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K^- Capture in He^4 in the Multiple-Impulse Approximation*

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The process $K^- + He^4 \rightarrow \Lambda + \pi^- + He^3$ is analyzed in the multiple-impulse approximation for both $1S$ and $2P$ atomic capture at rest. Reaction kinematics and rates, which are free of arbitrary constants, are calculated in both first and second order. The multichannel $\bar{K}n$ amplitudes of Kim are employed to incorporate the $Y_1^*(1385)$ resonance into the first-order (direct) term, and both the $Y_1^*(1385)$ and $Y_0^*(1405)$ resonances into the second-order (Σ-Λ conversion) term. Agreement with experiment is good, particularly when the conversion term is added.

I. INTRODUCTION

THE literature abounds in analyses of both Σ^- and K^- capture in He^4 and deuterium.[1] Many authors have confined themselves to the direct process, or, if investigating resonance or conversion processes, they have introduced arbitrary phases and constants in the absence of more specific knowledge. It is the aim of this paper to construct a general representation for impulse scattering and, in the subsequent application to the process $K^- + He^4 \rightarrow \Lambda + \pi^- + He^3$, to employ recent estimates of pertinent matrix elements.

In Sec. II, a very brief review of scattering theory is presented and the impulse approximation given explicit form. Sections III and IV carry the specific process $K^- + He^4 \rightarrow \Lambda + \pi^- + He^3$ through first and second orders, respectively. Parametrization of the scattering operators is given in Sec. V, and results compared with experiment in Sec. VI.

Owing to the number of particles in the reaction, Fourier analysis of the amplitude often becomes confusing in the proliferation of indices and variables. The following conventions may thus prove helpful to the reader. Fourier momentum variables will always carry a prime superscript, i.e., p_j', q_i', while physical states will carry no superscripts, i.e., p_j, q_i. Subscripts will be used to denote particles. Where confusion may arise as to whether a particle is in an initial, intermediate, or final state, it will be advantageous to use a double-primed subscript for the final state, a single-primed subscript for an intermediate state, and an unprimed subscript to designate a particle in the initial state.

II. MULTIPLE-SCATTERING IMPULSE APPROXIMATION

The complete T operator in a system of n scattering centers satisfies the operator equation

$$T = V + VG_0T = \sum_{i=1}^{n} T_i, \tag{1}$$

where T_i designates the individual transition operators for each of the n scatterers, and satisfies a similar equation in terms of the individual scattering potentials V_i:

$$T_i = V_i + V_iG_0T_i. \tag{2}$$

For a full Hamiltonian, $H = K + V$, where K is the unperturbed part of the $(n+1)$-body system; the scattering potential V and Green's function G_0 are defined as

$$V = \sum_{i=1}^{n} V_i, \tag{3}$$

$$G_0 = (K - E + i\epsilon)^{-1}, \tag{4}$$

while the full and unperturbed eigenvectors, or state vectors $|\psi\rangle$ and $|\phi\rangle$, satisfy the usual eigenvalue equation

$$H|\psi\rangle = E|\psi\rangle, \qquad K|\phi\rangle = E|\phi\rangle. \tag{5}$$

Using the individual transition operators T_i defined in Eq. (2), and iterating Eq. (1), we easily obtain the cluster expansion of T:

$$T = (T_1 + T_2 + \cdots + T_n) + (T_1 + T_2 + \cdots + T_n)G_0(T_1 + T_2 + \cdots + T_n) + \cdots. \tag{6}$$

In second order (see Fig. 1), the transition amplitude for state i to state f is given by

$$\langle f|T|i\rangle = \langle f|(T_1 + T_2 + \cdots + T_n)|i\rangle + \langle f|(T_1 + T_2 + \cdots + T_n)G_0(T_1 + T_2 + \cdots + T_n)|i\rangle. \tag{7}$$

In the impulse model, only one scattering center effects transitions at the various scattering stages so that[2]

$$\langle f|T|i\rangle = \langle f|T_1|i\rangle + \langle f|T_2G_0T_1|i\rangle. \tag{8}$$

The above has an obvious graphical parallel to relativistic Feynman graphs when $\langle f|$, $|i\rangle$ are identified with external "lines," G_0 with the propagator, and T_1, T_2 with coupling constants or form factors.

Applied to the process $K^- + He^4 \rightarrow \Lambda + \pi^- + He^3$, we

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[1] A short list of some important work is given here: D. E. Neville, Phys. Rev. **130**, 327 (1963); P. Said and J. Sawicki, *ibid.* **139**, B991 (1965); J. Sawicki, Nuovo Cimento **33**, 361 (1964); R. Chand, *ibid.* **31**, 1013 (1964); **34**, 1769 (1962); T. Kotani and M. Ross, *ibid.* **14**, 1282 (1959); R. Karplus and L. S. Rodberg, Phys. Rev. **115**, 1058 (1959); M. M. Block, Nuovo Cimento **20**, 715 (1961); J. Sawicki, Nucl. Phys. **B1**, 183 (1967).

[2] The following give a good account of formal scattering theory and the impulse approximation: G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952); M. Gell-Mann and M. L. Goldberger, *ibid.* **91**, 398 (1953); G. F. Chew and M. L. Goldberger, *ibid.* **87**, 778 (1952).

write, upon introducing complete sets of intermediate states, $\sum_n |n\rangle\langle n|$, where n refers to the complete set of attributes necessary to describe the intermediate states,

$$\langle f|T|i\rangle = \langle \pi^-\Lambda He^3|T_1|K^-He^4\rangle + \sum_n \langle \pi^-\Lambda He^3|T_2|n\rangle(E-E_n+i\epsilon)^{-1} \times\langle n|T_1|K^-He^4\rangle, \quad (9)$$

and E and E_n represent the total energy of the initial and intermediate state, respectively.

III. FIRST ORDER

Assuming plane-wave final π, Λ, and external[3] He^3 states, Coulomb K^--He^4 and either Gaussian or Hulthén n-He^3 initial states, and Gaussian internal states of the spectator He^3 nucleus,[4] the usual Fourier analysis of the first-order term takes the form (suppressing for the moment the spin and isospin labels)

$$\mathbf{M}_1 = \langle \pi^-\Lambda He^3|T_1|K^-He^4\rangle = \int d\xi d\xi'' \frac{d^9p'}{(2\pi)^9}\frac{d^9q'}{(2\pi)^9}\langle p_\pi p_\Lambda He^3|q_\pi' q_\Lambda' q_3'\xi''\rangle \times\langle \xi'' q_3' q_\Lambda' q_\pi'|T_1|p_K' p_n' p_3'\xi\rangle\langle \xi p_K' p_n' p_3'|K^-He^4\rangle, \quad (10)$$

where d^9p' and d^9q' denote integration over the Fourier sets $(p_K' p_n' p_3')$ and $(q_\pi' q_\Lambda' q_3')$, respectively. Both ξ and ξ'' denote internal coordinates of the spectator in the following manner. For a four-nucleon spectator with nucleon coordinates (y_1, y_2, y_3, y_4), ξ is the set of internal coordinates (x_3, x_2, x_1) such that, in the limit of equal-mass nucleons,

$$x_3 = y_2 - y_1,$$
$$x_2 = y_3 - \tfrac{1}{2}(y_1+y_2), \quad (11)$$
$$x_1 = y_4 - \tfrac{1}{3}(y_1+y_2+y_3).$$

As is probably evident, each successive internal coordinate x_i represents the relative separation of the $(i+1)$th nucleon from the center of mass of the preceding i nucleons. For a spectator of three nucleons, $\xi=(x_3, x_2)$, while for a two-nucleon spectator, $\xi=(x_3)$. Except for a configuration representation of internal degrees of the spectator, the factored, or product, state vector takes the form in momentum space

$$\langle p_\pi p_\Lambda He^3|q_\pi' q_\Lambda' q_3'\xi''\rangle = (2\pi)^9\delta(P_f - q_\pi' - q_\Lambda' - q_3') \times\delta(p_\pi - q_\pi')\delta(p_\Lambda - q_\Lambda')\psi_3^*(x_2'', x_3''), \quad (12)$$

[3] The label "external" applied to He^3 refers to the composite center of mass of the He^3 nucleus, in distinction to the two "internal" degrees of freedom.

[4] The Gaussian internal states referred to in the text are obtained from the product Gaussian wave functions in the shell model by transforming, using Eq. (11). In the shell-model representation, each nucleon is described by a wave function proportional to $\exp[-\sigma^2(y_i-y_0)^2/2]$, where y_i is the coordinate of the ith nucleon and y_0 is the center-of-mass coordinate of the nucleus as a whole.

$$\langle p_K' p_n' p_3'\xi|K^-He^4\rangle = (2\pi)^3\delta(P_i - p_K' - p_n' - p_3') \times\psi_{K-4}\left(\frac{m_4 p_K' - m_K p_n' - m_K p_3'}{m_4+m_K}\right) \times\phi_{n-3}\left(\frac{m_3 p_n' - m_n p_3'}{m_4}\right)\psi_4(x_2, x_3), \quad (13)$$

for initial and final total momenta P_i and P_f, given by

$$P_i = p_4 + p_K, \qquad P_f = p_\pi + p_\Lambda + p_3'',$$

where the physical momenta of the initial and final states are denoted by the sets $(p_4 p_K)$ and $(p_\pi p_\Lambda p_3'')$, respectively. The momentum wave functions, obtained by Fourier-transforming the normalized configuration-space wave functions mentioned previously, take the following explicit form[5]:

(a) $$\psi_{K-4}(q) = (2\pi)^3 N_{10}\delta(q) \quad (1S)$$
$$= (2\pi)^3 N_{21} e_i\cdot\nabla\delta(q) \quad (2P),$$

(b) $$\phi_{n-3}(q) = 4\pi N_4 \frac{m^2-n^2}{(q^2+m^2)(q^2+n^2)} \quad \text{(Hulthén)}$$
$$= N_G \exp(-2q^2/3\alpha^2) \quad \text{(Gaussian)}, \quad (14)$$

(c) $$\psi_3(x_2, x_3) = N_3 \exp[-\tfrac{1}{2}\beta^2(\tfrac{2}{3}x_2^2+\tfrac{1}{2}x_3^2)] \quad (He^3),$$

(d) $$\psi_4(x_1, x_2, x_3) = N_4 \exp[-\tfrac{1}{2}\alpha^2(\tfrac{2}{3}x_2^2+\tfrac{1}{2}x_3^2+\tfrac{3}{4}x_1^2)] \quad (He^4),$$

(e) $$\psi_I(x_2, x_3) = N_I \exp[-\tfrac{1}{2}\gamma^2(\tfrac{2}{3}x_2^2+\tfrac{1}{2}x_3^2)] \quad (H^3 \text{ or } He^3),$$

where the numerical values and normalization factors written above are given in Ref. 5. In $\psi_{K-4}(q)$, e_i denotes any one of the three possible azimuthal $2S$ K^--He^4

[5] The following is a complete list of constants and expressions appearing in the text:

Normalization factors:

$$N_{10} = (8/\pi a^3)^{1/2}, \qquad N_{21} = (1/\pi a^5)^{1/2},$$
$$N_H = [mn(m+n)/2\pi(m-n)^2]^{1/2},$$
$$N_G = (4/\sqrt{3}\alpha)^{1/2}, \qquad N_4 = (\alpha^3/4\pi^{3/2})^{3/2},$$
$$N_3 = (\beta^2/\sqrt{3}\pi)^{3/2}, \qquad N_I = (\gamma^2/\sqrt{3}\pi)^{3/2},$$
$$N_{n''-R} = (2\pi^{1/2}/\beta)^{3/2}, \qquad N_{n'-R} = (2\pi^{1/2}/\gamma)^{3/2}.$$

Constants:

$\alpha = 146$ MeV, $\beta = -155$ MeV, $\gamma = 157$ MeV,
$m = 250$ MeV, $n = 160$ MeV, $a = 0.34\times10^{-13}$ cm.

Overlap factors:

$$g_{I3} = [2\beta\gamma/(\beta^2+\gamma^2)]^{3/2}, \qquad g_{I4} = [2\alpha\gamma/(\alpha^2+\gamma^2)]^3,$$
$$g_{34} = [2\alpha\beta/(\alpha^2+\beta^2)]^3,$$
$$f_{I3} = N_{n''-R}N_{n'-R}\gamma^2\beta^3/(2\pi)^{3/2}(\gamma^2+\beta^2)^{3/2}.$$

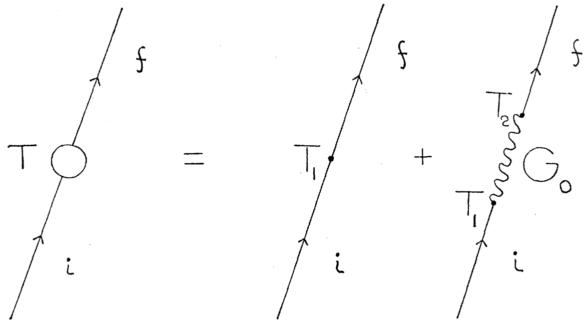

FIG. 1. Second-order impulse graphs.

states, i.e., for unit vectors in the relative momentum basis of K^--He^4 defined by k_x, k_y, and k_z,

$$e_i = -(k_x + ik_y)/\sqrt{2}$$
$$= k_z$$
$$= (k_x - ik_y)/\sqrt{2}. \quad (15)$$

Also, $\phi_{n-3}(q)$ is the Fourier transform of the x_1 part of $\psi_4(x_1, x_2, x_3)$, thereby yielding an effective form factor for the impulse scattering $K^- + n \to \Lambda + \pi^-$. The scattering nucleon n is hereafter referred to as the "knock-on" nucleon. Furthermore, the impulse approximation demands that the He^3 be a spectator to the reaction so that its state remain unaffected by the scattering operator:

$$\langle q_\pi' q_\Lambda' q_3' \xi'' \mid T_1 \mid p_K' p_n' p_3' \xi \rangle$$
$$= (2\pi)^3 \delta(q_3' - p_3') \delta(\xi - \xi'') \langle q_\pi' q_\Lambda' \mid T_1 \mid p_K' p_n' \rangle. \quad (16)$$

Integrating over all variables except d^3p_K', we obtain[6]

$$\mathbf{M}_1 = g_{34} \int \frac{d^3p_K'}{(2\pi)^3} \psi_{K-4}\left(p_K' - \frac{m_n}{m_4 + m_K} P_i\right)$$
$$\times \phi_{n-3}\left(p_{3''} - \frac{m_3}{m_4} p_K' - \frac{m_3}{m_4} P_i\right) \langle p_\pi p_\Lambda \mid T_1 \mid p_K' p_n^0 \rangle, \quad (17)$$

with g_{34} representing the spatial overlap of ψ_3^* and ψ_4, and $p_n^0 = P_i - p_K' - p_{3''}$. This result, as well as parametrizations of the matrix elements and wavefunctions, has been obtained by a number of authors, who vary somewhat in their approaches. More will be said in Sec. V about our choice for $\langle p_\pi p_\Lambda \mid T_1 \mid p_K' p_n^0 \rangle$.

IV. SECOND ORDER

The second-order term is important because it contains the so-called Σ-Λ conversion amplitude. Let us saturate the intermediate states $\mid n \rangle \langle n \mid$ with $\mid \Sigma \pi I \rangle \langle I \pi \Sigma \mid$ charge and spin states. Here I refers to the system of three intermediate nucleons formed in the process

$$K^- + He^4 \to \pi^- + \Sigma^a + I^b \to \pi^- + \Lambda + He^3. \quad (18)$$

The double-scattering conversion process, in the impulse model, requires two different knock-on nucleons n and n', and can be envisioned as occurring as pictured in Fig. 2 (the labeling given there also will carry through in the subsequent Fourier analysis in Appendix A), where R is a system of two nucleons, and the charge states are implicitly defined by a and b. This splitting facilitates parametrizing the scattering operator for $K^- + n \to \Sigma^a + \pi^-$ in the first stage, followed by $\Sigma^a + n' \to \Lambda + n''$ in the second stage. In second order, assuming plane-wave π, Σ intermediate states, the cluster amplitude becomes

$$\mathbf{M}_2 = \int \frac{d^9k}{(2\pi)^9} \langle p_\pi p_\Lambda He^3 \mid T_2 \mid k_\Sigma k_\pi I \rangle$$
$$\times (E - E_I + i\epsilon)^{-1} \langle k_\Sigma k_\pi I \mid T_1 \mid K^- He^4 \rangle, \quad (19)$$

where d^9k refers to integration over the set $(k_\Sigma k_\pi k_I)$ in the intermediate state $\Sigma \pi I$ having total energy E_I. Again the spin and isospin indices are suppressed until Sec. V. The details of the Fourier analysis, in analogy with Eqs. (10), (12), and (16), are given in the Appendix A, and we list the result here:

$$\mathbf{M}_2 = g_{I3} g_{I4} \int \frac{d^6k}{(2\pi)^6} \frac{d^3q_{n''}'}{(2\pi)^3} \frac{d^3p_K'}{(2\pi)^3} F(k, q, p)$$
$$\times \langle q_{n''}' p_\Lambda \mid T_2 \mid k_\Sigma h_n^1 \rangle (E - E_I + i\epsilon)^{-1}$$
$$\times \langle k_\Sigma p_\pi \mid T_1 \mid p_K' p_n^1 \rangle, \quad (20)$$

$$F(p, q, k) = \phi_{n-3}\left(k_I + \frac{m_3}{m_4} p_K'\right) \chi^*_{n''-R}\left(\frac{m_n}{m_I} p_{3''} - q_{n''}'\right)$$
$$\times \chi_{n'-R}\left(p_{3''} - q_{n''}' - \frac{m_n}{m_I} k_I\right) \psi_{K-4}\left(p_K' - \frac{m_n}{m_K + m_4} P_i\right),$$

with d^6k designating the set $(k_\Sigma k_I)$, and

$$p_n^1 = P_i - p_K' - k_I, \qquad h_n^1 = k_I - p_{3''} + q_{n''}';$$

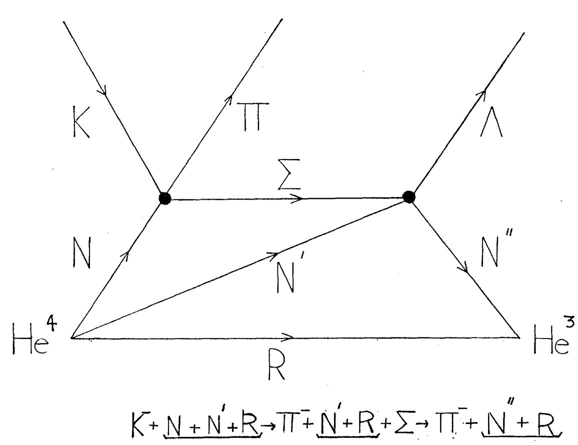

FIG. 2. Second-order scattering process.

[6] The overlap g_{34} is obtained from $g_{34} = \int d^3x_2 d^3x_3 \psi_3^*(x_2, x_3) \psi_4(x_2, x_3)$, and is given above in Ref. 5. Similar expressions hold for g_{I3} and g_{34}.

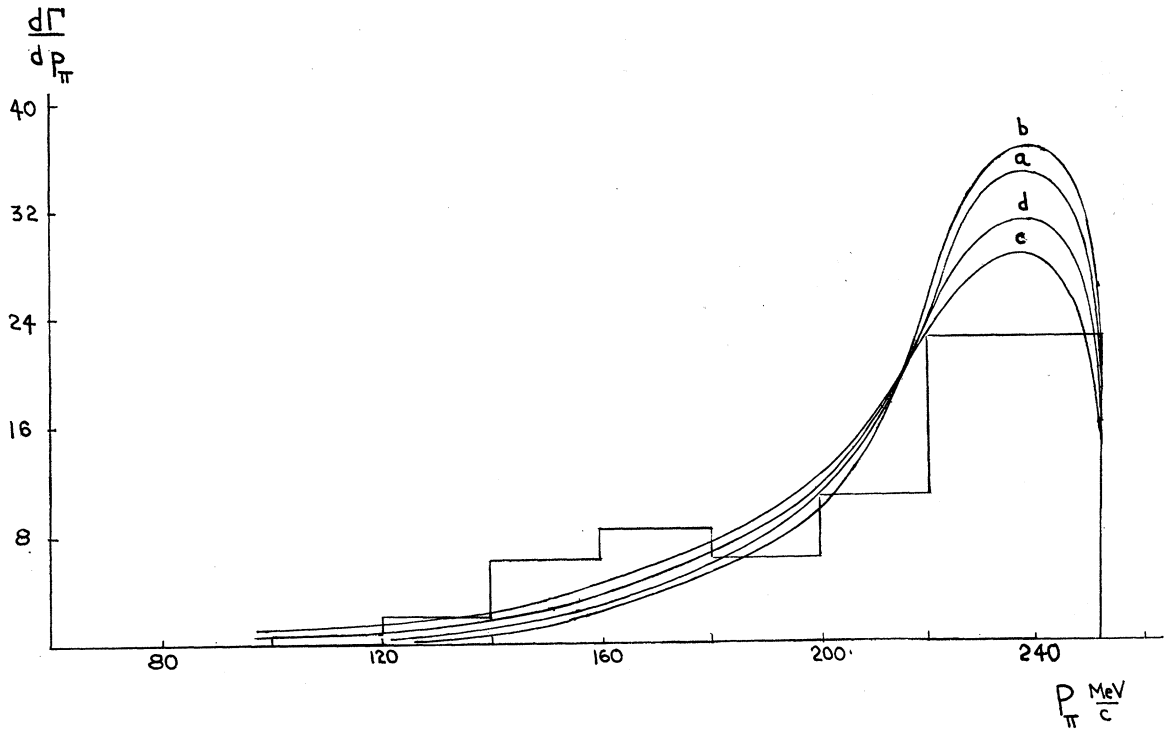

FIG. 3. Pion momentum distribution (Hulthén form factor). (a) $1S$ capture, no conversion; (b) $2P$ capture, no conversion; (c) $1S$ capture, conversion; (d) $2P$ capture, conversion.

the factors g_{I3} and g_{I4} represent the spatial overlap of the intermediate three-nucleon system with the initial He^4 and final He^3, while $\chi^*_{n''-R}$ and $\chi_{n'-R}$ represent effective form factors for the second-stage scattering $\Sigma+n'\rightarrow\Lambda+n''$.

V. MATRIX ELEMENTS

For the first-order matrix element, we parametrize the scattering in the relative, or center-of-mass, system and write

$$\langle p_\pi p_\Lambda | T_1 | p_K' p_n^0\rangle = (2\pi)^3\delta(p_\pi+p_\Lambda-p_K'-p_n^0)\langle q_f | T_1 | q_i\rangle, \quad (21)$$

and similarly in second order[7]

$$\langle q_{n''}' p_\Lambda | T_2 | k_\Sigma h_n^1\rangle = (2\pi)^3\delta(q_{n''}'+p_\Lambda-k_\Sigma-h_n^1)\langle h_f | T_2 | h_I\rangle, \quad (22)$$

$$\langle k_\Sigma p_\pi | T_1 | p_K' p_n^1\rangle = (2\pi)^3\delta(k_\Sigma+p_\pi-p_K'-p_n^1)\langle j_I | T_1 | j_i\rangle,$$

where q_f, q_i, h_f, h_I, j_I, and j_i are the relative momenta in the two-body states. For sake of generality, we merely list q_f with the understanding that the remaining momenta are similarly defined:

$$q_f=(m_\pi p_\Lambda-m_\Lambda p_\pi)/(m_\Lambda+m_\pi). \quad (23)$$

Using the operator identity

$$\lim_{\epsilon\to 0}(E-E_I\pm i\epsilon)^{-1}=P(E-E_I)^{-1}\mp i\pi\delta(E-E_I), \quad (24)$$

where P implies the principal value in any integrations over E_I, approximating $p_{3''}=k_I$ in the wave functions $\chi_{n''-R}$, $\chi_{n'-R}$, while at the same time neglecting the $q_{n''}'$ dependence of $\langle h_f | T_2 | h_I\rangle$ as well as the principle-value integral, we obtain for K^- capture at rest, i.e., $P_i=0$, in first order,

$$\mathbf{M}_1=g_{34}(2\pi)^3\delta(p_\Lambda+p_\pi+p_{3''})O_{K-4}\phi_{n-3}(p_{3''})\langle q_f | T_1 | q_i\rangle, \quad (25)$$

where, for capture from the $2P$ atomic state, $e_i\cdot\nabla\delta(p_K')$

TABLE I. First- and second-order capture rates.

Mode	Form factor	Γ_1 (sec^{-1})	Γ_2 (sec^{-1})
$1S$	Hulthén	1.1×10^{17}	1.8×10^{17}
$2P$	Hulthén	1.0×10^{13}	1.4×10^{13}
$1S$	Gaussian	1.0×10^{17}	1.5×10^{17}
$2P$	Gaussian	1.1×10^{13}	1.6×10^{13}

[7] The neglect of the $q_{n''}'$ dependence of $\langle h_f | T_2 | h_I\rangle$ anticipates replacing the matrix element with a complex scattering length. Using $k_I=-k_\Sigma-p_\pi=p_{3''}$ amounts to a neglect of the detailed structure of the form factors in the second stage.

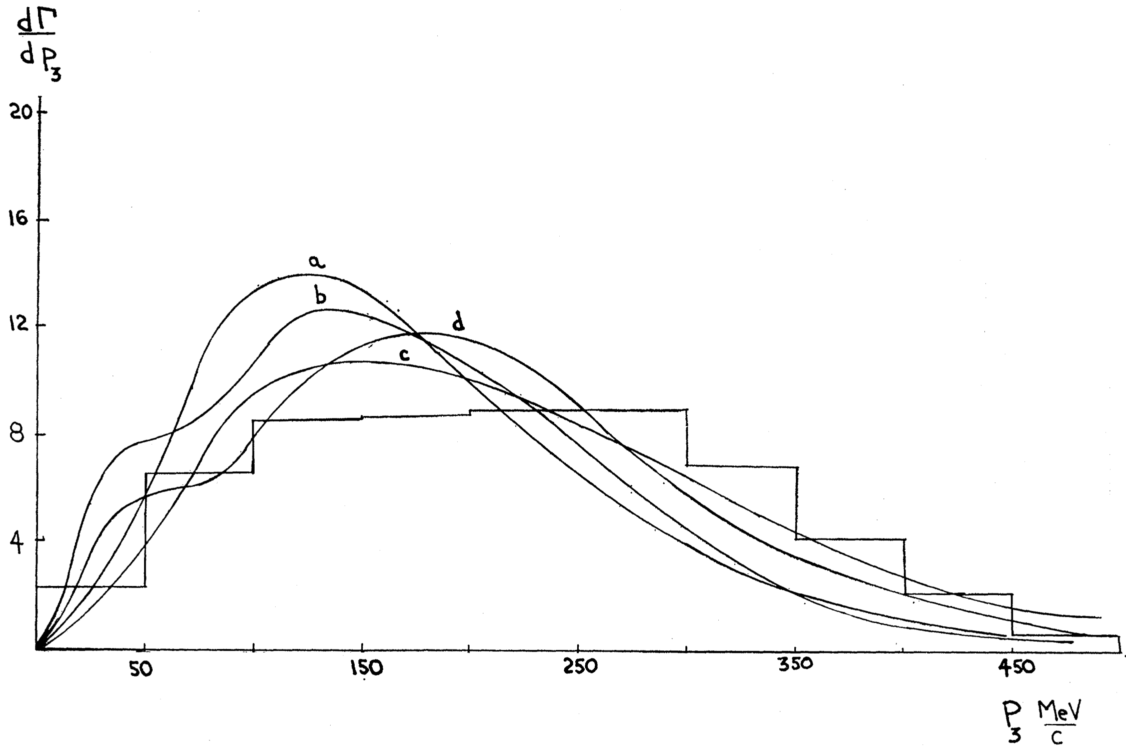

FIG. 4. He^3 momentum distribution (Hulthén form factor). (a) $1S$ capture, no conversion; (b) $2P$ capture, no conversion; (c) $1S$ capture, conversion; (d) $2P$ capture, conversion.

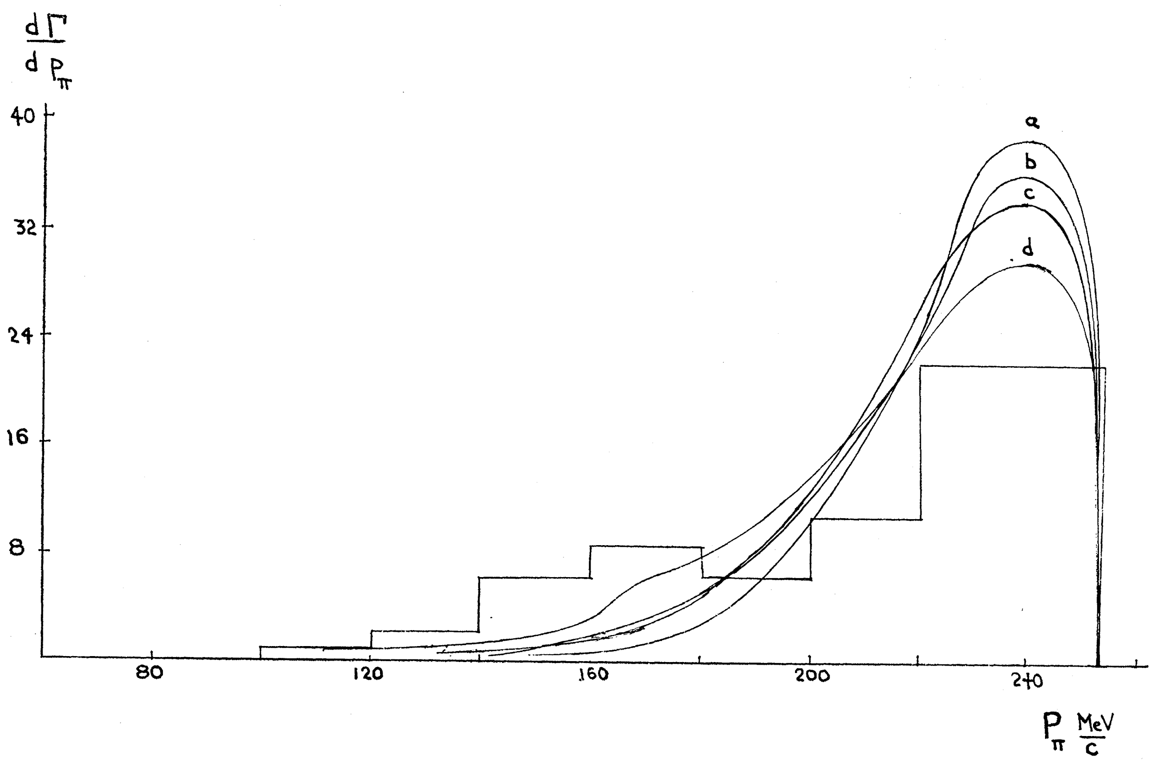

FIG. 5. Pion momentum distribution (Gaussian form factor). (a) $1S$ capture, no conversion; (b) $2P$ capture, no conversion; (c) $1S$ capture, conversion; (d) $2P$ capture, conversion.

operated in the usual sense of integration over a derivative of a δ function so that proper differentiation is implied before substituting $p_K'=0$. Other quantities not defined previously are given by

$$q_i=-[m_K/(m_K+m_n)]p_{3''},$$
$$O_{K-4}=N_{10} \quad (1S \text{ capture}) \tag{26}$$
$$=N_{21}e_i\cdot\nabla_{p_{K'}=0} \quad (2P \text{ capture}).$$

In second order, there similarly results

$$\mathbf{M}_2=-i\pi(2\pi)^3\delta(p_\Lambda+p_\pi+p_{3''})g_{I3}g_{I4}f_{I3} \times\int\frac{d^3k_I}{(2\pi)^3}O_{K-4}\phi_{n-3}(k_I)\delta(E-E_I) \times\langle h_f\mid T_2\mid h_I\rangle\langle j_I\mid T_1\mid j_i\rangle, \tag{27}$$

with relative momenta j_i and j_I given by

$$j_I=-[m_\pi/(m_\pi+m_\Sigma)]k_I-p_\pi, \tag{28}$$
$$j_i=-[m_K/(m_K+m_n)]k_I. \tag{29}$$

In view of the neglect of the $q_{n''}$ dependence of the matrix element, we let $h_f\rightarrow0$ and $h_I\rightarrow0$, in anticipation of using a complex scattering length for $\langle h_f\mid T_2\mid h_I\rangle$. Furthermore, f_{I3} represents the momentum overlap of $\chi_{n''-R}$ and $\chi_{n'-R}$ and is given in Ref. 5. The energy δ function fixes the magnitude of k_I as a function of E and s, for s, the center-of-mass energy of the Σ-π system.[8] For completeness we list the following expressions:

$$E_i=m_K+m_n=E=E_\pi+m_3 +(p_3^2/2m_3)+[p_3^2/2(m_\Lambda+m_\pi)]+t, \tag{30}$$
$$E_I=E_\pi+m_I+(k_I^2/2m_I)+[k_I^2/2(m_\Sigma+m_\pi)]+s, \tag{31}$$
$$|j_I|\,|j_i|\cos\theta=[m_K/(m_K+m_n)]k_Ip_\pi\cos\beta +[m_Km_\pi/(m_K+m_n)(m_\Sigma+m_\pi)]k_I^2, \tag{32}$$

where θ is the angle between j_I and j_i, t is the center-of-mass energy of the Λ-π final system, and β is the angle between k_I and p_π.

At this point, it is advantageous to introduce spin and isospin into the state vectors via the following notation and comments. The amplitudes $\mathbf{M}_1$ and $\mathbf{M}_2$ are generalized to include spin and isospin by forming the usual product representation of spin, isospin, and the dynamical particle space, so that

$$\mathbf{A}_1=\langle C_f\mid\mathbf{M}_1\mid C_i\rangle,$$
$$\mathbf{A}_2=\langle C_f\mid\mathbf{M}_2\mid C_i\rangle \sim\langle C_fh_f\mid T_2\mid h_IC_I\rangle\langle C_Ij_I\mid T_1\mid j_iC_i\rangle, \tag{33}$$

where C_i, C_I, and C_f refer to the initial, intermediate, and final spin and isospin states. Because the T operators are diagonal in the spin and isospin of spectators, Kronecker δ's are implicitly understood in Eqs. (33) for spectators in all stages of scattering. Appendix B gives explicit representation to Eqs. (33).

Kim[9] has parametrized the effective-range expansion of the T_1 operator with a multichannel fit to experimental K^-P and K^0P interactions. In the operator sense, T_1 is expanded on eigenstates of parity and total angular momentum, $j=l\pm\frac{1}{2}$, with l the orbital angular momentum of the $\bar{K}n$ system, such that

$$T_1=H+i\sigma\cdot(\hat{q}_i\times\hat{q}_f)G, \tag{34}$$

where σ are the Pauli spin operators, $\hat{q}_i$ and $\hat{q}_f$ are unit vectors in the direction of the initial and final channel momenta, $\hat{q}_i\times\hat{q}_f=\sin\theta$, and G and H take the following explicit expansions in Legendre polynomials:

$$H=(4\pi)^{-1}\sum_{l=0}^{\infty}[(l+1)T_{l+1/2}+lT_{l-1/2}]P_l(\cos\theta),$$
$$G=(4\pi)^{-1}\sum_{l=0}^{\infty}[T_{l+1/2}-T_{l-1/2}]P_l'(\cos\theta). \tag{35}$$

The partial-wave operators $T_{l\pm1/2}$ are defined in momentum space in an effective-range expansion with their matrix elements

$$\langle q_f\mid T_{l\pm1/2}\mid q_i\rangle=\langle q_f\mid k^l(M_{l\pm1/2}-ik^{2l+1})^{-1}k^l\mid q_i\rangle, \tag{36}$$

where the explicit range operator is given by[10]

$$M_{l\pm1/2}=M_{l\pm1/2}(E_0)+\tfrac{1}{2}C_lr^{1-2l}(k^2-k_0^2), \tag{37}$$

for r representing the range, E_0 representing the incident channel energy, k representing the channel momentum, k_0 representing the channel momentum corresponding to energy E_0, and $C_0=1$, $C_1=-3$. The $I=1$, $l=1$, $j=\frac{3}{2}$ matrix elements are fitted to include the $Y_1^*(1385)$ scattering resonance (though the coupling to $\bar{K}n$ is small), while the $I=0$, $l=0$, $j=\frac{1}{2}$ elements contain contributions from the virtual bound resonance $Y_0^*(1405)$. The $Y_1^*(1385)$ contributes to both $\mathbf{A}_1$ and $\mathbf{A}_2$ for the process $K^-+He^4\rightarrow\Lambda+\pi^-+He^3$, while the $Y_0^*(1405)$ only contributes to $\mathbf{A}_2$ because of isospin conservation.

Turning our attention to the stage $\Sigma n'\rightarrow\Lambda n''$, Neville has examined the hyperon-nucleon interaction in the reactions

$$\Sigma^-+d\rightarrow\Lambda+N+P$$
$$\rightarrow\Sigma+N+P \tag{38}$$

for capture at rest. Using the range expansions, Eqs. (36) and (37), in a global-symmetry model (which does not give good agreement with experiment for Σ^- capture in deuterium), Neville obtains for the spin-0 and spin-1 matrix elements at, or near, threshold (in

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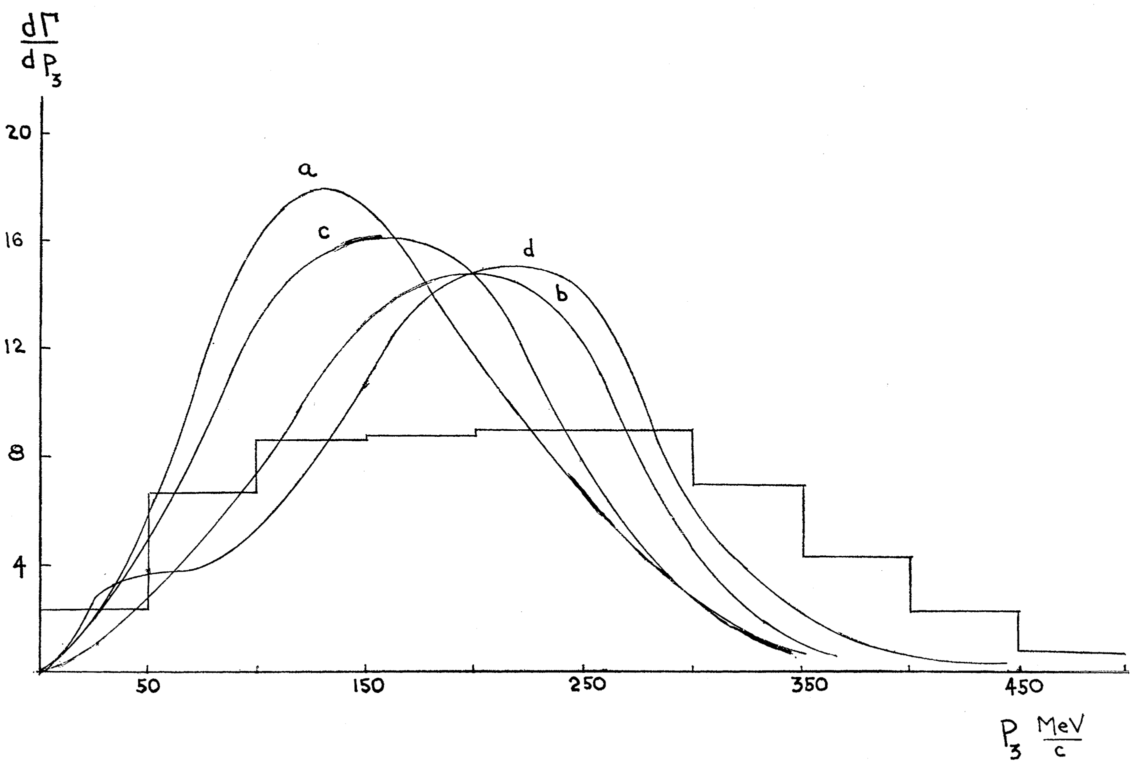
[8] See J. Sawicki, Nuovo Cimento **33**, 361 (1964), for alternative methods of intermediate integration.

[9] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967).

[10] The development of the range expansion is given by M. H. Ross and G. L. Shaw, Ann. Phys. (N.Y.) **13**, 147 (1961). The K-matrix analysis of $\bar{K}n$ interactions is introduced by R. H. Dalitz and S. F. Tuan, *ibid.* **3**, 307 (1960).



FGI. 6. He^3 momentum distribution (Gaussian form factor). (a) $1S$ capture, no conversion; (b) $2P$ capture, no conversion; (c) $1S$ capture, conversion; (d) $2P$ capture, conversion.

units of F),

$$(-i\pi)^{-1}\langle\Lambda n''\mid T_2^{1/2}\mid\Sigma n'\rangle=0.328+i0.534 \qquad (S=1)$$
$$=-0.244-i0.072 \qquad (S=0). \qquad (39)$$

In view of our neglect of spin for this stage, it will suffice to take the spin average of Eq. (39) as a first estimate. In neglecting the detailed structure of $\langle\Lambda n''\mid T_2^{1/2}\mid\Sigma n'\rangle$, we are effectively replacing the conversion process by a simple conversion amplitude. This will suffice, though, to exhibit the effects of the Σ-Λ conversion process on the momentum distributions of the final products.

VI. RESULTS

The rate to all final spin and momentum states of $\Lambda\pi^-He^3$ takes the form

$$\Gamma=(2\pi)^4\int\frac{d^3p_{3''}}{(2\pi)^3}\frac{d^3p_\Lambda}{(2\pi)^3}\frac{d^3p_\pi}{(2\pi)^3}\times\delta(E-E_i)\delta(p_\Lambda+p_\pi+p_{3''})\mathbf{A}^\dagger\mathbf{A}, \qquad (40)$$

where $\mathbf{A}^\dagger\mathbf{A}$ and implicit averages and sums are given in Appendix B. For completeness, we list $\mathbf{A}^\dagger\mathbf{A}$:

$$\mathbf{A}^\dagger\mathbf{A}=\mathrm{tr}(\mathbf{A}_1^\dagger+\mathbf{A}_2^\dagger)(\mathbf{A}_1+\mathbf{A}_2) \qquad (1S)$$
$$=\tfrac{1}{3}\sum_{i=1}^{3}\mathrm{tr}(\mathbf{A}_1^\dagger+\mathbf{A}_2^\dagger)(\mathbf{A}_1+\mathbf{A}_2) \qquad (2P) \qquad (41)$$

for $\mathbf{A}_1$, $\mathbf{A}_2$ the spin-isospin generalizations of amplitudes $\mathbf{M}_1$, $\mathbf{M}_2$, explicitly listed in Eq. (B3) of Appendix B, and e_i given in Eqs. (15). The He^3 and π momentum distributions are obtained by integrating over all variables except He^3, or π momentum. The kinetic energy distributions are obtained in the same fashion after transforming variables. As mentioned in Ref. 8, the integrations implied above are easily done following Sawicki, and we do not repeat them.

In Table I the rates are tabulated for $1S$ and $2P$ capture, in first and second order for both Gaussian and Hulthén form factors.[11]

Though the addition of the Σ-Λ conversion term has little effect on the rate, the effects in the momentum distributions are more noticeable. Figures 3 and 4 give the final pion and He^3 distributions for the Hulthén form factor, while Figs. 5 and 6 show the corresponding

[11] It should be pointed out that relativistic normalizations were used for actual calculations. We have adopted the normalization used by Dalitz and Tuan (Ref. 10) for the $\bar{K}n$ matrix elements, and that used by Neville (Ref. 1) for the hyperon-nucleon matrix elements. More simply, this implies that an additional factor of $(m/e)^{1/2}$ appears with each fermion and a factor of $(2\pi/\omega)^{1/2}$ with each boson in the $\bar{K}n$ matrix elements of the text, while an additional factor of $(E^{1/2}/e)^{1/2}$ appears for each fermion in the hyperon-nucleon matrix elements of the text. In the above, e and ω are the relativistic particle energies and E is the total energy in the center-of-mass system. For the sake of simplicity, the text uses the normalization $\langle k\mid k'\rangle=(2\pi)3\delta(k-k')$.

distributions for the Gaussian case.[12,13] All curves are normalized to the total area under the histogram. The labels *a* and *b* refer to 1*S* and 2*P* capture in first order, while *c* and *d* refer to 1*S* and 2*P* capture in second order.

Obviously the first-order term, corresponding to directly produced $\Lambda\pi^- He^3$ with a small amount of $Y_1^*(1385)$ production, is incapable of accounting for the momentum tails of both the pion and He^3 momentum distributions. The addition of the Σ-Λ conversion term helps to describe the He^3 and pion distributions to a better degree. Though some semblance of the Σ-Λ conversion bump (at 170 MeV, approximately) appears in the pion distribution with Gaussian form factor, both the absolute size of the hyperon-nucleon scattering length (which effectively weights the second-order term with reference to the first-order term) and the form factor (which falls off rapidly with increasing He^3 momentum) tend to negate its contribution when the second-order and first-order terms are added. In view of the approximation of the second-stage process, the results obtained in second order are noticeably improved over those obtained in first order. The theoretical distributions do not possess enough dissimilarity to allow determination of the capture orbit.

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APPENDIX A

Analogously to Eq. (10), we Fourier-analyze the amplitude in the following fashion (suppressing the decomposition of plane waves with its ultimate integration over δ-function momentum states $p_\pi p_\Lambda$ and $k_\pi k_\Sigma$):

$$\langle p_\pi p_\Lambda He^3 \mid T_2 \mid k_\pi k_\Sigma I\rangle = \int d\epsilon' d\epsilon'' \frac{d^6q'}{(2\pi)^6}\frac{d^6h'}{(2\pi)^6}\langle He^3 \mid q_{n''}' q_R' \epsilon''\rangle \times \langle q_{n''}' q_R' p_\pi p_\Lambda \epsilon'' \mid T_2 \mid \epsilon' k_\pi k_\Sigma h_{n'}' h_R'\rangle \langle h_{n'}' h_R' \epsilon' \mid I\rangle,$$

$$\langle k_\Sigma k_\pi I \mid T_1 \mid K^- He^4\rangle = \int d\zeta' d\zeta \frac{d^9p'}{(2\pi)^9}\frac{d^3h_I'}{(2\pi)^3}\langle I \mid \zeta' h_I'\rangle \times \langle h_I' k_\Sigma k_\pi \zeta' \mid T_1 \mid \zeta p_K' p_n' p_3'\rangle \langle p_K' p_n' p_3' \zeta \mid K^- He^4\rangle, \quad \text{(A1)}$$

with factored state vectors

$$\langle He^3 \mid q_{n''}' q_R' \epsilon''\rangle = (2\pi)^3 \delta(p_{3''} - q_{n''}' - q_R') \times \chi^*_{n''-R}[(m_n q_R' - m_R q_{n''}')/m_3]\psi_3^*(x_3''),$$

$$\langle h_{n'}' h_R' \epsilon' \mid I\rangle = (2\pi)^3 \delta(k_I - h_{n'}' - h_R') \times \chi_{n'-R}[(m_n h_R' - m_R h_{n'}')/m_I]\psi_I(x_3'),$$

$$\langle I \mid \zeta' h_I'\rangle = (2\pi)^3 \delta(k_I - h_I')\psi_I(x_3', x_2'), \quad \text{(A2)}$$

$$\langle p_K' p_n' p_3' \zeta \mid He^4\rangle = (2\pi)^3 \delta(P_i - p_K' - p_n' - p_3') \times \psi_{K-4}[(m_4 p_K' - m_K p_n' - m_K p_3')/(m_4 + m_K)] \times \phi_{n-3}[(m_3 p_n' - m_n p_3')/m_4]\psi_4(x_3, x_2).$$

Again, $\chi_{n''-R}$, $\chi_{n'-R}$ represent the Fourier transforms of the x_2'', x_2' coordinates of ψ_3, ψ_I, where ψ_I takes the Gaussian form

$$\psi_I(x_2, x_3) = N_I \exp[-\tfrac{1}{2}\gamma^2(\tfrac{2}{3}x_2^2 + \tfrac{1}{2}x_3^2)] \quad \text{(A3)}$$

and $\chi_{n''-R}$, $\chi_{n'-R}$ are explicitly given for $S = n'' - R$, $n' - R$:

$$\chi_S(q) = N_S \exp(-3q^2/4\gamma^2). \quad \text{(A4)}$$

In the impulse model for $K^- + n \to \Sigma + \pi$, followed by $\Sigma + n' \to \Lambda + n''$,

$$\langle q_{n''}' q_R' p_\pi p_\Lambda \epsilon'' \mid T_2 \mid \epsilon' k_\pi k_\Sigma h_{n'}' h_R'\rangle = (2\pi)^3 \delta(k_\pi - p_\pi)\delta(q_R' - h_R')\delta(\epsilon'' - \epsilon') \times \langle q_{n''}' p_\Lambda \mid T_2 \mid k_\Sigma k_{n'}'\rangle,$$

$$\langle h_I' k_\Sigma k_\pi \zeta' \mid T_1 \mid \zeta p_K' p_n' p_3'\rangle = (2\pi)^3 \delta(h_I' - p_3')\delta(\zeta' - \zeta)\langle k_\Sigma k_\pi \mid T_1 \mid p_K' p_n'\rangle, \quad \text{(A5)}$$

so that substituting Eqs. (A2) and (A5) into Eq. (A1) yields

$$\langle p_\pi p_\Lambda He^3 \mid T_2 \mid k_\pi k_\Sigma I\rangle = g_{I3} \int \frac{d^3 q_{n''}'}{(2\pi)^3} \chi^*_{n''-R}\left(\frac{m_n}{m_3} p_{3''} - q_{n''}'\right) \times \chi_{n'-R}\left(p_3'' - q_{n''}' - \frac{m_R}{m_I} k_I\right)\langle q_{n''}' p_\Lambda \mid T_2 \mid k_\Sigma h_n^1\rangle,$$

$$\langle k_\Sigma k_\pi I \mid T_1 \mid K^- He^4\rangle = g_{I4} \int \frac{d^3 p_K'}{(2\pi)^3} \psi_{K-4}\left(p_K' - \frac{m_n}{m_4 + m_K} P_i\right) \times \phi_{n-3}\left(k_I - \frac{m_3}{m_4} p_K' - \frac{m_3}{m_4} P_i\right)\langle k_\Sigma k_\pi \mid T_1 \mid p_K' p_n^1\rangle, \quad \text{(A6)}$$

[12] The experimental histograms are given in P. Said and J. Sawicki, Phys. Rev. **139**, B991 (1965). They represent the weighted sum of the data of the Helium Bubble Chamber Collaboration Group, Nuovo Cimento **20**, 724 (1961); J. Auman *et al.*, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 330; cf. also *Proceedings of the Tenth Annual International Conference on High-Energy Physics, Rochester, 1960*, edited by E. C. G. Sudarshan, J. J. Tinlot, and A. C. Melissinos (Interscience, New York, 1960), p. 426. The histograms and theoretical curves are normalized to the same, but arbitrary area.

[13] Virtual processes in any given channel are included in the calculation by following the usual prescription of replacing the channel center-of-mass momentum *k* with *ik* for energies below threshold in that channel (see Dalitz and Tuan, Ref. 10).

with $(2\pi)^3\delta(p_{3''}+p_\pi+p_\Lambda)$ suppressed in the above, and with the definitions

$$p_n{}^1=P_i-p_K{}'-k_I,\qquad h_n{}^1=k_I-p_{3''}+q_{n''}{}',\quad \text{(A7)}$$

and g_{I3}, g_{I4} the spatial overlaps of ψ_I and ψ_3, and ψ_I and ψ_4.

APPENDIX B

The scattering $K^-+\mathrm{He}^4\to\Lambda+\pi^-+\mathrm{He}^3$ can, through second order, take place in any of the following ways. Letting N and P designate knock-on neutrons and protons, respectively, we may have

$$K^-N+\mathrm{He}^3 \begin{cases}\nearrow \Lambda\pi^-\mathrm{He}^3\\ \searrow \Sigma^0\pi^-\mathrm{He}^3\to\Lambda\pi^-\mathrm{He}^3,\end{cases}\quad \text{(B1)}$$
$$K^-P+\mathrm{H}^3\to\Sigma^+\pi^-\mathrm{H}^3\to\Lambda\pi^-\mathrm{He}^3.$$

Other possible intermediate charge states occurring to the reader are ruled out by the impulse approximation and the resulting diagonality of the T operators with respect to the spectators. For the initial He^4 nucleus a $4\otimes4$ Slater determinant in spin$\otimes$isospin space is written, while the intermediate three-nucleon state and final He^3 are $3\otimes3$ Slater determinants. This insures proper antisymmetrization of the nucleon spin$\otimes$isospin wave functions. Specifically, we write

$$|\,\mathrm{He}^4\rangle=(4!)^{-1/2}\begin{vmatrix}P_1{}^+ & P_2{}^+ & P_3{}^+ & P_4{}^+\\ P_1{}^- & P_2{}^- & P_3{}^- & P_4{}^-\\ N_1{}^+ & N_2{}^+ & N_3{}^+ & N_4{}^+\\ N_1{}^- & N_2{}^- & N_3{}^- & N_4{}^-\end{vmatrix},\quad \text{(B2)}$$

which, in terms of similarly defined $3\otimes3$ Slater determinants representing He^3 and H^3, can be cast into the form

$$|\,\mathrm{He}^4\rangle=\tfrac{1}{2}(|\,P^+\mathrm{H}^-\rangle-|\,P^-\mathrm{H}^+\rangle+|\,N^+\mathrm{He}^-\rangle+|\,N^-\mathrm{He}^+\rangle),\quad \text{(B3)}$$

which $\pm$ refer to the spin states of the particles, i.e., $\pm\frac{1}{2}$, and H and He refer to the H^3 and He^3 Slater determinants. The processes $K^-n\to\Lambda\pi^-$ or $K^-n\to\Sigma\pi^-$ are parametrized by $I=0$, 1 isospin amplitudes, while $\Sigma n'\to\Lambda n''$ could be parametrized by an $I=\frac{1}{2}$ amplitude. Using Eq. (B3), coupling K^-N, K^-P, $\Sigma^0\pi^-$, and $\Sigma^+\pi^-$ through the usual Clebsch-Gordan expansions into $I=0$, 1 channels, while neglecting any spin or isospin dependence in the second-stage $\Sigma n'\to\Lambda n''$ by writing [see Eq. (33)]

$$\langle C_fh_f\,|\,T_2\,|\,h_IC_I\rangle=\delta_{s_\Lambda s_\Sigma}\delta_{s_{3'}s_{3''}}\langle h_f\,|\,T_2{}^{1/2}\,|\,h_I\rangle,\quad \text{(B4)}$$

where s_i refers to the spin of the ith particle, $3''$ and $3'$ refer to the final He^3 and intermediate He^3 or H^3, respectively, and $T_2{}^{1/2}$ refers to the $I=\frac{1}{2}$ isospin amplitude, we obtain with some algebra (again suppressing the δ function)

$$\begin{aligned}\mathbf{A}_1&=\tfrac{1}{2}g_{34}O_{K-4}\phi_{n-3}(p_{3''})\sum_{s_n}\langle s_\Lambda q_f\,|\,T_1{}^1\,|\,q_is_n\rangle Q,\\ \mathbf{A}_2&=-\tfrac{1}{2}i\pi g_{I3}g_{I4}f_{I3}O_{K-4}\\ &\quad\times\int\frac{d^3k_I}{(2\pi)^3}\delta(E-E_I)\phi_{n-3}(k_I)\sum_{s_n}\langle h_f\,|\,T_2{}^{1/2}\,|\,h_I\rangle\\ &\quad\times\{[(1+\sqrt{2})/\sqrt{2}]\langle s_\Lambda j_I\,|\,T_1{}^1\,|\,j_is_n\rangle\\ &\qquad-\sqrt{3}^{-1}\langle s_\Lambda j_I\,|\,T_1{}^0\,|\,j_is_n\rangle\}Q,\\ Q&=\delta_{s_{3''}-\frac{1}{2}}\delta_{s_n\frac{1}{2}}-\delta_{s_{3''}\frac{1}{2}}\delta_{s_n-\frac{1}{2}}.\end{aligned}\quad \text{(B5)}$$

In the above, n represents the initial knock-on nucleon, and $T_1{}^0$ and $T_1{}^1$ are the isoscalar and isovector amplitudes, respectively. The rate to all final states involves a summation over final Λ, He^3 spin states, and for a $2P$ capture, an average over initial K^- orientations, so that

$$\begin{aligned}\mathbf{A}^\dagger\mathbf{A}&=\sum_{s_\Lambda s_{3''}}|\,\mathbf{A}_1+\mathbf{A}_2\,|^2 &&(1S)\\ &=\tfrac{1}{3}\sum_{ei}\sum_{s_\Lambda s_{3''}}|\,\mathbf{A}_1+\mathbf{A}_2\,|^2 &&(2P),\end{aligned}\quad \text{(B6)}$$

which, upon performing the summation over $s_{3''}$, becomes

$$\begin{aligned}\mathbf{A}^\dagger\mathbf{A}&=\sum_{s_\Lambda s_n}|\,\mathbf{A}_1+\mathbf{A}_2\,|^2 &&(1S)\\ &=\tfrac{1}{3}\sum_{ei}\sum_{s_\Lambda s_n}|\,\mathbf{A}_1+\mathbf{A}_2\,|^2 &&(2P).\end{aligned}\quad \text{(B7)}$$