Breit-Wigner formula should not be added. But from our numerical results, we can say that to add the two kinds of anticlockwise loops produces no difficulty since the partial-wave cross sections obtained from the Regge-pole exchanges exhibit essentially the characteristic of nonresonant behavior except in the unitarity-violating threshold region as is shown in Figs. 7–10 and 14. In this connection we note that the variation of the Argand locus with respect to energy obtained from the Regge-pole exchanges is extremely weak compared to that of a phenomenological phase-shift analysis. This fact means that even when we add the anticlockwise loop obtained from Regge-pole exchanges to that of the Breit-Wigner amplitude, it produces only a small modification in the latter loop.

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Lepton-Hadron Deep-Inelastic Scattering, Gluon Model, and Reggeized Symmetry Breaking*

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We numerically estimate the expectation value between baryons of the equal-time commutator $[\dot{J}_i^a, J_j^b]$ of $U(3)\otimes U(3)$ currents on the gluon model by using a symmetry-breaking theory based on Regge-pole dominance. The result is in excellent agreement with the electroproduction data.

THE scaling behavior predicted by Bjorken[1] for deep-inelastic electron and neutrino scattering from hadrons has received considerable experimental support.[2–4] One of the most interesting consequences of scaling is the Callan-Gross[5] relation between certain integrals of the electroproduction structure functions and the infinite-momentum limit of the commutator $[\dot{J}_i(x),J_j(0)]\delta(x_0)$ of electromagnetic currents. The relation therefore enables one to determine which models for this commutator are in agreement with experiment. The recent experimental results[2–4] are in agreement with models in which the constituents of J_μ have spin $\frac{1}{2}$ and seem to be in disagreement with other models. We therefore propose in this paper to take a spin-$\frac{1}{2}$ model seriously and try to evaluate *numerically* matrix elements of current commutators by making some reasonable assumptions about the way in which the $\frac{1}{2}^+$ baryon octet deviates from an exactly $SU(3)$-symmetric multiplet. We find the resulting (generalized) Callan-Gross relation to be in excellent agreement with experiment.

The model we shall study is the so-called gluon model with Hamiltonian density[6]

$$\mathcal{H}=\psi^\dagger(-i\boldsymbol{\alpha}\cdot\nabla+\beta M+g\beta\gamma^\mu B_\mu)\psi+\mathcal{H}_B\,, \tag{1}$$

where ψ is a spin-$\frac{1}{2}$ quark field, B_μ is a neutral vector meson, $\mathcal{H}_B$ is the Hamiltonian of B_μ, and[7]

$$M=\alpha_0\lambda^0+\alpha_8\lambda^8 \tag{2}$$

is the quark mass term. Thus the chiral $SU(3)\otimes SU(3)$ symmetry breaking is due entirely to the explicit quark mass term. It is the very smooth nature of this symmetry-breaking mechanism which will enable us to proceed.

It is perhaps appropriate to mention that this model has already been shown to have very desirable features in problems connected with radiative corrections to weak interactions[8] and nonleptonic weak interactions.[9]

The $U(3)$ vector currents are given in the gluon model by

$$J_\mu{}^a=\tfrac{1}{2}\bar{\psi}\gamma_\mu\lambda^a\psi\,,\quad a=0,\ldots,8. \tag{3}$$

We can write, in general, that

$$-\int d^4x\,\delta(x_0)\langle p|[\dot{J}_i{}^a(x),J_j{}^b(0)]|p\rangle \equiv iE^{ab}p_ip_j+\bar{R}^{ab}\delta_{ij}+G^{ab}\epsilon_{ijk}p_k\,, \tag{4}$$

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[1] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[2] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); M. Breidenbach *et al.*, *ibid.* **23**, 935 (1969).

[3] W. Albrecht *et al.* (unpublished).

[4] I. Budagov *et al.*, Phys. Letters **30B**, 364 (1969).

[5] C. Callan and D. J. Gross, Phys. Rev. Letters **22**, 156 (1969).

[6] In the following we use (1) in a formal way along with canonical commutation relations, ignoring all perturbation-theoretic subtleties.

[7] We use the λ matrices of M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); in particular, $\lambda_0=(\frac{2}{3})^{1/2}1$.

[8] C. Callan, Phys. Rev. **169**, 1175 (1968); G. Preparata and W. I. Weisberger, *ibid.* **175**, 1965 (1968).

[9] S. Nussinov and G. Preparata, Phys. Rev. **175**, 2180 (1968).

where $|p\rangle$ is a $\frac{1}{2}^{+}$ baryon physical single-particle state and a spin average is understood. In the gluon model, we find

$$d^{abc}\langle p|\bar{\psi}V_i\gamma_j\lambda^c\psi|p\rangle = E^{ab}p_ip_j + R^{ab}\delta_{ij}\,, \tag{5}$$

where $V_i \equiv -i\partial_i + gB_i$. Since the left-hand side is a second-rank tensor, by Lorentz covariance we can write

$$d^{abc}\langle p|\bar{\psi}V_\mu\gamma_\nu\lambda^c\psi|p\rangle = E^{ab}p_\mu p_\nu - R^{ab}g_{\mu\nu}\,, \tag{6}$$

and we learn that E^{ab} and R^{ab} are constants, independent of $\mathbf{p}^2$. It further follows from (5) that E^{ab} and R^{ab} have the forms

$$E^{ab} = d^{abc}E^c\,, \quad R^{ab} = d^{abc}R^c\,, \tag{7}$$

for some constants E^c and R^c. The constants E^c and R^c may, of course, depend on the particular state $|p\rangle$ in question.

We write the absorptive part of the forward spin-averaged current-baryon scattering amplitude as

$$(p_0/m)(2\pi)^{-1}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu{}^a(x),J_\nu{}^b(0)]|p\rangle = p_\mu p_\nu W_2{}^{ab}(q^2,\nu) + \cdots\,, \tag{8}$$

where $\nu = q\cdot p$. In the scaling limit ($\nu\to\infty$, $\omega\equiv -q^2/\nu$ fixed), we define

$$F_2{}^{ab}(\omega) = \lim \nu W_2{}^{ab}(q^2,\nu)\,. \tag{9}$$

The sum rule which follows from (4)–(9) is then

$$2\int_0^2 d\omega\, F^{ab}(\omega) = d^{abc}E^c\,. \tag{10}$$

For electron scattering, $a=b=Q=[(3)+(\frac{1}{3})^{\frac{1}{2}}(8)]$ and (10) becomes

$$L^Q \equiv \int_0^2 d\omega\, F^{QQ}(\omega) = \tfrac{1}{2}d^{QQc}E^c = \tfrac{1}{3}E^Q + (\tfrac{2}{3})^{3/2}E^0\,, \tag{11}$$

and for neutrino or antineutrino scattering (with $\cos^2\theta_c = 1$), it becomes

$$L^W \equiv \int_0^2 d\omega\, F_2{}^{W\bar{W}}(\omega) = \tfrac{1}{2}d^{W\bar{W}c}E^c = (2/\sqrt{3})E^8 + (\sqrt{\tfrac{2}{3}})E^0\,. \tag{12}$$

Our problem now is to estimate numerically the E^c. To this end, we put (7) into (6) and take the trace in $\mu\nu$ and then use the field equations which follow from (1) to obtain

$$\tfrac{1}{2}\langle p|\bar{\psi}\{M,\lambda^a\}\psi|p\rangle = E^a m^2 - 4R^a\,, \tag{13}$$

where $m^2 = p^2$ is the mass of the state $|p\rangle$. Now, effectively,

$$m^2 = \langle p|M^0 + \alpha_8 S^8|p\rangle\,, \tag{14}$$

where $S^a \equiv \frac{1}{2}\bar{\psi}\lambda^a\psi$ is a scalar algebraic nonet and M^0 is a $SU(3)$ scalar to second order in symmetry breaking.[10] We now label the state $|p\rangle$ with the $SU(3)$ index e which runs over the octet (N,Ξ,Σ,Λ) and use a subscript e to label the expectation value of the corresponding operator in the state e. Thus (13) becomes

$$\alpha_0 d^{a0b}S_e{}^b + \alpha_8 d^{a8b}S_e{}^b = E_e{}^a(M_e{}^0 + \alpha_8 S_e{}^8) - 4R_e{}^a\,. \tag{15}$$

This is our master equation which we want to use to evaluate $E_e{}^a$ for substitution in (10). Note that although $S_e{}^b$, $E_e{}^b$, and $R_e{}^b$ are all algebraic nonets in the sense that they are matrix elements of operators which commute like nonets with the charges $\int d^3x\, J_0{}^a(x)$, because of symmetry breaking we cannot *a priori* assume that their matrix elements behave like octets when the labels b and e are varied.

For purposes of orientation, we first consider the free field limit $g=0$. Then α_8 is a free parameter so that (15) gives

$$E_e{}^a = d^{8ab}S_e{}^b/S_e{}^8 \quad (g=0)\,. \tag{16}$$

Furthermore, since the baryon octet states are simply direct products of three free quark states, the matrix elements $S_e{}^b$ at rest can be obtained by simple quark counting. Thus, for the proton $e=P$ we obtain $E_P{}^0 = \sqrt{\frac{2}{3}}$, $E_P{}^3 = \frac{1}{3}$, and $E_P{}^8 = \sqrt{\frac{1}{3}}$, and substitution into (11) gives

$$\int_0^2 d\omega\, F_2{}^{QQ}(\omega) = \tfrac{2}{3} \quad (g=0)\,. \tag{17}$$

We thus reproduce the well-known free field result that

$$\int_0^2 d\omega\, F_2{}^{QQ} = \tfrac{2}{3}\sum Q_i{}^2 = \tfrac{2}{3}\,.$$

Experimentally,[2] (17) is about $\frac{1}{3}$ and this fact has led some people[11] to abandon the *simple* three-quark structure for baryons. We shall see below, however, that the reduction of (17) to $\frac{1}{3}$ can easily be accounted for by solving (14) using our knowledge of symmetry breaking in the assumed real world with $g\neq 0$.

In order to use Eq. (15), we must determine how $S_e{}^a$, $E_e{}^a$, and $R_e{}^a$ deviate from exact $U(3)$ nonets to order $\alpha_8{}^2$. We emphasize that, although we shall neglect terms of order $\alpha_8{}^2$, we do *not* assume that coefficients of 1 and of α_8 can be equated in (15). We know empirically, from the success of mass-splitting calculations,[12] that $\alpha_8 S_e{}^a$ is a good octet. These calculations give for S^a a D-to-F ratio $D/F = -0.31\pm 0.02$. To study $E_e{}^a$ and $R_e{}^a$, however, we require an explicit theory of

[10] By an adaptation of an argument by G. Preparata and W. Weisberger (Ref. 8), $\theta_{\mu\nu}{}^{(0)}$, the energy-momentum tensor for $\alpha_8 = 0$, is seen to be nonrenormalized in first order in α_8.

[11] R. P. Feynman (unpublished); S. D. Drell, D. Levy, and T. M. Yan, Phys. Rev. Letters **22**, 744 (1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).

[12] See, for example, K. Kikkawa, Progr. Theoret. Phys. (Kyoto) **35**, No. 2 (1966).

symmetry breaking. We propose to make use of the relation

$$\delta\langle p|O(0)|p\rangle=-i\alpha_8\int d^4x\langle p|\tilde{T}(S^8(x)O_{(0)}(0))|p\rangle, \quad (18)$$

which follows from (1) and (2). Here $\delta O\equiv\alpha_8 O_{(1)}$ is the change of the local operator O from its value $O_{(0)}$ in the absence of symmetry breaking to its value to first order in symmetry breaking, and $\tilde{T}$ denotes the usual T product but with the Born term removed.[13] Note that, because we use physical states, we cannot conclude, e.g., from $O=0$ that $O_{(0)}=O_{(1)}=0$ but only that $O_{(0)}+\alpha_8 O_{(1)}\simeq 0$. Thus

$$\delta\langle p|O(0)|p\rangle=T(p,0), \quad (19)$$

where

$$T(p,q)=-i\alpha_8\int d^4x\, e^{iq\cdot x}\langle p|\tilde{T}(S^8(x)O_{(0)}(0))|p\rangle \quad (20)$$

is a spin-averaged forward scattering amplitude with its Born terms removed.

The invariant amplitudes $T_i(q^2,\nu)$ comprising $T(q,p)$ will be assumed to satisfy fixed-q^2 dispersion relations in ν with a number of subtractions specified by the Regge behavior of $\mathrm{Im}T_i$. The subtraction terms will have the algebraic structure of the leading Regge terms. We further assume that the dispersive integral itself, at least for $\nu=q^2=0$, has the same algebraic structure as the Regge (subtraction) term, as is suggested by finite energy sum rules. In particular, unsubtracted dispersion integrals will be comparatively small. This final assumption seems quite reasonable, in view of the fact that in (20) the Born terms are explicitly removed and the decuplet resonances are strongly suppressed by phase space because S^8 is a Lorentz scalar.

We now apply our symmetry-breaking theory to Eq. (6). We write (6) as

$$\langle p|O^a{}_{\mu\nu}|p\rangle=E^a p_\mu p_\nu-R^a g_{\mu\nu}, \quad (21)$$

where $O^a{}_{\mu\nu}=\bar{\psi}V_\mu\gamma_\nu\lambda^a\psi$ and application of (18) gives

$$\begin{aligned}\delta\langle p|O^a{}_{\mu\nu}|p\rangle&=-i\alpha_8\int d^4x\langle p|T(S^8(x)O_{(0)}{}^a{}_{\mu\nu})|p\rangle\\&=\delta E^a p_\mu p_\nu-\delta R^a g_{\mu\nu}. \quad (22)\end{aligned}$$

Now the scattering amplitude $T_2{}^a(\nu)$ corresponding to δE^a satisfies $\mathrm{Im}T_2{}^a(\nu)\sim\nu^{\alpha_a-2}$, and the one $T_1{}^a(\nu)$ corresponding to δR^a satisfies $\mathrm{Im}T_1{}^a(\nu)\sim\nu^{\alpha_a}$ for $\nu\to\infty$, where α_a is the $t=0$ intercept of the leading contributing Regge trajectory. By charge-conjugation invariance, the relevant trajectories comprise the tensor nonet (f,A,K^{**},f') with $\alpha<\frac{1}{2}$. Thus $T_2{}^a$ satisfies an unsubtracted dispersion relation, so that, in accordance with our remarks above, we shall neglect δE^a. This conclusion is in agreement with what one would conclude from asymptotic symmetry.

$T_1{}^a$, on the other hand, requires a subtraction, and so we shall use some known properties of the assumed dominant Regge subtraction term to learn about δR^a. The trace of (18) now gives

$$-4\delta R^a=-i\alpha_8\int d^4x\langle p|T(S^8(x)O_{(0)}{}^a{}_\mu{}^\mu)|p\rangle, \quad (23)$$

and since $O_{(0)}{}^a{}_\mu{}^\mu$ is a scalar, to order α_8, δR^a can only involve an $SU(3)$ singlet, symmetric octet, and 27-plet.[14] The same is true for δS^a,[15] and so we can write

$$-\alpha_0 d^{0ab}\delta S^b-4\delta R^a=\alpha_8 d^{8ab}\bar{T}^b+\alpha_8 R_{(27)}{}^a, \quad (24)$$

where $\bar{T}^a$ is an octet and $R^a{}_{(27)}$ is the appropriate 27-plet operator. The first term on the right in (24) corresponds to the tensor nonet Regge exchange and so $\langle p|\bar{T}^a|p\rangle$ has the ratio $d/f=-(0.5_{-0.1}{}^{+0.2})$.[16]

Because of $f-f'$ mixing, however, and because $\alpha_f\sim\frac{1}{2}$, whereas $\alpha_{f'}<0$ (so that only the f-exchange contribution requires a subtraction), the f and d for $a=8$ exchange are reduced by $\frac{1}{3}=\cos^2\theta$ compared with those for $a=3$. Thus we have $\bar{T}^a=\eta_a T^a$, where T^a is a good octet and $\eta_8=\frac{1}{3}\eta_3$. We note finally that because of the tensor meson nonet symmetry, (24) should be valid for $a=0,\ldots,8$.

The second term in (24) must correspond to continuum intermediate states (or Regge cuts) and so has an effective $\alpha<0$. We therefore must consistently have its contribution rather smaller than the first terms, although possibly larger than the neglected δE^a with $\alpha-2\sim-\frac{3}{2}$.

We incorporate these symmetry-breaking conclusions into (15) and obtain

$$d^{8ab}(S_e{}^b-\eta_b T_e{}^b)=E_e{}^a S_e{}^8+K_e{}^a+R^a{}_{(27)e}, \quad (25)$$

where S^a, T^a, E^a, and K^a are all good nonets and $R_{(27)}{}^a$ is a good 27-plet to order $\alpha_8{}^2$. We associate with each octet an F and D as follows: $S\leftrightarrow(F,D)$, $T\leftrightarrow(f,d)$, $E\leftrightarrow(\alpha,\beta)$, $K\leftrightarrow(\gamma,\delta)$. The tensor $R_{(27)}$ can be characterized by a single parameter A.

Our next step is to vary e through the baryon octet. For $a=3$, we obtain three independent equations which can be taken as

$$\alpha D=\beta F, \quad (26)$$

$$3\alpha F-\beta D=-(10/3)A, \quad (27)$$

and an uninteresting relation which simply fixes S^0-T^0. We see that E and S have the same D/F ratio.[17] For

[13] The $\tilde{T}$ product is involved in (15) because mass insertions are not made in the external legs which are already the exact physical states.

[14] The suppression of a $10+\overline{10}$ contribution follows from an assumed scalar meson dominance and Bose symmetry generalized to $SU(3)$.

[15] A $10+\overline{10}$ contribution is here rigorously absent.

[16] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

[17] The light-cone behavior assumed in R. Brandt, Phys. Rev. Letters **23**, 1260 (1969), is therefore consistent with this model.

$\alpha=8$, we obtain four independent equations which, using (26) and (27), can be taken as

$$2D-\tfrac{4}{3}d=3\alpha F\,, \tag{28}$$

$$2F-\tfrac{4}{3}f=4\alpha D\,, \tag{29}$$

and two uninteresting relations which simply fix γ and δ.

Taking $R\equiv D/F=-\frac{1}{3}$ in (26) gives $\beta=-\frac{1}{3}\alpha$, and taking $r\equiv d/f=-\frac{1}{2}$ in (28) and (29) then gives

$$\alpha=1/7\,,\quad \beta=-1/21\,. \tag{30}$$

These give $A/F=-9\alpha/10$ in (27) so that our consistency condition that A be small is satisfied. We note that small changes δr in r give $\alpha=1/7+\delta r$.

The $a=0$ equation gives

$$(\sqrt{\tfrac{2}{3}})(S_e{}^8-\tfrac{1}{3}T_e{}^8)=E^0S_e{}^8 \tag{31}$$

and $K^0=0$. This equation is, however, extremely sensitive to r, giving very different values for E^0 for $r\neq R$ and for $r=R$, independently of the magnitude of the symmetry breaking. Our procedure will therefore be to choose the value of E^0 which minimizes the differences in the left- and right-hand sides of (31) as e varies. A more physical procedure, which gives essentially the same result, is to evaluate E^0 in the limit $r=R$. In this limit, (28) and (29) require that α, and hence β and A, vanish. The smallness of these parameters thus suggests the reliability of this procedure. We now learn from (28) and (29) that $T^a=\frac{3}{2}S^a$ so that we obtain from (31) the result

$$E^0=\tfrac{1}{2}\sqrt{\tfrac{2}{3}}\,. \tag{32}$$

The above minimization procedure gives this same result to within 5%.

We now consider the experimental implication of these values. From (11) we obtain for $e=P=$ proton

$$L_P{}^Q=\tfrac{1}{9}(6\alpha+2\beta)+(\tfrac{2}{3})^{3/2}E^0\simeq 0.31\,. \tag{33}$$

This is in excellent agreement with the experimental[2] value of ~ 0.32. Of course, because of the uncertainties in R and r and because we are working to only first order in α_8, our theoretical prediction (33) should be uncertain by $\sim 15\%$. For $e=N=$ neutron, we obtain

$$L_N{}^Q=-4\beta/9+(\tfrac{2}{3})^{3/2}E^0\simeq 0.24\,. \tag{34}$$

Finally, from (12), for neutrino or antineutrino scattering from protons or neutrons, we predict

$$L_{N,P}{}^W=\tfrac{1}{3}(6\alpha-2\beta)+2(\tfrac{2}{3})^{3/2}E^0\simeq 0.98\,. \tag{35}$$

To compare (35) with experiment, we note that in our model

$$\sigma\to\frac{G^2ME}{3\pi}\int_0^2 d\omega\,\mathbf{F}_2(\omega)\,, \tag{36}$$

where $\sigma=\frac{1}{2}(\sigma_{\nu P}+\sigma_{\nu N})$, $E=$ neutrino energy $\to\infty$, and $\mathbf{F}_2$ is the appropriate structure function. The quoted experimental value of σ is $(0.6\pm0.2)G^2ME/\pi$, whereas (35) and (36) give $(0.33)G^2ME/\pi$. We consider this agreement to be satisfactory because the quoted experimental error requires the use of small-E points and because of the relatively small values of q^2 and ν for the observed neutrino events.

We conclude that the experimental values of the integrals (10) are in agreement with the theoretical values obtained from the gluon model and our Regge theory of symmetry breaking. This gives further support to this model and suggests its relevance to understanding the structure of hadrons.