Meson Form Factors and the Veneziano Model

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The K_{l4}, K_{l3}, and K_{l2} form factors, as well as the pion electromagnetic form factor, are derived from the matrix element $\langle\pi|A_\mu|\pi K\rangle$ by using partial conservation of axial-vector current, current algebra, and the field-current identity, in addition to the Veneziano representation for the scattering amplitudes. This model contains essentially only one free parameter, by which all the quantities are expressed. The results agree with the present experimental data excellently.

I. INTRODUCTION

THE Veneziano model[1] is the only one that succeeds in expressing the idea of the duality of strong interactions in a simple mathematical form. Characteristic features of meson-meson scattering are particularly well understood in the framework of this model.[2–4] Relevant amplitudes show the expected Regge behavior at high energies and have poles corresponding to the observed resonances. It is also found that the results at low energies are compatible with those from current algebra. Therefore this model provides us with a conviction that a unified understanding of the low- and high-energy phenomena in elementary-particle physics will be attained along this line. A number of attempts have been made to study meson form factors within this model.[5–7] None of them, however, succeeds in explaining several form factors simultaneously.

In this paper we investigate several pion and kaon form factors in a systematic fashion. We adopt the hypothesis of partially conserved axial-vector current (PCAC)[8] both for the pion and the kaon, current algebra,[9] and the field-current identity,[10] in addition to the Veneziano representation for the scattering amplitudes. Let us consider the matrix element of the strangeness-changing axial-vector current taken between a one-pion state and a one-pion-plus-one-kaon state,

$$\langle\pi|A_\mu|\pi K\rangle, \tag{1.1}$$

where A_μ is related to the kaon and axial-vector meson fields through the field-current identity assumption,

$$A_\mu=\sqrt{2}f_K\partial_\mu K+\sqrt{2}f_{K_A}K^A{}_\mu. \tag{1.2}$$

Then the matrix element is expressed in terms of the scattering amplitudes for $\pi K\to\pi K$ and $\pi K\to\pi K_A$ through the reduction formula. The K_{l4} axial-vector form factors are defined in terms of the matrix element (1.1). We next reduce one (both) of the pions in (1.1) and use the PCAC and current algebra to obtain the K_{l3} (K_{l2}) form factors. On the other hand, if we reduce the kaon in (1.1), we have the pion electromagnetic form factor by the same procedure. In this way, once the scattering amplitudes for $\pi K\to\pi K$ and $\pi K\to\pi K_A$ are given, we obtain various form factors mentioned above.

It is well known that the scattering amplitude for $\pi K\to\pi K$ involves only one parameter,[3] while there are four[7] in the scattering amplitude for $\pi K\to\pi K_A$, provided in both cases that no satellite term exists. We obtain relations between these parameters by imposing Adler's PCAC consistency condition[2,11] on the scattering amplitudes and by assuming that the relation due to the hard-meson method of Schnitzer and Weinberg[12–14] holds at a certain point. With these conditions, all the form factors are expressed in terms of only one parameter. In our model, Callan-Treiman relations[15] hold naturally, and one of them is equivalent to the modified Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[3,16] Further, the ratio G_s/G_d is also determined by this parameter, where G_s and G_d are constants describing s- and d-wave decays of $K_A\to K^*\pi$. If we fix the parameter by the condition $f_+(0)=1$, we obtain all the numerical values of the relevant quantities. Here we summarize the results of our calculation. As for the K_{l4} form factors, we obtain $|f_1|=5.0m_K^{-1}$, $|f_2|=8.1m_K^{-1}$, and $|f_3|=0.15m_K^{-1}$ as the average values. The K_{l3} form factors are calculated: $f_-(0)/f_+(0)=-0.18$, $\lambda_+=0.018$, and $\lambda_-=-0.049$. The

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weak coupling constant for $K\to\mu\nu$ decay is determined: $f_K=0.112$ GeV. For the pion electromagnetic form factor, we obtain $\sqrt{\langle r_\pi^2\rangle}=0.85\times10^{-13}$ cm. The s- to d-wave decay ratio is predicted: $G_s/G_d=-0.45$ GeV^2.

In Sec. II the scattering amplitudes for $\pi K\to\pi K$ and $\pi K\to\pi K_A$ are given. Several relations between the parameters are found. In Sec. III the K_{l4}, K_{l3}, and K_{l2} form factors as well as the pion electromagnetic form factor are derived from the matrix element (1.1) by reducing appropriate mesons and using PCAC and current algebra. The final section is devoted to the numerical results and discussions.

II. SCATTERING AMPLITUDES FOR $\pi K\to\pi K$ AND $\pi K\to\pi K_A$

We give here the scattering amplitudes for $\pi K\to\pi K$ and $\pi K\to\pi K_A$ which are necessary for the subsequent discussions. The Veneziano amplitude with no satellite terms for the reaction $\pi_\alpha(p_1)+K(p_3)\to\pi_\beta(p_2)+K(p_4)$ is written as[3]

$$T_K^{(\pm)}(s,t,u)=-\tfrac{1}{2}f_\rho^2[F(\alpha^{(1)}(s),\alpha^{(2)}(t))\pm F(\alpha^{(1)}(u),\alpha^{(2)}(t))],$$

$$s=-(p_1+p_3)^2,\quad t=-(p_1-p_2)^2,\quad u=-(p_1-p_4)^2,\quad (2.1)$$

where $F(X,Y)=\Gamma(1-X)\Gamma(1-Y)/\Gamma(1-X-Y)$. The functions $\alpha^{(1)}(s)$ and $\alpha^{(2)}(s)$ are the Regge trajectories for K^* and ρ, respectively. They are assumed to be real and linear with respect to s, namely,

$$\alpha^{(1)}(s)=0.89s+0.28,\quad (2.2)$$

$$\alpha^{(2)}(s)=0.89s+0.48,\quad (2.3)$$

which are determined by requiring (1) Adler's PCAC consistency condition: $T_K^{(\pm)}(m_K^2,m_\pi^2,m_K^2)=0$, and (2) the universal slope for the Regge trajectories: $\alpha'=0.89$ (GeV/c)$^{-2}$. The normalization factor f_ρ^2 is a constant which is to be determined from experiment or from a theoretical consistency condition. The scattering amplitude for the process $\pi_\alpha(p_1)+K(p_3)\to\pi_\beta(p_2)+K_A(p_4,\epsilon)$ is written in the following form:

$$T(s,t,u)=A(s,t,u)(p_1+p_2)\cdot\epsilon+B(s,t,u)(p_1-p_2)\cdot\epsilon,\quad (2.4)$$

where ϵ is the polarization vector of K_A. According to the Veneziano formula, A and B are expressed in terms of four parameters[7] $g^{(a)}$, $g^{(b)}$, a, and b:

$$A^{(\pm)}(s,t,u)=g^{(a)}(1+at)[G(\alpha^{(1)}(s),\alpha^{(2)}(t))\mp G(\alpha^{(1)}(u),\alpha^{(2)}(t))],\quad (2.5)$$

$$B^{(\pm)}(s,t,u)=g^{(b)}[(1-bs-b_1t)G(\alpha^{(1)}(s),\alpha^{(2)}(t))\pm(1-bu-b_1t)G(\alpha^{(1)}(u),\alpha^{(2)}(t))],\quad (2.6)$$

where $g^{(a)}a=g^{(b)}b_1$ and $G(X,Y)=\Gamma(1-X)\Gamma(1-Y)/\Gamma(2-X-Y)$. The plus and minus signs in Eqs. (2.1), (2.5), and (2.6) refer to the usual description of the isospin structure, i.e.,

$$A=A^{(+)}\delta_{\alpha\beta}+A^{(-)}\tfrac{1}{2}[\tau_\alpha,\tau_\beta].\quad (2.7)$$

The expressions (2.5) and (2.6) show, of course, the expected Regge behavior in all channels: $A\to s^{\alpha(t)-1}$, $B\to s^{\alpha(t)}$ for $s\to\infty$ and t fixed, for example.

Let us impose Adler's PCAC consistency condition[11] on the amplitude (2.4) in order to obtain the relationship between the parameters.

(i) In the soft-pion limit $p_1\to0$, the PCAC consistency condition requires that

$$A-B=0,\quad (2.8)$$

which leads to

$$A^{(\pm)}=B^{(\pm)}\ \text{at}\ s=m_K^2,\ t=m_\pi^2,\ \text{and}\ u=m_{K_A}^2.\quad (2.9)$$

From Eq. (2.9) we obtain two relations. One is

$$g^{(a)}(1+am_\pi^2)=g^{(b)}(1-bm_K^2-b_1m_\pi^2),\quad (2.10)$$

and the other is either

$$G(\alpha^{(1)}(m_{K_A}^2),\tfrac{1}{2})=0\quad (2.11a)$$

or

$$g^{(a)}(1+am_\pi^2)+g^{(b)}(1-bm_{K_A}^2-b_1m_\pi^2)=0.\quad (2.11b)$$

(ii) In the soft-pion limit $p_2\to0$, the PCAC consistency condition requires that

$$A+B=0.\quad (2.12)$$

The relations obtained from Eq. (2.12) are the same as Eqs. (2.10), (2.11a), and (2.11b).

(iii) In the soft-kaon limit[17] $p_3\to0$, the PCAC consistency condition requires that

$$A^{(\pm)}=B^{(\pm)}=0$$

$$\text{at}\quad s=m_\pi^2,\quad t=m_{K_A}^2,\quad \text{and}\quad u=m_\pi^2.\quad (2.13)$$

From Eq. (2.13) we obtain the relations

$$G(\alpha^{(1)}(m_\pi^2),\alpha^{(2)}(m_{K_A}^2))=0\quad (2.14a)$$

or

$$1+am_{K_A}^2=0,\quad 1-bm_\pi^2-b_1m_{K_A}^2=0.\quad (2.14b)$$

Simple manipulation shows that

$$G(\alpha^{(1)}(m_\pi^2),\alpha^{(2)}(m_{K_A}^2))=G(\alpha^{(1)}(m_{K_A}^2),\alpha^{(2)}(m_\pi^2)),\quad (2.15)$$

so that Eq. (2.14) is equivalent to Eq. (2.11).

So far we are concerned with the restrictions obtained

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[17] It is well known that the soft-kaon limit (PCAC for the kaon) does not give good results as compared with the soft pion. We are encouraged, however, by the fact that there are some examples in which the soft-kaon method works well in the framework of the Veneziano model. See Y. Oyanagi and N. Tokuda, Progr. Theoret. Phys. (Kyoto) **42**, 430 (1969). See also Ref. 3.

under the PCAC consistency condition. As for the limit $p_4 \to 0$ (or $p_4^2 \to 0$), we adopt the hard-kaon method of Schnitzer and Weinberg.[12,13]

(iv) In the hard-meson limit $p_4^2 \to 0$, we define the kaon field operator $K(x)$ as

$$\partial_\lambda A_\lambda^{\pm(s=1)}(x) = \sqrt{2} f_K m_K^2 K^{\mp}(x)\,, \tag{2.16}$$

where $A_\lambda^{(s=1)}$ is the strangeness-changing axial-vector current, and f_K is the kaon decay constant. We assume the following field-current identity:

$$A_\lambda^{\pm(s=1)}(x) = \sqrt{2} f_K \partial_\lambda K^{\mp}(x) + \sqrt{2} f_{K_A} K_\lambda^{A\mp}(x)\,, \tag{2.17}$$

where K_λ^A is the K_A meson field operator and f_{K_A} is a constant. The matrix element of $A_\lambda^{-(s=1)}$ between a one-pion state and a one-pion-plus-one-kaon state is given by

$$\langle \pi^-(p_2) | A_\lambda^{-(s=1)}(0) | \pi^-(p_1) K^+(p_3) \rangle = \bigg[\frac{i\sqrt{2} f_K}{p_4^2 + m_K^2} p_{4\lambda} (T_K^+ + T_K^-) + \frac{\sqrt{2} f_{K_A}}{p_4^2 + m_{K_A}^2} \left(\delta_{\lambda\nu} + \frac{p_{4\lambda} p_{4\nu}}{m_{K_A}^2} \right) [(A^+ + A^-)(p_1+p_2)_\nu + (B^+ + B^-)(p_1 - p_2)_\nu] \bigg] (8 p_{10} p_{20} p_{30})^{-1/2}, \tag{2.18}$$

by using the reduction formula. This is a basic equation for our subsequent discussions on the form factors. If we take the divergence of Eq. (2.18) and use Eq. (2.16), we obtain the equation

$$T_K^+ + T_K^- = i f_{K_A} f_K^{-1} m_{K_A}^{-2} p_{4\lambda} [(A^+ + A^-)(p_1+p_2)_\lambda + (B^+ + B^-)(p_1 - p_2)_\lambda]\,. \tag{2.19}$$

At a first glance we notice that both sides of Eq. (2.19) vanish in the limit $p_4 \to 0$. Here we assume that Eq. (2.19) holds at $p_4^2 = 0$.[13,18] Then a simple calculation leads to

$$g^{(b)} b = 2 g^{(a)} a\,, \tag{2.20}$$

$$g^{(a)} [1 + 2a(m_\pi^2 + m_K^2)] = g^{(b)}\,, \tag{2.21}$$

and

$$-f_\rho^2 / i f_{K_A} f_K^{-1} m_{K_A}^{-2} = (2 g^{(a)}/\alpha')(1 + a m_K^2)\,. \tag{2.22}$$

Here we consider all the relations between the parameters. Combining the relations obtained in (i)–(iv), we find that the choice of Eqs. (2.11b) and (2.14b) must be rejected, and there remain only four independent equations, Eqs. (2.10), (2.11a), (2.20), and (2.22). Equation (2.11a) implies that the K_A meson rides on the K trajectory.

[18] If we make a stricter assumption that Eq. (2.19) should hold irrespective of the values of p_4^2, we have an additional relation $a=0$. Then the results are changed, and the asymptotic behavior of the pion electromagnetic form factor becomes essentially the same as that obtained in the previous paper (Ref. 6) of one of the present authors (Y. O.). As pointed out by Geffen, the previous result is derived from the assumption that no σ term exists in the matrix element considered, which assumption, however, leads to inconsistency. Therefore we do not adopt this strict assumption. See D. A. Geffen, Phys. Rev. Letters **23**, 897 (1969). See also R. Jengo and E. Remiddi, Nucl. Phys. **B15**, 1 (1970).

III. FORM FACTORS

The K_{l4} form factors are defined by

$$\langle \pi^-(p_2) | A_\mu^{-(s=1)}(0) | \pi^-(p_1) K^+(p_3) \rangle = [f_1(s,t,u)(p_1 - p_2)_\mu + f_2(s,t,u)(p_1+p_2)_\mu + f_3(s,t,u)(p_1 + p_3 - p_2)_\mu] (8 p_{10} p_{20} p_{30})^{-1/2}. \tag{3.1}$$

From Eqs. (2.18) and (3.1), we obtain the following expressions for the f_i's:

$$f_1(s,t,u) = [\sqrt{2} f_{K_A} / (p_4^2 + m_{K_A}^2)](B^+ + B^-)\,,$$

$$f_2(s,t,u) = [\sqrt{2} f_{K_A} / (p_4^2 + m_{K_A}^2)](A^+ + A^-)\,,$$

$$f_3(s,t,u) = \frac{i\sqrt{2} f_K}{p_4^2 + m_K^2} (T_K^+ + T_K^-) + \frac{\sqrt{2} f_{K_A}}{p_4^2 + m_{K_A}^2} \frac{1}{m_{K_A}^2} [(A^+ + A^-) p_4 \cdot (p_1 + p_2) + (B^+ + B^-) p^4 \cdot (p_1 - p_2)]\,, \tag{3.2}$$

where $T_K^\pm$, $A^\pm$, and $B^\pm$ are given by Eqs. (2.1), (2.5), and (2.6), respectively.

The K_{l3} form factors are defined as follows:

$$\langle \pi^-(p_2) | V_\mu^{-(s=1)}(0) | K^0(p_3) \rangle = -[f_+(u)(p_3 + p_2)_\mu + f_-(u)(p_3 - p_2)_\mu] (4 p_{20} p_{30})^{-1/2}, \tag{3.3}$$

where the left-hand side is related to the original matrix element through PCAC, current algebra, and the reduction formula,

$$\langle \pi^-(p_2) | A_\mu^{-(s=1)}(0) | \pi^0(p_1) K^0(p_3) \rangle \xrightarrow[p_1 \to 0]{} \frac{i}{2 f_\pi} \times \langle \pi^-(p_2) | V_\mu^{-(s=1)}(0) | K^0(p_3) \rangle (2 p_{10})^{-1/2}. \tag{3.4}$$

From Eqs. (2.18), (3.3), and (3.4) we obtain the following expressions for $f_\pm$:

$$f_+(u) = [2 f_\pi i f_{K_A} / (m_{K_A}^2 - u)](B^- - A^-)\,,$$

$$f_-(u) = \frac{4 f_\pi f_K}{m_K^2 - u} T_K^- - \frac{2 i f_\pi f_{K_A}}{m_{K_A}^2 - u} \left(1 - \frac{u + m_\pi^2 - m_K^2}{m_{K_A}^2} \right) (B^- - A^-)\,, \tag{3.5}$$

where $s = m_K^2$, $t = m_\pi^2$, and $u = -(p_2 - p_3)^2$.

The K_{l2} form factor is defined by

$$\langle 0|A_\mu^{-(s=1)}(0)|K^+(p_3)\rangle=\sqrt{2}f_K i p_{3\mu}(2p_{30})^{-1/2}. \quad (3.6)$$

The left-hand side of Eq. (3.6) is related to the original matrix element through

$$\langle\pi^0(p_2)|V_\mu^{-(s=1)}(0)|K^+(p_3)\rangle \xrightarrow[p_2\to 0]{} \frac{i}{2f_\pi} \times\langle 0|A_\mu^{-(s=1)}(0)|K^+(p_3)\rangle(2p_{20})^{-1/2} \quad (3.7)$$

and Eq. (3.4). Therefore f_K is expressed as

$$f_K=-[2f_\pi^2 f_K/(p_3^2+m_K^2)](T_K^+ + T_K^-), \quad (3.8)$$

where $s=m_K^2$, $t=0$, and $u=m_K^2$. The terms $A^\pm$ and $B^\pm$ do not appear on the right-hand side of Eq. (3.8), as they are multiplied by $p_1 \pm p_2$ $(=0)$. Neglecting the pion mass, we obtain from Eq. (3.8)

$$f_\pi^2 f_\rho^2/m_\rho^2=1/\pi, \quad (3.9)$$

which is the modified KSRF relation.[3] It is easily seen that our model satisfies the following Callan-Treiman relations[15]:

$$f_1=f_2,\quad f_3=0 \quad \text{when } p_1\to 0, \quad (3.10)$$

$$\left.\begin{aligned} f_1+f_2+f_3&=(i/\sqrt{2}f_\pi)[f_+(s)-f_-(s)]\\ f_3&=(-i/\sqrt{2}f_\pi)[f_+(s)+f_-(s)]\end{aligned}\right\}\ \text{when } p_2\to 0, \quad (3.11)$$

$$f_K/f_\pi=f_+(u)+f_-(u) \quad \text{when } p_2\to 0. \quad (3.12)$$

We note that one of the Callan-Treiman relations, (3.12) is equivalent to the modified KSRF relation if we neglect the pion mass.

Next we consider the pion electromagnetic form factor, defined as

$$\langle\pi^-(p_2)|V_\mu^0(0)|\pi^-(p_1)\rangle=-F_\pi(t)(p_1+p_2)_\mu \times(4p_{10}p_{20})^{-1/2}. \quad (3.13)$$

Here V_μ^0 is the electromagnetic current and is conserved. The quantity corresponding to the left-hand side of Eq. (3.13) is obtained from the original matrix element by using PCAC for the kaon and current algebra:

$$\langle\pi^-(p_2)|A_\mu^{-(s=1)}(0)|\pi^-(p_1)K^+(p_3)\rangle \xrightarrow[p_3\to 0]{} -\frac{i}{\sqrt{2}f_K} \times\langle\pi^-(p_2)|V_\mu^{0\prime}(0)|\pi^-(p_1)\rangle(2p_{30})^{-1/2}. \quad (3.14)$$

The current $V_\mu^{0\prime}$ defined in Eq. (3.14) is not the same as V_μ^0 in Eq. (3.13). The latter has a nonconserved part which corresponds to the so-called σ term. We explain the relation between V_μ^0 and $V_\mu^{0\prime}$ in the following way. Let us take the divergence of both sides of Eq. (3.14); then the left-hand side represents the scattering amplitude for the $\pi K\to\pi K$ process, which does not vanish automatically. On the other hand, the right-hand side of Eq. (3.14) becomes the matrix element of $\partial_\mu V_\mu^{0\prime}$ between the pions. Thus the vector current $V_\mu^{0\prime}$ defined by Eq. (3.14) contains the nonconserved part. We take the electromagnetic current V_μ^0 in Eq. (3.13) as the conserved part of $V_\mu^{0\prime}$ in Eq. (3.14). Then the form factor is given by

$$F_\pi(t)=-4if_K f_{K_A}\alpha' g^{(a)}(1+at) \times\frac{\Gamma(1-\alpha^{(1)}(m_\pi^2))\Gamma(1-\alpha^{(2)}(t))}{\Gamma(\frac{5}{2}-\alpha^{(1)}(t))}. \quad (3.15)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Here we summarize all the parameters introduced in our model and the restrictions among them. There are seven parameters: f_π, f_K, f_ρ^2, $g^{(a)}$, $g^{(b)}$, a, and b. The parameter f_{K_A} is excluded from the set of the independent ones, since it always appears in the form of $f_{K_A}g^{(a)}$ or $f_{K_A}g^{(b)}$. Four equations have been found: three equations for the parameters in the $\pi K\to\pi K_A$ scattering amplitude, Eqs. (2.10), (2.20), and (2.22), and the modified KSRF relation, Eq. (3.9). Here we notice that $F_\pi(0)=+1$, since the pion charge is unity. We introduce as the scaling factor of our model, $f_\pi=0.093$ GeV, determined from $\pi\to\mu\nu$ decay. Thus only one free parameter remains. We fix it by the assumption $f_+(0)=1$, which is expected from the argument based on $SU(3)$ symmetry. The Ademollo-Gatto theorem[19] indicates that $f_+(0)=1$ up to the second order in $SU(3)$-symmetry breaking.

Now we can determine the numerical values of the form factors in Sec. III and predict the ratio G_s/G_d. The three K_{l4} axial-vector form factors averaged in the physical region are calculated and compared with the values obtained from experiments by assuming the Cabibbo angle $\theta_A=0.21$ (experimental values are listed in square brackets):

$$|f_1|=5.0m_K^{-1}\ [(5.7\pm0.4)m_K^{-1}]\ \text{(Ref. 20)}, \quad (4.1)$$

$$|f_2|=8.1m_K^{-1}\ [(7.5\pm1.1)m_K^{-1}], \quad (4.2)$$

$$|f_3|=0.15m_K^{-1}. \quad (4.3)$$

The following values are obtained for the quantities relevant to the K_{l3} form factors:

$$\lambda_+=0.018\ \ [0.023\pm0.008 \text{ from } K^+ \text{ decay},\ 0.013\pm0.009 \text{ from } K^0 \text{ decay}]\ \text{(Ref. 21)}, \quad (4.4)$$

$$\lambda_-=-0.049\ \ [-0.14], \quad (4.5)$$

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and

$$f_-(0)/f_+(0) = -0.18 \quad [0.00\pm0.25 \text{ from the branching ratios, } -1.35\pm0.3 \text{ from the } \mu \text{ polarization}], \quad (4.6)$$

where $\lambda_\pm$ are defined by

$$f_\pm(q^2) = f_\pm(0)(1+\lambda_\pm q^2/m_\pi^2). \quad (4.7)$$

The K_{l2} decay constant f_K is found to be[22]

$$f_K = 0.112 \text{ GeV} \quad [0.119 \text{ GeV}]. \quad (4.8)$$

The pion charge radius is calculated[23] to be

$$\sqrt{\langle r_\pi^2\rangle} = 0.85\times10^{-13} \text{ cm} \quad [(0.80\pm0.10)\times10^{-13} \text{ or } (0.86\pm0.14)\times10^{-13} \text{ cm}], \quad (4.9)$$

where $\langle r_\pi^2\rangle$ is related to the pion electromagnetic form factor in a form

$$\langle r_\pi^2\rangle = -6F_\pi'(0)/F_\pi(0). \quad (4.10)$$

The asymptotic behavior of $F_\pi(q^2)$ is seen from Eq. (3.15)[24] to be

$$F_\pi(q^2) \propto q^{-2[1-\alpha^{(1)}(m_\pi^2)]} = q^{-1.4}, \quad q\to\infty. \quad (4.11)$$

The ratio G_s/G_d is predicted to be

$$G_s/G_d = -\tfrac{1}{2}[1/a+(m_\pi^2+m_K^2-m_{K^*}^2)] = -0.45 \text{ GeV}^2, \quad (4.12)$$

where we define G_s and G_d by the effective Hamiltonian

$$G_s K^A{}_\mu K^*{}_\mu \pi + G_d K^A{}_\mu K^*{}_\nu \partial_\mu\partial_\nu \pi. \quad (4.13)$$

[22] The decay constant f_K is determined from the $K\to\mu\nu$ decay rate.

[23] C. W. Akerlof, W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Rev. **163**, 1482 (1967); C. Mistretta, D. Imrie, J. A. Appel, R. Budnitz, L. Carroll, M. Goitein, K. Hanson, and R. Wilson, Phys. Rev. Letters **20**, 1523 (1968).

[24] This asymptotic behavior is different from that obtained in Ref. 6, as pointed out before. We do not claim, however, that this expression should be valid in the large-momentum-transfer limit.

Here we note the following point. We calculate these numerical values by using one set of solution. The other set exists, since one of the equations is quadratic for the parameter a. When the pion mass is neglected, however, all the equations become linear, and only one set of solution exists. We discard the other set because it disappears when the pion mass is set equal to zero.

The numerical values obtained above agree with the present experimental data excellently. This fact is surprising if we consider that the starting point of our discussion does not rest on a firm basis. The kaon PCAC is not good approximation in the discussion of kaon physics. Nor does the one-term description of the πK scattering amplitude in the Veneziano model satisfy the Adler-Weisberger relation.[25] Moreover, in the field-current identity [Eq. (1.2)], we could have assumed that $A^K{}_\mu$ is coupled to the infinite series of pseudoscalar and axial-vector mesons, whose existence is specially characteristic of the Veneziano model. It can be said, however, that the cooperation among PCAC, current algebra, the field-current identity, and the Veneziano representation overcomes these difficulties and leads to good results. For instance, current algebra alone gives no information on the mass shell, while the Veneziano amplitude gives the unique on-shell extrapolation of the results from the current algebra. On the other hand, the Veneziano model itself deals only with the scattering amplitudes and is not suited for describing form factors. We conclude that PCAC, current algebra, the field-current identity, and the Veneziano representation play a cooperative role in our model.

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[25] N. Tokuda, Progr. Theoret. Phys. (Kyoto) **42**, 641 (1969).