\delta-k_{\gamma'}-k_{\delta'})$, as follows:

$$\sum_i\langle\delta_j{}'\gamma_i{}'\mid H_1\mid\gamma_i\delta_j\rangle\sim 6\delta_{\delta\delta'}\delta_{\gamma\gamma'}\times(\bar{U}(k_{\delta'})\Gamma U(k_\delta))(\bar{U}(k_{\gamma'})\Gamma U(k_\gamma))-2\delta_{\gamma\delta'}\delta_{\delta\gamma'}(\bar{U}(k_{\gamma'})\Gamma U(k_\delta))(\bar{U}(k_{\delta'})\Gamma U(k_\gamma)), \tag{11}$$

$$\sum\langle\delta_j{}'\gamma_i{}'\mid H_2\mid\gamma_i\delta_j\rangle\sim 2\delta_{\delta\delta'}\delta_{\gamma\gamma'}\times(\bar{U}(k_{\delta'})\Gamma U(k_\delta))(\bar{U}(k_{\gamma'})\Gamma U(k_\gamma))-6\delta_{\gamma\delta'}\delta_{\delta\gamma'}(\bar{U}(k_{\gamma'})\Gamma U(k_\delta))(\bar{U}(k_{\delta'})\Gamma U(k_\gamma)). \tag{12}$$

Therefore if we take such a combination as

$$H=H_2-\tfrac{1}{3}H_1, \tag{13}$$

the terms containing $\delta_{\delta\delta'}\delta_{\gamma\gamma'}$ drop out and we are left with only the term with $\delta_{\gamma\delta'}\delta_{\gamma'\delta}$, which leads to the process $\gamma_i\delta_j\to\gamma_j\delta_i$. In the same way, the quark-antiquark scattering $\sum_i\langle\delta_j\bar{\delta}_i{}'\mid -i\int H dt\mid\bar{\gamma}_i\gamma_j{}'\rangle$ contains only a term proportional to $\delta_{\gamma\gamma'}\delta_{\delta\delta'}$, which gives $\bar{\gamma}_i\gamma_j\to\bar{\delta}_i\delta_j$.

The Hamiltonian we obtained in Eq. (13) can be written in a form

$$H=\tfrac{1}{2}G\sum_{k=1}^{8}(\bar{t}\Gamma I\Lambda_k t)(\bar{t}\Gamma I\Lambda_k t), \tag{14}$$

where $I$ is the $3\times 3$ unit matrix operating in the space spanned by upper indices, and $\Lambda_1\cdots\Lambda_8$ are $3\times 3$ $SU(3)$ matrices for the lower indices of $t_i{}^\alpha$. If we take here the vector coupling for $\Gamma$, the Hamiltonian (14) is the same as the one proposed by Hori.[12] The potentials among quarks and antiquarks are given, from Eq. (14), by

$$V_{qq}=\sum_{k=1}^{8}\Lambda_k{}^{(1)}\cdot\Lambda_k{}^{(2)}V,\qquad V_{q\bar{q}}=-\sum_{k-1}^{8}\Lambda_k{}^{(1)}\cdot\Lambda_k{}^{(2)}V. \tag{15}$$

The negative sign in $V_{q\bar{q}}$ appears to be due to the vector coupling of the Hamiltonian. Since

$$\begin{aligned}\sum_{k=1}^{8}\Lambda_k{}^{(1)}\cdot\Lambda_k{}^{(2)}&=-8/3 &&\text{for } q-q \text{ in the } \mathbf{3^*} \text{ representation}\\ &=4/3 &&\text{for } q-q \text{ in the } \mathbf{6}\\ &=16/3 &&\text{for } q-\bar{q} \text{ in the } \mathbf{1}\\ &=-2/3 &&\text{for } q-\bar{q} \text{ in the } \mathbf{8},\end{aligned} \tag{16}$$

it is predicted by Hori that pairs of $q-q$ and $q-\bar{q}$ bind only when the former is in the $\mathbf{3^*}$ representation and the latter is in the singlet representation of the $SU(3)$ for lower suffices; namely, these are just cases given by Eqs. (2) and (1), respectively.

The Hamiltonian (14) also leads to the nonet-type meson-quark interactions, since a calculation similar to Eqs. (11) and (12) shows that

$$\langle\delta_{j'}{}'\mid H\mid\sum_i\delta_i\bar{\gamma}_i\gamma_j{}'\rangle\sim\delta_{jj'}\delta_{\delta\delta'}\delta_{\gamma\gamma'},$$

where $\sum_i\delta_i\bar{\gamma}_i$ is considered as a representative of the meson.

## IV. DISCUSSION

We have shown that the strange selection rule derived from meson decay can, indeed, be formulated consistently if we work with the paraquark or the three-triplet quark model instead of the usual quark. The rather good fit to the experiments of our $\alpha$, which is derived under the selection rule, seems to give evidence for the necessity of three degrees of extra freedom of the quark, as in the case of both models.

We are aware in these calculations that the selection

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[12] S. Hori, Progr. Theoret. Phys. (Kyoto) **36**, 131 (1966).

rule and the nonet character of the Hamiltonian are only true for the lowest-order calculation, so that we can only apply our restrictions to phenomena where the renormalization effects are supposed to be small.

The vector coupling in the Hamiltonian (14) is equivalent to

$$H_W=(iG/\sqrt{2})\bar{t}\gamma_\mu I\Lambda_k t W_{k,\mu}{}^0, \tag{17}$$

mediated by vector mesons $W_{k,\mu}{}^0$, which is in the singlet of the ordinary $SU(3)$ group and in the octet representation of the $SU(3)$ group for lower indices. Since here we have no mixing of the representations in the same $SU(3)$ group, and also we have no renormalization for the vector-meson coupling except for a common $Z_3$ factor in the limit of zero momentum transfer, the form (17) remains unchanged in this limit even if we take account of renormalization effects.

Therefore it may be reasonable to apply our restrictions to phenomena where the transferred momentum is small and the contribution from the diagram with one intermediate vector-boson exchange is dominant, namely, to phenomena such as low-energy scattering without nearby resonances, $p\bar{p}$ annihilations, and the forward or backward peak of the quasielastic scatterings. These will be discussed in forthcoming papers.

## ACKNOWLEDGMENT

The author thanks Professor Y. Takahashi for his encouragement.

## APPENDIX

In this appendix we shall show that the selection rules (a) and (b) follow from the same Hamiltonian (13) in the case of the three-triplet model,[6] too.

Let $a_i'^{\alpha}$ and $a_i'^{*\alpha}$ be an annihilation and a creation operator of a three-triplet quark model, and assume that they satisfy the Fermi commutation relation. Then the $a_i^{\alpha}$'s defined by

$$a_1'^{\alpha}=\eta_2 a_3{}^{\alpha},\qquad a_2'^{\alpha}=\eta_3 a_1{}^{\alpha},\qquad a_3'^{\alpha}=\eta_1 a_2{}^{\alpha} \tag{A1}$$

satisfy Green's[5] commutation relation of the parastatistics, where $\eta_i$ is an operator which anticommutes with $a_i$ and commutes with $a_j$ if $i\neq j$. The explicit form of $\eta_i$ is given by

$$\eta_i=(-1)N_i, \tag{A2}$$

$$N_i=\sum_k\sum_\alpha\left[a_i{}^{*\alpha}(k)a_i{}^{\alpha}(k)-b_i{}^{*\alpha}(k)b_i{}^{\alpha}(k)\right], \tag{A3}$$

and thus we have

$$(\eta_i)^2=1. \tag{A4}$$

Now, in general, the Hamiltonian in the paraquark model must be made up of products of the form

$$\sum_i \bar{t}_i{}^{\alpha}\Gamma t_i{}^{\beta}, \tag{A5}$$

in order to satisfy the causality[13] requirement. Indeed our Hamiltonian (9) is of this form, and (10) is also reducible to products of the type (A5) by the Fierz transformation, because $t_l{}^{\beta}$ commutes with $\sum_m t_m{}^{\alpha}\bar{t}_m{}^{\beta}$. Since from (A1) and (A4), we have

$$\sum_i \bar{t}_i{}^{\alpha}\Gamma t_i{}^{\beta}=\sum_i \bar{t}_i'^{\alpha}\Gamma t_i'^{\beta}, \tag{A6}$$

we see that the Hamiltonians (9) and (10) are transformed form-invariantly by the replacement $t_i{}^{\alpha}\to t_i'^{\alpha}$.

Because of the equality (A1), we have also

$$\sum_i a_i{}^{*\alpha}b_i{}^{*\beta}=\sum_i a_i'^{*\alpha}b_i'^{*\beta} \tag{A7}$$

for the meson operator, and

$$\sum_{ijk} a_i{}^{*\alpha}a_j{}^{*\beta}a_k{}^{*\gamma}f_{ijk}{}^{\alpha\beta\gamma}=\sum_{ijk} a_i'^{*\alpha}a_j'^{*\beta}a_k'^{*\gamma}\eta_1\eta_2\eta_3 g_{ijk}{}^{\alpha\beta\gamma} \tag{A8}$$

for the baryon operator, where $g_{ijk}{}^{\alpha\beta\gamma}$ has the same absolute value as $f_{ijk}$ and has a negative sign when $(ijk)$ is an odd permutation of (123). Using Eqs. (A6)–(A8) and also the relation $\eta_1\eta_2\eta_3|0\rangle=|0\rangle$, we are now ready to see that

$$\text{Eq. (8)}=\sum_{lmn}\sum_i \langle a_m'^{\eta'}a_i'^{\gamma'}|-i\textstyle\int H'dt\,|a_i'^{*\gamma}a_m'^{*\eta}\rangle g_{lmn}{}^{\xi\eta\zeta}\times(g_{lmn}{}^{\xi\eta'\zeta})^*, \tag{A9}$$

where $H'$ is the Hamiltonian obtained from (13) by replacing $t\to t'$.

The right-hand side of Eq. (A9) is nothing but the matrix element for meson-baryon scattering calculated in the three-triplet model, and this proves the equivalence of the two models.

[13] Y. Ohnuki and S. Kamefuchi, Phys. Rev. **170**, 1279 (1968).