Nonleptonic Hyperon Decays*

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Suzuka and Sugawara's theory of S-wave nonleptonic hyperon decay is reviewed to see to what extent their theory tests current algebra and the current×current form of interaction. The only argument for the current×current form in nonleptonic decays that it is all compelling is the vanishing of the S-wave amplitude $A(\Sigma_+^+)$ for the decay $\Sigma^+ \rightarrow n+\pi^+$. For the P-wave decay, to which the soft-pion formalism is inapplicable, an octet pole model is used. To fit the empirical P-wave amplitudes, strong-coupling shifts are required in both the S and the P waves. Including $SU(3)$ symmetry breaking leads to parity-violating Born terms in the S waves; to estimate the parity-violating spurion coupling, the kaon tadpole model and $K_{2\pi}$ decay rate are used. If pionic and kaonic strong-coupling constants are suppressed by $\frac{2}{3}$ and $\frac{1}{3}$, all the S- and P-wave amplitudes are given within 20%, but $A(\Sigma_+^+)$ now does not precisely vanish. If $A(\Sigma_+^+)=0$ exactly is insisted on, then the suppression of pionic strong-coupling constants by $\frac{2}{3}$ and $\frac{1}{3}$ is necessary.

I. INTRODUCTION

THE notion of the universality of the current×current form of weak interactions[1,2] has been a powerful tool in understanding leptonic and semileptonic decay processes. It is, therefore, tempting to extend this notion to nonleptonic weak interactions to encompass all weak interactions in the same dynamical theory. According to this idea, the nonleptonic weak-interaction Hamiltonian is proportional to the strangeness-changing part of the symmetric product of the Cabibbo current[2] with itself. Such a Hamiltonian contains the **27** part as well as the octet part. On the other hand, it is known experimentally that nonleptonic decays are governed by an octet selection rule, namely, the $\Delta I=\frac{1}{2}$ and the Lee-Sugawara sum rules.[3] Therefore, one must explain the octet rule in the presence of the **27** part in the interaction Hamiltonian. The first illuminating success along this line was achieved by Suzuki[4] and Sugawara[5] in their current-algebra study of S-wave hyperon decays: Using partial conservation of axial-vector current (PCAC), the algebra of currents,[6] and $SU(3)$ symmetry, the $\Delta I=\frac{1}{2}$ rule for Λ, Ξ decays, the pseudo-$(\Delta I=\frac{1}{2})$ rule for Σ decays, and the pseudo-Lee-Sugawara relation[4,5] were derived. Indeed, the best-fit values for the parameters of their theory gave rise to the notion of a universal spurion coupling.[7]

On the other hand, it has been shown[8–13] that in the soft-pion formalism, chiral invariance[8–12] and the W_1 invariance[12,13] of nonleptonic weak interactions are sufficient for the S-wave $\Delta I=\frac{1}{2}$ rule and the pseudo-Lee-Sugawara relation. Then the success of Suzuki and Sugawara's S-wave theory raises the following questions:

(1) To what extent are Suzuki and Sugawara's successes actually tests of the current×current form of the nonleptonic Hamiltonian and the current algebra?

(2) As we shall see, the P-wave amplitudes are not given by the soft-pion formalism but are model dependent. If the P-wave amplitudes are calculated in a pole model,[9–12] the following problems arise: (a) The Suzuki-Sugawara S-wave analysis makes $\alpha_W=\alpha_{MS}$, where α_W and α_{MS} are the weak and medium-strong spurion D/F ratios, respectively. For this value of α_W, the P-wave Lee-Sugawara relation is satisfied if the strong couplings are $SU(3)$ symmetric. However, the relation $\alpha_W=\alpha_{MS}$ makes the P-wave amplitude for $\Sigma^+ \rightarrow n+\pi^+$ vanish, in contradiction with experiment. (b) If one allows $\alpha_W \neq \alpha_{MS}$, the best-fit compromise solution to both S and P waves leaves the P-wave amplitudes too small by the factor of 2 or 3.[9–12] Can the symmetry breaking of the strong interaction account for the observed values of the P-wave amplitudes?

The above questions are what motivated us to the present study. In Sec. II, we discuss the application of PCAC. In particular, we are interested in the effects of mass splittings on the extrapolation of the S-wave amplitudes from the unphysical (zero pion four-momenta) point to the physical point. Section III is devoted to the discussion of the dynamical aspects of the S-wave theory, with attention focused on the role played by the current×current Hamiltonian. In Sec.

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IV, we introduce the pole model for the P-wave decays and estimate the amounts of coupling shifts required by the model. In Sec. V, we summarize our results and compare with the results of other calculations.

II. PION PRODUCTION IN NONLEPTONIC PROCESSES

A. Low-Energy Theorem

For nonleptonic hyperon decays in which an initial baryon with momentum P decays into a final baryon with momentum P' emitting a pion with momentum q and isospin j, we define the off-mass-shell decay amplitude

$$M(q;P',P)\equiv(q^2+\mu^2)\int d^4x \times \exp(-iq\cdot x)\langle P'\mid T\{\phi^j(x)\mathcal{H}_W(0)\}\mid P\rangle, \quad (2.1)$$

where our metric is such that $q^2=\mathbf{q}^2-q_0^2$. In the above expression, q and μ are the pion momentum and mass, and $\phi^j(x)$ is the off-mass-shell pion field operator defined through the relation

$$\partial_\nu A_\nu^j(x)\equiv F_\pi\mu^2\phi^j(x), \quad (2.2)$$

where $A_\nu^j(x)$ is the axial-vector current. F_π is the pion decay amplitude defined by

$$\langle 0\mid \partial_\nu A_\nu^j(0)\mid \pi^k(q)\rangle\equiv\delta^{jk}F_\pi\mu^2/(2VE_\pi)^{1/2}, \quad (2.3)$$

where V is a normalization volume and E_π is the pion energy. Expression (2.3) is written in a rectangular basis in isospin space, so that F_π has the empirical value $0.95\mu\times\sqrt{2}$. In expression (2.1), $\mathcal{H}_W(0)$ is the nonleptonic weak Hamiltonian density. The physical decay amplitude $\langle q;P'\mid\mathcal{H}_W(0)\mid P\rangle$ is related to the amplitude $M(q;P',P)$ by the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula

$$\langle q;P'\mid\mathcal{H}_W(0)\mid P\rangle =[i/(2VE_\pi)^{1/2}]M(q=P-P',q^2=-\mu^2;P',P). \quad (2.4)$$

We assume that $\mathcal{H}_W(0)$ is a local operator, so that the commutator $[A_0^j(\mathbf{x},0),\mathcal{H}_W(0)]$ is proportional to the δ function $\delta^3(\mathbf{x})$. With this assumption, expression (2.1) can be written in the following form[14]:

$$M(q;P',P)=F_\pi^{-1}iq_\nu N_\nu(q;P',P) -F_\pi^{-1}\langle P'\mid[Q_5^j(0),\mathcal{H}_W(0)]\mid P\rangle, \quad (2.5)$$

where $N_\nu(q;P',P)$ is

$$\int d^4x\exp(-iq\cdot x)\langle P'\mid T\{A_\nu^j(x)\mathcal{H}_W(0)\}\mid P\rangle$$

with its pion pole removed and $Q_5^j(0)$ is the axial charge operator defined by

$$Q_5^j(0)\equiv\int d^3x\, A_0^j(\mathbf{x},0). \quad (2.6)$$

[14] S. Weinberg, Phys. Rev. Letters **16**, 616 (1966).

The first term on the right-hand side of expression (2.5) has, by definition, no pion pole and the second term does not depend on the pion four-momentum. Thus, expression (2.5) is an exact formula valid on as well as off the pion mass shell. We call expression (2.5) the *master formula*.

The structures of the amplitudes $M(q;P',P)$ and $N_\nu(q;P',P)$ are generally unknown. In the soft-pion limit $q_\nu\to0$, however, only poles in $N_\nu(q;P',P)$ due to intermediate baryon states degenerating in mass with the initial or final baryon contribute, so that the master formula provides an exact theorem for the emission of the soft pion as

$$M(q\to0;P',P)=\text{“Born amplitude”} -F_\pi^{-1}\langle P'\mid[Q_5(0),\mathcal{H}_W(0)]\mid P\rangle. \quad (2.7)$$

This theorem is called the low-energy theorem.[14–17]

B. PCAC

In order to estimate the *physical* value of the amplitude $M(q;P',P)$, we must extrapolate $M(q;P',P)$ from $q_\nu=0$ to $q_\nu=P_\nu-P_\nu'$, $q^2=-\mu^2$. If we assume smoothness of the nonsingular part of the amplitude

$$M^{\text{nonsing}}(q=P-P',q^2=-\mu^2;P',P) \simeq M^{\text{nonsing}}(q=0;P',P), \quad (2.8)$$

then, using the low-energy theorem (2.7), we find the extrapolation formula given by Alessandrini, Bég, and Brown[16]:

$$\begin{aligned}M(q=P-P',q^2=-\mu\ ;P',P) &=\lim_{q_\nu\to0}[F_\pi^{-1}iq_\nu N_\nu(q;P',P)-M^{\text{sing}}(q;P',P)]\\ &\quad+M^{\text{sing}}(q=P-P;q^2=-\mu^2;P',P)\\ &\quad-F_\pi^{-1}\langle P'\mid[Q_5^j(0),\mathcal{H}_W(0)]\mid P\rangle, \quad (2.9)\end{aligned}$$

where $M^{\text{sing}}(q;P',P)$ is the baryon pole contribution.

The validity of the extrapolation formula depends on the smoothness assumption (2.8). The nonsingular amplitude with P^2 and P'^2 fixed can be written in the form

$$M^{\text{nonsing}}(q;P',P)=\bar{u}_N(P')[F_1^{\text{pv}}+i\boldsymbol{q}F^{\text{spv}} +(F_1^{\text{pc}}+i\boldsymbol{q}F_2^{\text{pc}})\gamma_5]u_Y(P), \quad (2.10)$$

where the F's are nonsingular in the invariant variables $s\equiv(P'+q)^2$, $u\equiv(P-q)$, and q^2, and slowly varying in q^2 because of PCAC. In the limit of degenerate baryon mass, s and u take on the same value $-M_Y^2=-M_N^2$ at both $q_\nu=0$ and $q_\nu=P_\nu-P_\nu'$, and the physical and nonphysical ($q_\nu=0$) values of the F's agree.

[15] S. L. Adler, Phys. Rev. Letters **14**, 1041 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).
[16] V. A. Alessandrini, M. A. Bég, and L. S. Brown, Phys. Rev. **144**, 1137 (1966).
[17] S. Weinberg, Phys. Rev. Letters **16**, 879 (1966).

The factor $\boldsymbol{iq}$ in the parity-violating (pv) part of the nonsingular amplitude becomes $M_N - M_Y$ at the physical point. In the $SU(3)$ limit, therefore, the pv part of the nonsingular amplitude satisfies condition (2.8), and the extrapolation formula (2.9) is applicable to the S-wave hyperon decays.

In the parity-conserving (pc) part of the nonsingular amplitude, the factor $\boldsymbol{iq}\gamma_5$ becomes $(M_N+M_Y)\gamma_5$ at the physical point. Therefore, the pc part of the nonsingular amplitude need not satisfy the smoothness condition (2.8). The extrapolation formula (2.9) is inapplicable to the P-wave hyperon decays, whether or not $SU(3)$ symmetry is assumed. In other words, the low-energy theorem gives no information on the P-wave hyperon decays, nor on the $SU(3)$ symmetry-breaking part of the S-wave amplitudes.

The pv part of the baryon pole contributions in the extrapolation formula (2.9) vanishes only in the limit of degenerate mass. The $\Delta M/M$ corrections are again model dependent.

III. S-WAVE HYPERON DECAYS IN SU(3) LIMIT

If $SU(3)$ symmetry is assumed, then for S-wave decays[4,5]

$$M(q=P-P', q^2=-\mu^2; P', P) = -F_\pi^{-1}\langle P' \,|\, [Q_5^j(0), \mathcal{H}_W^{pv}(0)] \,|\, P\rangle, \quad (3.1)$$

where $\mathcal{H}_W^{pv}$ is the pv part of $\mathcal{H}_W$. Aside from the locality of $\mathcal{H}_W$ and PCAC, we have not made any dynamical assumption. In the present section, we discuss the consequences of additional dynamical assumptions on the nonleptonic weak Hamiltonian going beyond locality and PCAC.

A. Chiral Invariance

It has been shown[8–12] that in the soft-pion formalism the chiral invariance of $\mathcal{H}_W$ is sufficient for the S-wave $\Delta I=\frac{1}{2}$ and psueodo-($\Delta I=\frac{1}{2}$) rules.[4,5] We remark that chiral invariance can be a common property of all weak interactions and $\mathcal{H}_W$ may contain members of any multiplet of $SU(3)$.

B. W_1 Invariance

We have so far used $SU(3)$ symmetry only to eliminate the model-dependent parts in the extrapolation of the amplitudes. In order to obtain the Lee-Sugawara relation among different hyperon decays, we must use further elements of $SU(3)$ symmetry. The charge-conserving character of $\mathcal{H}_W$ makes the U-spin formalism appropriate for this purpose.

Empirically, it is known that $\mathcal{H}_W$ changes hypercharge by one unit while preserving the electric charge. Therefore, the most general form of $\mathcal{H}_W$ must be

$$\mathcal{H}_W = \sum_u (U_{+1}{}^u + U_{-1}{}^u), \quad (3.2)$$

where $U_{+1}{}^u$ and $U_{-1}{}^u$ are the $U_z=1$ and $U_z=-1$ members of a U-spin $(2u+1)$-plet, respectively. In the above expression, the phase convention for the U-spin multiplets has been so chosen that

$$(U_{\pm 1}{}^u)^\dagger = U_{\mp 1}{}^u. \quad (3.3)$$

In the soft-pion formalism, the S-wave amplitudes are written in terms of the matrix elements of $\mathcal{H}_W^{pc}$ (parity-conserving part of $\mathcal{H}_W$) between single octet baryon states. Since the U spin of octet baryons is not greater than one, only $\mathcal{H}^{(1)} \equiv U_{+1}{}^1 + U_{-1}{}^1$ and $\mathcal{H}^{(2)} \equiv U_{+1}{}^2 + U_{-1}{}^2$ contribute to these matrix elements and, hence, to the S-wave amplitudes. To be more explicit, all the S-wave amplitudes are written in terms of the four reduced matrix elements $\langle 1 \| U^1 \| 1\rangle$, $\langle 1 \| U^1 \| 0\rangle$, $\langle \frac{1}{2} \| U^1 \| \frac{1}{2}\rangle$, and $\langle 1 \| U^2 \| 1\rangle$. Since only four of the seven amplitudes are independent because of the $\Delta I=\frac{1}{2}$ rule, we must make an assumption on these matrix elements in order to obtain more sum rules than the $\Delta I=\frac{1}{2}$ rule. Since only $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$ contribute to the S-wave amplitudes, a simple and natural assumption is to claim that $\mathcal{H}_W$ is simply either pure $\mathcal{H}^{(1)}$ or $\mathcal{H}^{(2)}$. If $\mathcal{H}_W = \mathcal{H}^{(2)}$, the wrong prediction $A(\Sigma_0{}^+)=0$ follows immediately from $\langle \frac{1}{2} \| U^2 \| \frac{1}{2}\rangle = 0$, where A denotes an S-wave invariant amplitude. Therefore, the empirical fact $A(\Sigma_0{}^+) \neq 0$ forces $\mathcal{H}^{(2)}$ to be absent in $\mathcal{H}_W$, if $\mathcal{H}_W$ is to be simply either $\mathcal{H}^{(1)}$ or $\mathcal{H}^{(2)}$. Thus, the "$\Delta U=1$ rule" emerges from the empirical data as a natural assumption.

The $\Delta U=1$ rule can be phrased as "W_1 invariance"[13] or U-spin charge symmetry[12] of $\mathcal{H}_W$, i.e., as the condition

$$W_1 \mathcal{H}_W W_1^{-1} = \mathcal{H}_W, \quad (3.4)$$

where W_1 is the 180° rotation through the first axis in U-spin space, or the Weyl reflection in $SU(3)$ space which reflects the weight diagram through the Q axis. Here, the direction of the first axis in U-spin space is chosen so as to coincide with the sixth axis in $SU(3)$ space. In the phase convention (3.3), $U_{u_z}{}^u$ transforms under W_1 as

$$W_1 U_{u_z}{}^u W_1^{-1} = (-1)^{u+u_z} U_{-u_z}{}^u. \quad (3.5)$$

Therefore, $\mathcal{H}^{(1)}$ is symmetric under W_1 and $\mathcal{H}^{(2)}$ is antisymmetric under the same operation. Although the W_1 invariance is a more general assumption than the $\Delta U=1$ rule, they are equivalent in the case of the S-wave hyperon decays, since only $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$ contribute to the S-wave amplitudes.

In the soft-pion formalism W_1 invariance is sufficient[12,13] for the pseudo-Lee-Sugawara relation.[4,5] Therefore, this sum rule is compatible with the presence of any multiplet of $SU(3)$ in $\mathcal{H}_W$.

C. Current$\times$Current Hamiltonian

We have so far assumed the chiral and W_1 invariances for $\mathcal{H}_W$ to obtain the $\Delta I=\frac{1}{2}$ rules and the pseudo-Lee-Sugawara relation. Because of these sum rules, only three of the seven amplitudes are now independent. These three amplitudes are determined, say, by the three matrix elements $\langle p \mid \mathcal{H}_W{}^{pc} \mid \Sigma^+\rangle$, $\langle n \mid \mathcal{H}_W{}^{pc} \mid \Sigma^0\rangle$, and $\langle n \mid \mathcal{H}_W{}^{pc} \mid \Lambda\rangle$. We will now consider the role played in the soft-pion formalism of S-wave hyperon decays by the current$\times$current Hamiltonian, which we have not yet assumed.

If the nonleptonic weak Hamiltonian is the strangeness-changing part of the symmetric product of the Cabibbo current[2] with itself, then it is W_1 invariant,[12,13] provided[18] that the polar Cabibbo angle θ_V is equal to the axial Cabibbo angle θ_A. If the current algebra or the charge-current algebra proposed by Gell-Mann[6] is assumed, then the current$\times$current Hamiltonian is also chirally invariant,[8–12] provided that $\theta_V=\theta_A$. In fact, in the $V-A$ theory of current$\times$current interactions, chiral invariance is equivalent to Gell-Mann's current algebra.

Besides chiral and W_1 invariances, the current$\times$current Hamiltonian has the $SU(3)$ property of transforming only as $\mathbf{8}_s\oplus\mathbf{27}$. If we do not assume the current$\times$current form for $\mathcal{H}_W$, then not only $\mathbf{8}_s$ and $\mathbf{27}$, but also $\mathbf{8}_a$, $\mathbf{10}$, and $\mathbf{10}^*$ parts of $\mathcal{H}_W$ can contribute to matrix elements between octet baryon states. Nevertheless, the S-wave amplitudes are completely determined by only three parameters $\langle p \mid \mathcal{H}_W{}^{pc} \mid \Sigma^+\rangle$, $\langle n \mid \mathcal{H}_W{}^{pc} \mid \Sigma^0\rangle$, and $\langle n \mid \mathcal{H}_W{}^{pc} \mid \Lambda\rangle$ when chiral and W_1 invariances are assumed. For this reason, the additional assumption of the current$\times$current form of interaction leads to no additional sum rule. However, it does have implications for the vanishing of $A(\Sigma_+{}^+)$.

D. $A(\Sigma_+{}^+)=0$ and Octet Dominance

Empirically, it is known that the value of $A(\Sigma_+{}^+)$ is finite but almost zero. In the soft-pion formalism, $A(\Sigma_+{}^+)$ is proportional to $\langle n \mid [Q^-, \mathcal{H}_W{}^{pc}] \mid \Sigma^+\rangle$, where Q^- is an isocharge operator. If we do not assume the current$\times$current form $\mathcal{H}_W$, then $\mathcal{H}_W$ can contain members of $\mathbf{8}_a$, $\mathbf{8}_s$, $\mathbf{10}$, $\mathbf{10}^*$, $\mathbf{27}$, and other representations. Since the isocharge operator Q^- does not mix members of different representations, and the members of $\mathbf{10}$ and $\mathbf{27}$ (but not of $\mathbf{8}_a$, $\mathbf{8}_s$, $\mathbf{10}^*$, and others) can contribute to the transition $\Sigma^+\to n$, the vanishing of $A(\Sigma_+{}^+)$ is not necessarily a result of the octet dominance but can be a result of the cancellation between the $\Delta I=\frac{3}{2}$ parts of the $\mathbf{10}$ and $\mathbf{27}$ of $\mathcal{H}_W$.

Within the current$\times$current formalism, $A(\Sigma_+{}^+)=0$ is equivalent to octet dominance. Therefore, in the current$\times$current picture, the empirical data $A(\Sigma_+{}^+)=0$ is a manifestation of the suppression of $\mathbf{27}$. This suppression of the $\mathbf{27}$ is apparently obtained when the current$\times$ current form is saturated by octet and decuplet baryon states.[19] Without the current$\times$current form for $\mathcal{H}_W$, the vanishing of $A(\Sigma_+{}^+)$ requires a peculiar cancellation between $\mathbf{10}$ and $\mathbf{27}$.

To summarize the present section:

(1) The assumption of the current$\times$current form with $\theta_V=\theta_A$ for the nonleptonic Hamiltonian is sufficient for the chiral and W_1 invariances, but not necessary, since these invariances are compatible with different theoretical structure for weak interactions. Although the assumption of the current$\times$current form (with $\theta_V=\theta_A$) is more restrictive in general than the assumption of the chiral and W_1 invariances, both assumptions lead to the same sum rules in S-wave hyperon decay.

(2) Generally, the octet dominance is sufficient but not necessary for $A(\Sigma_+{}^+)=0$. If the current$\times$current form is assumed, then $\mathbf{27}$ suppression is the necessary and sufficient condition for $A(\Sigma_+{}^+)=0$. In this sense, the current$\times$current Hamiltonian explains what would otherwise be a remarkable cancellation between $\mathbf{10}$ and $\mathbf{27}$.

IV. P-WAVE HYPERON DECAYS

In Sec. II, we have seen that the low-energy theorem gives no information on the P-wave hyperon decays. Therefore, we must use a model in order to calculate the P-wave amplitudes. In the present section, we use an octet pole model for the P-wave decays.

A. Octet Pole Model

The pole model which we consider consists of the s-, t-, and u-channel single-particle pole terms.[20] For the intermediate states, we consider only octet baryon and meson states.[12,20]

As for meson-baryon coupling, we use gradient (pseudovector) coupling, which is the only simple coupling consistent with PCAC. While all couplings are, of course, equivalent when all three particles at the vertex are on the mass shell, in the second-order diagrams, one of the particles is off the mass shell. The direct coupling theory is therefore not equivalent to the gradient coupling theory.

In the gradient coupling theory, the fundamental coupling constants are

$$f_{B'B}\equiv g_{B'B}/(M_{B'}+M_B), \tag{4.1}$$

where $g_{NN}{}^2/4\pi=14.6$.

[18] This was kindly pointed out by Professor S. P. Rosen.

[19] Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 710 (1967); Y. T. Chiu and J. Schechter, Phys. Rev. Letters **16**, 1022 (1966); S. Biswas, A. Kumar, and R. Saxena, *ibid.* **17**, 268 (1966); T. Y. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966); S. Nussinov and G. Preparata, *ibid.* **175**, 2180 (1968).

[20] G. Feldman, P. T. Matthews, and A. Salam, Phys. **121**, 302 (1961).

We parametrize the baryon-spurion and meson-spurion couplings as

$$\langle B'(P')\,|\,\mathcal{H}_W^{pc}(0)\,|\,B(P)\rangle = (1/V)\bar{u}_{B'}(P')s_{B'B}^{pc}u_B(P), \qquad (4.2a)$$

$$\langle M'(P')\,|\,\mathcal{H}_W(0)\,|\,M(P)\rangle = [1/(2VP_0')^{1/2}(2VP_0)^{1/2}]s_{M'M}^{pc}, \qquad (4.2b)$$

and the decay amplitude as

$$\langle N(P'),\pi(q)\,|\,\mathcal{H}_W(0)\,|\,Y(P)\rangle = [i/V(2VE_\pi)^{1/2}]\bar{u}_N(P')(A+\gamma_5 B)u_Y(P), \qquad (4.2c)$$

where

$$\gamma_5=\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Then we obtain for the P-wave amplitudes

$$\begin{aligned}
B(\Lambda_-^0) &= (M_\Lambda+M_N)\big(f_{P\Lambda K^-}(\mu_K^2-\mu_\pi^2)^{-1}s_{\Pi^-K^-}^{pc} \\
&\quad + f_{pn\pi^-}(M_N-M_\Lambda)^{-1}s_{N\Lambda}^{pc}+s_{p\Sigma^+}^{pc}(M_\Sigma-M_N)^{-1}f_{\Sigma^+\Lambda\Pi^-}\big),\\
B(\Sigma_-^-) &= (M_\Sigma+M_N)\big(f_{N\Sigma^-K^-}(\mu_K^2-\mu_\pi^2)^{-1}s_{\Pi^-K^-}^{pc} \\
&\quad + s_{n\Lambda}^{pc}(M_\Lambda-M_N)^{-1}f_{\Lambda\Sigma^-\Pi^-}+s_{N\Sigma^0}^{pc}(M_\Sigma-M_N)^{-1}f_{\Sigma^0\Sigma^-\Pi^-}\big),
\end{aligned} \qquad (4.3)$$

$$\begin{aligned}
B(\Sigma_+^+) &= (M_\Sigma+M_N)\big(f_{NP\Pi^+}(M_N-M_\Sigma)^{-1}s_{P\Sigma^+}^{pc} \\
&\quad + s_{n\Lambda}^{pc}(M_\Lambda-M_N)^{-1}f_{\Lambda\Sigma^+\Pi^+}+s_{N\Sigma^0}^{pc}(M_\Sigma-M_N)^{-1}f_{\Sigma^0\Sigma^+\Pi^+}\big),\\
B(\Xi_-^-) &= (M_\Xi+M_\Lambda)\big(f_{\Lambda\Xi^-K^-}(\mu_K^2-\mu_\pi^2)^{-1}s_{\Pi^-K^-}^{pc} \\
&\quad + f_{\Lambda\Sigma^-\pi^-}(M_\Sigma-M_\Xi)^{-1}s_{\Sigma^-\Xi^-}^{pc}+s_{\Lambda\Xi^0}^{pc}(M_\Xi-M_\Lambda)^{-1}f_{\Xi^0\Xi^-\Pi^-}\big).
\end{aligned}$$

Assuming $SU(3)$ symmetry and octet dominance for spurion couplings, we parametrize

$$s_{B'B}^{pc}=2F_{B'B}^{6}F+2D_{B'B}^{6}D, \qquad (4.4a)$$

$$s_{M'M}^{pc}=2D_{M'M}^{6}D', \qquad (4.4b)$$

where $F_{\alpha\beta}^{j}\equiv -if_{j\alpha\beta}$ and $D_{\alpha\beta}^{j}\equiv d_{j\alpha\beta}$. Using the notations $\alpha_W\equiv D/F$, $\alpha_{MS}\equiv -\frac{3}{2}(M_\Sigma-M_\Lambda)/(M_\Xi-M_\Lambda)=-0.307$, $F_{MS}\equiv\frac{1}{2}(M_\Xi-M_N)=190$ MeV, and $D_{MS}'\equiv\mu_K^2-\mu_\pi^2=2.25\times10^5$ MeV^2, and the Condon-Shortley phase conventions for the I- and V-spin multiplets, we write the expressions for the P-wave amplitudes[21] in the following way:

$$B(\Lambda_-^0)=(M_\Lambda+M_N)\frac{F}{F_{MS}}\left(f_{P\Lambda K^-}\frac{D'}{D_{MS}'}\frac{F_{MS}}{F} - \frac{\sqrt{3}}{\sqrt{2}}f_{pn\pi^-}\frac{1+\frac{1}{3}\alpha_W}{1+\frac{1}{3}\alpha_{MS}} - f_{\Sigma^+\Lambda\Pi^-}\frac{1-\alpha_W}{1-\alpha_{MS}}\right),$$

$$B(\Sigma_-^-)=(M_\Sigma+M_N)\frac{F}{F_{MS}}\left(f_{N\Sigma^-K^-}\frac{D'}{D_{MS}'}\frac{F_{MS}}{F} + \frac{\sqrt{3}}{\sqrt{2}}f_{\Lambda\Sigma^-\pi^-}\frac{1+\frac{1}{3}\alpha_W}{1+\frac{1}{3}\alpha_{MS}} - \frac{1}{\sqrt{2}}f_{\Sigma^0\Sigma^-\Pi^-}\frac{1-\alpha_W}{1-\alpha_{MS}}\right), \qquad (4.5)$$

$$B(\Sigma_+^+)=(M_\Sigma+M_N)\frac{F}{F_{MS}}\left(f_{NP\Pi^-}\frac{1-\alpha_W}{1-\alpha_{MS}} + \frac{\sqrt{3}}{\sqrt{2}}f_{\Lambda\Sigma^+\pi^-}\frac{1+\frac{1}{3}\alpha_W}{1+\frac{1}{3}\alpha_{MS}} - \frac{1}{\sqrt{2}}f_{\Sigma^0\Sigma^+\Pi^-}\frac{1-\alpha_W}{1-\alpha_{MS}}\right),$$

$$B(\Xi_-^-)=(M_\Xi+M_\Lambda)\frac{F}{F_{MS}}\left(f_{\Lambda\Xi^-K^-}\frac{D'}{D_{MS}'}\frac{F_{MS}}{F} - f_{\Lambda\Sigma^-\pi^-}\frac{1+\alpha_W}{1+\alpha_{MS}} - \frac{\sqrt{3}}{\sqrt{2}}f_{\Xi^0\Xi^-\Pi^-}\frac{1-\frac{1}{3}\alpha_W}{1-\frac{1}{3}\alpha_{MS}}\right).$$

The $SU(3)$-symmetric values of $f_{B'BM}$ can be written in terms of the parameters f and d as

$$f_{B'BM}=(r/F_\pi)(2F_{B'B}^{M}f+2D_{B'B}^{M}d), \qquad (4.6)$$

where

$$r\equiv(g_{NN\pi}/2M_N)F_\pi\simeq1.3 \qquad (4.7)$$

and

$$f+d=1. \qquad (4.8)$$

The simple relation

$$2B(\Xi_-^-)[2M_N/(M_\Xi+M_\Lambda)]-B(\Lambda_-^0)[2M_N/(M_\Lambda+M_N)] = \sqrt{3}\,B(\Sigma_0^+)[2M_N/(M_\Sigma+M_N)] \qquad (4.9)$$

is satisfied among amplitudes (4.5), if the strong meson-baryon coupling is $SU(3)$ symmetric and $\alpha_W=\alpha_{MS}$. Now relation (4.9) is experimentally satisfied equally as well as the Lee-Sugawara relation for the P-wave amplitudes. Therefore, if $SU(3)$ symmetry is assumed, the empirical relation (4.9) would force us to choose $\alpha_W=\alpha_{MS}$, which in the pole model leads to $B(\Sigma_+^+)=0$. Since the observed value of $B(\Sigma_+^+)$ is not zero, $SU(3)$ symmetry is incompatible with the octet pole model.

B. Universal pc Spurion

It has been observed[9–12] that in the Suzuki-Sugawara theory[4,5] the best fit of the pc baryonic $\alpha_W=D/F$ to the S-wave amplitudes is remarkably close to the value of the baryonic mass splitting $\alpha_{MS}=D_{MS}/F_{MS}$. We call the hypothesis $\alpha_W=\alpha_{MS}$ that of *common baryon* pc *spurion*. From this remarkable equality in the S waves, it has been further conjectured[7] that the entire pc non-

[21] A. Kumar and J. C. Pati, Phys. Rev. Letters **18**, 1230 (1967). Our amplitudes agree with Eqs. (14)–(20) of Kumar and Pati. However, their Eq. (21) and their tabulated results use a value of F_π which is $\sqrt{2}$ too large so that they overestimate all the P-wave amplitudes by $\sqrt{2}$. This slip is acknowledged (private communication by J. Pati to S.A.B.).

leptonic weak Hamiltonian and the medium-strong Hamiltonian belong to the same octet. We will call this hypothesis that of the *universal* pc *spurion*. The universal pc spurion is a stronger assumption than that of common baryon pc spurion called for by the Suzuki-Sugawara S-wave fit. Hara and Nambu[7] found that the universal spurion theory is also consistent with $K_{2\pi}$ decays within 10–20%.

Let us consider, however, further implications of the universal spurion theory. The Coleman-Glashow theorem[22] states that in the universal spurion theory there would be no pc nonleptonic $K_{3\pi}$ and P-wave hyperon decays if the strong interaction is $SU(3)$ symmetric. The theorem requires that the pc nonleptonic decays take place only through the symmetry breaking of the strong interaction.

Motivated by the results of the calculations of the $K_{2\pi}$ and S-wave hyperon-decay amplitudes, we will assume, in the present subsection, the universal pc spurion for the P-wave hyperon decays, allowing, however, symmetry breaking in the strong-coupling constants.

Let us estimate how large the symmetry-breaking factor $\xi_{B'BM}$ must be such that $f_{B'BM}\equiv\xi_{B'BM}\times[SU(3)$ value of $f_{B'BM}]$. In calculating the $SU(3)$ values of $f_{B'BM}$, we use the Brene ratio[23] $d/(f+d)=\frac{2}{3}$. Then, all the P-wave amplitudes are written in terms of the parameters F and $\xi_{B'BM}$. We use the best-fit value[12] $F/F_\pi=-1.016\times10^5$ sec$^{-1/2}(\mu c^2/\hbar)^{-1/2}$ obtained in the Suzuki-Sugawara S-wave theory and try $\frac{1}{3}<\xi<1$. With this constraint $B(\Sigma_+{}^+)$ can be as large as 14.0, when $\xi_{\Sigma\Sigma\pi}$ and $\xi_{\Lambda\Sigma\pi}$ are as small as $\frac{1}{3}$. The observed value of $B(\Sigma_+{}^+)$ is 19.1, so that the calculated value is reasonably close to the observed value. Indeed, the observed values for other amplitudes are obtained exactly when $\xi_{\Sigma\Sigma\pi}=\frac{1}{3}$, $\xi_{\Lambda\Sigma\pi}=\frac{1}{3}$, $\xi_{\Xi\Xi\pi}=1$, $\xi_{N\Sigma K}=\frac{1}{3}$, $\xi_{N\Lambda K}\simeq\frac{4}{5}$, and $\xi_{\Lambda\Xi K}\simeq\frac{1}{3}$.

The above estimate shows that the P-wave pole model will require considerable suppression of the meson coupling to the heavier-mass channels. Since little is known about the hyperon-meson coupling constants experimentally, the question of whether such large coupling shifts are reasonable must be answered by theoretical or semitheoretical calculations. *A priori* one would expect coupling shifts comparable to the observed mass shifts. In fact, bootstrap calculations[24] do suggest a considerable reduction of heavier-mass meson-hyperon coupling constants. On the other hand, Kim[25] used K-p scattering data in a dispersion-theoretical sum rule[26] and found baryon-kaon coupling constants consistent with $SU(3)$ symmetry.

[22] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).
[23] N. Brene *et al.*, Phys. Rev. **149**, 1288 (1966).
[24] R. H. Dashen, Y. Dothan, S. C. Frautschi, and D. H. Sharp, Phys. Rev. **143**, 1185 (1966); B. Diu, H. R. Rubinstein, and R. P. Van Royen, Nuoto Cimento **43**, 961 (1966).
[25] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967).
[26] P. T. Matthews and A. Salam, Phys. Rev. **110**, 565 (1958); **110**, 569 (1958).

C. Renouncing Universal pc Spurion

In Sec. IV B, we have seen that a large suppression of the hyperon-meson coupling constants is required in order for the octet pole model to be compatible with the universal pc spurion. If, in the light of the success of approximate symmetry in other calculations, such large coupling shifts turn out to be unreasonable, then the universal pc spurion must be abandoned. If the universal pc spurion is abandoned, then the Suzuki-Sugawara S-wave theory must be modified by including correction terms due to broken symmetry. These corrections are expected in any case, but as remarked in Sec. II, such corrections are model dependent. In introducing a model, we assume that the Born amplitude $B(q;P',P)$ with only octet baryon intermediate states is the only strongly varying part of the S-wave amplitudes. Then, the S-wave amplitudes are given by formula (2.9) with $M^{\rm sing}$ replaced by B. For the strong baryon-meson coupling, we use gradient (pseudovector) coupling in accord with our P-wave model.

Then, the expressions for S-wave amplitudes[21] are

$$\begin{aligned}
A(\Lambda_-{}^0)=&-(1/F_\pi)[\sqrt{3}\,F+(1/\sqrt{3})D]\\
&-(M_\Lambda-M_N)(f_{pn\pi^-}(M_N+M_\Lambda)^{-1}s_{n\Lambda}{}^{\rm pv}\\
&\qquad-s_{p\Sigma^+}{}^{\rm pv}(M_\Sigma+M_N)^{-1}f_{\Sigma^+\Lambda\pi^-}),\\
A(\Sigma_-{}^-)=&-(1/F_\pi)\sqrt{2}\,(F-D)\\
&-(M_\Sigma-M_N)(-s_{n\Lambda}{}^{\rm pv}(M_\Lambda+M_N)^{-1}f_{\Lambda\Sigma^-\pi^-}\\
&\qquad-s_{n\Sigma^0}{}^{\rm pv}(M_\Sigma+M_N)^{-1}f_{\Sigma^0\Sigma^-\pi^-}),\qquad(4.10)\\
A(\Sigma_+{}^+)=&-(M_\Sigma-M_N)\,(f_{np\pi^+}(M_N+M_\Sigma)^{-1}s_{p\Sigma^+}{}^{\rm pv}\\
&\qquad-s_{n\Lambda}{}^{\rm pv}(M_\Lambda+M_N)^{-1}f_{\Lambda\Sigma^+\pi^+}\\
&\qquad-s_{n\Sigma^0}{}^{\rm pv}(M_\Sigma+M_N)^{-1}f_{\Sigma^0\Sigma^+\pi^+}),\\
A(\Xi_-{}^-)=&-(1/F_\pi)[\sqrt{3}\,F-(1/\sqrt{3})D]\\
&-(M_\Xi-M_\Lambda)(f_{\Lambda\Sigma^-\pi^-}(M_\Sigma+M_\Xi)^{-1}s_{\Sigma^-\Xi^-}{}^{\rm pv}\\
&\qquad-s_{\Lambda\Xi^0}{}^{\rm pv}(M_\Xi+M_\Lambda)^{-1}f_{\Xi^0\Xi^-\pi^-}),
\end{aligned}$$

where $s^{\rm pv}$ is the parity-violating counterpart of $s^{\rm pc}$.

If $SU(3)$ symmetry is assumed for the strong coupling, $\mathcal{H}_W{}^{\rm pv}$ transforms like the sixth or seventh component of an octet, and $\alpha_W=\alpha_{\rm MS}$, then

$$\begin{aligned}
2A(\Xi_-{}^-)[2M_N/(M_\Xi-M_\Lambda)]&\\
-A(\Lambda_-{}^0)[2M_N/(M_\Lambda-M_N)]&\\
=\sqrt{3}\,A(\Sigma_0{}^+)[2M_N/(M_\Sigma-M_N)]&\qquad(4.11)
\end{aligned}$$

is satisfied among the S-wave amplitudes (4.10). The above relation is experimentally satisfied as well as the Lee-Sugawara relation. Once $\alpha_W=\alpha_{\rm MS}$ is given up, the empirical relation (4.11), like (4.9), requires breaking of the symmetry of the strong baryon-meson coupling.

D. pv Spurion Coupling Estimated from Kaon Tadpole Model

We will consider two estimates for $s_{B'B}{}^{\text{pv}}$ in the kaon tadpole modil[21,27]:

$$s_{B'B}{}^{\text{pv}} = i f_{B'BK_1{}^0}(M_{B'}+M_B)A(K_1{}^0\to 0)/\mu_K{}^2, \quad (4.12)$$

where

$$A(K_1{}^0\to 0) \equiv (2E_K V)^{1/2}\langle 0|\mathcal{H}_W{}^{\text{pv}}|K_1{}^0\rangle. \quad (4.13)$$

(i) Using PCAC and the chiral invariance of $\mathcal{H}_W$,

$$A(K_1{}^0\to 0) = iF_\pi D' = -F_\pi{}^2 A(K_1{}^0\to 2\pi^0). \quad (4.14)$$

Numericilly,

$$\left|\frac{D'}{F_\pi}\frac{F_{\text{MS}}}{D_{\text{MS}}'}\right| = 1.0\times 10^5\ \text{sec}^{-1/2}\left(\frac{\mu c^2}{\hbar}\right)^{-1/2}, \quad (4.15)$$

where we have used $K_{2\pi}$ data $|A(K_1{}^0\to 2\pi^0)| = 2.7\times 10^4$ MeV to evaluate D'. The sign of D' is, of course, not determined by the $K_{2\pi}$ decay rate. If the universal pc spurion were used, the relative sign of D' to F would be fixed. Since the universal pc spurion must be discarded, we now regard the sign of D' as adjustable.

(ii) As a second estimate, we take $A(K_1{}^0\to 0)=0$, which would be the case in the $SU(3)$ limit of a CP-invariant current$\times$current form for $\mathcal{H}_W$. The S-wave amplitudes then reduce to those of Suzuki and Sugawara.

E. Effect of pv Spurion on *P*- and *S*-Wave Amplitudes

We have pointed out that the P-wave amplitudes are model dependent. If nevertheless, the soft-pion formalism were applied to P-wave amplitudes, matrix elements of equal-time commutators between single-baryon states would be obtained.[9–12,21] Such matrix elements are proportional to $s_{B'B}{}^{\text{pv}}$. When these pv spurion coupling constants are evaluated using Eqs. (4.12) and (4.14), such equal-time commutator terms would exactly replicate the kaon pole terms considered by Bludman.[12]

Neglecting $\mu_\pi{}^2$ relative to $\mu_K{}^2-\mu_\pi{}^2 = D_{\text{MS}}'$ and substituting Eqs. (4.12) and (4.14) into Eqs. (4.10), we have

$$\begin{aligned}
A(\Lambda_-{}^0) &= -(1/F_\pi)[\sqrt{3}\,F+(1/\sqrt{3})D] \\
&\quad + F_\pi D'(F_{\text{MS}}/D_{\text{MS}}')(1+\tfrac{1}{3}\alpha_{\text{MS}}) \\
&\quad \times(f_{pn\pi^-}f_{n\Lambda K_1{}^0} - f_{p\Sigma^+K_1{}^0}f_{\Sigma^+\Lambda\pi^-}), \\
A(\Sigma_-{}^-) &= -(1/F_\pi)\sqrt{2}(F-D) \\
&\quad + F_\pi D'(F_{\text{MS}}/D_{\text{MS}}')(1-\alpha_{\text{MS}}) \\
&\quad \times(-f_{n\Lambda K_1{}^0}f_{\Lambda\Sigma^-\pi^-} - f_{n\Sigma^0K_1{}^0}f_{\Sigma^0\Sigma^-\pi^-}), \qquad (4.16) \\
A(\Sigma_+{}^+) &= F_\pi D'(F_{\text{MS}}/D_{\text{MS}}')(1-\alpha_{\text{MS}}) \\
&\quad \times(f_{np\pi^+}f_{p\Sigma^+K_1{}^0} - f_{n\Lambda K_1{}^0}f_{\Lambda\Sigma^+\pi^+} - f_{n\Sigma^0K_10}f_{\Sigma^0\Sigma^+\pi^+}), \\
A(\Xi_-{}^-) &= -(1/F_\pi)[\sqrt{3}\,F-(1/\sqrt{3})D] \\
&\quad + F_\pi D'(F_{\text{MS}}/D_{\text{MS}}')(1-\tfrac{1}{3}\alpha_{\text{MS}}) \\
&\quad \times(f_{\Lambda\Sigma^-\pi^-}f_{\Sigma^-\Xi^-K_1{}^0} - f_{\Lambda\Xi^0K_1{}^0}f_{\Xi^0\Xi^-\pi^-}).
\end{aligned}$$

[27] A. K. Mohanti, Nuovo Cimento **52A**, 1 (1968).

F. Suppression of Strong Meson-Baryon Coupling

Coupling shifts are expected, in general, to be comparable to mass shifts. Since among octet baryons and pseudoscalar mesons the largest mass shift is that of kaons, we will allow rather larger coupling shifts for kaonic coupling constants than for pionic coupling constants. For simplicity, we assume one suppression factor ξ_K for all kaon couplings and one suppression factor ξ_π for all pion couplings. Then from Eqs. (4.16), (4.15), and (4.6) our final formulas, including a kaon-tadpole estimate of $s_{B'B}{}^{\text{pv}}$ and ξ_K, ξ_π strong coupling shifts, are

$$\begin{aligned}
A(\Lambda_-{}^0) &= -(1/F_\pi)[\sqrt{3}\,F+(1/\sqrt{3})D] \\
&\quad + (D'/F_\pi)(F_{\text{MS}}/D_{\text{MS}}')(1+\tfrac{1}{3}\alpha_{\text{MS}})r^2\xi_K \\
&\quad \times[\sqrt{3}\,f+(1/\sqrt{3})d - \xi_\pi(f-d)(2/\sqrt{3})d], \\
A(\Sigma_-{}^-) &= -(1/F_\pi)\sqrt{2}(F-D) \\
&\quad + (D'/F_\pi)(F_{\text{MS}}/D_{\text{MS}}')(1-\alpha_{\text{MS}})r^2\xi_K\xi_\pi \\
&\quad \times\sqrt{2}\,(f^2-2fd-\tfrac{1}{3}d^2), \\
A(\Sigma_+{}^+) &= (D'/F_\pi)(F_{\text{MS}}/D_{\text{MS}}')(1-\alpha_{\text{MS}})r^2\xi_K \\
&\quad \times\sqrt{2}\,[f-d-\xi_\pi(f^2-\tfrac{1}{3}d^2)], \\
A(\Xi_-{}^-) &= -(1/F_\pi)[\sqrt{3}\,F-(1/\sqrt{3})D] \\
&\quad + (D'/F_\pi)(F_{\text{MS}}/D_{\text{MS}}')(1-\tfrac{1}{3}\alpha_{\text{MS}})r^2\xi_K\xi_\pi \\
&\quad \times[\sqrt{3}\,f^2-(2/\sqrt{3})fd+\sqrt{3}\,d^2]; \qquad (4.17)
\end{aligned}$$

$$\begin{aligned}
B(\Lambda_-{}^0) &= \frac{M_\Lambda+M_N}{2M_N}\frac{2M_N}{F_{\text{MS}}}\,r\left[\xi_K\left(\sqrt{3}\,f+\frac{1}{\sqrt{3}}\,d\right)\frac{D'}{F_\pi}\frac{F_{\text{MS}}}{D_{\text{MS}}'}\right. \\
&\quad \left. -\sqrt{3}\,\frac{1}{F_\pi}\frac{F+\frac{1}{3}D}{1+\frac{1}{3}\alpha_{\text{MS}}} + \frac{2}{\sqrt{3}}\,\xi_\pi d\,\frac{1}{F_\pi}\frac{F-D}{1-\alpha_{\text{MS}}}\right], \\
B(\Sigma_-{}^-) &= \frac{M_\Sigma+M_N}{2M_N}\frac{2M_N}{F_{\text{MS}}}\,r\left[\sqrt{2}\,\xi_K(f-d)\,\frac{D'}{F_\pi}\frac{F_{\text{MS}}}{D_{\text{MS}}'}\right. \\
&\quad \left. +\sqrt{2}\,\xi_\pi d\,\frac{1}{F_\pi}\frac{F+\frac{1}{3}D}{1+\frac{1}{3}\alpha_{\text{MS}}} - \sqrt{2}\,\xi_\pi f\,\frac{1}{F_\pi}\frac{F-D}{1-\alpha_{\text{MS}}}\right], \qquad (4.18) \\
B(\Sigma_+{}^+) &= \frac{M_\Sigma+M_N}{2M_N}\frac{2M_N}{F_{\text{MS}}}\,r\left[-\sqrt{2}\,\frac{1}{F_\pi}\frac{F-D}{1-\alpha_{\text{MS}}}\right. \\
&\quad \left. +\sqrt{2}\,\xi_\pi d\,\frac{1}{F_\pi}\frac{F+\frac{1}{3}D}{1+\frac{1}{3}\alpha_{\text{MS}}} + \sqrt{2}\,\xi_\pi f\,\frac{1}{F_\pi}\frac{F-D}{1-\alpha_{\text{MS}}}\right], \\
B(\Xi_-{}^-) &= \frac{M_\Xi+M_\Lambda}{2M_N}\frac{2M_N}{F_{\text{MS}}}\,r\left[\xi_K\left(\sqrt{3}\,f-\frac{1}{\sqrt{3}}\,d\right)\frac{D'}{F_\pi}\frac{F_{\text{MS}}}{D_{\text{MS}}'}\right. \\
&\quad \left. -\frac{2}{\sqrt{3}}\,\xi_\pi d\,\frac{1}{F_\pi}\frac{F+D}{1+\alpha_{\text{MS}}} - \sqrt{3}\,\xi_\pi(f-d)\,\frac{1}{F_\pi}\frac{F-\frac{1}{3}D}{1-\frac{1}{3}\alpha_{\text{MS}}}\right].
\end{aligned}$$

Using Eq. (4.15) for D', $\alpha_{\text{MS}} = -0.307$, $r=1.3$,

TABLE I. Best-fit compromise solutions to both S- and P-wave amplitudes. (1) Experimental values in the units $(\mu^2/h)^{-1/2}\times 10^{-5}$ $\sec^{-1/2}$ taken from Berge.[a] More recently, the asymmetry parameters $\alpha(\Sigma_+^+)$ and $\alpha(\Sigma_-^-)$ have been measured as $\alpha(\Sigma_+^+)=0.072\pm0.026$ (Ref. 28), $\alpha(\Sigma_-^-)=-0.055\pm0.014$ (Ref. 28), $\alpha(\Sigma_-^-)=-0.10\pm0.04$ (Ref. 29), which indicate that $A(\Sigma_+^+)$ and $B(\Sigma_-^-)$ are nonvanishing. Since these changes do not significantly affect the tabulated values of A and B, we retain the older values on which the present fit calculations are based. (2) Best fit by Brown and Sommerfield (Ref. 9) and by Bludman (Ref. 12). (3) Best fit with F, D, kaon coupling suppression ξ_K adjustable, and no pionic suppression ($\xi_\pi=1$). (4) Best fit with F, D, kaon coupling suppression ξ_K adjustable, and some pionic suppression $\xi_\pi=\frac{2}{3}$ fixed. (5) Best fit, excluding corrections from the S waves, with F, D, kaon coupling suppression ξ_K adjustable, and $\xi_\pi=\frac{2}{3}$ fixed. χ^2 values have been tabulated only in order to facilitate comparison of the various theoretical fits to the experimental amplitudes.

	(1)	(2)	(3)	(4)	(5)
$A(\Lambda^0)$	1.55±0.02	1.41	1.40	1.41	1.31
$A(\Sigma_-^-)$	1.86±0.02	1.52	1.90	1.95	2.08
$A(\Sigma_+^+)$	−0.01±0.03	0	−0.51	−0.47	0
$A(\Xi_-^-)$	2.02±0.03	2.16	2.24	2.09	1.93
$B(\Lambda_-^0)$	11.0±0.5	5.1	3.8	9.2	9.4
$B(\Sigma_-^-)$	−0.2±0.4	0.02	−1.1	−1.8	−1.1
$B(\Sigma_+^+)$	19.1±0.3	6.6	9.7	14.6	10.6
$B(\Xi_-^-)$	−6.6±0.6	−3.4	−9.8	−5.5	−2.4
F		−0.87	−0.80	−0.79	−0.94
$\alpha_W=D/F$		−0.59	−1.08	−1.02	−0.57
ξ_π		1	1	$\frac{2}{3}$	$\frac{2}{3}$
ξ_K		...	0.27	0.30	0.22
sign of D'		0	positive	positive	negative
χ^2		2500	1300	450	950

[a] P. Berge, in Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966 (unpublished).

$d/(f+d)=\frac{2}{3}$ and observed values of baryon mass, we have

$$F_\pi A(\Lambda_-^0)=-1.73F-0.58D-0.39\xi_\pi\xi_K-1.46\xi_K,$$
$$F_\pi A(\Sigma_-^-)=-1.41F+1.41D+1.56\xi_\pi\xi_K,$$
$$F_\pi A(\Sigma_+^+)=0.81\xi_\pi\xi_K+1.04\xi_K,$$
$$F_\pi A(\Xi_-^-)=-1.73F+0.58D-1.31\xi_\pi\xi_K,$$
$$F_\pi B(\Lambda_-^0)=(-27.2+8.4\xi_\pi)F+(-9.1-8.1\xi_\pi)D-13.6\xi_K, \quad (4.19)$$
$$F_\pi B(\Sigma_-^-)=10.1\xi_\pi F+10.4\xi_\pi D+6.9\xi_K,$$
$$F_\pi B(\Sigma_+^+)=(-16.0+20.7\xi_\pi)F+(16.0-0.2\xi_\pi)D,$$
$$F_\pi B(\Xi_-^-)=-9.7\xi_\pi F-21.6\xi_\pi D-3.3\xi_K.$$

In the above expressions the sign of D' has been chosen negative.

Choosing F, D, ξ_K, and the sign of D' as adjustable parameters, and using fixed values $\xi_\pi=1$ and $\frac{2}{3}$, we found the best-fit compromise solution to both S and P waves. The results are tabulated in columns (3) and (4) of Table I.[28,29] The values in column (3) are what Kumar and Pati[21] would have found, if there had not been an error by the factor $\sqrt{2}$ in Eq. (21) of their paper. The values of column (2) have been obtained by Brown and Sommerfield[9]; similar values have been obtained also by Badier and Bouchiat,[10] by Hara, Nambu, and Schechter,[11] and by Bludman.[12] Comparing column (4) to columns (3) and (2), we can see that with both pionic and kaonic coupling suppression ($\xi_\pi=\frac{2}{3}$ $\xi_K=\frac{1}{3}$) a compromise fit to both S and P waves is obtained that is considerably improved over that of Refs. 9–12. The factor of 2 improvement in $B(\Sigma_+^+)$ and $B(\Lambda_-^0)$ is gained at the expense of making $A(\Sigma_+^+)$ and $B(\Sigma_-^-)\neq 0$.

The major difficulty with including Born correction terms in the S waves is that they make $A(\Sigma_+^+)$ small but nonvanishing. If the Born correction terms are omitted from the S waves, so that $A(\Sigma_+^+)=0$ exactly, the results tabulated in column (5) are obtained. Comparison with column (4) shows that keeping the Born correction terms in the S waves, while making $A(\Sigma_+^+)$ nonvanishing, does lead to a better over-all fit.

We summarize the results obtained in the present section:

(1) The universal pc spurion requires the suppression of both pionic and kaonic coupling constants by factors of as large as 3 in order for the P-wave octet pole model to be consistent with experiment.

(2) As a symmetry-breaking effect, the correction terms due to the Born amplitude have been included in the S waves. The kaon tadpole model and $K_{2\pi}$ data have been used for the pv spurion coupling. If the universal pc spurion is abandoned and the pionic and

[28] R. Bangerter *et al.*, in *Proceedings of the Heidelberg International Conference on Elementary Particles, Heidelberg, 1967* (North-Holland, Amsterdam, 1968).

[29] D. Berley *et al.* in *Proceedings of the Heidelberg International Conference on Elementary Particles, Heidelberg, 1967* (North-Holland, Amsterdam, 1968).

kaonic coupling constants are suppressed by the factors $\frac{2}{3}$ and $\frac{1}{3}$, respectively, the best-fit compromise solution to both S and P waves is considerably improved over that of Brown and Sommerfield and of Bludman. The fit is far from perfect, however. In particular, this best over-all fit makes $A(\Sigma_+^+)\neq 0$; the fit with $A(\Sigma_+^+)=0$ is not much poorer than the best fit.

V. CONCLUDING DISCUSSION

In this paper we have investigated two questions: (1) the extent to which nonleptonic S-wave hyperon decay tests the current$\times$current form of interaction and current algebra and (2) the role of $SU(3)$-symmetry breaking in improving the octet-pole-model predictions for P-wave hyperon decay.

We will now summarize our conclusions and discuss the possibilities of improving on the octet pole model by (a) departing from octet dominance and (b) including decuplet intermediate states.

(1) Because nonleptonic hyperon decay involves the emission of a single pion, not current algebra, but only the pion threshold theorem is involved. Within the soft-pion formalism, the S-wave $\Delta I=\frac{1}{2}$ and pseudo-($\Delta I=\frac{1}{2}$) rules depend only on the chiral invariance of the weak Hamiltonian.[8] The S-wave Lee-Sugawara relation depends on W_1 invariance,[12,13] for which the current$\times$ current form (with Cabibbo angles $\theta_A=\theta_V$) is sufficient but not necessary. This sufficiency explains why other models[30,31] not of the current$\times$current form successfully reproduce the Suzuki-Sugawara result. The only argument for current$\times$current form in nonleptonic decays that is, in our opinion, at all compelling is the vanishing of $A(\Sigma_+^+)$: In a general chiral and W_1 invariant theory,[13] $A(\Sigma_+^+)=0$ can come about because of cancellation between the **27** and **10** parts of $\mathcal{H}_W$; in a current$\times$current theory, $\mathcal{H}_W(\mathbf{27})$ is absent and $A(\Sigma_+^+)=0$ is simply an expression of octet dominance.

(2) In an octet-dominant theory, the $SU(3)$-symmetric analysis of S waves leads to a common pc baryon spurion $[(D/F)_W=(D/F)_{MS}]$. Since the P-wave amplitudes are not given by the soft-pion formalism, we use an octet pole model for the P-wave decay. If the strong coupling is $SU(3)$ symmetric, the empirical P-wave Lee-Sugawara relation also requires a common pc baryon spurion. This common pc baryon spurion would however, make $B(\Sigma_+^+)$ vanish, in contradiction with experiment. For this reason, coupling shifts are needed for the P-wave amplitudes.

Once $SU(3)$-symmetry breaking is considered in the P waves it is logical to consider its effects on the S waves. Brown and Sommerfield[9] and Badier and Bouchat[10] had obtained a compromise fit to both S and P waves by renouncing the universal pc spurion and allowing mass shifts but not coupling shifts. We included the correction terms due to the Born amplitude in the S waves and used the kaon tadpole model along with the $K_{2\pi}$ data to calculate the pv spurion coupling. We found that if the pionic and kaonic strong-coupling constants are suppressed by $\frac{2}{3}$ and $\frac{1}{3}$, a considerably improved fit is obtained [column (4) in Table I].

(a) In this fit, all the S- and P-wave amplitudes are given within 20%, but $A(\Sigma_+^+)$ was nonvanishing. Now, in the $SU(3)$-symmetric soft-pion analysis, $A(\Sigma_+^+)=0$ was an expression of octet dominance of $\mathcal{H}_W$. We have emphasized that any treatment of $SU(3)$ symmetry breaking is model dependent. In our generalization of the Suzuki-Sagawara treatment, $SU(3)$ coupling shifts lead to pv Born terms which make $A(\Sigma_+^+)\neq 0$, so long as only octet intermediate states are considered. By including $\mathcal{H}_W(\mathbf{27})$ along with $\mathcal{H}_W(\mathbf{8})$, Chan[32] was able to improve the over-all fit. The $\mathcal{H}_W(\mathbf{27})$ needed to make $A(\Sigma_+^+)=0$, then, leads to appreciable deviations from the $\Delta I=\frac{1}{2}$ rule in P-wave Σ decay. Thus, at least in Chan's analysis, including $\mathcal{H}_W(\mathbf{27})$ achieves $A(\Sigma_+^+)\approx 0$ at the price of departure from the $\Delta I=\frac{1}{2}$ rule.

(b) The $SU(3)$-symmetric S-wave analysis also suggested the common pc baryon spurion or the attractive stronger possibility of the universal pc spurion. Our analysis has shown that in the octet pole model for P-wave decays, the common pc baryon spurion is possible only at the price of unreasonably large hyperon-pion coupling shifts. Including decuplet pole terms leads to considerable improvement in the P-wave pole model[32,33] and in the saturation of the current algebra.[19] We can only wonder what would happen if other higher-mass baryon states were included. By considering only octet intermediate states, we have considered the simplest possibility, putting the entire burden of the fit on the hyperon-meson coupling shifts.

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