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Broken Chiral $SU(3)\times SU(3)$ Symmetry. I. Meson Decays and K_{l3} Form Factors*

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Broken chiral $SU(3)\times SU(3)$ symmetry is considered by extending the Glashow-Weinberg-Berdeen-Lee scheme to include spin-1 mesons. Vector-meson decays and K_{l3} decays are treated on the basis of the resultant Lagrangian. Numerical estimates are made on the assumption of nonet symmetry, at the $SU(3)$ level, for the spin-1^- mesons. It is found that $F_K/F_\pi \simeq 1.10$, $f_+(0)\simeq 0.96$, $\xi = f_-(0)/f_+(0) \simeq -0.048$, $\lambda_+ \simeq 0.022$, and $\xi\lambda_- \simeq -0.002$.

I. INTRODUCTION

Chiral $SU(2)\times SU(2)$ dynamics,[1] which incorporates the notion of chiral $SU(2)\times SU(2)$ symmetry, the hypothesis of a partially conserved axial-vector current (PCAC), and vector-meson dominance in a phenomenological Lagrangian, has been extremely useful in correlating the low-energy parameters of the low-lying "particle" states. It reproduces all previous low-energy results of current-algebra calculations in a compact and convenient way and provides a suitable basis for further dynamical calculations. An attempt[2] has been made to attach a fundamental meaning to such a Lagrangian by assuming that it is actually the basic Lagrangian, valid to arbitrarily short distances. We have no doubt that an attempt like this, firstly, is interesting and valuable, and secondly, will provide a simple and concrete field-theoretical model over the whole energy range. Nevertheless, we shall hold the more conservative point of view that the various Lagrangians in chiral dynamics are only partial and approximate representations of the real physical world in the low-energy region.

Generalization of $SU(2)\times SU(2)$ chiral dynamics to the case of $SU(3)\times SU(3)$ is naturally called for. But, in this case, one is somewhat plagued by one's ignorance of how to introduce symmetry breaking. While there are the Goldberger-Treiman relations,[3] the Adler self-consistency condition,[4] and the Adler-Weisberger relation[5] as built-in guarantees for the approximate validity of the PCAC hypothesis in the case of $SU(2)\times SU(2)$, one has no comparable guide in introducing $SU(3)\times SU(3)$-symmetry breaking. Even if one is willing to assume PCAC, say, for the strangeness-changing axial-vector current (with its divergence dominated by the K meson), for which there is some evidence,[6] one is still confronted with difficult questions in regard to the strangeness-changing vector current: Is a partial conservation of vector current (PCVC) hypothesis to be adopted? Also, does there exist a $T=\frac{1}{2}$ strange scalar meson?

Broken chiral $SU(3)\times SU(3)$ Lagrangians have been discussed by various authors,[7–9] mostly on the basis of the $SU(3)$ σ model. Of particular interest to us is the work of Bardeen and Lee.[9] One of the schemes obtained by Bardeen and Lee assumes the existence of a pseudoscalar octet and a $T=\frac{1}{2}$ strange scalar meson κ. This scheme is essentially the one previously considered by Glashow and Weinberg.[10] In both of these works, spin-1 mesons were not directly brought into the system. The purpose of the present work is to generalize the Glashow-Weinberg-Bardeen-Lee scheme to include the spin-1 mesons, similar to what is done in Ref. 11. By doing this, we expect to obtain more detailed results than these authors did. The Lagrangian so obtained will then provide a suitable basis for a *systematic* and *correlated* discussion of the meson decays and K_{l3} and K_{l4} form factors. It could also be used for further dynamical calculations, such as the decay rate of $K^+\to\pi^+\pi^0$. In the present paper, we shall report the results on meson decays and K_{l3} form factors. In a subsequent paper, calculations on K_{l4} form factors will be reported.

The inclusion of the spin-1 mesons has been discussed by Gasiorowicz and Geffen,[8] and others.[12] In our present consideration, the parametrization of symmetry breaking is guided by the principle of simplicity, and is

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[4] S. L. Adler, Phys. Rev. **137**, B1022 (1965); **139** B1638 (1965).

[5] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); S. L. Adler, *ibid.* **14**, 1051 (1965).

[6] See, e.g., W. I. Weisberger, Phys. Rev. **143**, 1302 (1966); H. T. Nieh, Phys. Rev. Letters **20**, 1254 (1968).

[7] M. Levy, Nuovo Cimento **52**, 23 (1967).

[8] S. Gasiorowicz and D. Geffen, Argonne Lecture Notes, 1968 (unpublished). This work has considerably influenced our notation in this paper. An excellent review article on the subject of phenomenological Lagrangians by these authors has recently appeared; S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. **41**, 531 (1969).

[9] W. A. Bardeen and B. W. Lee, Phys. Rev. **177**, 2389 (1969).

[10] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968). See also L. N. Chang and Y. C. Leung, *ibid.* **21**, 122 (1968).

[11] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1506 (1968).

[12] References can be found in the review article by Gasiorowicz and Geffen (Ref. 8).

introduced to reflect the underlying chiral symmetry and the octet breaking implied by the Gell-Mann–Okubo mass formula. On the basis of the quantum action principle,[13] a simple derivation of the characteristic relations of the so-called "field algebra" is presented. The absence of the "mass-mixing"-type symmetry breaking for spin-1 mesons is seen to be related to the requirement that the space components of the currents transform as octets. After redefinition (or renormalization) of the fields has been carried out, explicit expressions for the various coupling and decay constants are obtained. We shall also demonstrate the Ademollo-Gatto theorem[14] and obtain an explicit expression for the K_{l3} renormalization factor $f_+(0)$.

Numerical estimates of the various parameters are obtained on the basis of the assumed nonet symmetry, at the level of $SU(3)$, for the vector mesons. Using the masses as inputs, we estimate that

$$F_K/F_\pi \simeq 1.10, \qquad f_+(0) \simeq 0.96,$$

in reasonable agreement with the present empirical result[15] (on the basis of the Cabibbo theory[16])

$$F_K/F_\pi \simeq 1.19 f_+(0).$$

Our finding indicates that the $SU(3)$-symmetry-breaking effect on $f_+(0)$ is significant but not intolerably large.

In Sec. IX, we express the ratios

$$\Gamma(K^* \to K+\pi)/\Gamma(\rho\to 2\pi), \qquad \Gamma(A_1\to\rho+\pi)/\Gamma(\rho\to 2\pi),$$

and

$$\Gamma(K_A\to K^*+\pi)/\Gamma(\rho\to 2\pi)$$

in terms of parameter δ. With $\delta=-\frac{3}{4}$, the ratios are, respectively, 0.39, 0.60, and 0.85. In Sec. X, we present the results of a detailed calculation of the K_{l3} form factors based on the Lagrangian and currents we have obtained. Using as inputs the values estimated in the earlier sections for the various parameters, we obtain

$$\xi(0)=f_-(0)/f_+(0)\simeq -0.048, \quad \lambda_+\simeq 0.022, \quad \lambda_-\simeq 0.051.$$

Since our main interests, at the moment, are the dynamical properties of physical systems involving the hadrons π, K, A_1, K_A, ρ, and K^*, and also because of the singlet-octet mixing complication, we shall in the present paper ignore the isoscalar 0^- and 1^+ mesons. However, we shall assume the nonet symmetry, at the level of $SU(3)$, for the vector mesons.

II. TRANSFORMATIONS AND GENERATORS

The quantum action principle can be considered as the starting point of any Lagrangian field theory. It contains both the field equations and the canonical commutation relations. While the field equations are obtained by requiring that the action be stationary with respect to field variations within the boundary, the canonical commutation relations are inferred through the quantum-mechanical interpretation by identifying the (time) surface term as the infinitesimal generator for the unitary transformation corresponding to the freedom of changing description of the quantum-mechanical system in question. The action principle, therefore, appears suitable for discussing the chiral transformations and the related commutation relations. In particular, the currents and their divergences can be easily and naturally identified. As an illustration, we consider in this section the massive $SU(2)$ Yang-Mills fields.

The Lagrangian for the massive $SU(2)$ Yang-Mills field[17] can be written in the form

$$\mathfrak{L}=\mathfrak{L}_0-\tfrac{1}{2}m^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu, \tag{1}$$

where $\mathfrak{L}_0$, which, in general, contains fields other $\boldsymbol{\rho}_\mu$, is invariant under the infinitesimal isotopic-spin gauge transformation

$$\delta\boldsymbol{\rho}_\mu(x)=-\delta\boldsymbol{\omega}(x)\times\boldsymbol{\rho}_\mu(x)+(1/g)\partial_\mu\delta\boldsymbol{\omega}(x) \tag{2}$$

and the corresponding isotopic-spin transformations for other fields. The change induced by these transformations in the action

$$W\equiv\int d^4x\, \mathfrak{L}(x)$$

is given by

$$\begin{aligned}\delta W_{12}&=-\int_{t_2}^{t_1} d^4x\ (m^2/g)\,\boldsymbol{\rho}^\mu(x)\cdot\partial_\mu\delta\boldsymbol{\omega}(x)\\ &=\left(-\int d^3x\ (m^2/g)\,\boldsymbol{\rho}^0(x)\cdot\delta\boldsymbol{\omega}(x)\right)\Bigg|_{t_2}^{t_1}\\ &\quad+\int_{t_2}^{t_1} d^4x\ (m^2/g)\,\partial_\mu\boldsymbol{\rho}^\mu(x)\cdot\delta\boldsymbol{\omega}(x),\end{aligned} \tag{3}$$

where the spatial surface term has been neglected. Firstly, the principle of stationary action implies the field equation

$$\partial_\mu\boldsymbol{\rho}^\mu(x)=0. \tag{4}$$

Secondly, the generator which induces the transformation (2) for the *independent field variables* $\boldsymbol{\rho}_k(x)$ ($k=1, 2, 3$) is identified as

$$G(t)=-\int d^3x\ (m^2/g)\,\boldsymbol{\rho}^0(x)\cdot\delta\boldsymbol{\omega}(x). \tag{5}$$

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[13] J. Schwinger, Phys. Rev. **82**, 914 (1951); **91**, 713 (1953). For a brief introduction to the quantum action principle, see J. Schwinger, in *Lectures on Particles and Field Theory, Summer School Proceedings, Brandeis University, 1964* (Prentice-Hall, Englewood Cliffs, N.J., 1965), Vol. II.

[14] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964); C. Bouchiat and Ph. Meyer, Nuovo Cimento **24**, 1122 (1964); S. Fubini and G. Furlan, Physics **1**, 229 (1964).

[15] The decay rates for $K^+\to\pi^0+e^++\nu$, $K^+\to\mu^++\nu$, and $\pi^+\to\mu^++\nu$ which we adopt are those of N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Matts Roos, A. H. Rosenfeld, Paul Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969). We also take $\lambda_+=0.02$, where λ_+ is the usual slope parameter for the K_{l3} form factor f_+.

[16] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[17] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

This, of course, means that ($x^0=t$)

$$\delta\varrho_k(x)=i[G(t),\varrho_k(x)] \quad (6)$$

or, with $x^0=x^{0\prime}$,

$$i\int d^3x' [-(m^2/g)\varrho^0(x')\cdot\delta\boldsymbol{\omega}(x'),\varrho_k(x)] = -\delta\boldsymbol{\omega}(x)\times\varrho_k(x)+(1/g)\partial_k\delta\boldsymbol{\omega}(x), \quad (7)$$

which implies that

$$[(m^2/g)\rho_\alpha{}^0(\mathbf{x}',t),(m^2/g)\rho_\beta{}^k(\mathbf{x},t)] = i\epsilon_{\alpha\beta\gamma}(m^2/g)\rho_\gamma{}^k(x)\delta(\mathbf{x}-\mathbf{x}')+i\delta_{\alpha\beta}(m^2/g)\partial^k\delta(\mathbf{x}-\mathbf{x}'). \quad (8)$$

By specifying $\delta\boldsymbol{\omega}(x)$ to be a constant in (5), the isospin charge is identified to be

$$Q_\alpha=\int d^3x\,(m^2/g)\varrho^0(x), \quad (9)$$

which satisfies the $SU(2)$ Lie algebra

$$[Q_\alpha,Q_\beta]=i\epsilon_{\alpha\beta\gamma}Q_\gamma. \quad (10)$$

The isospin current is thus identified to be

$$j_\alpha{}^\mu(x)=(m^2/g)\rho_\alpha{}^\mu(x), \quad (11)$$

which is also consistent with (8). In virtue of the field equation (4), the current is conserved:

$$\partial_\mu j_\alpha{}^\mu(x)=0. \quad (12)$$

The charge algebra (10) implies the following equal-time commutation relation for the charge density:

$$[j_\alpha{}^0(\mathbf{x},t),j_\beta{}^0(\mathbf{x}',t)]=i\epsilon_{\alpha\beta\gamma}j_\gamma{}^0(x)\delta(\mathbf{x}-\mathbf{x}')+\tau_{\alpha\beta}{}^{00}(x,x'), \quad (13)$$

where $\tau_{\alpha\beta}{}^{00}(x,x')$ is antisymmetric with respect to the interchange $\alpha\to\beta$, $\mathbf{x}\to\mathbf{x}'$, and satisfies

$$\int d^3x\,d^3x'\,\tau_{\alpha\beta}{}^{00}(x,x')=0. \quad (14)$$

The exact form of $\tau_{\alpha\beta}{}^{00}(x,x')$ is dependent on dynamics, i.e., on the equation of motion for $\rho_\alpha{}^0(x)$.

The commutation relation (8), and the usual equal-time commutation relations for independent field variables ($i,j=1,2,3$),

$$[\rho_\alpha{}^i(\mathbf{x},t),\rho_\beta{}^j(\mathbf{x}',t)]=0, \quad (15)$$

or, on account of field-current identity (11),

$$[j_\alpha{}^i(\mathbf{x},t),j_\beta{}^j(\mathbf{x}',t)]=0, \quad (16)$$

are characteristic of what is known as "field algebra,"[2] which includes, in addition to (8) and (16), the following equal-time commutation relations for the charge densities:

$$[j_\alpha{}^0(\mathbf{x},t),j_\beta{}^0(\mathbf{x}',t)]=i\epsilon_{\alpha\beta\gamma}j_\gamma{}^0(x)\delta(\mathbf{x}-\mathbf{x}'). \quad (17)$$

The unique feature of this model lies in its complete specification of the Schwinger terms as well as the space-space current-density commutation relations. While these latter commutation relations are due to the dynamical independence of the corresponding field variables, it is clear from our derivation of (8) that the structure of the Schwinger term is a reflection of the structure of the gauge term in (2).

We might mention that one advantage of our derivation of (8) lies in its bypassing the field equations and the canonical quantization rules for the field variables, which could be quite involved for complicated interaction terms contained in $\mathcal{L}_0$ of (1). (In our later consideration, we do have complicated interaction terms.)

III. BROKEN CHIRAL SYMMETRY FOR SPIN-0 MESONS

For completeness, we shall briefly recount some of the results obtained by Bardeen and Lee.[9] This also serves to introduce some of the notation we need in the present paper. These authors discuss the breakdown of the chiral $SU(3)\times SU(3)$ symmetry on the basis of a generalized σ model. In this model, nonets of the scalar and pseudoscalar fields are assigned to the $(3,\bar{3})+(\bar{3},3)$ representation of the chiral $SU(3)\times SU(3)$:

$$M_{\alpha\beta}=(\Sigma+i\Pi)_{\alpha\beta}, \qquad M_{\alpha\beta}{}^\dagger=(\Sigma-i\Pi)_{\alpha\beta}, \quad (18)$$

where Σ and Π are the usual 3×3 Hermitian matrices for the scalar and pseudoscalar nonets, respectively. The Lagrangian considered by Bardeen and Lee is

$$\mathcal{L}=-\tfrac{1}{2}\,\mathrm{Tr}\partial^\mu M\partial_\mu M^\dagger-\tfrac{1}{2}\mu^2[\mathrm{Tr}MM^\dagger+H] +\mathrm{Tr}A(M+M^\dagger), \quad (19)$$

where H is an arbitrary polynomial in chiral $SU(3)\times SU(3)$ invariants. By assuming that the vacuum expectation values of the scalar fields

$$\langle\Sigma\rangle_0=F \quad (20)$$

are not identically zero, the model is studied in the $\mu^2\to\infty$ limit. With the help of a canonical transformations, the Lagrangian, in the limit $\mu^2\to\infty$, can be *effectively* brought to the form

$$\mathcal{L}=-\tfrac{1}{2}\,\mathrm{Tr}\partial^\mu M\partial_\mu M^\dagger+\mathrm{Tr}A(M+M^\dagger)$$

or, equivalently,

$$\mathcal{L}=-\tfrac{1}{2}\,\mathrm{Tr}(\partial^\mu\Pi\partial_\mu\Pi+\partial^\mu\Sigma\partial_\mu\Sigma)+2\,\mathrm{Tr}(A\Sigma), \quad (21)$$

where M is expressed in terms of the newly introduced scalar and pseudoscalar octet fields $S_{\alpha\beta}$ and $P_{\alpha\beta}$:

$$M=\Sigma+i\Pi=\exp(iP)\,\exp(iS)F\,\exp(-iS)\,\exp(iP). \quad (22)$$

The number and nature of the particles present in (22) depend upon the form of the numerical matrix F. Among all the possible cases enumerated by Bardeen and Lee, two are of particular physical relevance:

(i) The octet of pseudoscalar mesons is present. The $SU(3)$ symmetry is *intrinsically* broken. The corresponding F is proportional to the unit matrix

$$F=(f/\sqrt{2})1 \tag{23}$$

and M contains only P,

$$M=(f/\sqrt{2})\exp(2iP). \tag{24}$$

(ii) The octet of pseudoscalar mesons and a $T=\frac{1}{2}$ strange scalar meson κ are present. The $SU(3)$ symmetry is both *intrinsically* and *spontaneously* broken. The corresponding F is of the form

$$F=(f/\sqrt{2})\begin{pmatrix}1 & & \\ & 1 & \\ & & w\end{pmatrix}, \tag{25}$$

where $w\neq 1, 0, -1$.

In both of these cases, $\sqrt{2}fP$ and $\sqrt{2}fS$ are to be interpreted as the usual 3×3 Hermitian matrices for the octets. In case (ii), a wave-function renormalization of the appropriate fields is further required. In the following sections we shall treat these two schemes on the same basis by not restricting the value of w.

IV. BROKEN CHIRAL SYMMETRY FOR SPIN-0 AND SPIN-1 MESONS

The dynamics of spin-0 and spin-1 mesons are closely correlated, as is revealed by the study of the $SU(2)\times SU(2)$ chiral dynamics. In this section we shall consider the system, in the framework of a broken $SU(3)\times SU(3)$ symmetry, of spin-0 and spin-1 particles.

The simplest $SU(3)\times SU(3)$ symmetric Lagrangian for the massive Yang-Mills fields and the spin-0 mesons can be immediately written down[8]:

$$\mathcal{L}_0=-\tfrac{1}{4}\operatorname{Tr}(F_{\mu\nu}F^{\mu\nu}+G_{\mu\nu}G^{\mu\nu})-\tfrac{1}{2}m^2\operatorname{Tr}(V_\mu V^\mu+A_\mu A^\mu)-\tfrac{1}{2}\operatorname{Tr}(\Delta_\mu\Pi\Delta^\mu\Pi+\Delta_\mu\Sigma\Delta^\mu\Sigma), \tag{26}$$

where, with the obvious 3×3 matrix notation,

$$F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu-i(g/\sqrt{2})[V_\mu, V_\nu]-i(g/\sqrt{2})[A_\mu, A_\nu], \tag{27}$$

$$G_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-i(g/\sqrt{2})[V_\mu, A_\nu]-i(g/\sqrt{2})[A_\mu, V_\nu] \equiv D_\mu A_\nu-D_\nu A_\mu, \tag{28}$$

$$\Delta_\mu\Pi=\partial_\mu\Pi-i(g/\sqrt{2})[V_\mu, \Pi]-(g/\sqrt{2})\{A_\mu, \Sigma\} \equiv D_\mu\Pi-(g/\sqrt{2})\{A_\mu, \Sigma\}, \tag{29}$$

$$\Delta_\mu\Sigma=\partial_\mu\Sigma-i(g/\sqrt{2})[V_\mu, \Sigma]+(g/\sqrt{2})\{A_\mu, \Pi\} \equiv D_\mu\Sigma+(g/\sqrt{2})\{A_\mu, \Pi\}. \tag{30}$$

Under infinitesimal $SU(3)\times SU(3)$ local gauge transformations, the various field variables undergo the following changes:

$$\begin{aligned}\delta_\omega V_\mu&=(i/\sqrt{2})[\omega, V_\mu]+(1/g)\partial_\mu\omega,\\ \delta_\omega A_\mu&=(i/\sqrt{2})[\omega, A_\mu],\\ \delta_\omega\Pi&=(i/\sqrt{2})[\omega, \Pi],\\ \delta_\omega\Sigma&=(i/\sqrt{2})[\omega, \Sigma],\end{aligned} \tag{31}$$

and

$$\begin{aligned}\delta_\lambda V_\mu&=(i/\sqrt{2})[\lambda, A_\mu],\\ \delta_\lambda A_\mu&=(i/\sqrt{2})[\lambda, V_\mu]+(1/g)\partial_\mu\lambda,\\ \delta_\lambda\Pi&=(i/\sqrt{2})\{\lambda, \Sigma\},\\ \delta_\lambda\Sigma&=-(1/\sqrt{2})\{\lambda, \Pi\}.\end{aligned} \tag{32}$$

The variables $F_{\mu\nu}$, $G_{\mu\nu}$, $\Delta_\mu\Sigma$, and $\Delta_\mu\Pi$ have been constructed in such a way that they undergo changes in exactly the same manner as the corresponding field variables listed above, except that the inhomogeneous gauge terms are absent.

There is another $SU(3)\times SU(3)$ symmetric term one can easily construct:

$$\mathcal{L}_0'=i(2\sqrt{2})^{-1}(\delta/m^2)g\operatorname{Tr}(F_{\mu\nu}[\Delta^\mu\Pi, \Delta^\nu\Pi]+F_{\mu\nu}[\Delta^\mu\Sigma, \Delta^\nu\Sigma]-2iG_{\mu\nu}\{\Delta^\mu\Sigma, \Delta^\nu\Pi\}). \tag{33}$$

This is the counterpart of the δ term in Ref. 18.

The chiral $SU(3)\times SU(3)$ symmetry is broken into the ordinary $SU(3)$ symmetry when a term $\mathcal{L}_1$ is added to $\mathcal{L}_0+\mathcal{L}_0'$. To have the PCAC equation at the $SU(3)$ level, $\mathcal{L}_1$ is simply chosen to be

$$\mathcal{L}_1=2a\operatorname{Tr}\Sigma. \tag{34}$$

Next, another piece $\mathcal{L}_8$ is introduced to break the $SU(3)$ symmetry. Octet dominance together with our intention of preserving the PCAC conditions at the $SU(2)$ level suggest the following simple parametrization:

$$\mathcal{L}_8=-\tfrac{1}{2}\xi(F_{\mu\nu}F^{\mu\nu}+G_{\mu\nu}G^{\mu\nu})_{33}-\xi'm^2(V_\mu V^\mu+A_\mu A^\mu)_{33}-\eta(\Delta_\mu\Pi\Delta^\mu\Pi+\Delta_\mu\Sigma\Delta^\mu\Sigma)_{33}+2a'\Sigma_{33}, \tag{35}$$

where we have also invoked the underlying symmetry between the 1^+ and 1^- states to assume the same symmetry-breaking parameters for both.

Our consideration of the broken $SU(3)\times SU(3)$ symmetry is based on the Lagrangian

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_0'+\mathcal{L}_1+\mathcal{L}_8. \tag{36}$$

Before going on, we remark that in order to account for the phenomenological findings about the singlet-octet mixing in the case of 0^- and 1^+ mesons, an additional piece has to be introduced into the Lagrangian, thus increasing the number of parameters. Since this mixing is a less interesting problem, and since we are presently primarily interested in the dynamical properties of

[18] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

bosons with nonvanishing isotopic spin, we shall not in this paper be concerned with the addition piece in $\mathfrak{L}$. For the 1^- particles, it is well known that the nonet symmetry, at the level of $SU(3)$, is good. This nonet symmetry for 1^- particles we shall assume throughout this paper, although most results of the ensuing sections are independent of this assumption. The numerical estimates in Sec. VIII, however, depends on this assumption.

According to the quantum action principle, the (time-) surface term in the action variation δW is to be identified with the generator for the corresponding infinitesimal transformation. The change induced in the Lagrangian by the $SU(3)\times SU(3)$ local gauge transformations (31) and (32) is given by

$$\delta\mathfrak{L}=\delta_\omega\mathfrak{L}+\delta_\lambda\mathfrak{L}, \tag{37}$$

where

$$\delta_\omega\mathfrak{L}=-(m^2/g)\ \mathrm{Tr}(V^\mu\partial_\mu\omega)-\xi'(m^2/g)\{V^\mu,\partial_\mu\omega\}_{33} +(\text{term proportional to }\delta\omega), \tag{38}$$

$$\delta_\lambda\mathfrak{L}=-(m^2/g)\ \mathrm{Tr}(A^\mu\partial_\mu\lambda)-\xi'(m^2/g)\{A^\mu,\partial_\mu\lambda\}_{33} +(\text{term proportional to }\delta\lambda). \tag{39}$$

For simplicity, we have not given explicit expressions for the terms proportional to $\delta\omega$ and $\delta\lambda$. Identifying the time-surface terms in δW gives, in analogy with (5), rise to the following expressions for the generators of the infinitesimal $SU(3)\times SU(3)$ local gauge transformations:

$$G_\omega(t)=-\int d^3x\ (m^2/g) \times\mathrm{Tr}[V^0(x)\omega(x)+\xi'\Delta_8\{V^0(x),\omega(x)\}], \tag{40}$$

$$G_\lambda(t)=-\int d^3x\ (m^2/g) \times\mathrm{Tr}[A^0(x)\lambda(x)+\xi'\Delta_8\{A^0(x),\lambda(x)\}], \tag{41}$$

where Δ_8 is the 3×3 numerical matrix

$$\Delta_8=\begin{pmatrix}0&&\\&0&\\&&1\end{pmatrix}. \tag{42}$$

The $SU(3)$ charges and axial charges are identified by considering constant ω and λ in (40) and (41):

$$\begin{aligned}Q(t)&=(m^2/g)\int d^3x\ [V^0(x)+\xi'\{\Delta_8,V^0(x)\}],\\ Q^5(t)&=(m^2/g)\int d^3x\ [A^0(x)+\xi'\{\Delta_8,A^0(x)\}],\end{aligned} \tag{43}$$

with obvious 3×3 matrix notiatons. The generators must necessarily satisfy the Lie algebra of the $SU(3)\times SU(3)$ transformation group. That is, the charges must satisfy the equal-time commutation relations ($\alpha,\beta,\gamma=1,2,\ldots,8$)

$$\begin{aligned}[][Q_\alpha,Q_\beta]&=if_{\alpha\beta\gamma}Q_\gamma,\\ [Q_\alpha,Q_\beta{}^5]&=if_{\alpha\beta\gamma}Q_\gamma{}^5,\\ [Q_\alpha{}^5,Q_\beta{}^5]&=if_{\alpha\beta\gamma}Q_\gamma,\end{aligned} \tag{44}$$

where (λ_α being the canonical Gell-Mann matrices)

$$Q_\alpha=(1/\sqrt{2})\ \mathrm{Tr}\lambda_\alpha Q,\text{ etc.} \tag{45}$$

The currents are then identified to be

$$\begin{aligned}j^\mu&=(m^2/g)[V^\mu+\xi'\{\Delta_8,V^\mu\}],\\ j^{5\mu}&=(m^2/g)[A^\mu+\xi'\{\Delta_8,A^\mu\}].\end{aligned} \tag{46}$$

We shall now see that j^μ and $j^{5\mu}$ given above can transform as $SU(3)$ octets only if $\xi'=0$.[19] Since G_ω and G_λ are the generators which induce transformations (31) and (32), respectively, for the *independent field variables*, we have

$$i[G_\omega(t),V^k(x)]=(i/\sqrt{2})[\omega,V^k]+(1/g)\partial^k\omega, \tag{47}$$

$$\begin{aligned}i[G_\omega(t),A^k(x)]&=(i/\sqrt{2})[\omega,A^k],\\ i[G_\lambda(t),V^k(x)]&=(i/\sqrt{2})[\lambda,A^k],\\ i[G_\lambda(t),A^k(x)]&=(i/\sqrt{2})[\lambda,V^k]+(1/g)\partial^k\lambda.\end{aligned} \tag{48}$$

In particular, for $\omega=$ const, we have from (47) that

$$[Q_\alpha,V_\beta{}^k]=if_{\alpha\beta\gamma}V_\gamma{}^k. \tag{49}$$

This expresses the fact that V^k transforms as an octet. It is then clear from (47) that j^k will transform as an octet if and only if $\xi'=0$. We would like to emphasize two points: (i) j^0 should transform as an octet with or without assuming $\xi'=0$. (ii) For a symmetry which is broken, Q_α depends on time and there is no *a priori* reason to require j^k to have the same internal transformation property as j^0. Notwithstanding, it is attractive, and a common practice, to assume the same transformation property for j^0 and j^k. That is, we will require j^k to transform as an octet and take

$$\xi'=0; \tag{50}$$

then the currents become

$$\begin{aligned}j^\mu&=(m^2/g)V^\mu,\\ j^{5\mu}&=(m^2/g)A^\mu.\end{aligned} \tag{51}$$

As is clear from (46)–(48), a by-product of $\xi'=0$ is the equality of all Schwinger terms for vector and axial-vector currents. [Compare with the derivation of (8) from (7).] This, in turn, implies[20] the validity of the

[19] This has previously been recognized by I. Kimel, Phys. Rev. Letters **21**, 177 (1968); K. Kang, Phys. Rev. **177**, 2439 (1969). Our conclusion is more general than those of these author's in that the invariant Lagrangian L_0 in (36) can be arbitrarily general in so far as it is invariant under the $SU(3)\times SU(3)$ local gauge transformations (31) and (32).

[20] See, e.g., H. T. Nieh, Phys. Rev. **163**, 1769 (1967).

first Weinberg sum rules[21] for $SU(3)\times SU(3)$. We shall explicitly verify these sum rules later.

V. IDENTIFICATION OF PHYSICAL PARTICLES

Because of the coupling between spin-1 and spin-0 mesons and the symmetry-breaking effects, a renormalization and diagonalization process is, in general, required to cast the Lagrangian $\mathcal{L}$ into a form having the usual structure of the kinetic-energy term and the mass term. Since the procedure is familiar,[22] we will only present the results without giving the details.

In terms of the unrenormalized field variables appearing in the Lagrangian $\mathcal{L}$, the physical ρ, A_1, K^*, and K_A particles are represented by the renormalized fields $\tilde{V}_\rho{}^\mu$, $\tilde{A}_{A_1}{}^\mu$, $\tilde{V}_{K^*}{}^\mu$, and $\tilde{A}_{K_A}{}^\mu$, respectively:

$$\tilde{V}_\rho{}^\mu = V_\rho{}^\mu, \tag{52}$$

$$\tilde{A}_{A_1}{}^\mu = A_{A_1}{}^\mu - (gf/M_{A_1}{}^2)(D^\mu\Pi)_\pi, \tag{53}$$

$$\tilde{V}_{K^*}{}^\mu = Z_{K^*}{}^{-1/2}\{V_{K^*}{}^\mu - Z_{K^*}(gf/M_{K^*}{}^2)(1+\eta) \times[(w-1)/2i](\mathfrak{D}^\mu\Sigma)_\kappa\}, \tag{54}$$

$$A_{K_A}{}^\mu = Z_{K_A}{}^{-1/2}\{A_{K_A}{}^\mu - Z_{K_A}(gf/M_{K_A}{}^2)(1+\eta) \times[\tfrac{1}{2}(1+w)](D^\mu\Pi)_K\}, \tag{55}$$

where

$$D^\mu\Pi \equiv \partial^\mu\Pi - (ig/\sqrt{2})[V^\mu, \Pi],$$
$$\mathfrak{D}^\mu\Sigma \equiv \partial^\mu\Sigma + (g/\sqrt{2})\{A^\mu, \Pi\}, \tag{56}$$

and the expressions for the masses and the wave-function renormalization constants will be given below. With the definition

$$\tilde{\Phi} \equiv Z^{-1/2}\Phi \tag{57}$$

for the spin-0 fields, we have

$$Z_{K^*} = Z_{K_A} = (1+\xi)^{-1}, \tag{58}$$

$$Z_\pi = [m^2 + (gf)^2]/m^2 = M_{A_1}{}^2/M_\rho{}^2, \tag{59}$$

$$Z_K = \{m^2 + (1+\eta)[\tfrac{1}{2}(1+w)]^2(gf)^2\} \times\{(1+\eta)[\tfrac{1}{2}(1+w)]^2 m^2\}^{-1}, \tag{60}$$

$$Z_\kappa = \{m^2 + (1+\eta)[\tfrac{1}{2}(w-1)]^2(gf)^2\} \times\{(1+\eta)[\tfrac{1}{2}(w-1)]^2 m^2\}^{-1}. \tag{61}$$

The masses of the various particles are given by

$$M_\rho{}^2 = M_\omega{}^2 = m^2, \tag{62}$$

$$M_\phi{}^2 = m^2/(1+2\xi), \tag{63}$$

$$M_{K^*}{}^2 = \{m^2 + (1+\eta)[\tfrac{1}{2}(w-1)]^2(gf)^2\}/(1+\xi), \tag{64}$$

$$M_{A_1}{}^2 = m^2 + (gf)^2, \tag{65}$$

$$M_{K_A}{}^2 = \{m^2 + (1+\eta)[\tfrac{1}{2}(w+1)]^2(gf)^2\}/(1+\xi), \tag{66}$$

$$M_\pi{}^2 = Z_\pi 2\sqrt{2}a/f, \tag{67}$$

$$M_K{}^2 = (Z_K/Z_\pi)[1 + (a'/2a)][\tfrac{1}{2}(1+w)]M_\pi{}^2, \tag{68}$$

$$M_\kappa = (Z_\kappa/Z_\pi)[a'/(2a)][\tfrac{1}{2}(w-1)]M_\pi{}^2. \tag{69}$$

We note that as $w\to 1$, $Z_\kappa{}^{-1/2}$ goes to zero and $M_\kappa{}^2\to\infty$. We shall come back to the mass formulas later. We also note that the mass formulas for ω and ϕ are obtained on the basis of nonet symmetry for the vector mesons.

VI. ADEMOLLO-GATTO THEOREM

That the isospin current is conserved implies that all isospin charge vertices are unchanged by the $SU(3)$-symmetry-breaking interactions. We would like to assure ourselves of this important property within our scheme. The conservation of isospin charge, then, does not set any further restriction on the parameters ξ, η, and w. We shall also demonstrate the Ademollo-Gatto theorem[14] and, as a by-product, obtain an explicit expression for the renormalization factor for the K_{l3}-decay coupling constant. We shall present a more detailed calculation of the K_{l3} form factors in Sec. X.

It is convenient to make use of the field equations. In terms of the unrenormalized field variables, the Lagrangian $\mathcal{L}$ gives rise to, among others, the following field equations:

$$\partial_\nu F_{12}{}^{\mu\nu} + m^2 V_{12}{}^\mu = (ig/\sqrt{2})[\Pi(\Delta^\mu\Pi) - (\Delta^\mu\Pi)\Pi]_{12} + \eta(ig/\sqrt{2})[\Pi_{13}(\Delta^\mu\Pi)_{32} - (\Delta^\mu\Pi)_{13}\Pi_{32}] + \cdots, \tag{70}$$

$$(1+\xi)\partial_\nu F_{13}{}^{\mu\nu} + m^2 V_{13}{}^\mu = (ig/\sqrt{2})[\Pi(\Delta^\mu\Pi) - (\Delta^\mu\Pi)\Pi]_{13} + \eta(ig/\sqrt{2})\Pi_{12}(\Delta^\mu\Pi)_{23} + (1+\eta)(1-w)(f/\sqrt{2})(ig/\sqrt{2})(\Delta^\mu\Sigma)_{13} + \cdots, \tag{71}$$

where we have neglected in both (70) and (71) terms that are not relevant for our purpose here. $\Delta^\mu\Pi$ and $\Delta^\mu\Sigma$ are defined according to (29) and (30). When the right-hand sides of (70) and (71) are expressed in terms of the renormalized field variables, (70) and (71) become (with $\tilde{\Pi}_{12} = \tilde{\pi}^+$, $\tilde{\Pi}_{13} = \tilde{K}^+$, etc.)

$$\partial_\nu F_{12}{}^{\mu\nu} + m^2 V_{12}{}^\mu = (ig/\sqrt{2})[\tilde{\Pi}(\partial^\mu\tilde{\Pi}) - (\partial^\mu\tilde{\Pi})\tilde{\Pi}]_{12} + \cdots, \tag{72}$$

$$(1+\xi)[\partial_\nu F_{13}{}^{\mu\nu} + M_{K^*}{}^2 V_{13}{}^\mu] = (ig/\sqrt{2})C_+(\tilde{\pi}^+\partial^\mu\tilde{K}^0 - \tilde{K}^0\partial^\mu\tilde{\pi}^+) + \cdots, \tag{73}$$

where

$$C_+ = \tfrac{1}{2}(\alpha^{-1}+\alpha) - [(gf)^2/2m^2](1+\eta)\tfrac{1}{2}(w-1) \times[\tfrac{1}{2}(1+w)]^{1/2}(\beta^{-1}-\beta), \tag{74}$$

$$\alpha^2 = [\tfrac{1}{2}(1+w)]^2 Z_K/Z_\pi, \tag{75}$$

$$\beta^2 = [\tfrac{1}{2}(1+w)]Z_K/Z_\pi. \tag{76}$$

In obtaining (72) and (73), use has been made of the expressions

$$(\Delta^\mu\Pi)_{12} = Z_\pi{}^{-1/2}\partial^\mu\tilde{\pi}^+ + \cdots, \text{ etc.}, \tag{77}$$

$$(\Delta^\mu\Pi)_{13} = \{(1+\eta)[\tfrac{1}{2}(1+w)]Z_K{}^{1/2}\}^{-1}\partial^\mu\tilde{K}^+ + \cdots, \text{ etc.}, \tag{78}$$

and a similar, but more complicated, expression for $(\Delta^\mu\Sigma)_{13}$. These expressions follow from (29), (30),

[21] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); S. L. Glashow, H. Schnitzer, and S. Weinberg, *ibid.* **19**, 139 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967).

[22] See, e.g., Refs. 8 and 11.

(55), and (60). In (73), we have also used the identity

$$AB+CD=\tfrac{1}{2}(A+C)(B+D)+\tfrac{1}{2}(A-C)(B-D) \quad (79)$$

and dropped a term of the form $\partial^\mu(\tilde{\pi}\tilde{K})$.

A spatial integration of the time component of (72) yields, in virtue of the field-current identity, the following expression for the isospin charge Q_{12}:

$$Q_{12}=(i/\sqrt{2})\int d^3x\,[\tilde{\Pi}(\partial^0\tilde{\Pi})-(\partial^0\tilde{\Pi})\tilde{\Pi}]_{12}+\cdots. \quad (80)$$

The isospin charge vertices for the 0^- mesons are thus seen to be unaltered by the $SU(3)$-symmetry-breaking interaction. One can similarly check that the hypercharge vertices are also not renormalized. Concerning the strangeness-changing vector vertices, we infer from (73) and the field-current identity that the $K\pi$ vertex, *at zero momentum transfer*, is effectively of the form

$$m^2[(1+\xi)M_{K^*}{}^2]^{-1}C_+(i/\sqrt{2})(\tilde{\pi}^+\partial^\mu\tilde{K}^0-\tilde{K}^0\partial^\mu\tilde{\pi}^+). \quad (81)$$

At zero momentum transfer, the renormalization factor is then given by

$$f_+(0)=C_+m^2/[(1+\xi)M_{K^*}{}^2]. \quad (82)$$

Since $w-1$, $\alpha-1$, and $\beta-1$ are all of first order in $SU(3)$-symmetry breaking, one easily sees that both factors in (78), i.e.,

$$C_+ \quad \text{and} \quad m^2/[(1+\xi)M_{K^*}{}^2],$$

are equal to 1 up to the first order in $SU(3)$-symmetry breaking, and, consequently, $f_+(0)$ is not renormalized up to the same order. This is the Ademollo-Gatto theorem.[14]

VII. TWO-BODY LEPTONIC DECAY CONSTANTS

The two-body leptonic decay constants can be obtained by expressing the weak currents in terms of the renormalized field variables. Using the relations (52)–(55), we obtain

$$j_{(\rho)}{}^\mu=(m^2/g)\tilde{V}_\rho{}^\mu, \quad (83)$$

$$j_{(K^*)}{}^\mu=(m^2/g)\{Z_{K^*}{}^{1/2}\tilde{V}_{K^*}{}^\mu+Z_{K^*}(gf/M_{K^*}{}^2)(1+\eta)\times[\tfrac{1}{2}(w-1)]^2Z_\kappa{}^{1/2}\partial^\mu\tilde{\kappa}+\cdots\}, \quad (84)$$

$$j_{(A_1)}{}^{5\mu}=(m^2/g)[\tilde{A}_{A_1}{}^\mu+(gf/M_{A_1}{}^2)Z_\pi{}^{1/2}\partial^\mu\tilde{\pi}+\cdots], \quad (85)$$

$$j_{(K_A)}{}^{5\mu}=(m^2/g)\{Z_{K_A}{}^{1/2}\tilde{A}_{K_A}{}^\mu+Z_{K_A}(gf/M_{K_A}{}^2)(1+\eta)\times[\tfrac{1}{2}(1+w)]^2Z_K{}^{1/2}\partial^\mu\tilde{K}+\cdots\}. \quad (86)$$

The various two-body leptonic decay constants are seen to be

$$F_\pi=(m^2/g)(gf/M_{A_1}{}^2)Z_\pi{}^{1/2}, \quad (87)$$

$$F_K=(m^2/g)Z_{K_A}(gf/M_{K_A}{}^2)(1+\eta)[\tfrac{1}{2}(1+w)]^2Z_K{}^{1/2}, \quad (88)$$

$$F_\kappa=(m^2/g)Z_{K^*}(gf/M_{K^*}{}^2)(1+\eta)[\tfrac{1}{2}(w-1)]^2Z_\kappa{}^{1/2}, \quad (89)$$

$$g_\rho=g_{A_1}=m^2/g, \quad (90)$$

$$g_{K^*}=g_{K_A}=(m^2/g)Z_{K^*}{}^{1/2}. \quad (91)$$

One can easily verify that the first Weinberg sum rules[21] of *all* possible combinations, e.g.,

$$g_\rho{}^2/M_\rho{}^2-g_{A_1}{}^2/M_{A_1}{}^2=F_\pi{}^2, \quad (92)$$

$$g_\rho{}^2/M_\rho{}^2-g_{K^*}{}^2/M_{K^*}{}^2=F_\kappa{}^2, \quad (93)$$

$$g_\rho{}^2/M_\rho{}^2-g_{K_A}{}^2/M_{K_A}{}^2=F_K{}^2, \text{ etc.}, \quad (94)$$

are explicitly satisfied. As we have remarked near the end of Sec. IV, all first Weinberg sum rules must necessarily be satisfied. They are the necessary consequences[19] of the singlet transformation property of the Schwinger terms. On the other hand, *not* all the second Weinberg sum rules are satisfied. Since Z_{K^*}, which is essentially determined by the K^* and ρ mass ratio, is different from 1, the valid ones are

$$g_\rho=g_{A_1},$$

$$g_{K^*}=g_{K_A}.$$

This demonstrates the dynamical nature of the second Weinberg sum rules. In a model where the field-current identity is satisfied, as in our present one, these sum rules are not necessarily valid.

It can be easily checked that[23]

$$F_\pi Z_\pi{}^{1/2}=F_KZ_K{}^{1/2}=F_\kappa Z_\kappa{}^{1/2}=f, \quad (95)$$

and

$$F_K/F_\pi=Z_\pi{}^{1/2}/Z_K{}^{1/2}, \quad (96)$$

$$F_\kappa/F_\pi=Z_\pi{}^{1/2}/Z_\kappa{}^{1/2}. \quad (97)$$

It is convenient, at this point, to make contact with the Glashow-Weinberg formula[10,24]

$$f_+(0)=(F_\pi{}^2+F_K{}^2-F_\kappa{}^2)/(2F_\pi F_K). \quad (98)$$

It is straightforward to show that, up to the second order in $SU(3)$-symmetry breaking, the relation (82) can be written in the form

$$f_+(0)=\frac{1}{2}\left(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\right)-\frac{1}{2}\left(\frac{w-1}{2}\right)\left[(1+\eta)\left(\frac{1+w}{2}\right)^2-1\right], \quad (99)$$

where use has been made of (59), (60), (95), and the well-satisfied relation

$$M_{A_1}{}^2=2M_\rho{}^2. \quad (100)$$

It can also be easily verified that to the same order in $SU(3)$-symmetry breaking,

$$F_\kappa{}^2/2F_\pi F_K=[\tfrac{1}{2}(w-1)]^2. \quad (101)$$

We combine (99) and (101) to obtain

$$f_+(0)=[(F_\pi{}^2+F_K{}^2-F_\kappa{}^2)/2F_\pi F_K]-\frac{1}{2}\left(\frac{w-1}{2}\right)\left[(1+\eta)\left(\frac{1+w}{2}\right)^2-1-2\left(\frac{w-1}{2}\right)\right]. \quad (102)$$

---

[23] Our difinitions for the Z's are different from those of Glashow and Weinberg (Ref. 10). This accounts for the conflicting appearences of our Eq. (91) and their Eq. (20).

[24] The formula (94) was also independently obtained by L. H. Chan (private communication).

This is our counterpart of the Glashow-Weinberg relation (98). It reduces to (98), up to the second order in $SU(3)$-symmetry breaking, if we set $\eta=0$. Thus if there is no "vector-mixing"-type symmetry breaking for the spin-0 mesons, i.e., $\eta=0$, the Glashow-Weinberg relation is obtained.

VIII. F_K/F_π AND $f_+(0)$

We shall invoke the mass relations (62)–(66) for an *approximate* determination of the parameters involved. We recall that the mass formula (63) for ϕ is obtained on the basis of the nonet symmetry, at the level of $SU(3)$, for the nine vector mesons. From (63), we estimate that

$$1+\xi\simeq 0.784. \tag{103}$$

It then follows from (64) and (65) that

$$(1+\eta)[\tfrac{1}{2}(w-1)]^2\simeq 0.07, \tag{104}$$

and from (65) and (66) that

$$(1+\eta)[\tfrac{1}{2}(1+w)]^2\simeq 1.20 \quad \text{for } M_{K_A}=1240\text{ MeV}$$
$$\simeq 1.52 \quad \text{for } M_{K_A}=1330\text{ MeV}. \tag{105}$$

The relation

$$F_K^2/F_\pi^2=Z_\pi/Z_K$$
$$=(1+\eta)[\tfrac{1}{2}(1+w)]^2[M_{A_1}^2/(1+\xi)M_{K_A}^2], \tag{106}$$

together with (99) and (101), then implies

$$F_K/F_\pi\simeq 1.04 \quad \text{for } M_{K_A}=1240\text{ MeV}$$
$$\simeq 1.10 \quad \text{for } M_{K_A}=1330\text{ MeV}. \tag{107}$$

It then follows from (95) that, up to the second order in $SU(3)$-symmetry breaking,

$$f_+(0)\simeq 0.99 \quad \text{for } M_{K_A}=1240\text{ MeV}$$
$$\simeq 0.96 \quad \text{for } M_{K_A}=1330\text{ MeV}. \tag{108}$$

To compare with experiment, we combine (103) with (104) to obtain

$$[F_K/F_\pi][1/f_+(0)]\simeq 1.05 \quad \text{for } M_{K_A}=1240\text{ MeV}$$
$$\simeq 1.15 \quad \text{for } M_{K_A}=1330\text{ MeV}. \tag{109}$$

Experimentally, the $K^+\to\pi^0+e^++\nu$ decay rate[15] implies

$$f_+(0)\ \sin\theta=0.226, \tag{110}$$

while the $K^+\to\mu^++\nu$ and $\pi^+\to\mu^++\nu$ decay rates[15] imply

$$(F_K/F_\pi)\ \tan\theta=0.275, \tag{111}$$

where θ is the Cabibbo angle, which, according to the Cabibbo theory, is a universal parameter. Combining (110) with (111) we obtain

$$(F_K/F_\pi)[1/f_+(0)]\simeq 1.19, \tag{112}$$

which is to be compared with our result (109). With $M_{K_A}=1330$ MeV, the agreement is good.

Our numerical estimate for $f_+(0)$ differs somewhat from those of Glashow and Weinberg.[10] Our result indicates that there is no excessive renormalization effect on $f_+(0)$, and the second-order $SU(3)$-symmetry-breaking effect is not intolerably large. This, certainly, is comforting. Otherwise, one would have to wonder why the Gell-Mann–Okubo mass formulas work so well.

From now on, we shall identify K_A to be $K_A(1320)$ and, correspondingly,

$$F_K/F_\pi\simeq 1.10, \qquad f_+(0)\simeq 0.96. \tag{113}$$

Corresponding to this identification, we also have

$$\tfrac{1}{2}(1+w)\simeq 1.28, \tag{114}$$

$$1+\eta\simeq 0.93, \tag{115}$$

and

$$F_\kappa^2/F_\pi^2\simeq 0.13. \tag{116}$$

The mass relations (68) and (69) then imply[25]

$$M_\kappa\simeq 660\text{ MeV}. \tag{117}$$

Since M_κ depends sensitively on the value of $\frac{1}{2}(w-1)$, the above value for M_κ cannot be taken literally. We note that our estimate of the value of $\frac{1}{2}(w-1)$ depends upon the assumed nonet symmetry for the vector mesons, as well as the value of M_ρ, which is not accurately known. A small violation of the nonet symmetry and a shift of the experimental value of M_ρ would considerably change the estimated value of M_κ. However, our estimates for F_K/F_π and $f_+(0)$ are not so sensitive to the value of $\frac{1}{2}(w-1)$.

Finally, the decay constants g_{K^*} and g_{K_A} are given by

$$g_{K^*}=g_{K_A}\simeq 1.13 g_\rho. \tag{118}$$

IX. VECTOR-MESON DECAYS

We shall give expressions for the effective vertices corresponding to the decays $\rho\to 2\pi$, $K^*\to K+\pi$, $A_1\to\rho+\pi$, and $K_A\to K^*+\pi$. These vertices can be obtained straightforwardly, although tediously, from the Lagrangian $\mathcal{L}$ by expressing it in terms of the renormalized field variables. We shall leave out the details and only present the final results.

On the mass shell, the effective Lagrangian term for

[25] A κ meson around the mass value of 1080 MeV was recently suggested by experiment. See T. G. Trippe, C. Y. Chien, E. Malamud, J. Mellema, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Letters **28B**, 203 (1968). Since we are unable to assess the significance of the interpretation of this experiment (having in mind the uncertainties with regard to the previously reported κ meson at 725 meV), we will refrain from taking it seriously. If the κ at 1080 MeV is confirmed by furture experiments, $M_\kappa=1080$ MeV can be used at imput for estimating the other parameters. In such a case, the $SU(3)$-symmetry-breaking effects will be still smaller than what are estimated in the present paper.

$\rho\to 2\pi$ can be written in the form

$$\mathcal{L}_{\rho\to 2\pi}=-g_{\rho\pi\pi}\tilde{\rho}^{\mu}\cdot\tilde{\pi}\times\partial_{\mu}\tilde{\pi}, \qquad (119)$$

with $g_{\rho\pi\pi}$ given by

$$g_{\rho\pi\pi}=g[1-(M_{A_1}^2-M_{\rho}^2)/2M_{A_1}^2-\delta(M_{\rho}^2/2M_{A_1}^2)]. \qquad (120)$$

Similarly, the coupling constant $g_{K^*K\pi}$, which is normalized so that $g_{K^*K\pi}\to g_{\rho\pi\pi}$ in the $SU(3)$-symmetry limit, is given by

$$g_{K^*K\pi}=[g/(1+\xi)^{1/2}]\times\left[C_{+}-\frac{F_K}{F_{\pi}}\frac{(1+\xi)M_{K^*}^2}{M_{\rho}^2}\frac{M_{A_1}^2-M_{\rho}^2}{2M_{A_1}^2}-\delta(1+\eta)^{-1}[\tfrac{1}{2}(1+w)]^{-1}\left(\frac{F_K}{F_{\pi}}\right)\left(\frac{M_{K^*}^2}{M_{\rho}^2}\right)\frac{M_{\rho}^2}{2M_{A_1}^2}\right], \qquad (121)$$

where C_{+} is defined by (74) and is equal to 1 up to the first order in $SU(3)$-symmetry breaking.

On the mass shell, the effective Lagrangian term for $A_1\to\rho+\pi$ can be written in the form

$$\mathcal{L}_{A_1\to\rho+\pi}=g_{A_1\rho\pi}M_{A_1}\tilde{\rho}_{\mu}\cdot\tilde{\pi}\times\tilde{A}^{\mu}+(g'_{A_1\rho\pi}/M_{A_1})(\partial_{\mu}\tilde{\rho}_{\nu}-\partial_{\nu}\tilde{\rho}_{\mu})\cdot\tilde{\pi}\times\partial^{\mu}\tilde{A}^{\nu}, \qquad (122)$$

where

$$g_{A_1\rho\pi}M_{A_1}=g(M_{\rho}/M_{A_1})(M_{A_1}^2-M_{\rho}^2)^{1/2}(1-\delta), \qquad (123)$$

$$g'_{A_1\rho\pi}/M_{A_1}=-g(M_{\rho}M_{A_1})^{-1}(M_{A_1}^2-M_{\rho}^2)^{1/2}\delta. \qquad (124)$$

Similarly, for $K_A\to K^*+\pi$ we have[26]

$$g_{K_AK^*\pi}M_{K_A}=g(M_{\rho}/M_{A_1})[(M_{A_1}^2-M_{\rho}^2)^{1/2}/(1+\xi)]\times w\{(1+\eta)-\delta/(1+\xi)-[\delta/(1+\xi)](1+\eta)\tfrac{1}{2}(w-1)\times\tfrac{1}{2}(w+1)[(M_{A_1}^2-M_{\rho}^2)/M_{\rho}^2]\}, \qquad (125)$$

$$g'_{K_AK^*\pi}/M_{K_A}=-g(w/M_{\rho}M_{A_1})[\delta/(1+\xi)](M_{A_1}^2-M_{\rho}^2)^{1/2}, \qquad (126)$$

which are normalized in such a way that they reduce to the corresponding expressions for $A_1\to\rho+\pi$ in the $SU(3)$ limit. The decay rates are given by the following formulas:

$$\Gamma(\rho\to 2\pi)=\tfrac{2}{3}(g_{\rho\pi\pi}^2/4\pi)\,|p|^3/M_{\rho}^2, \qquad (127)$$

$$\Gamma(K^*\to K+\pi)=(\tfrac{3}{4})(\tfrac{2}{3})(g_{K^*K\pi}^2/4\pi)\,|p|^3/M_{K^*}^2, \qquad (128)$$

$$\Gamma(A_1\to\rho+\pi)=\tfrac{1}{3}|p|\left(\frac{g_{A_1\rho\pi}^2}{4\pi}\right)\times\left\{2+\frac{(q_{\rho}\cdot q_{A_1})^2}{M_{A_1}^2M_{\rho}^2}+6\left(\frac{g'_{A_1\rho\pi}}{g_{A_1\rho\pi}}\right)\left(\frac{q_{\rho}\cdot q_{A_1}}{M_{A_1}^2}\right)+\left(\frac{g'_{A_1\rho\pi}}{g_{A_1\rho\pi}}\right)^2\left[\frac{M_{\rho}^2}{M_{A_1}^2}+2\frac{(q_{\rho}\cdot q_{A_1})^2}{M_{A_1}^4}\right]\right\}, \qquad (129)$$

$$\Gamma(K_A\to K^*+\pi)=(\tfrac{3}{8})\tfrac{1}{3}|p|\left(\frac{g_{K_AK^*\pi}^2}{4\pi}\right)\times\left\{2+\frac{(q_{K^*}\cdot q_{K_A})^2}{M_{K_A}^2M_{K^*}^2}+6\left(\frac{g'_{K_AK^*\pi}}{g_{K_AK^*\pi}}\right)\frac{q_{K^*}\cdot q_{K_A}}{M_{K_A}^2}+\left(\frac{g'_{K_AK^*\pi}}{g_{K_AK^*\pi}}\right)^2\left[\frac{M_{K^*}^2}{M_{K_A}^2}+2\frac{(q_{K^*}\cdot q_{K_A})^2}{M_{K_A}^4}\right]\right\}, \qquad (130)$$

where $|p|$ is the center-of-mass momentum of the daughter particles in the respective reactions, and the q are the four-momenta of the indicated particles.

Using the numerical estimates of the parameters obtained in Sec. VIII, we can express all the coupling constants of this section in terms of g and δ. We then obtain the following ratios of the decay widths:

$$\frac{\Gamma(K^*\to K+\pi)}{\Gamma(\rho\to 2\pi)}=0.344\left(\frac{1-0.43\delta}{1-0.33\delta}\right)^2, \qquad (131)$$

$$\frac{\Gamma(A_1\to\rho+\pi)}{\Gamma(\rho\to 2\pi)}=5.87\left(\frac{1-\delta}{3-\delta}\right)^2\times\left[3.13+4.50\left(\frac{2\delta}{1-\delta}\right)+1.63\left(\frac{2\delta}{1-\delta}\right)^2\right], \qquad (132)$$

$$\frac{\Gamma(K_A\to K^*+\pi)}{\Gamma(\rho\to 2\pi)}=7.46\left(\frac{1-1.2\delta}{3-\delta}\right)^2\times\left[3.09+4.35\left(\frac{3.25\delta}{1-1.2\delta}\right)+1.43\left(\frac{3.25\delta}{1-1.2\delta}\right)^2\right]. \qquad (133)$$

For $\Gamma(\rho\to 2\pi)=120$ MeV, a reasonable choice for δ is $\delta=-0.75$. The corresponding decay widths are the following:

$$\Gamma(\rho\to 2\pi)=120\text{ MeV},$$
$$\Gamma(K^*\to K+\pi)=47\text{ MeV},$$
$$\Gamma(A_1\to\rho+\pi)=72\text{ MeV},$$
$$\Gamma(K_A\to K^*+\pi)=102\text{ MeV}.$$

The ratios of the decay widths for various values of δ can be found in Table I.

TABLE I. Decay-rate ratios for a few different values of δ.

$-\delta$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
$\Gamma(K^*\to K+\pi)/\Gamma(\rho\to 2\pi)$	0.40	0.39	0.37	0.36	0.34
$\Gamma(A_1\to\rho+\pi)/\Gamma(\rho\to 2\pi)$	0.38	0.60	0.93	1.39	2.04
$\Gamma(K_A\to K^*+\pi)/\Gamma(\rho\to 2\pi)$	0.57	0.85	1.24	1.79	2.56

[26] One can easily write down the corresponding decay constants for $K_A\to\rho+K$. However, since the final phase space for this decay is small, one expects its branching ratio to be insignificant.

X. K_{l3} FORM FACTORS

In this section we present the calculation of the K_{l3} form factors, in the "tree approximation," on the basis of the Lagrangian and currents constructed in the previous sections. A detailed calculation of the K_{l4} form factors will be presented in a separate publication.

The interaction term responsible for the K_{l3} decay is of the form

$$\mathfrak{L}_{K_{l3}}=(G/\sqrt{2})\,\sin\theta l_\mu^{(-)}\sqrt{2}j_{(K^{*+})}{}^\mu+\text{H.c.},\qquad(134)$$

where G is the usual weak coupling constant, θ the Cabibbo angle, and l_μ the lepton current. According to the field-current identity (51), the strangeness-changing vector current is given by

$$j_{(K^*)}{}^\mu=(m^2/g)\,V_{K^*}{}^\mu,\qquad(135)$$

which, when expressed in terms of the renormalized fields representing the physical particles, becomes, according to (54),

$$j_{(K^*)}{}^\mu=(m^2/g)\{Z_{K^*}{}^{1/2}\tilde{V}_{K^*}{}^\mu+Z_{K^*}(gf/M_{K^*}{}^2)(1+\eta)\times[(w-1)/2i](\mathfrak{D}^\mu\Sigma)_\kappa\},\qquad(136)$$

where

$$\mathfrak{D}^\mu\Sigma\equiv\partial^\mu\Sigma+(g/\sqrt{2})\{A^\mu,\Pi\}.\qquad(137)$$

The field variables in (137) are still to be expressed in terms of the relevant renormalized fields, with the help of (22), (53), etc. For $K^+\to\pi^0+l^++\nu$, the relevant terms are

$$(g/m^2)j_\mu{}^{(K^{*\dagger})}=Z_{K^*}{}^{1/2}\tilde{K}_\mu{}^{*+}+(g/m^2)F_\kappa\partial_\mu\tilde{\kappa}^++(gf/m)[Z_\kappa\tfrac{1}{2}(w-1)i]^{-1}\Big[-\frac{1}{\sqrt{2}f}\tfrac{1}{4}(w+3)Z_\pi{}^{1/2}Z_K{}^{1/2}\partial_\mu\Big(\frac{\tilde{\pi}^0}{\sqrt{2}}\tilde{K}^+\Big)$$
$$+\Big(\frac{g}{\sqrt{2}}\Big)\Big(\frac{gf}{m^2}\Big)\Big(\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\tilde{K}^+\partial_\mu\frac{\tilde{\pi}^0}{\sqrt{2}}+\frac{F_K}{F_\pi}\frac{\tilde{\pi}^0}{\sqrt{2}}\partial_\mu\tilde{K}^+\Big)\Big]+\cdots.\qquad(138)$$

We also need the coupling terms for $K^*K\pi$ and $\kappa K\pi$ vertices, which are

$$\mathfrak{L}_{K^{*-}K^+\pi^0}=i\frac{g}{\sqrt{2}}Z_{K^*}{}^{1/2}\tilde{K}_\mu{}^{*-}\Big[\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}\frac{\tilde{\pi}^0}{\sqrt{2}}\partial^\mu\tilde{K}^+-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\tilde{K}^+j^\mu\frac{\tilde{\pi}^0}{\sqrt{2}}\Big]$$
$$+i\frac{g}{\sqrt{2}}\frac{Z_{K^*}{}^{1/2}}{m^2}\Big\{(1+\eta)[\tfrac{1}{2}(w-1)]\Big(\frac{gf}{m}\Big)^2\Big(\frac{m^2}{M_{K^*}{}^2}\Big)\Big(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\Big)$$
$$+\frac{m^2}{M_A{}^2}\frac{F_K}{F_\pi}\Big[(1+\xi)\Big(\frac{gf}{m}\Big)^2+\frac{\delta}{1+\eta}[\tfrac{1}{2}(1+w)]^{-1}\Big]\Big\}\tilde{K}_{\mu\nu}{}^{*-}\partial^\mu\frac{\tilde{\pi}^0}{\sqrt{2}}\partial^\nu\tilde{K}^+\qquad(139)$$

and

$$\mathfrak{L}_{\kappa^+K^-\pi^0}=\frac{i}{f^2}\tfrac{1}{2}(w-1)\tfrac{1}{2}(4a+a')Z_K{}^{1/2}Z_\pi{}^{1/2}Z_\kappa{}^{1/2}\tilde{\kappa}^-\tilde{K}^+\frac{\tilde{\pi}^0}{\sqrt{2}}$$
$$-\frac{i}{\sqrt{2}f}\tfrac{1}{2}(w-1)\Big(\frac{gf}{m}\Big)^2Z_\kappa{}^{1/2}Z_\pi{}^{-1/2}Z_K{}^{-1/2}[\tfrac{1}{4}(w+3)][\tfrac{1}{2}(1+w)]^{-1}\tilde{\kappa}^-\partial_\mu\tilde{K}^+\partial^\mu\frac{\tilde{\pi}^0}{\sqrt{2}}$$
$$-\frac{i}{\sqrt{2}f}Z_\kappa{}^{-1/2}[\tfrac{1}{2}(w-1)]^{-1}\partial^\mu\tilde{\kappa}^-\Big[\tfrac{1}{4}(w+3)Z_\pi{}^{1/2}Z_K{}^{1/2}\partial_\mu\Big(\frac{\tilde{\pi}^0}{\sqrt{2}}\tilde{K}^+\Big)-\Big(\frac{gf}{m}\Big)^2\Big(\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\tilde{K}^+\partial_\mu\frac{\tilde{\pi}^0}{\sqrt{2}}+\frac{F_K}{F_\pi}\frac{\tilde{\pi}^0}{\sqrt{2}}\partial_\mu\tilde{K}^+\Big)\Big]$$
$$+\frac{i}{\sqrt{2}f}\Big\{Z_\kappa{}^{1/2}\tfrac{1}{2}(w-1)\Big[\frac{F_\pi}{F_K}\partial_\mu\frac{\tilde{\pi}^0}{\sqrt{2}}\partial^\mu(\tilde{K}^+\tilde{\kappa}^-)+\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}\partial_\mu\tilde{K}^+\partial^\mu\Big(\frac{\tilde{\pi}^0}{\sqrt{2}}\tilde{\kappa}^-\Big)\Big]$$
$$+\Big(\frac{gf}{m}\Big)^2Z_\kappa{}^{-1/2}\partial_\mu\tilde{\kappa}^-\Big[\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}\frac{\tilde{\pi}^0}{\sqrt{2}}\partial^\mu\tilde{K}^+-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\tilde{K}^+\partial^\mu\frac{\tilde{\pi}^0}{\sqrt{2}}\Big]\Big\}.\qquad(140)$$

From the structure of the terms contained in (138), it is clear that the hadron part of the K_{l3}-decay matrix element consists of three terms: a K^*-pole term, a κ-pole term, and a contact term. With the usual definition of the $f_\pm(q^2)$ form factors

$$\langle\pi^0|\sqrt{2}j_{(K^{*+})}{}^\mu(0)|K^+\rangle=(1/\sqrt{2})[(p_K+p_\pi)^\mu f_+(q^2)+(p_K-p_\pi)^\mu f_-(q^2)],\qquad q\equiv p_K-p_\pi,\qquad(141)$$

the contributions from these terms to the form factors $f_\pm(q^2)$ can be calculated straightforwardly. They are listed in the Appendix. Collecting and making the usual linear approximation

$$f_\pm(q^2)=f_\pm(0)(1-\lambda_\pm q^2/M_\pi{}^2),\qquad(142)$$

we obtain

$$f_{\pm}(0)=Z_{K^*}(m^2/M_{K^*}{}^2)C_{\pm},$$

$$\lambda_+=\left(\frac{M_\pi}{M_{K^*}}\right)^2\left\{1-\frac{1}{2f_+(0)}\frac{F_K}{F_\pi}\left(\frac{m}{M_{A_1}}\right)^2\left[\left(\frac{gf}{m}\right)^2+\frac{\delta}{(1+\eta)(1+\xi)(1+w)/2}\right]\right\},$$

$$\lambda_-=\frac{\lambda_+}{\xi(0)}\frac{M_K{}^2-M_\pi{}^2}{M_\pi{}^2}\left[\left(\frac{M_\pi}{M_\kappa}\right)^2-\left(\frac{M_\pi}{M_{K^*}}\right)^2\right]+\left(\frac{M_\pi}{M_\kappa}\right)^2-\frac{1}{C_-}\left(\frac{M_\pi}{M_\kappa}\right)^2\left\{\frac{1}{2}\left[[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}-\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}\right](1+\xi)\left(\frac{M_{K_A}}{m}\right)^2\right.$$
$$\left.+\tfrac{1}{2}(w-1)\tfrac{1}{4}(w+3)[\tfrac{1}{2}(1+w)]^{-1}\left(\frac{gf}{m}\right)^2\left(\frac{M_{A_1}}{m}\right)^2\frac{F_K}{F_\pi}\right\},$$

$$\xi(0)=f_-(0)/f_+(0)=C_-/C_+,$$

$$C_+=\frac{1}{2}\left[[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}+\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}\right]-(1+\eta)\tfrac{1}{2}(w-1)\left(\frac{gf}{m}\right)^2\frac{1}{2}\left[\frac{F_K}{F_\pi}-\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}\right],$$

$$C_-=\frac{1}{2}\left[[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}-\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}\right]-C_+\lambda_+\frac{(M_K{}^2-M_\pi{}^2)}{M_\pi{}^2}+(1+\eta)\tfrac{1}{2}(w-1)\tfrac{1}{4}(w+3)\left(\frac{M_{A_1}}{m}\right)^2\frac{F_\pi}{F_K}$$
$$-(1+\eta)[\tfrac{1}{2}(w-1)]\left(\frac{gf}{m}\right)^2\left\{\frac{1}{2}\left[\frac{F_K}{F_\pi}+\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}\right]+\frac{M_K{}^2-M_\pi{}^2}{M_\kappa{}^2}\frac{1}{2}\left[\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\right]\right.$$
$$\left.+\left(\frac{M_K{}^2-M_\pi{}^2}{M_\kappa{}^2}\right)\tfrac{1}{2}(w-1)\frac{1}{2}\left[\tfrac{1}{2}(1+w)\frac{F_\pi}{F_K}+[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}\right]\right\}$$
$$+\tfrac{1}{2}(w-1)Z_{K^*}{}^{-1}\left(\frac{M_{K^*}}{m}\right)^2\left\{\frac{4a+a'}{4a}\left(\frac{M_\pi}{M_\kappa}\right)^2\frac{F_\pi}{F_K}-\left[\frac{F_\pi}{F_K}\left(\frac{M_\pi}{M_\kappa}\right)^2+[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}\left(\frac{M_K}{M_\kappa}\right)^2\right]\right.$$
$$\left.+\left(\frac{M_K{}^2+M_\pi{}^2}{M_\kappa{}^2}\right)\tfrac{1}{4}(w+3)[\tfrac{1}{2}(1+w)]^{-1}\left(\frac{gf}{m}\right)^2\left(\frac{M_{A_1}}{m}\right)^2\frac{F_K}{F_\pi}\right\}$$

The parameters in these equations are those defined in the previous sections. Their estimated values have also been given. With these as inputs, the following numerical values are obtained:

$$f_+(0)\simeq 0.96,\qquad \xi(0)\simeq -0.048,$$

$$\lambda_+\simeq 0.022,\qquad \lambda_-\simeq 0.051.$$

These values agree, in general, with previous calculations by Lee,[27] and Nieh.[28] Experimentally,[29] little is known about λ_-, and there is much controversy over the value of $\xi(0)$. The situation with λ_+ is somewhat better. The weighted averages from K^+ and K_L decays are[29]

$$\lambda_+=0.029\pm0.010\qquad (K^+\text{ decays}),$$

$$\lambda_+=0.019\pm0.008\qquad (K_L\text{ decays}).$$

From the present calculation, and from previous calculations based on various methods,[30] the parameter $\xi(0)$ invariably comes out small. It will be of great interest to see this confirmed eventually by experiment.

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APPENDIX

In this appendix we present the contributions to $f_{\pm}$ form factors from the K^*-pole term, the contact term, and the κ-pole term separately. The contribution from the K^*-pole term is

$$f_+:\quad Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2 \frac{1}{1+q^2/M_{K^*}{}^2}\bigg\{\frac{1}{2}\bigg[\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)+\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}\bigg]$$

$$+\frac{q^2}{m^2}\bigg[(1+\eta)\tfrac{1}{2}(w-1)\bigg(\frac{gf}{m}\bigg)^2\bigg(\frac{m}{M_{K^*}}\bigg)^2\frac{1}{2}\bigg(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg)\bigg]$$

$$+\frac{q^2}{m^2}(1+\xi)\bigg(\frac{m^2}{M_{A_1}{}^2}\bigg)\frac{F_K}{F_\pi}\bigg[\bigg(\frac{gf}{m}\bigg)^2+\frac{\delta}{(1+\eta)(1+\xi)(1+w)/2}\bigg]\bigg\};$$

$$f_-:\quad Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2\bigg\{\frac{1}{2}\bigg[\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg]$$

$$-\bigg(\frac{M_K{}^2-M_\pi{}^2}{M_{K^*}{}^2}\bigg)\frac{1}{1+q^2/M_{K^*}{}^2}\frac{1}{2}\bigg[\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)+\frac{F_K}{F_\pi}[\tfrac{1}{2}(1+w)]^{-1}\bigg]\bigg\}$$

$$+Z_{K^*}\bigg(\frac{M_K{}^2-M_\pi{}^2}{M_{K^*}{}^2}\bigg)\frac{1}{1+q^2/M_{K^*}{}^2}\bigg\{(1+\eta)\tfrac{1}{2}(w-1)\bigg(\frac{gf}{M_{K^*}}\bigg)^2\frac{1}{2}\bigg[\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg]$$

$$+(1+\xi)\bigg(\frac{m}{M_{A_1}}\bigg)^2\frac{F_K}{F_\pi}\bigg[\bigg(\frac{gf}{m}\bigg)^2+\frac{\delta}{(1+\eta)(1+\xi)(1+w)/2}\bigg]\bigg\}.$$

The contribution from the contact term is

$$f_+:\quad Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2(1+\eta)\tfrac{1}{2}(w-1)\bigg(\frac{gf}{m}\bigg)^2\frac{1}{2}\bigg[\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)-\frac{F_K}{F_\pi}\bigg];$$

$$f_-:\quad Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2(1+\eta)\tfrac{1}{2}(w-1)\bigg\{Z_\pi{}^{1/2}Z_K{}^{1/2}\tfrac{1}{4}(w+3)-\bigg(\frac{gf}{m}\bigg)^2\frac{1}{2}\bigg[\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)+\frac{F_K}{F_\pi}\bigg]\bigg\}.$$

The contribution from the κ-pole term is

f_+: 0;

$$f_-:\quad Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2(1+\eta)\tfrac{1}{2}(w-1)\frac{1}{1+q^2/M_\kappa{}^2}\bigg\{\bigg(\frac{gf}{m}\bigg)^2\bigg(\frac{M_\pi{}^2-M_K{}^2}{M_\kappa{}^2}\bigg)$$

$$\times\bigg[\frac{1}{2}\bigg(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg)+\tfrac{1}{4}(w-1)\bigg(\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)+\frac{F_K}{F_\pi}\frac{2}{1+w}\bigg)\bigg]$$

$$-\frac{q^2}{M_\kappa{}^2}\bigg[\tfrac{1}{4}(w+3)Z_\pi{}^{1/2}Z_K{}^{1/2}-\bigg(\frac{gf}{m}\bigg)^2\frac{1}{2}\bigg(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg)-\tfrac{1}{4}(w-1)\bigg(\frac{gf}{m}\bigg)^2\bigg(\frac{F_K}{F_\pi}\frac{2}{1+w}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg)\bigg]\bigg\}$$

$$+\tfrac{1}{2}(w-1)\frac{1}{1+q^2/M_\kappa{}^2}\bigg[\tfrac{1}{4}(w+3)[\tfrac{1}{2}(1+w)]^{-1}\bigg(\frac{gf}{m}\bigg)^2\frac{F_K}{F_\pi}\bigg(\frac{M_{A_1}}{m}\bigg)^2\bigg(\frac{M_K{}^2+M_\pi{}^2+q^2}{M_\kappa{}^2}\bigg)$$

$$+\bigg(\frac{a'}{4a}\bigg)\frac{F_\pi}{F_K}\bigg(\frac{M_\pi}{M_\kappa}\bigg)^2-[\tfrac{1}{2}(1+w)]^{-1}\frac{F_K}{F_\pi}\bigg(\frac{M_K}{M_\kappa}\bigg)^2\bigg].$$

Combining the contributions, we get

$$f_+(q^2)=Z_{K^*}\bigg(\frac{m}{M_{K^*}}\bigg)^2\frac{1}{1+q^2/M_{K^*}{}^2}\bigg\{\frac{1}{2}\bigg[\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)+\frac{F_K}{F_\pi}\bigg(\frac{2}{1+w}\bigg)\bigg]$$

$$-(1+\eta)\tfrac{1}{2}(w-1)\bigg(\frac{gf}{m}\bigg)^2\frac{1}{2}\bigg[\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\bigg]+\frac{1}{2}\frac{q^2}{M_{A_1}{}^2}\frac{F_K}{F_\pi}(1+\xi)\bigg[\bigg(\frac{gf}{m}\bigg)^2+\frac{\delta}{(1+\eta)(1+\xi)(1+w)/2}\bigg]\bigg\},$$

$$
\begin{aligned}
f_-(q^2) = {} & -\frac{M_K{}^2-M_\pi{}^2}{M_{K^*}{}^2} f_+(q^2) + Z_{K^*}\left(\frac{m}{M_{K^*}}\right)^2 \Bigg\{\frac{1}{2}\left[\frac{F_K}{F_\pi}\left(\frac{2}{1+w}\right) - \frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\right] \\
& + \frac{M_K{}^2-M_\pi{}^2}{2M_{A_1}{}^2}\frac{F_K}{F_\pi}(1+\xi)\left[\left(\frac{gf}{m}\right)^2 + \frac{\delta}{(1+\eta)(1+\xi)(1+w)/2}\right]\Bigg\} \\
& + Z_{K^*}\left(\frac{m}{M_{K^*}}\right)^2 (1+\eta)\tfrac{1}{2}(w-1)\,\frac{1}{1+q^2/M_\kappa{}^2}\Bigg\{\tfrac{1}{4}(w+3)Z_\pi{}^{1/2}Z_K{}^{1/2} - \left(\frac{gf}{m}\right)^2\frac{1}{2}\left(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\right) \\
& + \tfrac{1}{4}(w-1)\left(\frac{F_K}{F_\pi}\frac{2}{1+w} - \frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\right)\frac{q^2}{M_\kappa{}^2} - \left(\frac{M_K{}^2-M_\pi{}^2}{M_\kappa{}^2}\right)\left(\frac{gf}{m}\right)^2\Bigg[\frac{1}{2}\left(\frac{F_K}{F_\pi} - \frac{F_\pi}{F_K}\tfrac{1}{2}(1+w)\right) \\
& + \tfrac{1}{4}(w-1)\left(\frac{F_\pi}{F_K}\tfrac{1}{2}(1+w) + \frac{F_K}{F_\pi}\frac{2}{1+w}\right)\Bigg]\Bigg\} + \left(\frac{w-1}{2}\right)\frac{1}{1+q^2/M_\kappa{}^2}\Bigg[\left(\frac{a'}{4a}\right)\frac{F_\pi}{F_K}\left(\frac{M_\pi}{M_\kappa}\right)^2 - \frac{2}{1+w}\frac{F_K}{F_\pi}\left(\frac{M_K}{M_\kappa}\right)^2 \\
& + \tfrac{1}{4}(w+3)\left(\frac{2}{1+w}\right)\left(\frac{gf}{m}\right)^2\frac{F_K}{F_\pi}\left(\frac{M_{A_1}}{m}\right)^2\left(\frac{M_K{}^2+M_\pi{}^2+q^2}{M_\kappa{}^2}\right)\Bigg].
\end{aligned}
$$

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Regge-Pole Eikonal Theory for Small-Angle Nucleon-Nucleon and Antinucleon-Nucleon Scattering*

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We present an eikonal model for the small-angle high-energy scattering of nucleons and antinucleons on nucleon targets. The model uses a flat Pomeranchon with a residue function given by a squared dipole. We also use exchange-degenerate trajectories and residue functions for the three pairs of Regge poles (the ω and P', the ρ and A_2, and the π and B). Using previous work by Arnold and Blackmon and other work by the present authors to fix trajectories and relative sizes of residue functions, we find that the model satisfactorily describes elastic scattering. We give an interpretation of the secondary maximum in the differential cross section of $\bar{p}p$ scattering which occurs around $-t\simeq 0.9$ GeV^2. We find a crossover in the differential cross sections of pp and $\bar{p}p$ elastic scattering, and we show our results for various polarizations. We also discuss $np\to pn$ and $\bar{p}p\to\bar{n}n$, which are not described satisfactorily by the model.

I. INTRODUCTION

A LARGE amount of work on models for high-energy scattering which incorporate absorptive corrections to Regge poles has been reported.[1–6] (For a recent discussion of the difficulties of pure Regge-pole models and the improvements owing to absorptive corrections, see Ref. 7.) In this paper, we discuss an eikonal model for nucleon-nucleon and antinucleon-nucleon scattering. Previous work by Arnold and Blackmon[1,2] and by the present authors[3] is used to give constraints on the parameters of the Regge poles. In particular, the trajectories we use are fixed from previous calculations. The ratio of helicity-flip to helicity-nonflip couplings is also fixed.

Other papers[4,8,9] have discussed NN and $\bar{N}N$ reac-

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