Forward Charged-Pion Electroproduction from Fixed-Momentum-Transfer Dispersion Relations*

F. A. BERENDS
Cambridge Electron Accelerator, Harvard University, Cambridge, Massachusetts 02138
(Received 21 January 1970)

The s-channel approach to forward charged-pion photoproduction above the resonance region by means of fixed-t dispersion relations is known to be quite successful. In this paper we investigate the kind of behavior that can be expected for charged-pion electroproduction on the basis of such a model.

I. INTRODUCTION

CHARGED-PION photoproduction at very small momentum transfers is dominated by the isovector photon amplitudes at energies above the resonance region.[1] The dominant contribution to the isovector amplitudes comes from the Born term. In fact, up to $t=-\mu^2$ (μ being the pion mass), the forward cross section is qualitatively well described by the Born term alone. For larger t values, one needs corrections to the Born amplitude. As has been shown by Engels, Schwiderski, and Schmidt,[2] this can be achieved by the inclusion of fixed-t dispersion-relation contributions. The dispersion integrals are necessarily cut off at relatively low energies, but nevertheless the agreement between the model and experiment is good. The low-energy photoproduction data then determine the high-energy behavior at small t values, without the intervention of a specific model for the high-energy behavior. Recently this conclusion has also been obtained by Jackson and Quigg[3] in the context of finite-energy sum rules.

The success of the approach can be understood if for some reason the imaginary parts of the photoproduction amplitudes in the forward direction at higher energies are negligible with respect to the real parts. This makes it possible not only to neglect them in the evaluation of the cross sections, but also to cut off the dispersion integrals at relatively low energies, thus making a practical evaluation feasible.

Another simplification occurs in that the electric and magnetic multipoles of the second and third πN resonances are such that they cancel in the forward direction.[4] Therefore, the well-known magnetic dipole transition to N^* (1236) gives the most important contribution to the dispersion integral.

The model is, however, limited to the region $|t|<6\mu^2$ for two reasons. First, the rapid increase of the isoscalar amplitudes with t, as can be seen from the π^-/π^+ ratio of photoproduction on deuterium,[1] makes a calculation in terms of the isovector amplitude A^- alone not sensible. Second, the model starts to give a too large A^- at higher t values.

Because of this remarkably simple mechanism of charged photoproduction in the forward direction, it would be interesting to see whether this picture carries over to electroproduction. It is the purpose of this paper to investigate what type of behavior can be expected for charged-pion electroproduction in the forward region on the basis of this simple model.

In Sec. II we discuss the dispersion relations for electroproduction and the assumptions made for their evaluation. In Sec. III we present the results for the differential cross sections. In Appendix A the removal of kinematical singularities is discussed and in Appendix B some kinematics is given.

II. FIXED-t DISPERSION RELATIONS

Throughout the paper we use the notation of Ref. 5, where a rather explicit account of the electroproduction formalism is given. We refer to this reference for more details.

Introducing the Mandelstam variables

$$\begin{aligned} s&=-(K+P_1)^2,\\ t&=-(K-Q)^2,\\ u&=-(K-P_2)^2, \end{aligned} \tag{1}$$

where K, Q, P_1, and P_2 are the four-momenta of the virtual photon, pion, and in- and outgoing nucleon, the dispersion relations for the invariant isovector amplitudes A_i^- of Dennery[6] read

$$\mathrm{Re}\tilde{A}(s,t)=\{(s-m^2)^{-1}+[\xi](u-m^2)^{-1}\}\tilde{\Gamma}(t)+\frac{\tilde{\Gamma}_t}{t-\mu^2}+\frac{P}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\{(s'-s)^{-1}+[\xi](s'-u)^{-1}\}\,\mathrm{Im}\tilde{A}(s',t). \tag{2}$$

---

* Supported in part by the AEC under Contract-No.AT (30-1), 2076.

[1] B. Richter, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 3.

[2] J. Engels, G. Schwiderski, and W. Schmidt, Phys. Rev. **166**, 1343 (1968). See also W. Schmidt [*ibid.* **188**, 2458 (1969)] for an application for timelike K^2.

[3] J. D. Jackson and C. Quigg, Phys. Letters **29B**, 236 (1969).

[4] R. L. Walker, Phys. Rev. **182**, 1729 (1969).

[5] F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 1 (1967).

[6] Ph. Dennery, Phys. Rev. **124**, 2000 (1961).

The six amplitudes are written as one vector $\tilde{A}$; $[\xi]$ is a diagonal matrix, where the first, second, and fourth diagonal elements are -1 and the others are 1. The residues $\tilde{\Gamma}(t)$ are

$$\begin{aligned}\Gamma_1&=\tfrac{1}{2}gF_1{}^v,\\ \Gamma_2&=-gF_1{}^v/(t-\mu^2),\\ \Gamma_3&=\Gamma_4=-\tfrac{1}{2}gF_2{}^v,\\ \Gamma_5&=\tfrac{1}{2}\Gamma_2,\\ \Gamma_6&=0.\end{aligned} \quad (3)$$

$\tilde{\Gamma}_t$ is zero, except for its fifth element, which equals $(2g/K^2)[F_\pi-F_1{}^v]$. $F_1{}^v$ and $F_2{}^v$ are the isovector form factors for the nucleon normalized to e and $(e/2m)(\mu_p'-\mu_n')$, μ' being the anomalous magnetic moment. F_π is the pion electromagnetic form factor normalized to e.

The above pole terms and subtraction constant $\tilde{\Gamma}_t/(t-\mu^2)$ are obtained from the generalized Born approximation which is made gauge invariant (for a discussion of this point, see Ref. 7).

In the calculation we express the nucleon form factors in terms of the Sachs form factors, for which we assume scaling and the dipole fit, i.e.,

$$G_{Ep}=G_{Mp}/\mu_p=G_{Mn}/\mu_n=(1+K^2/0.71)^{-2}. \quad (4)$$

Furthermore, we take $G_{En}=0$. For the charge and πN coupling constant, the values $e^2/4\pi=1/137$ and $g^2/4\pi=14.4$ are used. For the pion form factor the result of Ref. 9 is taken, i.e.,

$$F_\pi=(1+K^2/0.31)^{-1}, \quad (5)$$

which was obtained through (model-dependent) fits to low-energy electroproduction data [up to $K^2=0.4$ $(\mathrm{GeV}/c)^2$].

The absorptive part has to be taken from low-energy electroproduction. The scarce data enforce necessarily rough approximations. If one cuts off the dispersion integral at an s value equivalent to a photon lab energy $E_\gamma\cong 1$ GeV for photoproduction, then the most prominent resonances are the first, second, and third πN resonances. The latter two are, however, negligible in the forward direction for $K^2=0$ as mentioned before. At present nothing is known on the ratio of electric, magnetic, and scalar multipoles of the excitations to these resonances for $K^2\neq 0$.[8] Therefore, we take the optimistic viewpoint that in electroproduction again one can neglect them in the forward direction.

This leaves us with the N^* (1236). Recent experimental results[9] indicate that the dispersion-theory calculations of Adler[10] and of Zagury[11] describe electroproduction reasonably well in this energy region, although the more refined photoproduction calculations achieve a better agreement with experiment.[12] We approximated the absorptive part by the $M_{1+}{}^3$, $E_{1+}{}^3$, and $S_{1+}{}^3$ multipoles as given by these theories (the integral now being cut off at an equivalent photon lab energy E_γ of 800 MeV). It turns out that the $M_{1+}{}^3$ contribution is the most important one, as expected from photoproduction. The results for the effects of $E_{1+}{}^3$ and $S_{1+}{}^3$ are ambiguous. The $S_{1+}{}^3$ contribution is the only one which can become important (up to 20% of the $M_{1+}{}^3$ contribution) but only for Zagury's multipoles. The two theories give contributions which are different in sign. This is due to the fact that Adler's multipoles go through zero at the resonance, whereas Zagury's do this between 400 and 600 MeV. Because of these uncertainties and because the $M_{1+}{}^3$ contribution is in any case the most important, only this multipole is taken into account. Since at higher K^2 the theories seem to underestimate the experiments, the calculations are carried out with a magnetic dipole of the form

$$M_{1+}{}^3(K^2)=(k/\bar{k})(1+K^2/0.71)^{-2}M_{1+}{}^3(0), \quad (6)$$

where an experimentally determined form-factor dependence[13,9] is attached to the photoproduction multipoles, which are taken from Ref. 14. k ($\bar{k}$) is the photon three-momentum in the πN c.m. system in electro- (photo-) production. For $K^2>1$ $(\mathrm{GeV}/c)^2$ one should use a more rapidly decreasing form factor,[15] e.g., the one suggested by Dufner and Tsai.[16]

It may be noticed that a variation of F_π in the Born term should be accompanied by an appropriate change in the absorptive part, because pion exchange is one of the forces for the multipoles. It is known,[10,11] however, that a variation of F_π affects mostly $E_{1+}{}^3$ and $S_{1+}{}^3$ but not $M_{1+}{}^3$. So when experimental data are available, one may try to adjust F_π without changing the dispersion integrals. A determination of the pion form factor in this way, of course, is highly model dependent.

There is one other point, which should be mentioned before presenting the results. Care has been taken to avoid spurious kinematical singularities in the cross section, which might be caused by the kinematical

[7] F. A. Berends and G. B. West, Phys. Rev. **188**, 2538 (1969).

[8] A. B. Clegg, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969*, p. 123 (Daresbury Nuclear Physics Laboratory, 1969).

[9] C. A. Mistretta, J. A. Appel, R. J. Budnitz, L. Carroll, J. Chen, J. R. Dunning, Jr., M. Goitlein, K. Hanson, D. C. Imrie, and R. Wilson, Phys. Rev. **184**, 1487 (1969).

[10] S. L. Adler, Ann. Phys. (N.Y.) **50**, 189 (1968).

[11] N. Zagury, Phys. Rev. **145**, 1112 (1966); **150**, 1406(E) (1966); **165**, 1934(E) (1968); Nuovo Cimento **52**, 506 (1967).

[12] H. Rollnik, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 400.

[13] W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemans, Phys. Letters **24B**, 165 (1967).

[14] F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 54 (1967).

[15] W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy, U. Meyer-Berkhout, W. Schmidt, V. Walther, and G. Weber, Phys. Letters **28B**, 148 (1968).

[16] A. J. Dufner and Y. S. Tsai, Phys. Rev. **168**, 1801 (1968).

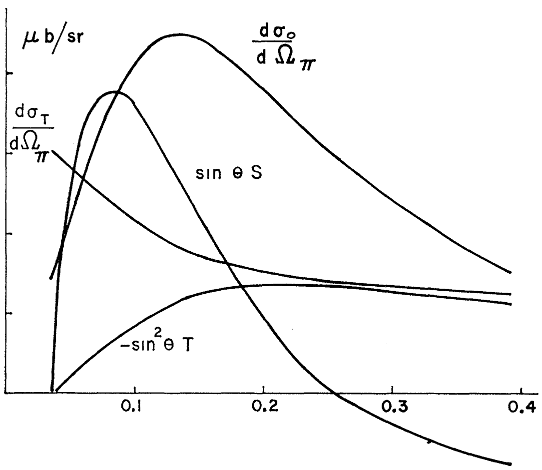

FIG. 1. Contributions of the four terms in Eq. (11) plotted for $W=3$ GeV and $K^2=0.3$ $(\mathrm{GeV}/c)^2$.

singularities at $t=\mu^2$ in A_2 and A_5. The procedure adopted is discussed in Appendix A.

III. RESULTS

The differential cross section for electroproduction can be written as[17]

$$d^3\sigma/dk_{20}{}^L d\Omega_e d\Omega_\pi = \Gamma_T (d\sigma/d\Omega_\pi)\,, \tag{7}$$

where[18]

$$\Gamma_T = \frac{\alpha}{4\pi^2}\frac{k_2{}^L}{k_1{}^L}\frac{\bar{k}^L}{K^2}\left[2+\frac{\cot^2(\frac{1}{2}\theta_e{}^L)}{1+(k_0{}^L)^2/K^2}\right]$$

$$= \frac{\alpha}{2\pi^2}\frac{k_2{}^L}{k_1{}^L}\frac{\bar{k}^L}{K^2}(1-\epsilon)^{-1}, \tag{8}$$

with

$$\epsilon = \{1+2[(k^L)^2/K^2]\tan^2(\tfrac{1}{2}\theta_e{}^L)\}^{-1}, \tag{9}$$

and where 1 and 2 denote the incoming and outgoing electron, $\theta_e{}^L$ is the electron scattering angle in the lab system, and Ω_e and Ω_π are the solid angles of the electron in the lab system and of the pion in the πN c.m. system. The quantity $\bar{k}^L$ is the equivalent photon lab energy,

$$\bar{k}^L = (W^2-m^2)/2m. \tag{10}$$

The virtual-photon differential cross section in the πN c.m. system consists of the following quantities (see, e.g., Ref. 9), depending on W, t, and K^2:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon\frac{d\sigma_0}{d\Omega_\pi} + \epsilon\cos 2\varphi\sin^2\theta\, T + [\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}\cos\varphi\sin\theta\, S, \tag{11}$$

[17] Four-vectors are represented by a capital letter. The corresponding three-vector, its magnitude, and the time component are denoted by the lower-case letter, e.g., $K=(\mathbf{k}, ik_0)$ and $k=|\mathbf{k}|$. These quantities are taken in the πN c.m. system except when labeled by a suffix L, when it denotes a lab quantity.

[18] L. N. Hand, Phys. Rev. **129**, 1834 (1963). This is the convention used in Ref. 9, but not in Ref. 13.

where θ and φ are the polar and azimuthal angle of the pion in the πN c.m. system, where a coordinate system is adopted with the xz plane the plane of $\mathbf{k}_1$ and $\mathbf{k}_2$, with the z axis along $\mathbf{k}$ and the y axis along $\mathbf{k}_1\times\mathbf{k}_2$. The explicit ϵ and φ dependence in (11) makes a separation of the four terms possible.

The first and third terms arise in photoproduction (with $\epsilon=1$). For $\varphi=0$ $(\pi/2)$, one obtains the cross section due to photons polarized parallel (perpendicular) to the scattering plane. The second term is the term due to the scalar photons, whereas the fourth term is caused by the interference of scalar and transverse photons.

In Appendix B these quantities are expressed in the traditional amplitudes[6] $\mathfrak{F}_i$ (and through them in the amplitudes A_i). The connection with the helicity amplitudes $f_{\lambda_2\lambda_1\lambda_\gamma}$ is also given.

In the t range considered $(|t|<6\mu^2)$, A_1, A_2, and A_5 are most important. In the evaluation of A_4 the Born term and dispersion integral almost cancel, making A_4 of the same size as A_3 and A_6 (a few percent of the other amplitudes). From Eqs. (2) and (3) it is evident that A_2 and A_5 are strongly t dependent, whereas A_1 is not. Moreover, the energy dependence of A_1 and A_2 is approximately $1/s$ and A_5 is constant. The form factors of course determine the K^2 dependence.

In Eq. (11) all terms have a strong t dependence. A

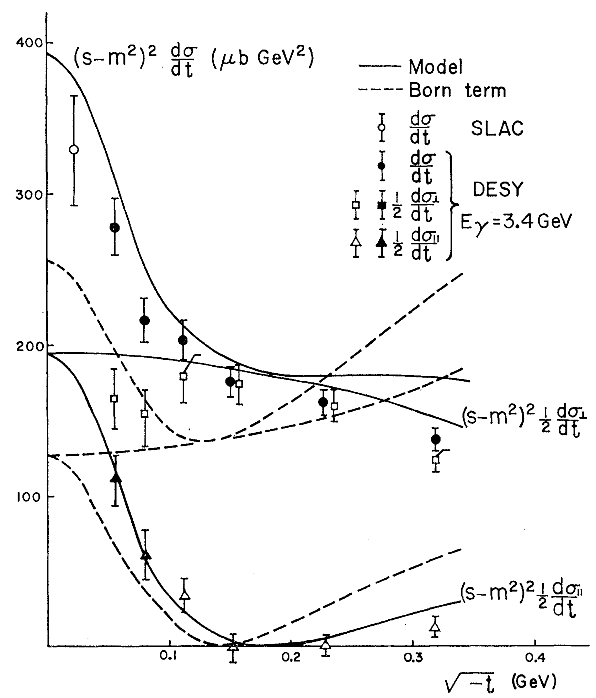

FIG. 2. Model (solid lines) and Born-term (dashed lines) predictions for the differential cross sections for unpolarized photons and for photons polarized perpendicular and parallel to the scattering plane as function of $(-t)^{1/2}$. The data are from Refs. 19 and 20 as compiled in Ref. 21.

typical example of their behavior is given in Fig. 1 for $W=3$ GeV and $K^2=0.3$ (GeV/c)^2.

The two photoproduction terms $d\sigma_T/d\Omega_\pi$ and $\sin^2\theta\, T$ behave as in photoproduction; i.e., the sum of the two (containing the pion pole) is decreasing sharply but the difference is almost constant. This gives the typical photoproduction behavior for photons polarized either parallel or perpendicular to the scattering plane. For completeness and for giving an idea of the quality of the model for $K^2=0$, the photoproduction result is given in Fig. 2, where the data[19,20] are taken from the review by Lübelsmeyer.[21] From the plotted Born-term cross section, it is seen how the dispersion-integral correction improves the cross section, in particular beyond $t=-\mu^2$. It is likely that a more complete treatment of the dispersion integral, including the smaller low-energy photoproduction multipoles, improves the agreement with experiment as the results of Ref. 3 indicate.

The two new terms $d\sigma_0/d\Omega_\pi$ and $\sin\theta\, S$ in electroproduction now change the typical photoproduction behavior (see Fig. 3). For $\varphi=\pi/2$ the cross section dips in the forward direction, which is the effect of the pion pole in $d\sigma_0/d\Omega_\pi$, which vanishes when $t\rightarrow 0$ (owing to the parity of the pion). For $\varphi=0$ and $\varphi=\pi$ the dramatic effect of the pion pole in the scalar transverse interference term is seen.

The energy behavior of $d\sigma/d\Omega_\pi$ for fixed t and K^2 is approximately as $1/s$, as in photoproduction. The effect of increasing K^2 for fixed t and W is a decrease in the cross section, although somewhat less han the form factors would suggest. Since $t_{\min}$ is increasing with K^2, the effect of the pion pole becomes less dramatic,

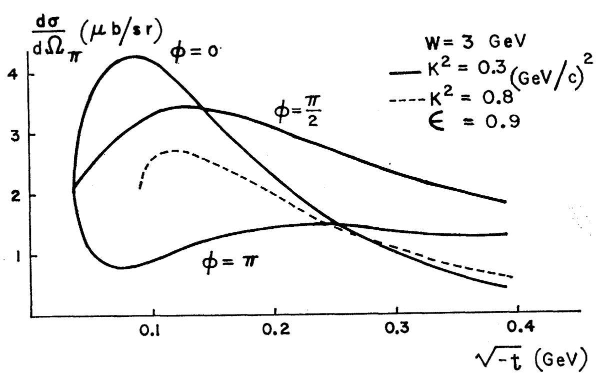

FIG. 3. Model prediction for $d\sigma/d\Omega_\pi$ for $\phi=0$, $\pi/2$, and π for $W=3$ GeV and $K^2=0.3$ (GeV/c)^2. The dashed line represents the $\phi=0$ cross section for $K^2=0.8$ (GeV/c)^2.

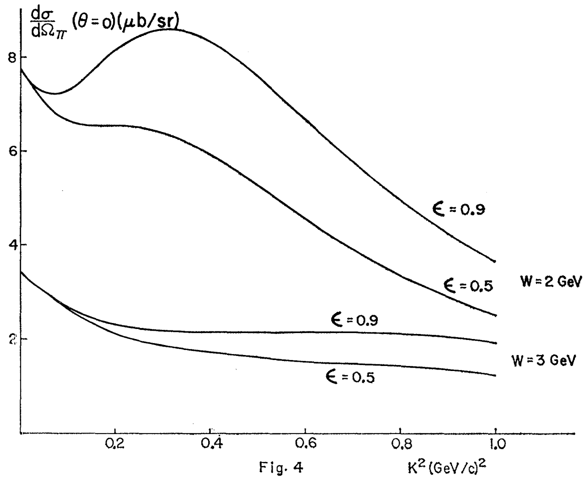

FIG. 4. Variation of the forward cross section with K^2 for $W=2$ and 3 GeV and $\epsilon=0.5$ and 0.9.

as is seen from the dashed curve in Fig. 3. In particular, the variation of $t_{\min}$ has a peculiar effect on the forward cross section: It keeps its size for a while before decreasing, in particular at higher W values. This is shown for $W=2$ and $W=3$ in Fig. 4, where the variation of ϵ gives an idea of the importance of the scalar term.

Generally $d\sigma_0/d\Omega_\pi$ is as important as $d\sigma_T/d\Omega_\pi$ in the t range considered here. The MIT-SLAC inelastic electron scattering experiment seems to indicate[22] that this cannot be the case for the complete t range of the cross sections of all channels.

ACKNOWLEDGMENTS

I would like to thank Dr. W. Schmidt for discussions on his work, and Professor F. M. Pipkin and L. Litt for their stimulating interest (their experiment prompted this investigation). The efforts of L. Litt to match his Born-term program with mine are gratefully acknowledged.

APPENDIX A: KINEMATICAL SINGULARITIES

The amplitudes A_2 and A_5 contain kinematical singularities at $t=\mu^2$. This can be seen from their relation to the amplitudes B_i of Ball,[23] which are free of kinematical singularities. They show up in the absorptive parts of Eq. (2), as can be seen explicitly from the expansion of Im$A_{2,5}$ in multipoles. This expansion is usually done via the amplitudes $\mathfrak{F}_i$ in which the T matrix is expressed when one uses Pauli spinors:

$$\tilde{A}=[B^{-1}][C]\mathfrak{F}, \qquad (A1)$$

where a matrix notation is adopted, the vector $\mathfrak{F}$ being

[19] C. Geweniger, P. Heide, U. Kötz, R. A. Lewis, P. Schmüser, H. J. Skronn, H. Wahl, and K. Wegener, Phys. Letters **29B**, 41 (1969).

[20] H. Burfeindt, G. Buschhorn, C. Geweniger, P. Heide, R. Kotthaus, M. Wahl, and K. Wegener, as quoted in Ref. 21.

[21] K. Lübelsmeyer, in *Proceedings Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969* (Daresbury Nuclear Physics Laboratory, 1969), p. 45.

[22] R. E. Taylor, in *Proceedings Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969* (Daresbury Nuclear Physics Laboratory, 1969), p. 251.

[23] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

$(\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \mathfrak{F}_7, \mathfrak{F}_8)$, and where the matrices $[B^{-1}]$ and $[C]$ are, e.g., given in Ref. 5. The $\mathfrak{F}_i$'s are easily expanded in terms of multipoles.[6,5] The matrix $[B^{-1}]$ has elements in its second and fifth rows which all have a factor $1/(t-\mu^2)$. Therefore, the dispersion integrals for A_2 and A_5 give rise to a kinematical singularity, irrespective of how one saturates the dispersion integral.

In principle, this should not lead to any singularity in the T matrix[5] where the combination $A_2M_2+A_5M_5$ occurs. Explicitly this reads

$$2i\gamma_5(P\cdot\epsilon)\times\tfrac{1}{2}(t-\mu^2)A_2+i\gamma_5(-2P\cdot KA_2-K^2A_5)Q\cdot\epsilon,$$

where $P=\frac{1}{2}(P_1+P_2)$ and ϵ is the electron vertex. The combination $-P\cdot KA_2-\frac{1}{2}K^2A_5$ is exactly Ball's amplitude B_3, which has no kinematical singularity at $t=\mu^2$. In practice, difficulties are likely to occur when A_2 and A_5 are calculated separately, since the singularities will only cancel approximately in the matrix element. Therefore, it is safer to evaluate a dispersion integral for A_2 and B_3 (i.e., $-P\cdot KA_2-\frac{1}{2}K^2A_5$), which is done here.

This procedure is equivalent to the evaluation of the dispersion relation for A_5 with an additional subtraction constant of the form $c/(t-\mu^2)$, where c is given by

$$c(t)=-\frac{2}{K^2\pi}\int_{(m+\mu)^2}^{\infty}ds'\,(t-\mu^2)\,\mathrm{Im}A_2(s't). \quad (A2)$$

It should be noted that this procedure is rather similar to the one adopted by Adler,[10] who takes instead of $c(t)$ the value $c(t=\mu^2)$. However, the simple connection with a dispersion relation for B_3 is then lost.

APPENDIX B: AMPLITUDES AND CROSS-SECTION FORMULAS

For dispersion-relation calculations the invariant amplitudes are the most convenient, whereas for the cross-section formulas or multipole expansions the traditional amplitudes $\mathfrak{F}_i$ (or the helicity amplitudes $f_{\lambda_2\lambda_1\lambda_\gamma}$) are preferable. The former arise when the T matrix is expanded in Dirac invariants, the latter when Pauli invariants in the πN c.m. system are used:

$$T_{fi}=\epsilon_\mu\langle\pi N_2|J_\mu|N_1\rangle=\sum_i A_iu(p_2)M_iu(p_1)=\chi^\dagger(2)\mathfrak{F}\chi(1), \quad (B1)$$

where the quantities M_i are given by Dennery, and where $\mathfrak{F}$ is of the form

$$\mathfrak{F}=i\boldsymbol{\sigma}\cdot\mathbf{b}\mathfrak{F}_1+i\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot(\hat{k}\times\mathbf{b})\mathfrak{F}_2 +i\boldsymbol{\sigma}\cdot\hat{k}\hat{q}\cdot\mathbf{b}\mathfrak{F}_3+i\boldsymbol{\sigma}\cdot\hat{q}\hat{q}\cdot\mathbf{b}\mathfrak{F}_4-i\boldsymbol{\sigma}\cdot\hat{q}b_0\mathfrak{F}_7-i\boldsymbol{\sigma}\cdot\hat{k}b_0\mathfrak{F}_8, \quad (B2)$$

where $\hat{q}=\mathbf{q}/q$, $\hat{k}=\mathbf{k}/k$, and b_μ is obtained from the electron vertex and photon propagator

$$\epsilon_\mu=eu(k_2)\gamma_\mu u(k_1)/K^2 \quad (B3)$$

by elimination of the longitudinal component, i.e.,

$$b_\mu=\epsilon_\mu-(\boldsymbol{\varepsilon}\cdot\hat{k}/k)K_\mu. \quad (B4)$$

The four terms of Eq. (11) are expressed in $\mathfrak{F}_i$ [and through the inverse of Eq. (A1) in A_i] as follows:

$$d\sigma_T/d\Omega_\pi=c[|\mathfrak{F}_1|^2+|\mathfrak{F}_2|^2-2\cos\theta\,\mathrm{Re}(\mathfrak{F}_1{}^*\mathfrak{F}_2)]+\sin^2\theta\,T,$$
$$d\sigma_0/d\Omega_\pi=c\frac{K^2}{k^2}[|\mathfrak{F}_7|^2+|\mathfrak{F}_8|^2+2\cos\theta\,\mathrm{Re}(\mathfrak{F}_7{}^*\mathfrak{F}_8)],$$
$$T=\tfrac{1}{2}c[|\mathfrak{F}_3|^2+|\mathfrak{F}_4|^2+2\,\mathrm{Re}(\mathfrak{F}_1{}^*\mathfrak{F}_4+\mathfrak{F}_2{}^*\mathfrak{F}_3+\cos\theta\,\mathfrak{F}_3{}^*\mathfrak{F}_4)], \quad (B5)$$
$$S=-2c(K^2/k^2)^{1/2}\,\mathrm{Re}[\mathfrak{F}_7{}^*(\mathfrak{F}_1+\mathfrak{F}_4+\cos\theta\,\mathfrak{F}_3)+\mathfrak{F}_8{}^*(\mathfrak{F}_2+\mathfrak{F}_3+\cos\theta\,\mathfrak{F}_4)],$$

where[24]

$$c=\frac{k^L}{\bar{k}^L}\frac{q}{k}\frac{m^2}{(4\pi W)^2}.$$

In terms of helicity amplitudes, we obtain

$$d\sigma_T/d\Omega_\pi=c\tfrac{1}{2}(|f_{\frac12\frac12-1}|^2+|f_{\frac12\frac12 1}|^2+|f_{\frac12-\frac12-1}|^2+|f_{\frac12-\frac12 1}|^2),$$
$$d\sigma_0/d\Omega_\pi=c\frac{K^2}{k^2}(|f_{\frac12\frac12 0}|^2+|f_{\frac12-\frac12 0}|^2), \quad (B6)$$
$$\sin^2\theta\,T=-c\,\mathrm{Re}(f_{\frac12\frac12-1}{}^*f_{\frac12\frac12 1}+f_{\frac12-\frac12-1}{}^*f_{\frac12-\frac12 1}),$$
$$\sin\theta\,S=-\sqrt{2}c(K^2/k^2)^{1/2}\,\mathrm{Re}[f_{\frac12\frac12 0}{}^*(f_{\frac12\frac12-1}-f_{\frac12\frac12 1})+f_{\frac12-\frac12 0}{}^*(f_{\frac12-\frac12-1}-f_{\frac12-\frac12 1})],$$

where the helicity amplitudes (for the virtual photon) are

$$f_{\frac12\frac12-1}=(i/\sqrt{2})\cos\tfrac12\theta\sin\theta\,(\mathfrak{F}_3+\mathfrak{F}_4),$$
$$f_{\frac12-\frac12 1}=(i/\sqrt{2})\sin\tfrac12\theta\sin\theta\,(\mathfrak{F}_3-\mathfrak{F}_4),$$
$$f_{\frac12-\frac12-1}=(i/\sqrt{2})2\cos\tfrac12\theta\,(\mathfrak{F}_1-\mathfrak{F}_2)-f_{\frac12-\frac12 1}, \quad (B7)$$
$$f_{\frac12\frac12 1}=(-i/\sqrt{2})2\sin\tfrac12\theta\,(\mathfrak{F}_1+\mathfrak{F}_2)-f_{\frac12\frac12-1},$$
$$f_{\frac12\frac12 0}=i\cos\tfrac12\theta\,(\mathfrak{F}_7+\mathfrak{F}_8),$$
$$f_{\frac12-\frac12 0}=i\sin\tfrac12\theta\,(\mathfrak{F}_7-\mathfrak{F}_8).$$

---

[24] The factor $k^L/\bar{k}^L$ arises from the convention adopted for Γ_T.