Exchange Degeneracy and Σ-Λ Splitting*†

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The hypothesis is made that the sum of the imaginary parts of the baryonic Regge contributions to exotic meson-baryon scattering amplitudes must vanish. This hypothesis is combined with the assumptions that all coupling constants satisfy exact $SU(3)$ symmetry and that the Σ-Λ mass splitting cannot be neglected, so that the Σ and Λ lie on different Regge trajectories. All possible partitions of the spin-parity-$\frac{3}{2}^-$ Λ's and Σ's (belonging to an octet, singlet, and decuplet) into the two trajectories are considered. A set of consistency equations is derived, and all solutions are obtained. If the Λ(1520) is the only odd-parity hyperon on the bottom [Λ(1115)] trajectory, and the Λ(1690) and one or two Σ's lie on the top trajectory, there is only one solution. In this solution, there is no odd-parity decuplet, the F/D ratios of the couplings to meson-baryon states of the $j^P=\frac{1}{2}^+$ and $\frac{3}{2}^-$ trajectories are 1, and the singlet-octet mixing angle of the $\frac{3}{2}^-$ Λ's is $\theta=\tan^{-1}\sqrt{\frac{1}{2}}$. This solution is in approximate agreement with experiment.

I. INTRODUCTION

RECENTLY, the author has applied the exchange-degeneracy (ED) hypothesis to various amplitudes involving PB and PD states, where P denotes the pseudoscalar meson octet, and B and D denote the spin-parity $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet.[1,2] The hypothesis states that in all amplitudes of exotic internal quantum numbers (quantum numbers for which no resonances occur), the sum of the imaginary parts of the baryon-exchange Regge contributions must vanish.[3] This usually requires cancellation from trajectories of opposite signature (and hence representing physical baryons of opposite parity).

There is some experimental evidence for the existence of three $SU(3)$ multiplets of spin-parity $\frac{3}{2}^-$ that are approximately ED with the nucleon octet; these are the singlet and octet that include the Λ particles at 1520 and 1690 MeV, and the decuplet that includes the 1690-MeV Δ^*.[4] This set of multiplets coincides with the predictions of the quark model or $SU(6)_W$ model, if the $SU(6)$ representations **56** and **70** are associated with baryon states of even and odd parity, respectively.[5] In the quark model, the increasing angular momentum along a trajectory is associated with increasing orbital angular momentum; thus the even- and odd-parity multiplets that are approximately exchange-degenerate should correspond to the same quark spin or $SU(6)$ spin. The only spin-$\frac{1}{2}$ multiplet in the $SU(6)$ representation **56** is an octet, while the **70** contains a spin-$\frac{1}{2}$ octet, singlet, and decuplet. Thus, the experimental and quark-model associations of ED multiplets are the same.

We denote the four $\frac{1}{2}^+$ and $\frac{3}{2}^-$ multiplets by $8^{(+)}$, $8^{(-)}$, $1^{(-)}$, and $10^{(-)}$, where the superscript is the parity. The consistency conditions that result from the ED hypothesis are not sufficient to determine the relative couplings of these four multiplets.[1] However, it was pointed out in Ref. 1 that if the Λ(1115) and Σ(1193) are assumed sufficiently different in mass so that their Regge trajectories cannot be considered degenerate, the ED conditions may be applied to the two trajectories separately, and more consistency conditions result. The purpose of this paper is to carry out this program completely, making all possible assumptions concerning the partitioning of the odd-parity Λ's and Σ's between the Λ(1115) and Σ(1193) trajectories. We continue to assume exact $SU(3)$ symmetry of the coupling constants.[6]

Of course, the Σ-Λ mass splitting is not extremely large. In terms of the mass squared, it is about half the ω-φ splitting. However, it seems to increase as one ascends the Regge trajectories, i.e., the splitting of the $\frac{5}{2}^+$ hyperons that are usually regarded as recurrences of the Σ and Λ is greater than the Σ-Λ splitting, even if one uses mass rather than the mass squared as the variable.[4] Furthermore, the assumption that the Λ(1115) and Λ(1520) lie on an ED trajectory that does not include the Λ(1690) is fairly conventional in Regge analyses of KN and $\bar{K}N$ scattering.[7]

If one assumes that all meson states other than singlet and octet states are exotic, the ED principle breaks down when applied to baryon-antibaryon amplitudes.[8] There are two different reasonable viewpoints concerning this problem. The first assumes that exotic meson resonances will be found that are not coupled strongly to meson-meson states. The second assumes that no

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[1] R. H. Capps, Phys. Rev. Letters **22**, 215 (1969).
[2] R. H. Capps, Phys. Rev. **186**, 2003 (1969).
[3] A recent discussion of the motivation of this hypothesis is given by Haim Harari, Phys. Rev. Letters **22**, 562 (1969), and by references cited therein.
[4] Except where otherwise noted, the experimental numbers are taken from the compilation of N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[5] The predictions of the $SU(6)_W$ model for odd-parity baryon resonances are given by R. H. Capps, Phys. Rev. **158**, 1433 (1967).
[6] A similar program, for the singlet-octet splitting of the mesons, has been carried out by C. B. Chiu and J. Finkelstein, Phys. Letters **27B**, 510 (1968). The baryon case is more complicated, because of the different set of $SU(3)$ multiplets of the even- and odd-parity particles.
[7] See, for example, V. Barger, Phys. Rev. **179**, 1371 (1969); K. Igi and J. K. Storrow, Nuovo Cimento **62A**, 972 (1969).
[8] For a discussion of this problem, see Jonathan L. Rosner, Phys. Letters **21**, 950 (1968); see also Ref. 2, Sec. II B.

exotic mesons will be found, and that the ED hypothesis cannot be applied to $B\bar{B}$ states. (In this paragraph only, B denotes either an octet or decuplet baryon.) One can find a theoretical reason why the second view might be correct, involving the fact that $B\bar{B}$ states (unlike MM, MB, or BB states) are coupled by unitarity to states of much lighter rest mass. In this paper we adopt the second viewpoint, using as justification only the experimental absence of exotic mesons. Hence, we do not apply the ED principle to the processes $PP \to B\bar{B}$. The extension of the principle to these processes has been discussed recently by Barger and Michael,[9] and by Mandula *et al.*[9]

The self-consistency equations are derived in Sec. II, and all the solutions are found in Sec. III. One of the solutions involves a spectrum of Σ and Λ particles that corresponds very well with the experimental spectrum. A comparison with experiment of some of the branching ratios predicted by this solution is given in Sec. IV.

II. CONSISTENCY EQUATIONS

The ED principle leads to two types of restrictions on Regge amplitudes: one referring to the trajectories and the other to the residues. We are concerned only with the residue conditions. As shown in several places, these conditions are simple, namely, the sum of the residues of even-parity (even-signature) Regge contributions to an exotic amplitude must be equal to the corresponding sum of odd-parity contributions.[2,6] Since the form of the conditions is the same for each spin state and each momentum transfer, we may omit spin and momentum-transfer variables in writing the conditions.

All PB states other than singlet, octet, and decuplet states are exotic. The I_z and Y quantum numbers of exotic PB states are shown in Fig. 1. These states are of two types, denoted by A and B. There are three types of exotic $PB \to PB$ processes; the crossed (u-channel)

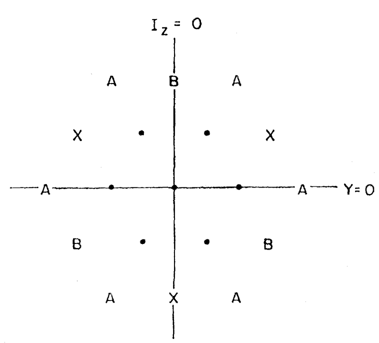

FIG. 1. I_z and Y quantum numbers of the A-type and B-type exotic PB states. The states labeled X are the decuplet states that are outside the octet.

[9] V. Barger and C. Michael, Phys. Rev. **186**, 1592 (1969); J. Mandula *et al.*, Phys. Rev. Letters **22**, 1147 (1969).

TABLE I. Three possible partitions of the $\Lambda^{(-)}$ and $\Sigma^{(-)}$ particles to the Regge trajectories of the $\Sigma(1193)$ and the $\Lambda(1115)$.

	Odd-parity Y's		
Trajectory	2 and 2	3 and 1	1 and 3
Top [$\Sigma^+(1193)$]	$\Sigma_t\Lambda_t$	$\Sigma_8\Sigma_{10}\Lambda_t$	Σ_t
Bottom [$\Lambda^+(1115)$]	$\Sigma_b\Lambda_b$	Λ_b	$\Sigma_b\Lambda_8\Lambda_1$

amplitudes corresponding to these are

$$A_{\text{el}}:\quad (K^-p)(\pi^-\Sigma^+)(K^0\Xi^0)\,, \tag{1a}$$

$$B_{\text{el}}:\quad (\bar{K}^0p)(K^0\Sigma^+)(\pi^-\Xi^0)\,, \tag{1b}$$

$$B_{\text{in}}:\quad \pi^-\Sigma^+\to K^0\Xi^0,\quad K^-p\to\bar{K}^0n\,. \tag{1c}$$

The states in parentheses denote elastic processes, and the subscripts "el" and "in" denote elastic and inelastic. Amplitudes related to the above by reflection around the line $I_z=0$ are omitted. The only baryon trajectories that contribute to the A_{el} and B_{in} amplitudes are the Λ and Σ^0 trajectories.

A. *PB* Equations for Two-and-Two Case

There are two $\Lambda^{(-)}$ and two $\Sigma^{(-)}$ particles in the odd-parity octet, singlet, and decuplet. (Superscript signs in parentheses denote parity.) The nature and number of the self-consistency equations depend on how these four particles are assumed partitioned to the bottom [$\Lambda(1115)$] and top [$\Sigma(1193)$] trajectories. We consider here the most complicated case, in which there is one $\Lambda^{(-)}$ and one $\Sigma^{(-)}$ on each trajectory. This is the two-and-two case shown in Table I. The subscripts t and b refer to the top and bottom trajectories. The equations for the other interesting partitions are obtained later by suitable modifications of the two-and-two equations.

The odd-parity particles on the bottom trajectory, $\Lambda_b^{(-)}$ and $\Sigma_b^{(-)}$, may be a singlet-octet mixture and an octet-decuplet mixture, respectively. We define two mixing angles α_Λ and α_Σ such that the state vectors of the odd-parity hyperons on the bottom trajectory are

$$\Lambda_b^{(-)}=\Lambda_1^{(-)}\cos\alpha_\Lambda+\Lambda_8^{(-)}\sin\alpha_\Lambda\,, \tag{2a}$$

$$\Sigma_b^{(-)}=-\Sigma_{10}^{(-)}\sin\alpha_\Sigma+\Sigma_8^{(-)}\cos\alpha_\Sigma\,, \tag{2b}$$

where the numerical subscripts denote $SU(3)$ multiplets. The state vectors for the particles on the top trajectory are orthogonal to these.

Since the consistency conditions to be used here apply only to the ratios of coupling constants, we normalize the coupling to PB states of the $8^{(+)}$ trajectory to unity. There are then seven constants to be determined in the two-and-two scheme, i.e.,

$$G_8, G_1, G_{10},\quad \theta_+, \theta_-,\quad \alpha_\Lambda, \text{ and } \alpha_\Sigma\,. \tag{3}$$

The G's are the coupling constants of the odd-parity trajectories. The normalization is such that $G_8{}^2$ is the sum of the squares of the couplings of one member of the $8^{(-)}$ trajectory octet with all PB states in the direct (u)

channel in which the trajectory particles exist. The θ's the $8^{(+)}$ and $8^{(-)}$ trajectories by the equation $\tan\theta_{\pm} = (9/5)^{1/2}(F/D)_{\pm}$. Exact $SU(3)$ symmetry of the coupling constants is assumed.

We write the conditions by referring to the u-channel amplitudes in Eqs. (1a) and (1c), the crossed exotic amplitudes. The residue of a $Y(\Lambda$ or $\Sigma^0)$ trajectory for the exotic amplitude is just the product of the couplings of the Y with the initial and final u-channel states. The consistency condition is that the sum of the $Y^{(-)}$ residues cancel (be equal to) the sum of the $Y^{(+)}$ residues.

We consider first the $\bar{K}N$ states. Because of the crossed exotic amplitude $K^-p \to \bar{K}^0 n$, and the crossed exotic amplitude $\bar{K}^0 p \to \bar{K}^0 p$ (coupled to charged Σ trajectories), the couplings of Λ trajectories to $\bar{K}N$ amplitudes must cancel, and the couplings of Σ trajectories to $\bar{K}N$ amplitudes must cancel also. We first apply these conditions, that the Λ and Σ couplings for K^-p scattering cancel separately, to the sum of the top and bottom trajectories. The results, obtained with the help of an $SU(3)$ Clebsch-Gordan table, are[10]

$$G_8^2(\tfrac{1}{5}^{1/2}c_- + s_-)^2 + \tfrac{1}{2}G_1^2 = (\tfrac{1}{5}^{1/2}c_+ + s_+)^2\,, \quad \text{(C1)}$$

$$G_8^2[(9/5)^{1/2}c_- - s_-]^2 + G_{10}^2 = [(9/5)^{1/2}c_+ - s_+]^2. \quad \text{(C2)}$$

For convenience we label all consistency conditions with capital C.

If we had followed the procedure of Refs. 1 and 2, and required cancellation in pure states of the exotic PB representations **27** and **10***, two independent linear combinations of Eqs. (C1) and (C2) would have resulted. Since adding the two trajectories gives the exact-$SU(3)$ results, no further independent PB conditions may be obtained by adding the trajectories.

The requirement that the couplings of the bottom trajectory Λ's cancel for K^-p states, and the corresponding requirement for Σ's, lead to the equations

$$G_8[(1/20)^{1/2}c_- + \tfrac{1}{4}^{1/2}s_-]\sin\alpha_\Lambda + \tfrac{1}{8}^{1/2}G_1\cos\alpha_\Lambda = (1/20)^{1/2}c_+ + \tfrac{1}{4}^{1/2}s_+\,, \quad \text{(C3)}$$

$$G_8[-(3/20)^{1/2}c_- + \tfrac{1}{12}^{1/2}s_-]\cos\alpha_\Sigma + \tfrac{1}{12}^{1/2}G_{10}\sin\alpha_\Sigma = 0. \quad \text{(C4)}$$

The fact that Eq. (C3) is not squared is discussed later. Sometimes it is convenient to replace Eq. (C3) by the condition that the $\Lambda^{(-)}$ on the top [$\Sigma(1193)$] trajectory is decoupled from K^-p states. This condition is

$$G_8[(1/20)^{1/2}c_- + \tfrac{1}{4}^{1/2}s_-]\cos\alpha_\Lambda - \tfrac{1}{8}^{1/2}G_1\sin\alpha_\Lambda = 0. \quad \text{(C3')}$$

We now turn to the $\pi^-\Sigma^+$ and $K^0\Xi^0$ states. We note from Eqs. (1a) and (1c) that the u-channel processes include elastic $\pi^-\Sigma^+$ and $K^0\Xi^0$ scatterings and the inelastic amplitude $\pi^-\Sigma^+ \to K^0\Xi^0$. If $G(i,j)$ is used to denote the coupling of the trajectory i to the state j, then application of the ED hypothesis to the elastic and inelastic processes leads to the conditions

$$G(\Lambda_b^{(-)},j) = pG(\Lambda^{(+)},j)\,, \quad (4)$$

$$G(\Sigma_b^{(-)},j) = qG(\Lambda^{(+)},j)\,, \quad (5)$$

where $p^2+q^2=1$, and j refers to either the $\pi^-\Sigma^+$ or $K^0\Xi^0$ state. If one writes the G's in terms of $SU(3)$ Clebsch-Gordan coefficients and the constants of Eq. (3), the result is[10]

$$-\tfrac{1}{5}^{1/2}G_8 c_-\sin\alpha_\Lambda + \tfrac{1}{8}^{1/2}G_1\cos\alpha_\Lambda = -p\tfrac{1}{5}^{1/2}c_+\,, \quad \text{(C5)}$$

$$\tfrac{1}{3}^{1/2}G_8 s_-\cos\alpha_\Sigma - \tfrac{1}{12}^{1/2}G_{10}\sin\alpha_\Sigma = -q\tfrac{1}{5}^{1/2}c_+\,, \quad \text{(C6)}$$

$$G_8[(1/20)^{1/2}c_- - \tfrac{1}{4}^{1/2}s_-]\sin\alpha_\Lambda + \tfrac{1}{8}^{1/2}G_1\cos\alpha_\Lambda = p[(1/20)^{1/2}c_+ - \tfrac{1}{4}^{1/2}s_+]\,, \quad \text{(C7)}$$

$$G_8[(3/20)^{1/2}c_- + \tfrac{1}{12}^{1/2}s_-]\cos\alpha_\Sigma + \tfrac{1}{12}^{1/2}G_{10}\sin\alpha_\Sigma = q[(1/20)^{1/2}c_+ - \tfrac{1}{4}^{1/2}s_+],\quad p^2+q^2=1. \quad \text{(C8)}$$

Here, Eqs. (C5) and (C6) are related to the $\pi^-\Sigma^+$ state, while Eqs. (C7) and (C8) are related to the $K^0\Xi^0$ state.

The conditions we have given are not all independent, but are complete, in the sense that any solution satisfying Eqs. (C1)–(C8) will lead to satisfaction of the residue conditions associated with every exotic PB process.

We now make some phase conventions. The condition that the $\Lambda^{(+)}$ and $\Lambda_b^{(-)}$ couplings cancel in the K^-p amplitude is actually the square of Eq. (C3), so that we should insert a $\pm$ before the right-hand side of the equation. However, any solution to the set of equations that corresponds to the insertion of a minus sign in Eq. (C3) is related to a solution involving the plus sign, if the signs of G_1, G_8, G_{10}, p, and q are changed. Hence, we lose no generality by omitting the $\pm$ sign. We also make the conventions that α_Λ and α_Σ are between 0° and 90°, while θ_+ and θ_- are between −90° and 90°. It is easy to show that any solution excluded by these conventions is related to an included solution by a change of sign of an appropriate subset of G_1, G_8, G_{10}, p, and q. These latter constants may be positive or negative.

B. Extension to *PD* States

In this section we write the extra consistency conditions for the two-and-two case of Table I that result from considering PD as well as PB states. Since the conditions involve only ratios of couplings to the same states, we normalize the coupling of the $8^{(+)}$ trajectory to PD states to be unity (assuming that this coupling is not zero). The constants of interaction of the $8^{(-)}$ and $10^{(-)}$ trajectories to the PD states are denoted by F_8 and F_{10}.

We first consider the exact $SU(3)$ conditions that result from summing the top and bottom trajectories. The Λ trajectories are not coupled to $\bar{K}\Delta$ states. The requirement that the sum of the $\Sigma^{(-)}$ couplings cancels the $\Sigma^{(+)}$ coupling to $K^-\Delta^+$ elastic scattering leads to

[10] Tables of $SU(3)$ Clebsch-Gordan coefficients are given by P. McNamee, S. J. Chilton, and Frank Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

the condition

$$F_8^2+\tfrac{5}{8}F_{10}^2=1\,. \qquad \text{(C9)}$$

This condition assures that cancellation occurs in both the exotic representations **27** and **35**, as shown in Ref. 2. Cancellation of the Σ couplings in the crossed-exotic inelastic process $K^-p\to K^-\Delta^+$ leads to the condition

$$F_8G_8[-\tfrac{2}{5}c_- +(4/45)^{1/2}s_-]-(1/18)^{1/2}F_{10}G_{10} = -\tfrac{2}{5}c_+ +(4/45)^{1/2}s_+\,. \qquad \text{(C10)}$$

This condition assures cancellation in the exotic representation **27**.

We now turn to the conditions resulting from requiring cancellation of bottom-trajectory contributions. The list of crossed-exotic processes coupled to the Λ and Σ^0 trajectories may be extended to include PD states by making any number of the replacements, $\Delta^+\to p$, $\Delta^0\to n$, $\Sigma_{10}\to\Sigma$, and $\Xi_{10}\to\Xi$ in the $A_{\rm el}$ and $B_{\rm in}$ amplitudes of Eqs. (1a) and (1c). The subscript 10 denotes a decuplet member. All of the states involving baryons of nonzero strangeness are connected by inelastic as well as elastic crossed-exotic amplitudes, so that the conditions involving these states may be written in the form of Eqs. (4) and (5), with j referring either to the $\pi^-\Sigma_{10}^+$ or $K^0\Xi_{10}^0$ state. The conditions related to the $\pi^-\Sigma_{10}^+$ state are

$$F_8\sin\alpha_\Lambda=p\,, \qquad \text{(C11)}$$

$$(1/15)^{1/2}F_8\cos\alpha_\Sigma+\tfrac{1}{6}^{1/2}F_{10}\sin\alpha_\Sigma=\tfrac{1}{5}^{1/2}q\,, \qquad \text{(C12)}$$

where the p and q are the same as in Eqs. (C5)–(C8). The conditions that result from considering the $K^0\Xi_{10}^0$ state are the same as these, rather than being independent.

There is one further condition, involving the bottom trajectory and $K^-\Delta^+$ states. This is the condition that $\Sigma_b^{(-)}$ decouples from the $K^-\Delta^+$ state, i.e.,

$$-(4/15)^{1/2}F_8\cos\alpha_\Sigma+\tfrac{1}{6}^{1/2}F_{10}\sin\alpha_\Sigma=0\,. \qquad \text{(C13)}$$

The constants F_8 and F_{10} may be positive or negative. This completes the list of consistency conditions for the two-and-two case.

III. SOLUTIONS

In one solution to the two-and-two case (and to some other cases also), $G_1=G_{10}=0$, $G_8=1$, $\theta_+=\theta_-$, and the $\Lambda^{(-)}$ and $\Lambda^{(+)}$ cancel each other, as do the $\Sigma^{(-)}$ and $\Sigma^{(+)}$. We call this solution the trivial solution, and ignore it.

The consistency equations lead to simple relations for the mixing angles α_Λ and α_Σ. Combination of Eqs. (C1) and (C3′) yields a simple expression for $(G_1\sec\alpha_\Lambda)^2$ in terms of c_+ and s_+. One may use Eqs. (C3), (C5), and (C7) to write $G_1\cos\alpha_\Lambda$ in terms of c_+ and s_+. If $G_1\neq0$, it may be eliminated from these relations, yielding

$$\cos\alpha_\Lambda=[\tfrac{1}{3}(1-p)]^{1/2}, \quad \text{if} \quad G_1\neq0\,. \qquad (6)$$

In a similar fashion, one may use Eqs. (C2), (C4), (C6), and (C8) to write the following equation for α_Σ:

$$\sin^2\alpha_\Sigma\cos^2\alpha_\Sigma=\tfrac{1}{3}q^2\,, \quad \text{if} \quad G_{10}\neq0\,. \qquad (7)$$

These equations are used in the following sections.

A. Three-and-One and One-and-Three Solutions

It is reasonable to assume that the two known $j^P=\tfrac{3}{2}^-$ Λ's at 1520 and 1690 MeV are on the bottom and top trajectories, respectively. Experimentally, there is evidence for one (and possible two) $\tfrac{3}{2}^-$ Σ's at around 1660 MeV. Thus, the partition of the four $Y^{(-)}$'s that is suggested by the data is not that of the two-and-two solution of Table I, but rather that of the three-and-one solution, with one $\Lambda^{(-)}$, alone of the $Y^{(-)}$ on the bottom trajectory. The modification of the $PB\to PB$ consistency equations of Sec. II A that is necessary to describe the three-in-one case is fairly simple. There is no α_Σ, and the $\Sigma_b^{(-)}$ consistency equations must be dropped. The remaining equations are (C1), (C2), (C3) [or (C3′)], (C5), and (C7). The quantity p must be ±1. The choice $p=-1$ means that the relative $\pi\Sigma/\bar{K}N$ phases of the $\Lambda^{(+)}$ and $\Lambda_b^{(-)}$ are opposite.

If $p=1$, it is easy to show that the only solution is the trivial one mentioned at the beginning of this section. Hence, we choose $p=-1$. The value of $\cos\alpha_\Lambda$ is $\tfrac{2}{3}^{1/2}$, as seen from Eq. (6). Using this value of α_Λ, one may solve Eqs. (C3), (C5), and (C7) for G_1, G_8c_-, and G_8s_- in terms of c_+ and s_+, and substitute these expressions into Eq. (C2). The result of this procedure is

$$G_{10}^2=-2[(9/5)^{1/2}c_+-s_+]^2. \qquad (8)$$

Since G_{10}^2 cannot be negative, this equation determines that $G_{10}^2=0$ and $\tan\theta_+=(9/5)^{1/2}$. All the PB constants are then determined to have the values

$$G_8=\tfrac{1}{3}^{1/2},\quad G_1=(32/21)^{1/2},\quad G_{10}=0,$$
$$\tan\theta_+=\tan\theta_-=(9/5)^{1/2},\quad \cos\alpha_\Lambda=\tfrac{2}{3}^{1/2}. \qquad (9)$$

Since one of the $\Sigma^{(-)}$'s is decoupled, this should be called the two-and-one solution.

One can attempt to extend this solution to processes involing PD states by considering Eqs. (C9)–(C11), with $p=-1$. A contradiction results; the solution cannot be extended unless all coupling to PD states vanishes. On the other hand, if one considers only those equations obtained by adding together the contributions of the two trajectories, Eqs. (C9) and (C10), no contradiction results, provided the F's are chosen so that Eq. (C9) is satisfied. It may be that the Σ-Λ splitting must be neglected when considering PD states.[11] At any rate, since the experimental Y spectrum seems to be of the two-and-one type, we compare the predictions of this solution with experiment in Sec. IV.

[11] The results of Barger and Michael, Ref. 9, may be used to show that this solution also leads to equality of the even- and odd-parity trajectory contributions for the exotic $PP\to B\bar{B}$ (t-channel) amplitudes *provided that one adds the contributions of the top and bottom trajectories.*

For completeness we include here another case very similar to the three-and-one case. This is the one-and-three case of Table I, in which only one $\Sigma^{(-)}$ lies on the top trajectory, and the other $\Sigma^{(-)}$ and two $\Lambda^{(-)}$ lie on the bottom trajectory. In this case, it is simpler to use consistency conditions that emphasize the top trajectory. If one does this, and carries out a procedure analogous to that described above for the three-and-one case, one nontrivial solution results, with the constants given by

$$G_1=0,\quad G_8=\tfrac{1}{3}^{1/2},\quad G_{10}=\tfrac{4}{3},\qquad \tan\theta_+=\tan\theta_-=-\tfrac{1}{5}^{1/2},\quad \cos\alpha_\Sigma=\tfrac{2}{3}^{1/2}. \tag{10}$$

This solution may be extended to satisfy all the *PD* conditions also, if the following couplings are chosen:

$$F_8=\tfrac{1}{3}^{1/2},\quad F_{10}=(16/15)^{1/2}. \tag{11}$$

This solution does not correspond to experiment.

B. Two-and-Two Solutions

We now return to the equations for the two-and-two case, Eqs. (C1)–(C13). If one solves Eqs. (C4), (C6), and (C8) for $G_{10}\sin\alpha_\Sigma$, $G_8 s_-\cos\alpha_\Sigma$, and $G_8 c_-\cos\alpha_\Sigma$, and takes the ratio of the latter two quantities, then the following equation relating θ_+ and θ_- results:

$$\tan\theta_-=[\tan\theta_+ + (9/5)^{1/2}]/(5^{1/2}\tan\theta_+ - 1). \tag{12}$$

If one uses Eqs. (C3), (C5), and (C7) to write equations for $G_8 s_-\sin\alpha_\Lambda$ and $G_8 c_-\sin\alpha_\Lambda$, and takes the ratio of these, another equation for $\tan\theta_-$ may be obtained. Setting this equation equal to Eq. (12) leads to the equation

$$(1+2p)(\tan\theta_+ + \tfrac{1}{5}^{1/2})[\tan\theta_+ - (9/5)^{1/2}]=0. \tag{13}$$

Setting each of these factors in turn equal to zero leads to three solutions of the *PB* equations, each a function of one continuous variable. These three solutions are shown in Table II. We have chosen $\cos\alpha_\Sigma$ (denoted by $x^{1/2}$) to be the arbitrary parameter for solution II, and $\cos\alpha_\Lambda$ (denoted by $y^{1/2}$) to be the arbitrary parameter for solution III. Solutions I and II may be extended to satisfy the *PD* equations, as shown in Table II. The variables $x^{1/2}$ and $y^{1/2}$ are limited to ranges for which all cosines and sines of angles, and p and q, are of magnitudes not less than zero nor greater than 1.

We discuss these three solutions briefly. In solution II, for which $\tan\theta_+=\tan\theta_-=-\tfrac{1}{5}^{1/2}$, the singlet does not interact with *PB* states and it cannot interact with *PD* states. Effectively, one sees an octet split into two parts of different masses. If we require that all existing trajectories couple to either the *PB* or *PD* states, so that no singlet exists in this solution, then we should require that the octet Λ is a single particle at a single mass. Solution II reduces to a solution with one $\Lambda^{(-)}$ only if $x=0$ or $\tfrac{2}{3}$. The value $x=0$ leads to the trivial two-octet solution. The value $x=\tfrac{2}{3}$ leads to the one-and-three solution of Eqs. (10) and (11); this solution is in fact a one-and-two solution since one $\Lambda^{(-)}$ vanishes, and the other lies on the bottom trajectory.

TABLE II. Solutions of the consistency equations for the two-and-two case.

Parameter	Solution I	Solution II	Solution III
$\tan\theta_+$	arbitrary	$-\tfrac{1}{5}^{1/2}$	$(9/5)^{1/2}$
$\tan\theta_-$	$\dfrac{\tan\theta_+ + (9/5)^{1/2}}{(5)^{1/2}\tan\theta_+ - 1}$	$-\tfrac{1}{5}^{1/2}$	$(9/5)^{1/2}$
$\cos\alpha_\Lambda$	$\tfrac{1}{2}^{1/2}$	$\left(\dfrac{2x-3x^2}{1-x}\right)^{1/2}$	$y^{1/2}$
$\cos\alpha_\Sigma$	$\tfrac{1}{2}^{1/2}$	$x^{1/2}$	$\left(\dfrac{2y-3y^2}{1-y}\right)^{1/2}$
$G_8 s_-$	$\tfrac{1}{8}^{1/2}[s_+ + (9/5)^{1/2}c_+]$	$-[\tfrac{1}{6}(1-x)]^{1/2}$	$[(9/14)(1-y)]^{1/2}$
G_1	$s_+ + \tfrac{1}{5}^{1/2}c_+$	0	$(16y/7)^{1/2}$
G_{10}	$\tfrac{1}{2}^{1/2}[s_+ - (9/5)^{1/2}c_+]$	$(8x/3)^{1/2}$	0
p	$-\tfrac{1}{2}$	$(1-3x+3x^2)^{1/2}$	$1-3y$
q	$-\tfrac{3}{4}^{1/2}$	$[3x(1-x)]^{1/2}$	$-[3y(2-3y)]^{1/2}$
F_8	$-\tfrac{1}{2}^{1/2}$	$(1-x)^{1/2}$	...
F_{10}	$-\tfrac{4}{5}^{1/2}$	$(8x/5)^{1/2}$	...

In solution III, the decuplet trajectory is not coupled to *PB* states, but may be coupled to *PD* states. However, it can be shown that the solution can be extended to include the *PD* equations, Eqs. (C9)–(C13), only if $p=1$ $(y=0)$, in which case the trivial solution results, or if $p=-\tfrac{1}{2}$, in which case the solution is a special case of solution I.

Solution I is an interesting solution. Both mixing angles are 45°. It can be shown that Eq. (12), the relation between θ_+ and θ_- in all two-and-two solutions, is the condition that these angles are conjugate in the sense defined in Ref. 1, that is, the ratios of forces in the two exotic representations **10*** and **27**, produced by the exchange of two octets with these θ values, are the same. The conjugate values coincide only in the cases $\tan\theta=(9/5)^{1/2}$ and $\tan\theta=-\tfrac{1}{5}^{1/2}$. If one chooses $\tan\theta_+=\tfrac{4}{5}^{1/2}$ in solution I, then the values of the coupling constants G and the angles θ correspond to the quark or $SU(6)_W$ model.[5] This solution can correspond to experiment only if a $\Sigma^{(-)}$ particle is discovered in the vicinity of 1520 MeV.

C. Other Possible Solutions

Although we have described only a few of the possible partitions of the $\Lambda^{(-)}$ and $\Sigma^{(-)}$ particles to the top and bottom trajectories, we have investigated all possibilities. No partition leads to a solution that is different from all those mentioned here. It is interesting that one of the partitions that seems simplest *a priori*, namely, that of assigning both $\Lambda^{(-)}$'s to the $\Lambda^{(+)}$ trajectory and both $\Sigma^{(-)}$'s to the $\Sigma^{(+)}$ trajectory, leads to no solution other than the trivial two-octet solution.

IV. COMPARISON OF THREE-AND-ONE SOLUTION WITH EXPERIMENT

The experimental spectrum of $j^P=\frac{3}{2}^-$ hyperons includes the Λ(1690), Σ(1660), and, at a lower mass, the Λ(1520). No other $\frac{3}{2}^-$ Y's are known in this mass region. The spectrum corresponds to that predicted in the three-and-one solution, Eq. (9), since the decuplet Σ is not coupled in this solution. In this section, we compare this solution with experiment. In making this comparison, it is assumed that the ratios of interactions of the same spin structure (such as F/D ratios) are the same in the Regge-exchange region as in the positive-u region of the resonance decays.

The prediction that $G_{10}=0$ is violated by the Δ*(1690), if this particle is a member of the decuplet in question. However, the PB coupling of this Δ* is weak.[12] Since no Σ* member of this decuplet has been identified as yet, we assume that such a Σ* is also coupled weakly to PB states (if it exists at all).

An interesting property of this solution is that it disagrees with the quark model [or the equivalent $SU(6)_W$ model]. In the quark model, the F/D ratios of the $\frac{1}{2}^+$ and $\frac{3}{2}^-$ octets are $\frac{2}{3}$ and 5/3, respectively, rather than both being 1, as in the solution of Eq. (9).[5] The analysis of $\frac{3}{2}^-$-octet data by Tripp yields the value $F/D\sim1.2$.[13] The value $F/D=1[\tan\theta=(9/5)^{1/2}]$ is the value for which Σ particles are decoupled from $\bar{K}N$ states. The fact that F/D is fairly close to 1 for the $\frac{5}{2}^+$ multiplet which is the recurrence of the nucleon octet helps explain the puzzle that Tripp's branching-ratio analysis for this multiplet yields the result $F/D\sim1.2$,[13] while the phase argument of Kernan and Smart yields the result $F/D<1$.[14,15]

In order to avoid assuming the Gell-Mann–Okubo mass formula, we will not check the predicted mixing angle α_Λ directly, but rather will check the related Λ* branching ratios. The ED hypothesis, together with the assumption that one $\Lambda^{(-)}$ is "alone" of the $Y^{(-)}$ on the $\Lambda^{(+)}$ trajectory, implies that the $\pi\Sigma/\bar{K}N$ coupling ratio should be the same for all Λ's on this trajectory. The ratio predicted by the parameters of Eq. (9) is $\frac{3}{8}$.

[12] Data concerning this resonance are given by A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[13] R. D. Tripp, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), pp. 173–191. The α parameter of this reference is related to F/D by the formula $\alpha=D/(F+D)$.

[14] Anne Kernan and Wesley B. Smart, Phys. Rev. Letters **17**, 832 (1966).

[15] The author would like to thank Mr. V. V. Dixit for a useful conversation concerning this point.

TABLE III. Experimental $\pi\Sigma/\bar{K}N$ coupling ratios of Λ particles on the bottom trajectory.

Λ mass, spin, and parity	$1520(\frac{3}{2}^-)$	$1815(\frac{5}{2}^+)$	$2100(\frac{7}{2}^-)$
$\pi\Sigma/\bar{K}N$ ratio	0.79	0.35	0.30

We have calculated these experimental ratios from the data of Ref. 4, using the popular phase-space factor $\rho=k^{2l+1}/[(W+m)^2-\mu^2]$, where W, m, and μ are the masses of the resonance, baryon, and meson, respectively.[16] The results are shown in Table III. The experimental ratio for the Λ(1520) is between our prediction of $\frac{3}{8}$ and the $\frac{3}{2}$ value that corresponds to a pure singlet, but the Λ(1815) and Λ(2100) ratios are in very good agreement with the predictions. By contrast, the experimental $\pi\Sigma/\bar{K}N$ ratio for the Λ(1690), which lies on the top trajectory, is 2.4. The predicted $\bar{K}N$ width for this particle is zero. It should be noted that while the experimental data concerning the Λ's of high mass are not very accurate, the data concerning the Λ(1520) are sufficiently accurate to rule out definitely the $\pi\Sigma/\bar{K}N$ ratio that corresponds to a pure $SU(3)$ singlet.[17]

As pointed out in Sec. III A, the predicted relative $\pi\Sigma/\bar{K}N$ phases of the even- and odd-parity Λ's on the bottom trajectory are opposite. This predicted phase for the Λ(1520) has been verified experimentally.[18] (The predicted $\pi\Sigma/\bar{K}N$ phase of the $\Lambda^{(-)}$ is that of the singlet.) It will be interesting to see if the phase predictions are correct for the heavier Λ's on this trajectory.

We conclude by noting that while deviations from the predictions of the three-and-one solution are not negligible, the solution is in fair agreement with experiment. The data support the contention than the ED hypothesis applies approximately to the "bottom" Y trajectory, which includes the Λ(1115) and the Λ* particles of Table III. The ED principle may be the reason that the even- and odd-parity Λ's on this trajectory have similar properties, despite having quite different $SU(3)$ assignments.

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[16] See J. G. Rushbrooke, Phys. Rev. **143**, 1345 (1966).

[17] See the analysis of G. B. Yodh, Phys. Rev. Letters **18**, 810 (1967).

[18] R. D. Tripp, R. O. Bangerter, A. Barbaro-Galtieri, and T. S. Mast, Phys. Rev. Letters **21**, 1721 (1968).