inelastic effects in pion-deuteron scattering are probably negligible near the forward direction at very high energies. At large angles, however, the inelastic contributions significantly increase the differential cross section and smooth out the structure (dips in our calculations) that would be present in pion-deuteron scattering, due to the structure in pion-nucleon scattering, if only elastic intermediate states contributed. Including corrections from only low-lying excited states of the pion gives considerably more accurate results at intermediate angles and seems to eliminate most of the spurious structure.

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Nonleptonic Decays of the Ω^- in the Current-Current Model*

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The decuplet parity-conserving weak spurion has been evaluated in the Sugawara-Suzuki model by assuming that the relevant matrix elements, in a current×current model, are saturated by baryon octet and decuplet single-particle intermediate states. $SU(6)$ symmetry has been used to determine the decuplet form factors at zero momentum transfer. In this approach, octet dominance for the decuplet spurion emerges in a natural manner. The P-wave Ω^- decay amplitudes are obtained with the baryon octet and decuplet spurion strengths as input. By assuming that the D-wave amplitudes for $\Omega^- \to \Lambda K$, $\Xi\pi$ decays are given by the K^*-pole model, we predict a zero asymmetry parameter for $\Omega^- \to \Lambda K$ and a maximal one ($\alpha \approx 1$) for $\Omega^- \to \Xi\pi$ decays. The total decay rate of the Ω^- is estimated to be $\Gamma(\Omega^-) \cong 8 \times 10^9$ sec^{-1}, which is in good agreement with experiment.

I. INTRODUCTION

THE use of current algebra and a current×current model for weak interactions has allowed an evaluation of S- and P-wave decay amplitudes for the nonleptonic decays of the baryons. For S waves the Sugawara-Suzuki[1] model relates the parity-violating (p.v.) amplitudes to single-particle matrix elements of the weak Hamiltonian. As suggested by Sugawara,[1] these matrix elements have been evaluated[2,3] by inserting a complete set of single-particle states between the currents. With the assumption that the sum is saturated by baryon octet and decuplet states, it is possible to show that (1) the amplitudes are in reasonable agreement with experiment, (2) the evaluated d/f ratio for the weak Hamiltonian is close to the phenomenologically determined value, and (3) octet dominance for the matrix elements emerges in a natural manner.

Here we wish to study the weak decays of the Ω^- particle in an analogous manner. In order to carry out a similar program of evaluation it is necessary to know the octet-decuplet baryon (B_8-B_{10}) and the decuplet-decuplet (B_{10}-B_{10}) form factors for vector and axial-vector currents. For the B_8-B_{10} form factors experimental data are scanty, and for the B_{10}-B_{10} form factors no experimental information is available at present. Guided by the work of Chiu, Schechter, and Ueda,[2] we use $SU(3)$ symmetry to generalize and to extend the experimental information on the B_8-B_{10} factors, and we use $SU(6)$ symmetry to predict the B_{10}-B_{10} form factors.

In Sec. II, we give the form factors used in this calculation. The decuplet spurion, by which we mean the matrix elements of symmetrized products of currents for decuplet initial and final particle states, has been evaluated in Sec. III. This has been done by the usual method of putting in B_8 and B_{10} single-particle intermediate states and assuming that these states alone saturate the matrix elements. We show that octet dominance for the decuplet spurion emerges from the calculation, without any assumptions regarding it being made at the beginning. This result parallels the corresponding one for the matrix elements for the octet baryon states, and arises because of a cancella-

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[1] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); M. Suzuki, *ibid.* **15**, 968 (1965).

[2] Y. T. Chiu and J. Schechter, Phys. Rev. Letters **16**, 1022 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966).

[3] S. Biswas, Aditya Kumar, and R. Saxena, Phys. Rev. Letters **17**, 268 (1966); Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 710 (1967).

tion between the B_8 and B_{10} intermediate-state contributions.

These results have been used in Sec. IV to predict the S-wave p.v. decay amplitudes for $\Omega^- \to \Xi^*\pi$ decays and the P-wave amplitudes for $\Omega^- \to \Lambda K^-$, $\Xi\pi$ decays. We obtain a value for $P(\Omega^- \to \Xi^0\pi^-)$ which is 20% of that for $P(\Omega^- \to \Lambda K^-)$. We evaluate the parity-conserving (p.c.) amplitudes with octet and decuplet baryon poles in the usual manner.[4]

The asymmetry parameters and decay rates of the nonleptonic Ω^- decay modes have been estimated by assuming that the D-wave p.v. amplitudes are given by the K^*-pole model. The estimate for the total decay rate is in good agreement with the experimental value. We obtain $\alpha(\Omega^-{}_K) \simeq 0$ and $\alpha(\Omega^-{}_-) \simeq 1$.

II. FORM FACTORS

The form factors used in our analysis are given below. The assumptions that have been made regarding the B_8-B_8 and the B_8-B_{10} form factors closely parallel those of Ref. 2.

The form factors of the octet baryons have been studied in considerable detail both experimentally and theoretically. The use of the conserved-vector-current (CVC) hypothesis[5] relates unknown vector form factors to the electromagnetic form factors. For the axial-vector currents, we use the Nambu[6] form of partial conservation of axial-vector current (PCAC) and assume that the momentum dependence of the one unknown form factor is the same as that for the vector form factors. In Ref. 2, use has been made of invariance under chiral $SU(3)\otimes SU(3)$ for the currents to relate the vector and axial-vector form factors. This assumption does not find strong experimental support.[7] We have assumed a simpler momentum dependence for the axial-vector form factor.

We thus have

$$\langle B_8(p^1)|V_{\mu b}{}^a|B_8(p)\rangle = \left(\frac{MM^1}{p_0p_0{}^1}\right)^{1/2} \bar{u}(p^1) \times \left\{ i\gamma\mu[F^V(q^2)]_b{}^a - \frac{(P^1+p)_\mu}{2M}[G^V(q^2)]_b{}^a \right\} u(p) \quad (1a)$$

and

$$\langle B_8(p^1)|A_{\mu b}{}^a|B_8(p)\rangle = \left(\frac{MM^1}{p_0p_0{}^1}\right)^{1/2} \bar{u}(p^1) \times \left(i\gamma_\mu\gamma_5 - \frac{2Mq_\mu\gamma_5}{q^2+m_\pi{}^2} \right) [G^A(q^2)]_b{}^a u(p) , \quad (1b)$$

where

$$[F^V(q^2)]_b{}^a = G_E(q^2)\mu_p[(1-\tfrac{1}{2}\beta)F_b{}^a + \tfrac{3}{2}\beta D_b{}^a] , \quad (1c)$$

$$[G^V(q^2)]_b{}^a = (1+q^2/4M^2)^{-1}G_E(q^2) \times\{[-1+\mu_p(1-\tfrac{1}{2}\beta)]F_b{}^a + \tfrac{3}{2}\mu_p\beta D_b{}^a\} , \quad (1d)$$

$$[G^A(q^2)]_b{}^a = 1.18G_E(q^2)(D_b{}^a+\beta F_b{}^a) , \quad (1e)$$

$$G_E(q^2) = (1+q^2/0.71)^{-2} , \quad (1f)$$

with $D_b{}^a$ and $F_b{}^a$ being the usual symmetric and antisymmetric $SU(3)$ matrices; the total magnetic moment of the proton μ_p has the value $\mu_p = 2.79$, and β is the f/d ratio for the currents. The $SU(6)$ prediction for the f/d ratio is $\beta = \frac{2}{3}$ and the same value is obtained from experiment. The form factor $G_E(q^2)$ is given by experiment.[8]

The B_8-B_{10} form factors are obtained from the Gourdin-Salin[9] isobaric model for pion-photoproduction in the case of vector currents, and from the analysis of neutrino-N^* production reactions by Albright and Liu[10] for the axial-vector currents.

With our normalization[11] for the decuplet $SU(3)$ wave functions, we have

$$\langle B_{10}|V_{\mu b}{}^a|B_8\rangle = \left(\frac{MM^*}{p_0p_0{}^1}\right)^{1/2} \bar{\psi}_\alpha(p^1)^{iak}\epsilon_{klb}\sqrt{3} \times[\delta_{\alpha\mu}f_1(q^2) + ip_\alpha\gamma_\mu f_2(q^2)]\gamma_5 B_i{}^l \quad (2a)$$

and

$$\langle B_{10}|A_{\mu b}{}^a|B_8\rangle = \left(\frac{MM^*}{p_0p_0{}^1}\right)^{1/2} \bar{\psi}_\alpha(p^1)^{iak}\epsilon_{klb} \times\sqrt{3}\delta_{\alpha\mu}g_1(q^2)B_i{}^l , \quad (2b)$$

where we have retained only the dominant form factors. We use

$$f_1(q^2) = 5.6(1+q^2/b)^{-2} , \quad (2c)$$

$$f_2(q^2) = \frac{-5.6}{M+M^*}(1+q^2/b)^{-2}(1+q^2/4M^2)^{-1} , \quad (2d)$$

$$g_1(q^2) = -0.87(1+q^2/b)^{-2} . \quad (2e)$$

The matrix elements of symmetrized products of currents for initial and final octet baryon states show octet dominance quite apart from the details of the form factors[2,3] used in evaluating them. In order to obtain a finite contribution with decuplet intermediate states, it is necessary to introduce a cutoff factor of $(1+q^2/4M^2)^{-1}$ in the vector form factor $f_2(q^2)$. The cutoff factor ensures that the major contribution to integrals comes from the region of low momentum

[4] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* **16**, 751 (1966); S. Badier and C. Bouchait, Phys. Letters **20**, 529 (1966).

[5] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. Gerstein and Ya. Zeldovich, Zh. Eksperim. i Teor. Fiz. **2**, 576 (1956) [English transl.: Soviet Phys.—JETP **29**, 698 (1955)].

[6] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

[7] W. A. Simmons, Phys. Rev. **164**, 1956 (1967).

[8] L. Chan, K. Chen, J. Dunning, Jr., N. Ramsey, J. Walker, and R. Wilson, Phys. Rev. **141**, 1298 (1966).

[9] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).

[10] C. H. Albright and L. S. Liu, Phys. Rev. **140**, B748 (1965); **140**, B1611 (1965).

[11] Our phase conventions and normalizations are the same as the ones given in A. Pais, Rev. Mod. Phys. **38**, 215 (1966.) Thus, we use $N^{*++} = \psi^{111}$, $N^{*+} = \sqrt{3}\psi^{112}$, $Y_1{}^{*0} = (\sqrt{6})\psi^{123}$, etc.

transfer. One expects the form factors to fall off to zero as $q^2 \to \infty$, and the inclusion of such a cutoff factor seems to be a reasonable assumption.

The parameter b in Eq. (2) is roughly 0.71, a value which is empirically found to be universal for all form factors. However, on evaluation of the nonleptonic baryon decay amplitudes[2] it is found that a lower value gives a better fit to the experimental data. We use $b=0.5$ and the central values for the octet and decuplet particle masses: $M=1.14$ BeV and $M^*=1.4$ BeV, respectively.

One could have used $SU(6)$ symmetry[12] in determining the above form factors at zero momentum transfer. When the baryons are assigned to the 56-dimensional respresentation of $SU(6)$, it is possible to correlate the properties of the octet and decuplet baryons. Static $SU(6)$ symmetry predicts the d/f ratio of the B_8-B_8 vector and axial-vector currents, the ratio of the magnetic moments of the proton and the neutron, and the vector and axial-vector B_8-B_{10} form factors at zero momentum transfer; all these predictions are close to the experimentally measured values. For the decuplet (B_{10}-B_{10}) form factors there is a complete absence of experimental data and the use of a higher symmetry group becomes essential. We use $SU(6)$ symmetry, specifically for estimating the B_{10}-B_{10} form factors at zero momentum transfer.

$SU(6)$ symmetry predicts[13] the magnetic moments of the decuplet particles to be $\mu_{B_{10}}=Q\mu_p$, where Q is the charge associated with the decuplet particle and μ_p is the total magnetic moment of the proton. Also, from $SU(6)$ we have $G_A^*(0)=0.708G_V$ for the axial-vector B_{10}-B_{10} form factor. We thus write, in analogy with the B_8-B_8 form factors,

$$\langle B_{10}(p^1)|V_{\mu b}{}^a|B_{10}(p)\rangle = 3\left(\frac{M_1{}^*M_2{}^*}{p_0p_0{}^1}\right)^{1/2}[\bar\psi_\zeta(p^1)]^{ija} \times\left\{i\gamma_\mu\left[F_1{}^*(q^2)+F_2{}^*(q^2)\frac{q_\zeta q_\lambda}{2M^{*2}}\right]-\frac{(p^1+p)_\mu}{2M^*} \times\left[F_3{}^*(q^2)+F_4{}^*(q^2)\frac{q_\zeta q_\lambda}{2M^{*2}}\right]\right\}[\psi_\lambda(p)]_{ijb} \quad (3a)$$

and

$$\langle B_{10}(p^1)|A_{\mu b}{}^a|B_{10}(p)\rangle = 3\left(\frac{M_1{}^*M_2{}^*}{p_0p_0{}^1}\right)^{1/2}[\bar\psi_\zeta(p^1)]^{ija}\left\{\left(i\gamma_\mu\gamma_5-\frac{2M^*q_\mu\gamma_5}{q^2+m_\pi{}^2}\right) \times\left[G_1{}^*(q^2)+G_2{}^*(q^2)\frac{q_\zeta q_\lambda}{2M^{*2}}\right]\right\}[\psi_\lambda(p)]_{ijb}. \quad (3b)$$

In order to obtain finite contributions to the decuplet spurion with decuplet intermediate state we include cutoff factors of the type $(1+q^2/4M^{*2})^{-n}$ in the above form factors. This is done in the same spirit as in earlier calculations.[2] We then have

$$F_1{}^*(0)=F_2{}^*(0)=\mu_p\,,\quad F_3{}^*(0)=F_4{}^*(0)=\mu_p-1\,, \quad (3c)$$

$$F_1{}^*(q^2)=(1+q^2/4M^{*2})^{-1}\mu_pG_E(q^2) =(1+q^2/4M^{*2})^{+1}F_2{}^*(q^2)\,, \quad (3d)$$

$$F_3{}^*(q^2)=(1+q^2/4M^{*2})^{-2}(\mu_p-1)G_E(q^2) =(1+q^2/4M^{*2})F_4{}^*(q^2)\,, \quad (3e)$$

and

$$G_1{}^*(q^2)=G_A{}^*(0)G_E(q^2)(1+q^2/4M^{*2})^{-1} =(1+q^2/4M^{*2})G_2{}^*(q^2)\,. \quad (3f)$$

Assumptions similar to the ones above have been made earlier in evaluating the decay rates for the semileptonic decays of the Ω^-, in $SU(6)$.[14] The form factors were taken at zero momentum transfer in these calculations. Thus the estimates for the decay rates of the semileptonic decays of the Ω^- made by Pakvasa and Rosen[14] would be valid in our model.

III. PARITY-CONSERVING DECUPLET SPURION

As a first step towards estimating the nonleptonic decay amplitudes for the Ω^- decays, we evaluate the matrix elements of symmetrized products of currents between decuplet states (the decuplet spurion). We insert a complete set of states between the currents and assume that the B_8 and B_{10} intermediate states alone saturate the sum.

The $SU(3)$ factors alone indicate that quite independent of the details of the form factors one may expect octet dominance in the matrix elements. This is due to the difference in sign between the contributions of the B_8 and B_{10} baryon intermediate states to the 27-plet spurion. This cancellation is explicitly shown in Eq. (9) below.

The integrals involved in summing over the B_8 intermediate states are the same as those obtained in Ref. 2. These and the ones that appear in the sum over the B_{10} intermediate states are given in the Appendix. In evaluating the matrix elements, we take the spin average over the initial and final decuplet particle spins.

We define[15] $SU(3)$ operators for the decuplet baryons as follows:

$$(\bar\psi\psi_8)_b{}^a=3\bar\psi^{ija}\psi_{ijb}-\delta_b{}^a\langle\bar\psi\psi_1\rangle\,, \quad (4a)$$

[12] For a review on $SU(6)$ symmetry and its applications see A. Pais (Ref. 11).

[13] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

[14] S. Pakvasa and S. P. Rosen, Phys. Rev. **147**, 1166 (1966); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965); I. J. Muzinich, Phys. Letters **14**, 252 (1965).

[15] These definitions are similar to those for octet baryons given by S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962). We have used the co- (contra-) gredient tensor notation for the decuplet wave functions which are totally symmetric on their $SU(3)$ indices.

$$(\bar{\psi}\psi_{27})_{bd}{}^{ac}=4\{3(\bar{\psi}^{iac}\psi_{ibd})-\tfrac{1}{5}[\delta_b{}^a(\bar{\psi}\psi_8)_d{}^c+\delta_d{}^c(\bar{\psi}\psi_8)_b{}^a+\delta_d{}^a(\bar{\psi}\psi_8)_b{}^c+\delta_b{}^c(\bar{\psi}\psi_8)_d{}^a]-\tfrac{1}{4}(\delta_b{}^a\delta_d{}^c+\delta_b{}^c\delta_d{}^a)\langle\bar{\psi}\psi_1\rangle\}, \quad (4b)$$

where $\langle\bar{\psi}\psi_1\rangle$, $(\bar{\psi}\psi_8)_b{}^a$, and $(\bar{\psi}\psi_{27})_{bd}{}^{ac}$ are the $SU(3)$ singlet, octet, and 27-plet operators formed out of the direct product $\mathbf{10}^*\otimes\mathbf{10}$.

The p.c. matrix element $\langle B_{10}|\{j_{\mu a}{}^b,j_{\mu d}{}^c\}|B_{10}\rangle$ may be written as

$$\langle B_{10}|\{j_{\mu b}{}^a,j_{\mu d}{}^c\}|B_{10}\rangle=\langle B_{10}|\{V_{\mu b}{}^a,V_{\mu d}{}^c\}+\{A_{\mu b}{}^a,A_{\mu d}{}^c\}|B_{10}\rangle=S^*{}_{bd}{}^{ac} \quad (5a)$$

$$=\tau^*[27]+\phi^*[8]+\sigma^*[1], \quad (5b)$$

where τ^* and σ^* are the strengths of the 27-plet, the octet, and the singlet parts of the decuplet spurion, and

$$[27]\equiv(\bar{\psi}\psi_{27})_{bd}{}^{ac}, \quad (6)$$

$$[8]=(\delta_d{}^a(\bar{\psi}\psi_8)_b{}^c+\delta_b{}^c(\bar{\psi}\psi_8)_d{}^a)-\tfrac{2}{3}(\delta_b{}^a(\bar{\psi}\psi_8)_d{}^c+\delta_d{}^c(\bar{\psi}\psi_8)_b{}^a), \quad (7)$$

$$[1]=(\delta_d{}^a\delta_b{}^c-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c)\langle\bar{\psi}\psi_1\rangle. \quad (8)$$

The space-time parts of the matrix elements B_8 and B_{10} intermediate states (see Appendix) give $\mathfrak{B}^*=46\times10^{-4}M_p{}^3$ and $\mathfrak{D}^*=36.6\times10^{-4}M_p{}^3$, respectively. We therefore obtain

$$\tau^*=\tau_{10}{}^*+\tau_8{}^*=\mathfrak{D}^*-\tfrac{1}{2}\mathfrak{B}^*=13.6\times10^{-4}M_p{}^3,$$
$$\phi^*=\phi_{10}{}^*+\phi_8{}^*=(9/5)\mathfrak{D}^*+\tfrac{3}{5}\mathfrak{B}^*=93.2\times10^{-4}M_p{}^3, \quad (9)$$
$$\sigma^*=\sigma_{10}{}^*+\sigma_8{}^*=3\mathfrak{D}^*+\tfrac{3}{2}\mathfrak{B}^*=179.0\times10^{-4}M_p{}^3.$$

In Eqs. (9) we have separated the contributions from B_8 and B_{10} particle intermediate states. They show that τ^*, the strength of the 27-plet spurion, is much smaller than ϕ^*, and we thus have octet dominance.

The above result is interesting in that one could have considered octet dominance in the case of baryon octet matrix elements to be "accidental" because of the specific choice of the d/f ratio for the B_8-B_8 currents. The experimentally determined d/f ratio for the vector and axial-vector currents is quite close to the value $\sqrt{3}$, which gives the lowest value for the 27-plet spurion.[7] In the case of the decuplet particle matrix elements, such a possibility is ruled out owing to the absence of a d/f ratio: The direct products $\mathbf{10}^*\otimes\mathbf{10}$ and $\mathbf{8}\otimes\mathbf{10}$ contain only one $\mathbf{8}$ representation in their decomposition into irreducible representations.

We shall also need the matrix elements of symmetrized products of currents between initial and final octet baryon states in our calculation of the Ω^- decays. With our modifications in the treatment of the form factors, we have[2]

$$\langle B_8|\{j_{\mu b}{}^a,j_{\mu d}{}^c\}|B_8\rangle=S_{bd}{}^{ac}=\tau T_{bd}{}^{ac}+\delta[(\delta_d{}^aD_b{}^c+\delta_b{}^cD_d{}^a)-\tfrac{2}{3}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a)]+\phi[(\delta_b{}^cF_d{}^a+\delta_d{}^aF_b{}^c)-\tfrac{2}{3}(\delta_b{}^aF_d{}^c+\delta_d{}^cF_b{}^a)]+\sigma(\delta_d{}^a\delta_b{}^c-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c)\langle\bar{B}B\rangle, \quad (10)$$

with

$$\tau=-32.2\times10^{-4}M_p{}^3,\quad \delta=-334.5\times10^{-4}M_p{}^3,\quad \phi=576\times10^{-4}M_p{}^3,$$

and

$$\sigma=681\times10^{-4}M_p{}^3. \quad (11)$$

For the sake of completeness we note that these values lead to the S-wave nonleptonic decay amplitudes of the baryons which are in reasonable agreement with experiment.[2,3] With $f_\pi=0.95m_\pi$ for the pion decay constant, we obtain

$$A(\Lambda^0{}_-)=3.3\times10^{-7},\quad A(\Xi^-{}_-)=4.66\times10^{-7},\quad A(\Sigma^+{}_+)=0.36\times10^{-7}, \quad (12a)$$

to be compared with the experimental values of[16]

$$A_{\rm expt}(\Lambda^0{}_-)=3.3\times10^{-7},\quad A_{\rm expt}(\Xi^-{}_-)=4.4\times10^{-7},\quad A_{\rm expt}(\Sigma^+{}_+)=0.001\times10^{-7}\ . \quad (12b)$$

For the $\Delta S=0$ p.v. amplitude $A(n\to p\pi^-)=A(n^0{}_-)$, we predict

$$A(n^0{}_-)=0.38\times10^{-7}, \quad (12c)$$

which may be compared with the value $A(n^0{}_-)=0.414\times10^{-7}$ obtained by Fischbach[17] by using experimental numbers for the $\Delta S=1$ decay amplitudes as input. The sum rules given by Fischbach would naturally be valid here.

IV. WEAK DECAYS OF Ω^-

The weak-interaction Hamiltonian in the current$\times$current model is given by

$$H_W=(G_W/2\sqrt{2})\{J_\mu,J_\mu{}^\dagger\}, \quad (13)$$

where $G_W=1.05\times10^{-5}/M_p{}^2$, and the currents are given in the Cabibbo hypothesis[18] to be of the form

$$J_\mu=l_\mu+\cos\theta\,(V_\mu+A_\mu)_2{}^1+\sin\theta\,(V_\mu+A_\mu)_3{}^1. \quad (14)$$

The hadronic currents $(V_\mu+A_\mu)_b{}^a$ have the $SU(3)$ indices a and b, and the angle $\theta=0.25$.

The terms in Eq. (13) that give rise to the nonleptonic decays are

$$H_W{}^{\rm NL}=(G_W/2\sqrt{2})\sin\theta\cos\theta[\{(V_\mu+A_\mu)_2{}^1,(V_\mu+A_\mu)_1{}^3\}+(2\leftrightarrow3)]. \quad (15)$$

For Ω^- decays,[19,20] we write the decay amplitudes in the form

$$m(\Omega^-\to B(p_\beta);\pi(k))=-i\bar{u}(p_\beta)[B-C\gamma_5](p_\beta)_\mu u_\mu(\Omega), \quad (16)$$

[16] H. Filthuth, in Proceedings of the Topical Conference on Weak Interactions, CERN, Geneva, 1969 (unpublished).
[17] E. Fischbach, Phys. Rev. **170**, 1398 (1968).
[18] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1965).
[19] The nonleptonic decays of the Ω^- have been investigated earlier by, among others, M. Suzuki, Progr. Theoret. Phys. (Kyoto) **32**, 138 (1964); S. Pakvasa and S. P. Rosen (Ref. 14); Y. Hara, Phys. Rev. **150**, 1175 (1966); A. P. Balachandran, M. G. Gundzik, and S. Pakvasa, *ibid.* **153**, 1553 (1967); K. Ohya, M. Imoto, and S. Kawasaki, Progr. Theoret. Phys. (Kyoto) **39**, 1268 (1968).
[20] X. Y. Pham and R. Zaoui, Phys. Rev. **167**, 1319 (1968).

where B and C correspond to the P- and D-wave amplitudes, respectively. The decay rate obtained from Eq. (16) is

$$\Gamma(\Omega^- \to B\pi) = \frac{|p_\beta|^3}{24\pi M_\Omega{}^2}[(M_\Omega+M_B)^2-m_\pi{}^2] \times\left\{|B|^2+|C|^2\frac{(M_\Omega-M_B)^2-m_\pi{}^2}{(M_\Omega+M_B)^2-m_\pi{}^2}\right\} \quad (17a)$$

$$=C_1[|B|^2+C_2|C|^2]. \quad (17b)$$

We denote the amplitudes corresponding to $\Omega^- \to \Xi^0\pi^-$, $\Xi^-\pi^0$, ΛK^- by $B(\Omega^-_-)$, $B(\Omega^-_0)$, and $B(\Omega^-_K)$, etc.

The decay modes $\Omega^- \to \Xi^*\pi$ would be greatly inhibited by the small phase space available for these decays. We shall therefore consider only the S-wave p.v. amplitudes to be at all relevant, and we have

$$m(\Omega^- \to \Xi^*(p^1); \pi(k)) = \bar{u}_\mu(\Xi^*)\{A\}u_\mu(\Omega^-), \quad (18)$$

$$\Gamma(\Omega^- \to \Xi^*\pi) = (|\mathbf{p}^1|/8\pi M_\Omega{}^2) \times[(M_\Omega+M_{\Xi^*})^2-m_\pi{}^2]|A|^2. \quad (19)$$

The use of the well-known reduction technique, current algebra, the PCAC hypothesis, and the soft-pion limit leads to[1,5]

$$\lim_{k_\mu\to 0}\{-i(2k_0)^{1/2}\langle B(p_\beta), \pi_b{}^a(k)|H_W(0)|\Omega^-(p_\alpha)\rangle\} = -\frac{m_\pi{}^2}{C_\pi}\langle B|[F_{5b}{}^a, H_W(0)]|\Omega^-\rangle + \lim_{k_\mu\to 0}\frac{m_\pi{}^2+k^2}{C_\pi}\int d^4x\, e^{-ik\cdot x}\theta(x_0) \times\langle B(p_\beta)|[ik_\mu A_{\mu b}{}^a, H_W{}^{(0)}]|\Omega^-(p_\alpha)\rangle, \quad (20)$$

where C_π is the PCAC constant.

We denote the two terms on the right-side of Eq. (20) by the equal-time-commutator (ETC) and the retarded-time-commutator (RTC) terms, respectively. In the following we evaluate the amplitudes of the Ω^- decay modes.

A. Ω^- Decay into $\Xi^*\pi$

First consider the simpler problem of the decays $\Omega^- \to \Xi^*\pi$. The Ω^- and Ξ^* particles belong to the same $SU(3)$ multiplet and have the same intrinsic parity. In the limit of zero pion momentum, the ETC term of Eq. (20) contributes to the amplitudes $A(\Omega^-_-{}^*)$ and $A(\Omega^-_-{}^*)$. The RTC term reduces to p.v. pole terms for which the p.v. weak spurions are assumed to be zero.[20] This is analogous to the Sugawara-Suzuki[1] treatment of the p.v. amplitudes for the nonleptonic decays of the octet baryons. The use of current algebra in evaluating the ETC term allows us to relate $A(\Omega^{-*})$ to the p.c. decuplet spurion obtained in Sec. III. Pham and Zaoui[20] have expressed these amplitudes in terms of unknown reduced matrix elements; here we are able to numerically evaluate the reduced matrix elements (the decuplet spurion) and to estimate the amplitudes.

We have

$$A(\Omega^- \to \Xi^{*0}\pi^-) = A(\Omega^-_-{}^*) = (1/f_\pi) \times\langle\Xi^{*0}|[(F_5)_2{}^1, H_W(0)]|\Omega^-\rangle = \frac{G_W \sin\theta\cos\theta}{f_\pi 2\sqrt{2}} \times(S^*{}_{22}{}^{13}+S^*{}_{32}{}^{12}-S^*{}_{31}{}^{11}), \quad (21)$$

where $S^*{}_{bd}{}^{ac}$ have been defined in Eq. (5a). With $f_\pi = 0.95m_\pi$ we obtain

$$A(\Omega^-_-{}^*) = -\sqrt{2}A(\Omega^-_0{}^*) = \frac{G_W\sin\theta\cos\theta}{f_\pi 2\sqrt{2}}\{\tfrac{4}{5}\sqrt{3}\tau^*-\sqrt{3}\phi^*\} = -0.815\times10^{-7}. \quad (22)$$

We may compare this with the value obtained from the sum rule of Balachandran, Gundzik, and Pakvasa,[19] who used $SU(6)_W$ symmetry and current algebra to relate this amplitude with the octet baryon decay amplitudes. In *their phase convention* the sum rule, namely,

$$A(\Omega^-_-{}^*) = \sqrt{3}A(\Sigma^+{}_+)+\tfrac{1}{2}\sqrt{3}A(\Sigma^-{}_-)-\tfrac{3}{2}\sqrt{2}A(\Lambda^0{}_-), \quad (23a)$$

gives

$$A(\Omega^-_-{}^*) = 3.02\times10^{-7} \quad (23b)$$

when experimental numbers are used on the right-hand side of the sum rule.

The decay rate for $\Omega^- \to \Xi^{*0}\pi^-$, using Eqs. (18), (19), and (22), is

$$\Gamma(\Omega^-_-{}^*) \cong 0.74\times10^8\ \text{sec}^{-1}. \quad (24)$$

Earlier estimates for this decay rate are larger: Pakvasa and Rosen,[14] Glashow and Socolow,[21] and Ohya *et al.*[19] obtain 22×10^8 sec^{-1}, 3.8×10^8 sec^{-1}, and $4.77 \sim 12.3\times10^8$ sec^{-1}, respectively.

B. P-Wave Amplitudes for Ω^- Decays into $\Xi\pi$ and ΛK^-

For Ω^- decay into a baryon and a meson we again assume that the matrix elements of the p.v. weak Hamiltonian between decuplet initial states and decuplet or octet baryon final states are zero. Then the ETC term of Eq. (20) alone contributes to the p.v. D-wave amplitudes, since the RTC terms reduce to p.v. baryon pole terms. The matrix elements $\langle B_8|H_W{}^{\text{p.c.}}(0)|B_{10}\rangle$ vanish in the limit $k_\mu \to 0$, since the transition involves a change of spin, so that in the soft-meson limit it is not possible to obtain the p.v. amplitudes from current algebra. In Sec. IV C we consider an

[21] S. L. Glashow and R. Socolow, Phys. Letters 10, 143 (1964).

alternative mechanism in the K^*-pole model in order to make some estimate of the p.v. amplitudes.

The P-wave p.c. amplitudes are given by the RTC terms of Eq. (20) in the soft-meson limit. In the limit $k_\mu \to 0$, they include decuplet and octet baryon poles and hence do not vanish. The phenomenological analyses of Chan[22] and of Ram Mohan[23] for the P-wave nonleptonic decay amplitudes of the octet baryons indicate that the matrix elements of the p.c. weak Hamiltonian between decuplet and octet states are much smaller than the corresponding matrix elements between octet-octet and decuplet-decuplet states. We shall therefore consider only those pole terms which involve octet-octet and decuplet-decuplet spurions.

We then have

$$(\mathrm{RTC})=\left(\frac{m_\pi^2+k^2}{C_\pi}\right)\left[\sum_{\gamma_{10}}\int\frac{d^3p_\gamma}{2E_\gamma}\frac{i\delta^3(\mathbf{p}_\gamma-\mathbf{p}_\beta-\mathbf{k})}{k_0+E_\beta-E_\gamma}\times\langle B(p_\beta)|ik\cdot A(0)_b{}^a|\gamma\rangle\langle\gamma|H_W(0)|\Omega^-\rangle\right.$$
$$\left.-\sum_{\delta_8}\int\frac{d^3p_\delta}{2E_\delta}\frac{i\delta^3(\mathbf{p}_\delta+\mathbf{k}-\mathbf{p}_\alpha)}{k_0+E_\delta-E_\alpha}\langle B(p_\beta)|H_W|\delta\rangle\times\langle\delta|ik\cdot A(0)_b{}^a|\Omega^-\rangle\right], \quad (25)$$

where the intermediate states $|\gamma\rangle$ and $|\delta\rangle$ are restricted to the decuplet and octet baryon states, respectively.

In the limit $k_\mu \to 0$ these terms may be well approximated by the corresponding Born-approximation amplitudes,[4,24] and the quantity $(\mathrm{RTC})-(B_\gamma+B_\delta)$, where B_γ and B_δ are the Born terms, has a well-defined limit when $m_\gamma-m_\Omega$ and $m_\delta-m_B$ tend to zero[20] (i.e., in the symmetry limit).

When physical masses are used, the inclusion of symmetry-breaking effects in the masses leads to

$$(\mathrm{RTC})=\left(1-\frac{M_\gamma{}^2-M_B{}^2}{2E_B{}^2}\right)B_\gamma+\left(1-\frac{M_\delta{}^2-M_\Omega{}^2}{2M_\Omega{}^2}\right)B_\zeta. \quad (26)$$

In the Ω^- decay, these correction factors arise due to the mass differences between the octet and the decuplet particles; they were not considered in Ref. 20.

Using the Born diagrams of Fig. 1, the amplitudes are evaluated with an $SU(3)$-symmetric strong B_8-B_{10}-π

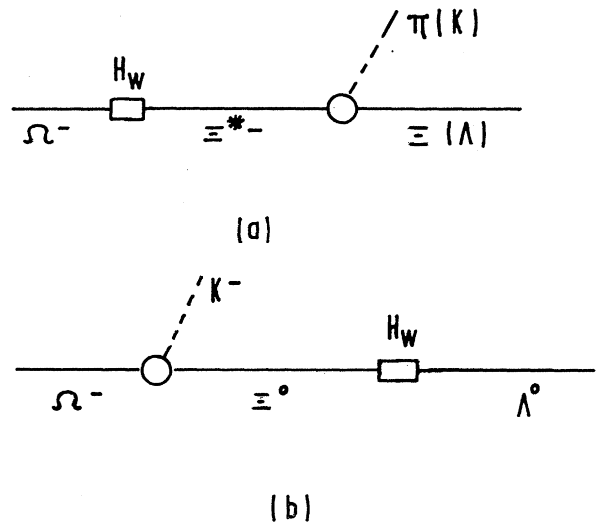

FIG. 1. Pole diagrams for P-wave Ω^- decay amplitudes.

coupling of the form

$$H_{B_8B_{10}\pi}=(\lambda_D/m_\pi)\bar{B}_l{}^i\partial_\mu P_m{}^j\epsilon^{klm}(\psi_\mu)_{ijk}. \quad (27)$$

The coupling constant λ_D is determined from the experimentally measured[25] decay width $\Gamma(N^{*++}\to p\pi^+)=120$ MeV to be

$$\lambda_D=2.14. \quad (28)$$

When Eqs. (9), (11), and (27) are used in Eq. (26), we have

$$B(\Omega^-{}_-)=-\sqrt{2}B(\Omega^-{}_0)=-\frac{\lambda_D}{m_\pi}\frac{G_W\sin\theta\cos\theta}{2\sqrt{2}(M_\Omega-M_{\Xi^*})}\times\left(1-\frac{M_{\Xi^*}{}^2-M_\Xi{}^2}{2M_\Omega{}^2}\right)(\tfrac{4}{5}\tau^*-\phi^*)$$
$$=-0.95\times10^{-7}/m_\pi. \quad (29)$$

The u-channel contribution to $B(\Omega^-{}_-)$ is absent because of the absence of a strong $(\Omega\Xi\pi)$ coupling. For $B(\Omega^-{}_K)$, we obtain

$$B(\Omega^-{}_K)=-\frac{\lambda_D}{m_\pi}\tfrac{1}{4}G_W\sin\theta\cos\theta\left\{\frac{1}{M_\Omega-M_{\Xi^*}}\times\left(1-\frac{M_{\Xi^*}{}^2-M_\Lambda{}^2}{2M_\Omega{}^2}\right)[\tfrac{4}{5}\sqrt{3}\tau^*-\sqrt{3}\phi^*]+\frac{1}{M_\Xi-M_\Lambda}\left(1+\frac{M_\Omega{}^2-M_\Xi{}^2}{2M_\Omega{}^2}\right)\times[\tfrac{1}{5}(\sqrt{6})\tau+(\sqrt{\tfrac{3}{2}})\phi-\delta/\sqrt{6}]\right\}$$
$$=5.1\times10^{-7}/m_\pi. \quad (30)$$

[22] F. C. P. Chan, Phys. Rev. **171**, 1543 (1968).
[23] L. R. Ram Mohan, Phys. Rev. **179**, 1561 (1969).
[24] Arvind Kumar and J. C. Pati, Phys. Rev. Letters **18**, 1230 (1967); M. Hirano, K. Fujii, and O. Terazawa, Progr. Theoret. Phys. (Kyoto) **40**, 114 (1968); for a recent review on the theory of nonleptonic decays, see B. W. Lee, in Proceedings of the Topical Conference on Weak Interactions, CERN, Geneva, 1969 (unpublished).
[25] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

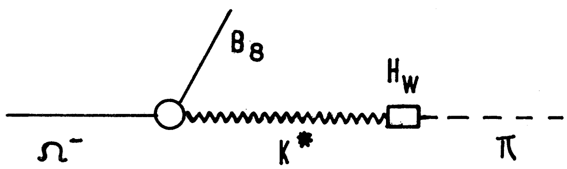

FIG. 2. Pole diagram for D-wave Ω^- decay amplitudes.

In Table I we compare the P-wave amplitudes of Eq. (26), with and without the correction factors, and give the P-wave contribution to the partial decay rates. The experimental value[25] quoted for the total decay rate of the Ω^- is $\Gamma_{\text{expt}}(\Omega^-)=(1.3\pm0.4)\times10^{-10}$ sec. With the central value for the decay rate and the experimentally observed branching ratios of 3:8:13 for the decay modes $\Omega^-{}_0$, $\Omega^-{}_-$, and $\Omega^+{}_K$, respectively, the partial rates are

$$\begin{aligned}\Gamma(\Omega^-{}_0)&=9.6\times10^8\ \text{sec}^{-1},\\ \Gamma(\Omega^-{}_-)&=26\times10^8\ \text{sec}^{-1},\\ \Gamma(\Omega^-{}_K)&=42\times10^8\ \text{sec}^{-1}.\end{aligned}\quad(31)$$

The values for the P-wave contribution to the partial decay rates indicate that the D-wave amplitudes are important. Also, evaluating the D-wave amplitudes would allow us to estimate the asymmetry parameters. A dynamical model has to be invoked to evaluate the D-wave amplitudes, since the current-algebra method fails to give these amplitudes in the soft-meson limit.

For this we assume that the K^*-pole model[26] is an effective respresentation of the current-algebra method. In the limit of octet dominance for the weak K^*-π effective Hamiltonian, the results are equivalent[27] to those obtained from current-algebra calculations. We pursue these considerations below.

C. K^*-Pole Model for D-Wave Amplitudes

It is assumed that the weak Hamiltonian describing the K^*-π transition dominates the p.v. amplitudes and is given by

$$H_W=g_{K^*-\pi}(K_\mu{}^*)_i{}^3(\partial_\mu\pi)_2{}^i+\text{H.c.}\quad(32)$$

The coupling constant $g_{K^*-\pi}$ may be obtained by using the experimental data[16] on p.v. baryon decay amplitudes. With the amplitudes given by Carlstone *et al.*,[27] we have

$$A(\Lambda^0{}_-)=-\frac{1}{\sqrt{6}}\frac{g_{K^*\pi}}{(M_{K^*})^2}(M_\Lambda-M_N)\times(D_W+3F_W)=3.3\times10^{-7},$$

$$A(\Xi^-{}_-)=\frac{1}{\sqrt{6}}\frac{g_{K^*-\pi}}{(M_{K^*})^2}(M_\Xi-M_\Lambda)\times(D_W-3F_W)=4.5\times10^{-7},\quad(33)$$

and

$$g_{K^*-\pi}/(M_{K^*})^2=-1.24\times10^{-9}\ \text{MeV}^{-1}.\quad(34)$$

In a manner analogous to the above calculation, in which exact current conservation is broken when physical masses are employed, we evaluate the Ω^- amplitudes using the Born diagram of Fig. 2. The decuplet-octet baryon coupling to vector-meson fields $\mathcal{V}_\mu$ is of the form

$$H_{B_8B_{10}\mathcal{V}}=\sqrt{3}\bar{B}_l{}^i\epsilon^{klm}\{\delta_{\mu\alpha}f_1(q^2)+iq_\alpha\gamma_\mu f_2(q^2)\}\times\gamma_5(\psi_\alpha)_{ijk}(\mathcal{V}_\mu)_m{}^j,\quad(35)$$

where $f_1(q^2)$ and $f_2(q^2)$ are the form factors of Eqs. (2c) and (2d) with M^* and M replaced by M_{N^*} and M_N.

The amplitude $C(\Omega^-{}_K)$ is zero, since the strong coupling for the $(\Omega\Lambda K^*)$ vertex has a vanishing matrix element. (No mesons with strangeness -2 are known to exist.) With physical masses for the initial and final baryons, and assuming that there is no momentum transfer across the weak vertex, we obtain

$$\begin{aligned}C(\Omega^-{}_K)&=0,\\ C(\Omega^-{}_-)&=-\sqrt{2}C(\Omega^-{}_0)=-6.3\times10^{-7}/M_\pi.\end{aligned}\quad(36)$$

Equations (29), (30), and (36) then yield the following decay rates:

$$\begin{aligned}\Gamma(\Omega^-{}_0)&=6.5\times10^8\ \text{sec}^{-1},\\ \Gamma(\Omega^-{}_-)&=13\times10^8\ \text{sec}^{-1},\\ \Gamma(\Omega^-{}_K)&=59\times10^8\ \text{sec}^{-1}.\end{aligned}\quad(37\text{a})$$

These results are to be compared with the "experimental" values of Eq. (31) which are obtained by assuming that the branching ratios are given by the ratios of the corresponding number of Ω^- decay events. The total number of Ω^- decay events observed so far is 24, and the branching ratios are not satistically reliable. On the other hand, the total decay rate may not turn out to be very different from the present experimental value with improved statistics.

Our total decay rate, after including the $\Omega^-\to\Xi^*\pi$ rates, is

$$\Gamma_{\text{theor}}(\Omega^-)=80.1\times10^8\ \text{sec}^{-1}.\quad(37\text{b})$$

The asymmetry parameters are given by

$$\alpha=\frac{2\,\text{Re}(C_2)^{1/2}B(\Omega^-)C(\Omega^-)}{|B(\Omega^-)|^2+C_2|C(\Omega^-)|^2},$$

TABLE I. P-wave amplitudes for Ω^- decays.

Amplitude	Born amplitude without corrections ($10^{-7}/m_\pi$)	P-wave contributions to decay rate (10^8 sec^{-1})	Born amplitude with corrections ($10^7/m_\pi$)	P-wave contribution to decay rate (10^8 sec^{-1})
$B(\Omega^-{}_-)$	-1.07	9.8	-0.95	7.5
$B(\Omega^-{}_K)$	4.0	41.6	5.1	59.5

[26] J. Schwinger, Phys. Rev. Letters **12**, 630 (1964); **13**, 355 (1964); B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964).
[27] D. S. Carlstone, S. P. Rosen, and S. Pakvasa, Phys. Rev. **174**, 1877 (1968).

where C_2 is defined in Eq. (17b); we predict the asymmetry parameters to be

$$\alpha(\Omega^-{}_K)=0\,,\qquad \alpha(\Omega^-{}_0)\cong\alpha(\Omega^-{}_-)\approx 1\,. \tag{38}$$

V. CONCLUDING REMARKS

With the simplest possible assumptions regarding the vector and axial-vector form factors of the baryons, we have shown that the decuplet spurions exhibit octet dominance. We have evaluated the P-wave amplitudes for the Ω^- decay by assuming that the matrix elements of the p.c. weak Hamiltonian between octet and decuplet states can be neglected in comparison with the matrix elements of the same Hamiltonian between the octet baryon states or the decuplet baryon states. This assumption can be justified by the following argument. The P-wave amplitudes for the nonleptonic decays of the octet baryons can be evaluated with octet baryon poles and octet meson poles only, to obtain fair agreement with experiment. Perhaps a selection rule may emerge from a study of the symmetry properties of the weak Hamiltonian which would allow the baryon resonance poles to be transformed away.[28]

We obtain the D-wave amplitudes for the Ω^- decays by using the K^*-pole model and we predict the asymmetry parameters to be $\alpha(\Omega^-{}_K)=0$ and $\alpha(\Omega^-{}_-)\simeq\alpha(\Omega^-{}_0)\approx 1$.

The use of $SU(3)$-symmetric $B_8B_{10}\pi$ strong-interaction coupling constants and our use of only one form factor in defining the matrix elements of the axial-vector current between octet and decuplet states [Eq. (2)] in the calculations are approximations. The steady increase in the observed number of Ω^- decay events should eventually allow a measurement of the partial decay rates and of the asymmetry parameters in the near future.

ACKNOWLEDGMENTS

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[28] That the meson poles can be transformed away has been shown by B. W. Lee, Phys. Rev. **140**, B152 (1965).

APPENDIX

The integrals appearing in the matrix elements of Eq. (5a) for octet and decuplet intermediate states are given below. We write

$$\langle B_{10}|\{j_{\mu b}{}^a, j_{\mu d}{}^c\}|B_{10}\rangle =\sum_n \langle B_{10}|j_\mu|n\rangle\langle n|j_\mu{}^\dagger|B_{10}\rangle[SU(3)\text{ factors}]\,.$$

For octet intermediate states the space-time part of the matrix elements is

$$\sum_n \langle B_{10}|j_\mu|(B_8)_n\rangle\langle(B_8)_n|j_\mu{}^\dagger|B_{10}\rangle=\mathcal{B}^*\,, \tag{A1}$$

$$=\frac{M^{*3}}{4\pi^2}\int_1^\infty dx(x^2-1)^{1/2}\{(x-1)(f_1)^2+(x^2-1)\times[\tfrac{2}{3}M^2(x+2)(f_2)^2+\tfrac{4}{3}Mf_1f_2]+(x+1)(g_1)^2\}\,, \tag{A2}$$

where $x=k_0/M$ and the form factors are functions of

$$q^2=-M^{*2}-2MM^*\,.$$

For the decuplet intermediate states, we have

$$\sum_n \langle B_{10}|j_\mu|(B_{10})_n\rangle\langle(B_{10})_n|j_\mu{}^\dagger|B_{10}\rangle =\mathcal{D}^*=(M^{*3}/2\pi^2)(\mathcal{I}+\mathcal{J}+\mathcal{K})\,, \tag{A3}$$

where the integrals $\mathcal{I}$, $\mathcal{J}$, and $\mathcal{K}$,after simplifications, are

$$\mathcal{I}=\frac{4\mu_p{}^2}{9}\int_1^\infty dx(x^2-1)^{1/2}\frac{[G_E(q^2)]^2}{(x+1)^2}\times[3(x-4)+8x(x-1)(x-2)-2(x-1)]\,,$$

$$\mathcal{J}=\frac{4(\mu_p{}^2-1)}{9}\int_1^\infty dx(x^2-1)^{1/2}\frac{[G_E(q^2)]^2}{(x+1)^2}\times[9+4(x-1)(2x+1)]\,, \tag{A4}$$

$$\mathcal{K}=\frac{4G_A{}^*(0)}{9}\int_1^\infty dx(x^2-1)^{1/2}[G_E(q^2)]^2\times(5-8x+8x^2)\,, \tag{A5}$$

where $x=k_0/M^*$ and the form factor $G_E(q^2)$ is a function of $q^2=-2M^{*2}+2k_0M^*$.