We have in any case a model in which the coherent inelastic effects can be evaluated completely, and, in fact, the total contribution from all coherent inelastic states is simpler to evaluate than that from any single state. It might be interesting to study in detail the magnitudes and phases of the individual contributions; this could probably be done using the methods developed in Ref. 11.

Insofar as our model is applicable to real elementary hadrons, it indicates that λ factors as large as 1.4 in the forward direction are quite likely, and that λ's as large as 2, as used in Ref. 3, are not completely unreasonable. Our model, however, gives little justification for choosing λ independent of momentum transfer.[22] We obviously have not proven that a constant λ is impossible, but we have shown that a not completely unrealistic model predicts a rather rapid variation.

ACKNOWLEDGMENTS

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[22] This conclusion was also reached, on somewhat different grounds, in Ref. 8.

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Reggeized $U(6)\otimes U(6)\otimes O(3)$ and the Absorption Model for $\gamma p\to\pi^+n$

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Preliminary results on the application of the Reggeized $U(6)\otimes U(6)\otimes O(3)$ absorptive peripheral model to photoproduction reactions are presented, with a detailed treatment for $\gamma p\to\pi^+n$. The agreement of the theory with experiment is most encouraging.

1. INTRODUCTION

In a series of papers,[1,2] the Reggeized $U(6)\otimes U(6)\otimes O(3)$ absorptive peripheral model has been applied with reasonable success to high-energy $0^-\frac{1}{2}^+\to 0^-\frac{1}{2}^+$ charge- and hypercharge-exchange reactions. The model has now been extended to photoproduction processes. The Reggeized $U(6)\otimes U(6)\otimes O(3)$ symmetry scheme is used to calculate the pole graph, thus providing significant constraints among the Regge residues. Regge-cut contributions are then introduced by applying absorption corrections to the pole amplitudes.

In Sec. 2 we present the general formalism for $\gamma\frac{1}{2}^+\to 0^-\frac{1}{2}^+$ reactions in terms of the s-channel helicity amplitudes and outline the derivation of the Reggeized supermultiplet amplitudes. Section 3 contains a discussion on the application of absorption corrections to elementary and Reggeized pion exchange. We conclude in Sec. 4 with a discussion of the differential cross section for the reaction $\gamma p\to\pi^+n$ obtained from our model.

2. FORMALISM

For photoproduction of mesons on baryons, the $SU(3)$ U-spin scalar transformation property of the photon in the vector-dominance model[3] gives for the covariant T matrix

$$T(\gamma B\to MB') = X_\rho[T(\rho^0 B\to MB') + (1/\sqrt{3})\,T(\omega_8 B\to MB')],$$

where the ρ-photon coupling X_ρ is given by

$$X_\rho = e/g_{\rho\pi\pi},\ \text{with}\ g_{\rho\pi\pi}^2/4\pi = 1.8\ \text{and}\ e^2/4\pi = 1/137.$$

The Reggeized $U(6)\otimes U(6)\otimes O(3)$ symmetry scheme[4] is used to calculate the pole graph for the reaction $0^-\frac{1}{2}^+\to 1^-\frac{1}{2}^+$. By applying time reversal and the vector-dominance model, we obtain the pole amplitudes for $\gamma\frac{1}{2}^+\to 0^-\frac{1}{2}^+$.

For meson-baryon scattering, which proceeds by the exchange of excitation number N, the relevant effective Lagrangians for the two $U(6)\otimes O(3)$ invariant 3-point

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[1] B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Phys. Rev. **187**, 1921 (1969); and Phys. Rev. D **1**, 954 (1970).

[2] P. A. Collins, B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Nucl. Phys. (to be published).

[3] For a review of the recent literature see, e.g., D. Schildknecht, DESY Report No. 69/10 (unpublished), in which further references can be found.

[4] R. Delbourgo, A. Salam, and J. Strathdee, Phys. Rev. **179**, 1487 (1968); **172**, 1727 (1968); **186**, 1516 (1969).

couplings are

(a) Meson-meson-meson supermultiplet vertex:

$$(6,\bar{6};0)_{\frac{1}{2}p+q'}-(6,\bar{6};0)_{\frac{1}{2}p-q'}-(6,\bar{6};N)_{-p},$$

$$\mathcal{L}_{(a)}=\mu^{1-N}\Phi_A{}^B(\tfrac{1}{2}p+q')\Phi_C{}^D(\tfrac{1}{2}p-q')\Bigg[h_0{}^{(-)}\delta_B{}^C\delta_D{}^A+h_0{}'^{(-)}\mu^{-2}q'_B{}^Aq'_D{}^C+\mu h_1{}^{(+)}\left(\delta_B{}^C\frac{\partial}{\partial q'_A{}^D}+\delta_D{}^A\frac{\partial}{\partial q'_C{}^B}\right)+\mu h_1{}^{(-)}\left(\delta_B{}^C\frac{\partial}{\partial q'_A{}^D}-\delta_D{}^A\frac{\partial}{\partial q'_C{}^B}\right)\Bigg]\Phi_{(N)}(-p,q').$$

(b) Baryon-baryon-meson supermultiplet vertex:

$$(56,1;0)_{-\frac{1}{2}p+q}-(\overline{56},1;0)_{\frac{1}{2}p+q}-(6,\bar{6};N)_p,$$

$$\mathcal{L}_{(b)}=m^{-N}\bar{u}^{(ACD)}(\tfrac{1}{2}p+q)u_{(BCD)}(-\tfrac{1}{2}p+q)\times(g_0\delta_A{}^B+mg_1\,\partial/\partial q_B{}^A)\Phi_{(N)}(p,q),$$

where m and μ are the masses associated with the (56, 1; 0) and (6, $\bar{6}$; 0) multiplets, respectively. The g_0, h_0, and h_0' couplings do not contribute to charge-exchange scattering.

The $U(6,6)$ fields are decomposed as

$$u_{(ABC)}(p)_{(\mu_1\cdots\mu_{2N})}=(6\sqrt{2}m)^{-1}\{[(\not{p}+m)\gamma_5C]_{\alpha\beta}\epsilon_{abd}N_{c\gamma}{}^d{}_{(\mu_1\cdots\mu_{2N})}+(\text{cyclic perm.})\}$$

and

$$\Phi_A{}^B(p)_{(\mu_1\cdots\mu_N)}=(2\sqrt{2}\mu)^{-1}\times\{(\not{p}+\mu)[\gamma_5P_{(\mu_1\cdots\mu_N)}-\gamma_\mu V_{\mu(\mu_1\cdots\mu_N))}]\}_A{}^B.$$

The covariant T matrix is

$$T_{MB}=\mathcal{L}_{(a)}\mathcal{L}_{(b)}$$

and

$$\Delta_{(N)}=(\Phi_{(N)}(p,q)\Phi_{(N)}(-p,q'))=(t-M^2)^{-1}(\mathbf{q}\cdot\mathbf{q}')^{N+1}$$

is the fully contracted propagator for the meson supermultiplet, where M is the mass associated with the exchange.

The Born diagram is evaluated at the pole $t=M^2$, and phase factors $\frac{1}{2}[1\pm\exp(-i\pi N)]$ extracted from $h^{(\pm)}$, respectively. Representing natural and unnatural parity exchange by n and u, respectively, we make the replacements for natural parity:

$$t-M^2\to\sin\pi(\alpha_n-1)=-\pi/\Gamma(\alpha_n)\Gamma(1-\alpha_n),\quad N\to\alpha_n-1,$$

and for unnatural parity

$$t-M^2\to\sin\pi\alpha_u=-\pi/\Gamma(1+\alpha_u)\Gamma(-\alpha_u),\quad N\to\alpha_u.$$

The Gell-Mann ghost-eliminating mechanism is introduced by dividing by $\Gamma(\alpha_n)$ for natural parity exchange and $\Gamma(1+\alpha_u)$ for unnatural parity exchange.

For $0^-\frac{1}{2}^+\to1^-\frac{1}{2}^+$ the T matrix is

$$T=(1+M/2\mu)2\bar{u}(\tfrac{1}{2}p+q)\times\{(1/8M\mu)[\not{p},[\not{q}',\gamma_\mu]\gamma_5]+(\mathbf{q}\cdot\mathbf{q}'/M^2\mu)p_\mu\gamma_5-(i\sigma_{\lambda\mu}\mathbf{q}_\lambda'\gamma_5/2\mu)\}u(-\tfrac{1}{2}p+q)\mathcal{A}+(1+M/2\mu)\times(1+2m/M)\bar{u}(\tfrac{1}{2}p+q)\gamma_5u(-\tfrac{1}{2}p+q)(p_\mu/M)\mathcal{B},$$

where

$$\mathcal{A}=\tfrac{1}{2}\beta_{n-}[1-\exp(-i\pi\alpha_{n-})]\Gamma(1-\alpha_{n-})(PV_\mu)_D\times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{n-}-1}+\tfrac{1}{2}\beta_{n+}\times[1+\exp(-i\pi\alpha_{n+})]\Gamma(1-\alpha_{n+})(PV_\mu)_F\times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{n+}-1}$$

is associated with natural parity exchange, and

$$\mathcal{B}=\tfrac{1}{2}\beta_{u-}[1-\exp(-i\pi\alpha_{u-})]\Gamma(-\alpha_{u-})(PV_\mu)_D\times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{u-}}+\tfrac{1}{2}\beta_{u+}\times[1+\exp(-i\pi\alpha_{u+})]\Gamma(-\alpha_{u+})(PV_\mu)_F\times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{u+}}$$

is associated with unnatural parity exchange. The $\pm$ refer to even and odd signatures, respectively.

The $U(6,6)$ spinor decomposition is carried out and "Gribov doubling" is used to remove the $\sqrt{t}$ kinematic singularities. All poles are made evasive by multiplying the residues by $t/4m\mu$, giving

$$T=\{-(t/8m\mu^2)(1+m/\mu)(1-t/4m^2)(\bar{N}\gamma_5[\gamma_\mu,\not{q}']N)_F-[(s-\tfrac{1}{2}\sum_i m_i^2+\tfrac{1}{2}t)/4m\mu^2]p_\mu[(1+t/4m\mu)\times(\bar{N}\gamma_5N)_{D+2F/3}-(t/4m^2)(1+m/\mu)(\bar{N}\gamma_5N)_F]+(t/4m^2\mu^2)[(1+t/4m\mu)\{q_\mu(\bar{N}\gamma_5\not{q}'N)_{D+2F/3}-\tfrac{1}{2}(s-\tfrac{1}{2}\sum_i m_i^2+\tfrac{1}{2}t)(\bar{N}\gamma_5\gamma_\mu N)_{D+2F/3}\}-(1+m/\mu)\times[q_\mu(\bar{N}\gamma_5\not{q}'N)_F-\tfrac{1}{2}(s-\tfrac{1}{2}\sum_i m_i^2+\tfrac{1}{2}t)(\bar{N}\gamma_5\gamma_\mu N)_F]]\}\mathcal{A}+(p_\mu/2\mu)(1+t/4m\mu)(1-t/4m^2)(\bar{N}\gamma_5N)_{D+2F/3}\mathcal{B}.$$

Defining T_μ to be the time-reversed T matrix with V_μ removed, gauge invariance implies that

$$(-\tfrac{1}{2}p+q')_\mu T_\mu=0,$$

where $(-\frac{1}{2}p+q')$ is the photon 4-momentum. The above T matrix is gauge invariant except for the pseudoscalar coupling, which can be made gauge invariant by the inclusion of a contribution from the s-channel graph. Since this contribution vanishes in the Coulomb gauge in the center-of-mass frame, the T matrix for $\gamma p\to\pi^+n$ is effectively gauge invariant.

We define m_i, E_i, and λ_i to be the mass, c.m. energy, and helicity of each particle, with $i=1, 2, 3, 4$ specifying the target baryon, the photon, and the outgoing baryon and meson, respectively. K and Q are the magnitudes of the incoming and outgoing 3-momenta in the c.m. frame. We also define

$$C=(E_1+m_1)(E_3+m_3),$$

$$D_\pm=C\pm QK,$$

$$H_\pm=K(E_3+m_3)\pm Q(E_1+m_1),$$

$$A=\tfrac{1}{2}\beta_{n-}[1-\exp(-i\pi\alpha_{n-})]\Gamma(1-\alpha_{n-})h_D \times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{n-}-1} +\tfrac{1}{2}\beta_{n+}[1+\exp(-i\pi\alpha_{n+})]\Gamma(1-\alpha_{n+})h_F \times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{n+}-1},$$

$$B=\tfrac{1}{2}\beta_{u-}[1-\exp(-i\pi\alpha_{u-})]\Gamma(-\alpha_{u-})h_D \times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{u-}}+\tfrac{1}{2}\beta_{u+} \times[1+\exp(-i\pi\alpha_{u+})]\Gamma(-\alpha_{u+})h_F \times[(s-m^2-\mu^2+\tfrac{1}{2}t)/2m\mu]^{\alpha_{u+}},$$

where h is the coupling constant at the meson-meson-Reggeon vertex and g is the coupling constant at the baryon-baryon-Reggeon vertex. For convenience we introduce

$$A_1=-\frac{t}{8\mu^2 m}\left(1+\frac{m}{\mu}\right)\left(1-\frac{t}{4m^2}\right)g_F A,$$

$$A_2=\frac{1}{2\mu}\left(1+\frac{t}{4m\mu}\right)\times\left[\left(1-\frac{t}{4m^2}\right)B-\frac{s-\frac{1}{2}\sum_i m_i^2+\frac{1}{2}t}{2m\mu}A\right]g_{D+2F/3} +\frac{t}{8m^2\mu}\left(1+\frac{m}{\mu}\right)\frac{s-\frac{1}{2}\sum_i m_i^2+\frac{1}{2}t}{2m\mu}g_F A,$$

$$A_3=\frac{t}{16m^2\mu^2}\left[\left(1+\frac{t}{4m\mu}\right)g_{D+2F/3}-\left(1+\frac{m}{\mu}\right)g_F\right]A.$$

Using the covariant boost convention, together with time reversal, the s-channel helicity amplitudes are, using the notation in Table I,

$$\phi_1=-(iX_\rho/\sqrt{2}\sqrt{C})\{A_1[-2\sin\tfrac{1}{2}\theta \times((E_2+E_4)D_- -(Q-K)H_-) -H_-Q\sin\theta\cos\tfrac{1}{2}\theta]+A_2H_-Q\sin\theta\cos\tfrac{1}{2}\theta +A_3[-Q\sin\theta\cos\tfrac{1}{2}\theta((E_2+E_4)H_+ +(Q+K)D_+) +2(2s+t-\sum_{i=1,4}m_i^2)D_+\sin\tfrac{1}{2}\theta]\},$$

$$\phi_2=(-iX_\rho/\sqrt{2}\sqrt{C})\{A_1[2\cos\tfrac{1}{2}\theta \times((E_2+E_4)D_+ +(Q+K)H_+) -H_+Q\sin\theta\sin\tfrac{1}{2}\theta]+A_2H_+Q\sin\theta\sin\tfrac{1}{2}\theta +A_3[Q\sin\theta\sin\tfrac{1}{2}\theta(-(E_2+E_4)H_- +(Q-K)D_-) -2(2s+t-\sum_{i=1,4}m_i^2)D_-\cos\tfrac{1}{2}\theta]\},$$

$$\phi_3=(-iX_\rho/\sqrt{2}\sqrt{C})Q\sin\theta\sin\tfrac{1}{2}\theta\{(-A_1+A_2)H_+ +A_3[-(E_2+E_4)H_- +(Q-K)D_-]\},$$

$$\phi_4=(-iX_\rho/\sqrt{2}\sqrt{C})Q\sin\theta\cos\tfrac{1}{2}\theta\{-(-A_1+A_2)H_- +A_3[(E_2+E_4)H_+ +(Q+K)D_+]\},$$

where θ is the c.m. scattering angle.

The ϕ_i's are expanded in partial-wave series and the absorption corrections applied using the traditional Watson formula[5]

$$T_{\lambda\mu}'^J=\tfrac{1}{2}(S^{\text{el }J}{}_{\text{initial}}+S^{\text{el }J}{}_{\text{final}})T_{\lambda\mu}{}^J,$$

where μ and λ are the initial and final helicities, respectively. The Gaussian model of elastic scattering is assumed:

$$S^J=1-c\exp[-J(J+1)/\nu^2p^2],$$

where J is the angular momentum, p is the c.m. 3-momentum, and ν the elastic radius of interaction. The partial-wave series are then resumed to give the modified helicity amplitudes ϕ_i', in terms of which

$$d\sigma/dt=(\pi/K^2)\times\tfrac{1}{2}\sum_{i=1,4}|\phi_i'/8\pi\sqrt{s}|^2.$$

The absorption coefficients used are shown in Table II.

TABLE I. Helicity dependence of amplitudes for $\gamma\frac{1}{2}^+\to0^-\frac{1}{2}^+$.

$\lambda_3\ \lambda_4$ \\ $\lambda_1\ \lambda_2$	$\frac{1}{2}$ 1	$-\frac{1}{2}$ 1	$\frac{1}{2}$ -1	$-\frac{1}{2}$ -1
$\frac{1}{2}$ 0	ϕ_1	ϕ_3	ϕ_4	$-\phi_2$
$-\frac{1}{2}$ 0	ϕ_2	ϕ_4	$-\phi_3$	ϕ_1

TABLE II. Absorption coefficients.

p_{lab} (GeV/c)	c_{initial}	ν_{initial} [(GeV/c)$^{-1}$]	c_{final}	ν_{final} [(GeV/c)$^{-1}$]
5.0	1	0.295	0.81	0.26
8.0	1	0.295	0.76	0.26
11.0	1	0.295	0.73	0.26
16.0	1	0.295	0.71	0.26
18.0	1	0.295	0.71	0.26

[5] H. D. D. Watson, Phys. Letters **17**, 72 (1965).

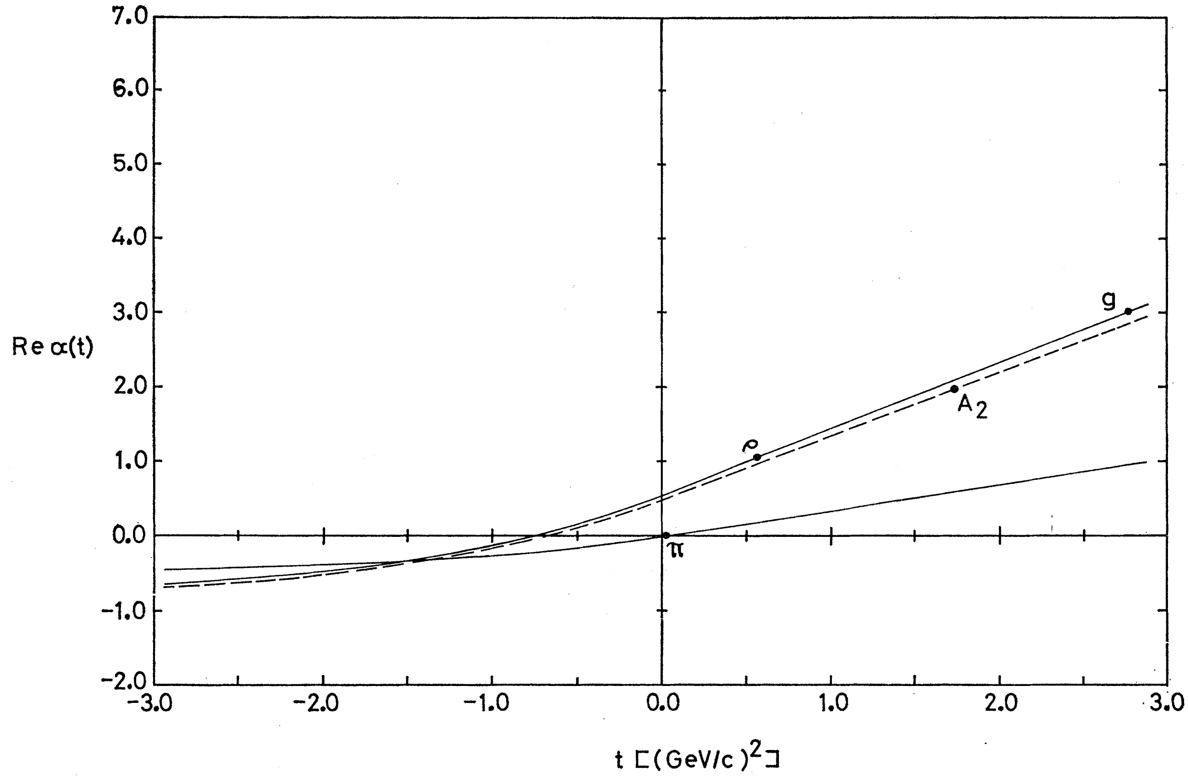

FIG. 1. Plot of $\alpha(t)$ against t for π, ρ, and A_2 trajectorics. Parameters from Table II.

3. PION EXCHANGE

The experimental data[6] on $\gamma p\rightarrow\pi^+n$ exhibit a sharp forward peak of width in t of the order of the square of the pion mass. This is suggestive of pion exchange.

Now it is well known that the elementary π is evasive and hence its contribution to $d\sigma/dt$ at $t=0$ vanishes. However, with the application of absorption corrections, a forward peak of the correct width can be obtained. In particular, for $\gamma p\rightarrow\pi^+n$,

$$\phi_2=\langle -\tfrac{1}{2}0\mid T\mid \tfrac{1}{2}1\rangle$$

does not involve any net helicity flip. Hence it need not vanish in the forward direction. However,

$$\phi_2{}^\pi\sim t/(t-m_\pi{}^2) \qquad \text{for small } t$$

$$=1+m_\pi{}^2/(t-m_\pi{}^2)\,.$$

Now the first term is s-wave, which grossly violates unitarity at high energy. This contribution is largely removed by the absorption corrections, giving

$$\phi_2{}^\pi(\text{absorbed})\sim m_\pi{}^2/(t-m_\pi{}^2)\,,$$

which peaks in the forward direction. However, for nonzero spin exchange, both the energy and momentum-transfer dependence[7] of the absorption model are in disagreement with experiment.

As discussed below, for small momentum transfers an elementary and an evasive Reggeized pion exchange give essentially the same result for small t. Thus a Reggeized absorption model would give all the good results of the elementary spin-0-exchange absorption model but improved results for higher-spin exchange.

In a recent paper,[8] Leader has discussed minimal requirements for attributing a contribution to Reggeized π exchange. These conditions are as follows:

(i) The contribution should be associated with a pole at $t=m_\pi{}^2$, and the polelike denominator should be largely responsible for the rapid variation with t for $|t|<$ a few $m_\pi{}^2$.

TABLE III. $SU(3)$ couplings.

Reaction	Baryon vertex $D+\frac{2}{3}F$	Baryon vertex F	Meson vertex D	Meson vertex F
$\rho^0p\rightarrow\pi^+n$	$(5/3)\sqrt{2}$	$\sqrt{2}$	0	-2
$(1/\sqrt{3})\omega_8p\rightarrow\pi^+n$	$(5/3)\sqrt{2}$	$\sqrt{2}$	$\frac{2}{3}$	0

[6] A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Ecklund, G. E. Fischer, J. Rees, and B. Richter, Phys. Rev. Letters **20**, 300 (1968), and contribution to the Fourteenth International Conference on High-Energy Physics, Vienna, 1968, paper 430 (unpublished).

[7] F. D. Gault, B. J. Hartley, J. H. R. Migneron, and K. J. M. Moriarty, Nuovo Cimento **62**, 269 (1969), and references therein.

[8] E. Leader, invited paper, Institute of Physics and the Physical Society, Conference on Elementary Particle Physics, Cambridge, England, 1969 (unpublished).

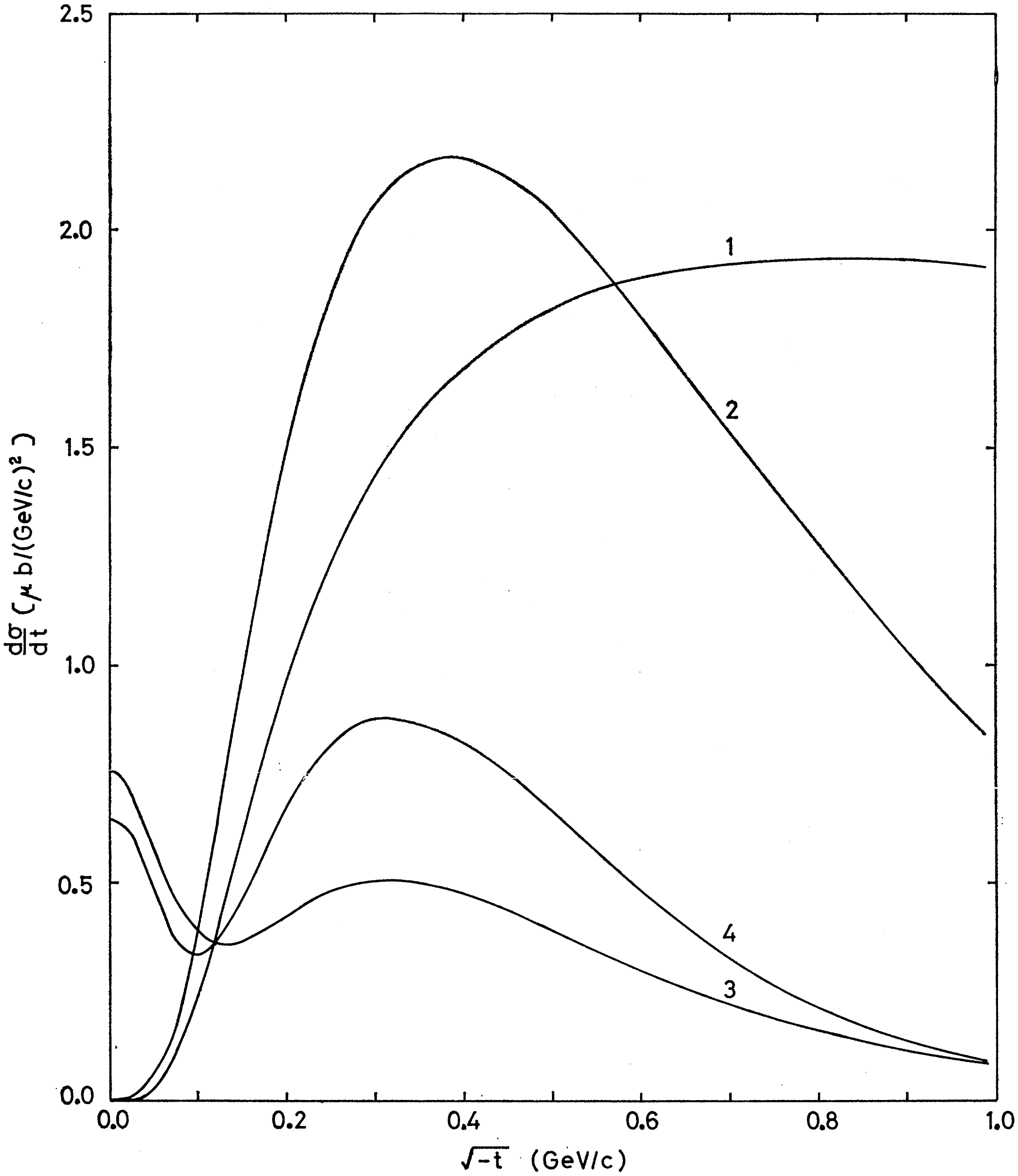

FIG. 2. Predictions for elementary π, Reggeized π, absorbed elementary π, and absorbed Reggeized π (indicated by curves 1–4, respectively) at 8 GeV/c.

(ii) The contribution, when continued to the region $t\simeq m_\pi^2$, should reduce to the result for elementary π exchange.

(iii) The energy dependence of $d\sigma/dt$ for small t should be approximately s^{-2}.

4. DISCUSSION AND RESULTS

Within the content of the vector-dominance model, the reaction $\gamma p\to\pi^+ n$ can be expressed in terms of the reactions

$$\rho^0 p\to\pi^+ n,$$

$$\omega_8 p\to\pi^+ n.$$

For $\rho^0 p\to\pi^+ n$, the possible t-channel exchanges are the A_2 (natural parity) and the π (unnatural parity), while for $\omega_8 p\to\pi^+ n$ the possible exchanges are the ρ (natural parity) and the B (unnatural parity). The relevant $SU(3)$ couplings are shown in Table III.

Little is known about the trajectory of the B meson, but it is generally believed that it is relatively low-lying. We therefore consider only π, ρ, and A_2 exchange.

We took the residues to be constants and parametrized the trajectory functions for negative t by

$$\alpha(t)=\alpha_0+\alpha_1\exp(\alpha_2 t)$$

in keeping with Ref. 1. This gives the conventional

TABLE IV. Regge parameters.

Trajectory	π	ρ	A_2
α_0	-0.520	-0.871 (not varied)	-0.936 (not varied)
α_1	0.513	1.415 (not varied)	1.420 (not varied)
α_2 [$(\mathrm{GeV}/c)^{-2}$]	0.674	0.632 (not varied)	0.607 (not varied)
β [$(\mathrm{GeV}/c)^{-1}$]	21.637	-33.318	35.040
No. of data points		99	
χ^2		255.0	

linear parametrization of a Regge trajectory in the peripheral region:

$$\alpha(t)\simeq(\alpha_0+\alpha_1)+\alpha_1\alpha_2 t.$$

The ρ- and A_2-trajectory parameters were taken from Ref. 1. The pion trajectory, in accordance with the first of Leader's conditions, was constrained to pass through the pion pole by extrapolating the linear form. In order to satisfy angular momentum conservation in the forward direction it is necessary to evade the pseudoscalar coupling. In the $U(6)\otimes U(6)\otimes O(3)$ symmetry scheme, this demands that we not only evade the π residue, but also the ρ and A_2 residues. The residues for the ρ and A_2 in Ref. 1 were nonevasive, so we cannot hope to carry over the residues from Ref. 1 to this work. We therefore have two trajectory parameters for the π, and three constant residues, giving a total of five parameters to be determined from the data. We used MINUITS (CERN Program Library No. D506) to obtain these parameters by a χ^2 fit of the differential cross section to the experimental data. The results of this minimization procedure are shown in Table IV.

A Chew-Frautschi plot of the π, ρ, and A_2 trajectories, using these parameters, is shown in Fig. 1. We note that the π trajectory has a slope for positive t of 0.35, which is to be contrasted with the ρ and A_2, which have slopes of 0.90 and 0.86, respectively.

The effect of absorption on elementary and Reggeized pion exchange is shown in Fig. 2. Unabsorbed elementary and Reggeized pion exchange are represented by curves 1 and 2, respectively. The result for the elementary pion exchange is taken from Ref. 9, which employed $U(6,6)$ symmetry to relate vertex couplings, and the Reggeized pion is the result of the present calculation. It should be noted that for both the differential cross section behaves as t for $t\to 0$. The application of absorption corrections to the elementary and Reggeized pion exchanges results in curves 3 and 4, respectively. A sharp forward peak of width $\sim m_\pi^2$, so characteristic of the data, is obtained in each case. Beyond the region in which we expect the pion to dominate, i.e., $|t|>m_\pi^2$, a "dip-bump" structure, not exhibited by the data, is present.

[9] D. G. Fincham, A. P. Hunt, J. H. R. Migneron, and K. Moriarty, Nucl. Phys. **B13**, 161 (1969).

Leader's first condition, that the contribution is associated with a pole at $t=m_\pi^2$, is ensured by constraining the pion trajectory to pass through the pion pole, i.e.,

$$\alpha_\pi(t=m_\pi^2)=0.$$

For small t our amplitude contains the factor

$$\Gamma(-\alpha_\pi)\simeq -1/(t-m_\pi^2)\alpha' \quad \text{near } t=m_\pi^2,$$

which is singular at $t=m_\pi^2$. This denominator is responsible for the rapid variation with t for $|t|\lesssim m_\pi^2$. In fact

$$m_\pi^2/(t-m_\pi^2)\simeq \exp(t/m_\pi^2)\simeq \exp(50t)$$

for small t. By comparing the elementary pion exchange of Ref. 9 with the Reggeized pion exchange of the present calculation, we obtain

$$\beta_\pi=6\,gh\mu\,\frac{(1-\mu^2/4m^2)(1+2m/\mu)}{(1+m_\pi^2/4m\mu)(1-m_\pi^2/4m^2)}\left.\frac{d\alpha}{dt}\right|_{t=m_\pi^2},$$

where g and h are the universal $U(6,6)$ couplings at the baryon-baryon-meson and meson-meson-meson vertices, respectively. This shows the correspondence of the elementary pion exchange and the Reggeized pion exchange when continued to the region $t=m_\pi^2$, as required by Leader's second condition.

In Fig. 3 is shown the energy variation of the momentum-transfer distributions for the reaction $\gamma p\to\pi^+ n$ obtained from our model. The s and t dependence of the experimental data is extremely well represented for $5.0\leq p_{\mathrm{lab}}\leq 18.0$ GeV/c and for $0\leq|t|\leq 1.0$ $(\mathrm{GeV}/c)^2$. Data exist at higher momentum transfers, but this was not included in our analysis since this is beyond the peripheral region. The addition of ρ and A_2 exchanges fills in the dip structure obtained from π exchange alone to give a "shoulder", and increases the forward normalization. For small t, where our trajectories are linear, the trajectory of the $\pi-P$ cut contribution is given by[10]

$$\alpha_c(t)=\alpha_\pi(0)+\alpha_{\mathrm{el}}(0)-1 +\{\alpha_\pi'(0)\alpha'_{\mathrm{el}}(0)/[\alpha_\pi'(0)+\alpha_{\mathrm{el}}'(0)]\}t.$$

In our case we have a fixed j pole

$$\alpha_{\mathrm{el}}(t)=1,$$

[10] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962); S. Mandelstam, *ibid.* **30**, 1127 (1963); **30**, 1148 (1963).

giving

$$\alpha_0(t)=\alpha_\pi(0)\simeq 0.$$

Therefore the cut contribution to the differential cross section is given by

$$d\sigma/dt\sim[s^{\alpha_c(t)-1}/(\ln s+\text{const})]^2\sim s^{-2}.$$

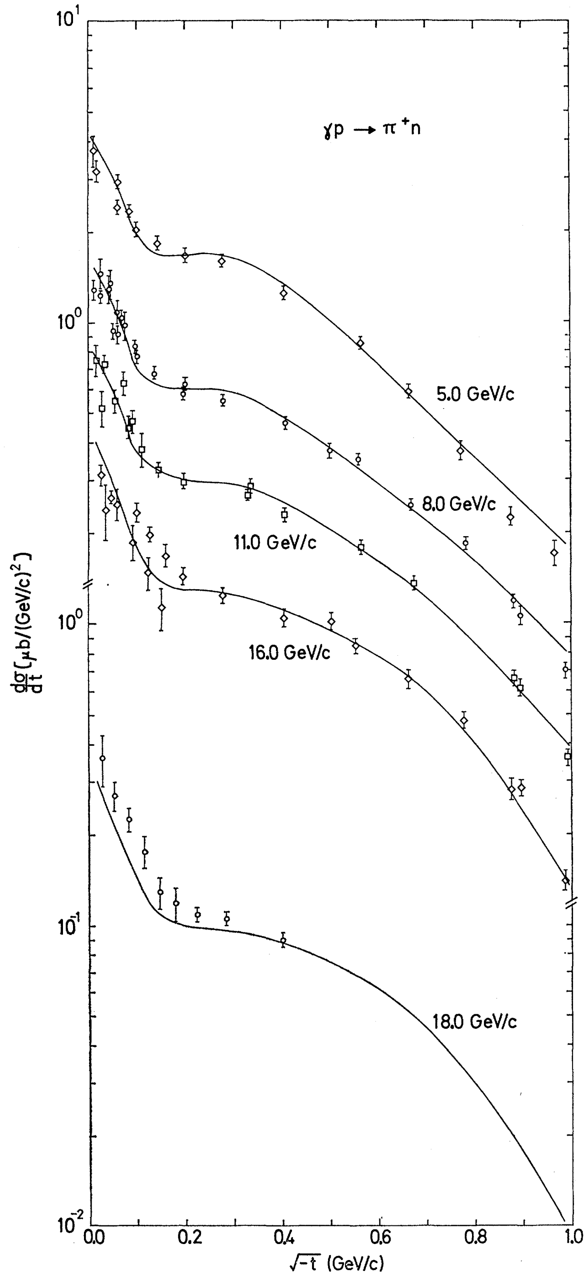

FIG. 3. Differential cross section for $\gamma p\rightarrow\pi^+n$. Data from Ref. 6.

Because of the slow logarithmic variation, this satisfies Leader's third condition.

The reaction $\gamma p\rightarrow\pi^+n$ has been treated in a pole+cut approach by Frøyland and Gordon,[11] Henyey *et al.*,[12,13] and Blackmon *et al.*[14] The model of Ref. 11 contained evasive π and ρ exchanges and their respective cut contributions, which were of adjustable strength. Using 18 free parameters, they were able to represent the differential cross section. We note that their differential cross section has a dip at $t\sim m_\pi^2$, the depth of which increases with increasing energy, in contrast with our model. Reference 13 used π, ρ, and A_2 exchanges with adjustable cuts generated by absorption. Twelve free parameters were used. As in the previous model of Frøyland and Gordon,[11] this gives a dip which becomes more prominent with increasing energy. The agreement for large t is somewhat unsatisfactory. The model of Blackmon *et al.*[14] used an elementary π exchange $[\alpha_\pi(t)=0]$ plus a Reggeized evasive A_2 exchange. Since their π is elementary, there were no free parameters for this exchange. However, the A_2 exchange involved five free parameters. The fit to the experimental data was reasonable.

Our model, which employs Reggeized π, ρ, and A_2 exchanges plus absorptive correction cuts, at the expense of only five free parameters, is able to reproduce the features of the experimental data. We feel that this work is additional evidence of the importance of Regge cuts especially for π-exchange reactions.[15]

For the calculations we used an exact partial-wave summation of 30 partial waves calculated with a 48-point Gaussian quadrature. Experience has shown that sufficient accuracy is obtained in this manner.

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