Measurement of p-p and p-d Total Cross Sections at 3.00 GeV/c*

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In a new measurement of proton total cross sections at 3.00 GeV/c, the p-d total cross section is found to be lower than a previous measurement by 1.17±0.09 mb. This implies a corresponding new value for the total cross section for $I=0$ which is 2.18±0.27 mb lower than the previous value. Possible sources of systematic error are discussed.

Several high-precision total cross-section measurements using hydrogen and deuterium targets have been reported recently.[1–8] The experiments were standard good-geometry transmission experiments in which conventional circular transmission counters were used, except for Foley *et al.*,[6] who employed a system of hodoscopes, and Bellettini *et al.*,[7] who employed spark chambers. The measurements usually exhibit very closely the same behavior versus energy but often disagree on the absolute scale. The discrepancies are particularly noticeable in the deuterium measurements. With present techniques, nonstatistical point-to-point errors, i.e., systematic and random errors that may vary significantly over a typical resonance width, may be comparable to the statistical error, which is typically ±0.1%. However, the systematic uncertainties on the absolute scale of the total cross sections are normally considerably larger.

We report a new measurement of proton-nucleon total cross sections, a comparison with previous data,[1,2,9] and a detailed analysis of the most important contributions to the systematic error. Our measured values of the p-p and p-d total cross sections at 3.00 GeV/c are 44.33±0.06 and 81.78±0.07 mb, respectively. These values are to be compared with 44.47±0.04 and 82.95±0.05 from the Cambridge-Rutherford (C-R) group.[9] While the p-p cross sections agree closely, the p-d cross sections disagree by 1.17±0.09 mb, or 1.4%. All stated errors are statistical standard deviations. A similar systematic discrepancy in $\sigma_T(p\text{-}d)$ is found in a comparison between the results of Refs. 1 and 2 in the region between 6 and 8 GeV/c.

The measurement was made during an experiment on $K^{\pm}$-nucleon and $\bar{p}$-nucleon total cross sections described in Ref. 4; the experimental arrangement is discussed in detail in Ref. 3. The partially separated beam at the alternating gradient synchrotron (AGS) was tuned to protons which were identified by a liquid Čerenkov counter. The transmission of the protons was measured alternately through a 3-ft-long liquid-hydrogen, liquid-deuterium, or dummy target by a series of circular scintillation counters subtending different solid angles at the center of the target. The partial cross sections obtained by each of the transmission counters, after being corrected for various small effects discussed below, are shown in Fig. 1, plotted as a function of $-t$, the negative of the square of the four-momentum transferred to the target nucleon.

A least-squares fit to the partial cross sections with a polynomial of the form $\sigma=A+Bt+Ct^2$ yields the solid curves shown in Fig. 1. The coefficients from the p-p fit are $A=44.32\pm0.06$ mb, $B=224\pm6$ mb (GeV/c)$^{-2}$, and $C=720\pm100$ mb (GeV/c)$^{-4}$. Elimination of the smallest transmission counter (S_9) from the fit does not change the values of the coefficients within the errors. This reaffirms confidence in the extrapolation procedure; it also indicates that the extrapolation procedure is not affected by multiple scattering and beam size effects. When a smaller counter [S_{10}, subtending a solid angle with $-t\leq0.005$ (GeV/c)^2] is included in the fit, both the p-p and the p-d extrapolated cross sections rise by about 0.5%. Although multiple scattering and beam size effects may be contributing to this rise, we have taken ±0.5% to be a conservative estimate of the over-all systematic uncertainty in the extrapolation procedure (see Table I).

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[1] D. V. Bugg, D. C. Salter, G. H. Stafford, R. F. George, K. F. Riley, and R. J. Tapper, Phys. Rev. **146**, 980 (1966).
[2] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).
[3] R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters **16**, 1228 (1966); **17**, 102 (1966); R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, J. Teiger, and C. Wilkin, Phys. Rev. D **1**, 1887 (1970); see this paper for further experimental details.
[4] R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, and D. N. Michael, Phys. Rev. Letters **18**, 1209 (1967); **19**, 259 (1967); **19**, 678 (1967); Phys. Rev. D **1**, 1917 (1970); see this paper together with last paper mentioned in Ref. 3 for further experimental details.
[5] A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. Rousset, and P. H. Sharp, Phys. Rev. **144**, 1101 (1966).
[6] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters **19**, 330 (1967); **19**, 857 (1967).
[7] G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethun, G. Matthiae, J. P. Scanlon, and A. M. Wetherell, Nucl. Phys. **79**, 609 (1966).
[8] A. A. Carter, K. F. Riley, R. J. Tapper, D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Salter, G. H. Stafford, E. J. N. Wilson, J. D. Davies, J. D. Dowell, P. M. Hattersley, J. J. Homer, and A. W. O'Dell, Phys. Rev. **168**, 1466 (1968).
[9] K. F. Riley, following paper, Phys. Rev. D **1**, 2481 (1970).

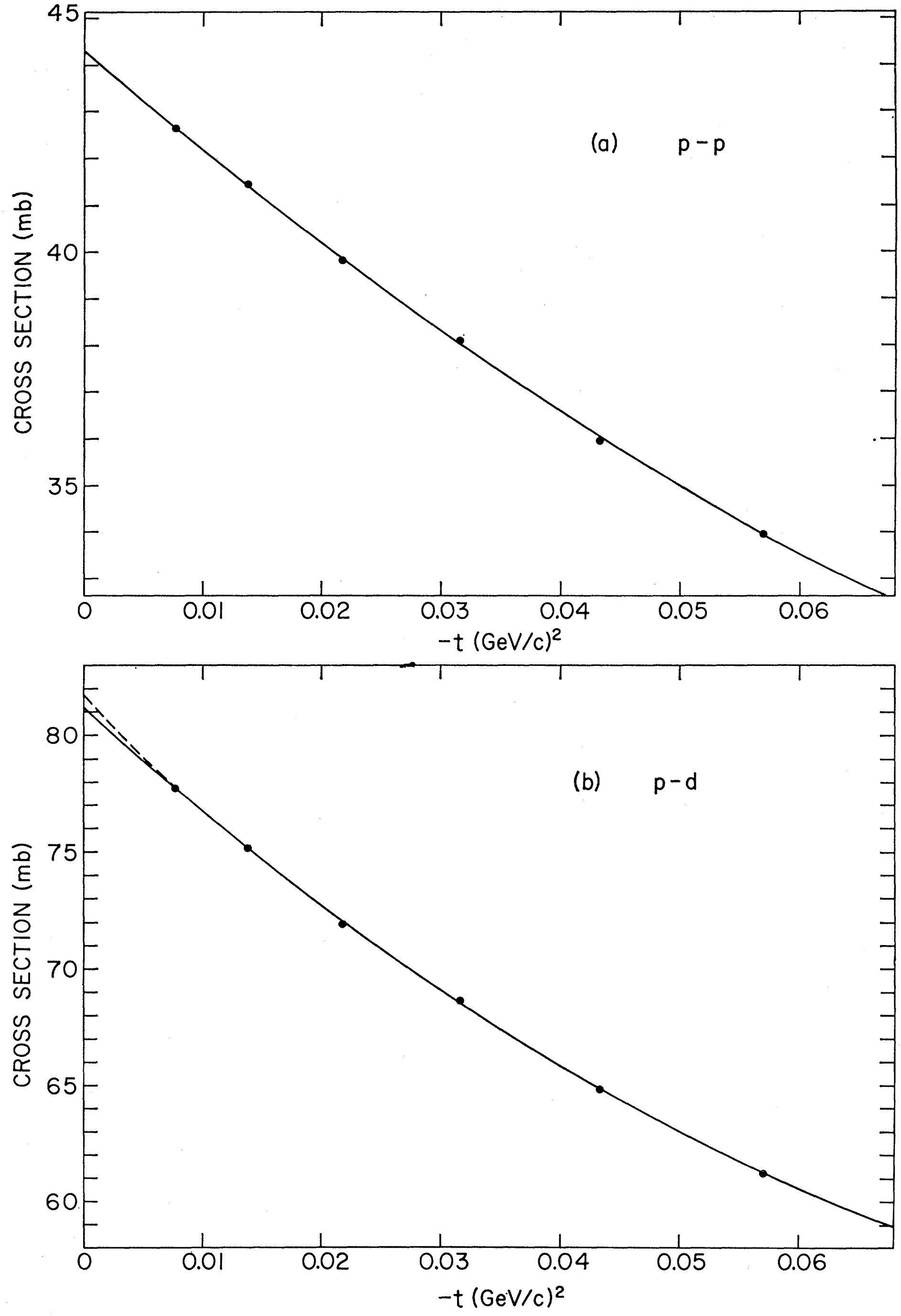

FIG. 1. Extrapolation to zero solid angle for *p-p* and *p-d* at 3.0 GeV/*c*. The points are the corrected experimental partial cross sections σ_i. The solid lines are the results of the least-squares fit to a quadratic curve in *t*. *t* is computed for scattering from free nucleons. In the *p-d* graph the dashed line shows an exponential extrapolation which includes the effect of coherent scattering.

The coefficients B and C can be related in the following way to the parameters deduced from elastic scattering. One may write the differential cross section for having a charged secondary in the forward direction as the sum of elastic and inelastic contributions:

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_{\rm el}+\left(\frac{d\sigma}{dt}\right)_{\rm in}. \tag{1}$$

The elastic contribution at small t values is known to have an exponential form as ae^{bt}. Assuming the same form for the inelastic contribution, we obtain

$$d\sigma/dt = ae^{bt} + ce^{et} \simeq a(1+bt) + c(1+et). \quad (2)$$

The partial cross section σ_i measured by a counter of finite size is then given by

$$\sigma = \sigma_T - \int_{t_i}^{0} \frac{d\sigma}{dt} dt \simeq \sigma_T + (a+c)t + \tfrac{1}{2}a\left(b + \frac{ce}{a}\right)t^2. \quad (3)$$

From the p-p total cross sections, using the optical theorem and dispersion relations, one infers the value $a = 128$ mb $(\mathrm{GeV}/c)^{-2}$.[10] Since $B = a + c = 224 \pm 6$ mb, there is an important contribution from inelastic processes. A value of $b = 5$ $(\mathrm{GeV}/c)^{-2}$ is given by elastic scattering data measured in the range $-0.5 < t < -0.1$ $(\mathrm{GeV}/c)^2$.[10] From C and a, we compute $b + ce/a = 11.2 \pm 0.8$ $(\mathrm{GeV}/c)^{-2}$, which again indicates the importance of inelastic effects.

Before proceeding with the above extrapolations, it was necessary to apply the following corrections to the partial cross sections:

(a) *Single Coulomb scattering and Coulomb-nuclear interference.* These were computed in Ref. 3. We have used $e^{2.3t}$ and $(0.47e^{3.7t} + 0.53e^{24.6t})$ as approximations for the proton[11] and deuteron[12] form factors, respectively, $\rho_p = -0.30$ and $\rho_n = -0.51$ for the ratio of real to imaginary part of the scattering amplitude at $t = 0$ for p-p and p-n, respectively.[1] For the angular dependence of the scattering amplitudes we have taken as slopes of p-p and p-n exponentials $b_p = b_n = 5$ $(\mathrm{GeV}/c)^{-2}$.[10] The single Coulomb scattering correction is -0.05 mb; the Coulomb-nuclear interference correction amounts to -0.34 mb for p-p and -0.65 mb for p-d.

(b) *HD contamination.* Mass-spectrograph analysis of the deuterium used in the experiment showed only 0.6 mole % of HD,[13] resulting in a $+0.15\%$ correction to the deuterium total cross section. Contamination from other substances was negligible.

(c) *Target length.* The target vessel is made of an aluminum cylinder with 0.36-mm Mylar walls at both ends.[3] Because of the inner pressure, the Mylar walls bow by about 1 cm. The lengths of the targets were measured to about ± 0.5 mm when warm. The uncertainties in the correction required to go to liquid-hydrogen temperature raise the error to ± 2 mm.[14]

(d) *Dummy target.* The dummy target was not exactly identical to the empty hydrogen and deuterium targets.[4]

[10] M. N. Focacci and G. Giacomelli, CERN Report No. CERN 66-18, 1966 (unpublished).

[11] R. Hofstadter, F. Bumiller, and M. Croissiaux, Phys. Rev. Letters **5**, 263 (1960).

[12] J. A. McIntyre and G. R. Burleson, Phys. Rev. **112**, 2077 (1958).

[13] R. Gibbs, Cryogenic Group of Brookhaven National Laboratory (private communication).

[14] H. M. Roder, D. E. Miller, L. A. Weber, and R. D. Goodwin, Cryogenics **3**, 16 (1963).

TABLE I. Estimate of the percentage errors for the largest systematic uncertainties in the p-p and p-d total cross sections. We have attempted to give the meaning of standard deviations to these errors. Most other uncertainties are less than $\pm 0.1\%$ as stated in the text.

	In hydrogen %	In deuterium %
Extrapolation procedure	±0.50	±0.50
Coulomb-nuclear interference	±0.30	±0.30
Target density	±0.04	±0.25
Target length	±0.20	±0.20
Target contamination	···	±0.10
Uncertainty in dummy target correction	±0.25	±0.30

The correction for this effect was measured by comparing the dummy target rate with the empty target rates at 1.9 GeV/c with an incident K^+ beam and at 2.5 GeV/c with an incident π^- beam. These corrections were then scaled according to the behavior of the carbon total cross section to give the dummy corrections for the 3.0-GeV/c incident proton beam: $+0.14 \pm 0.10$ mb for the hydrogen target, and -0.11 ± 0.24 mb for the deuterium target. The quoted systematic errors in the corrections were taken from the amount of inconsistency between the 1.9- and the 2.5-GeV/c measurements.

The quadratic approximations (2) and (3) were not really adequate in our t range for large values of b such as, for example, coherent deuteron scattering. A correction for this effect of $+0.51$ mb [the dashed curve of Fig. 1(b)] was applied to $\sigma_T(p\text{-}d)$ and $+0.01$ mb to $\sigma_T(p\text{-}p)$ as computed by Riley.[9]

Table I lists what should be conservative estimates of the systematic errors which have been discussed above. Other errors are due to contributions from a number of effects, like multiple scattering, beam size, efficiencies of transmission counters, Čerenkov counts in light guides, randoms and various electronic effects, spread in solid angle due to finite target length, beam momentum resolution, and the result of any systematic uncertainty in beam momentum. Each of these effects should contribute a scale error of less than $\pm 0.1\%$. The total systematic error in this experiment is therefore estimated to be $\pm 0.7\%$ for hydrogen and $\pm 0.8\%$ for deuterium if the errors are added in quadrature. The quoted systematic error of the C-R experiment was $\pm 0.3\%$.[1] Since the C-R and the present experiment were very similar with regard to both experimental equipment and data reduction, the systematic error in comparing one with the other should be less than $\pm 0.5\%$. The existing discrepancy between the two measurements of $\sigma_T(p\text{-}d)$ at 3.00 GeV/c is 1.4%. This discrepancy may simply indicate that systematic errors were underestimated.

As pointed out by Riley,[9] a significant error can be made in the determination of the deuterium density, hence in $\sigma_T(p\text{-}d)$. The hydrogen and deuterium targets used in the two experiments were in thermal equilibrium with the surrounding liquid-hydrogen jackets. One would expect that the temperature of the inner target would be determined by that of the jacket. This proved to be the case for the hydrogen target. Instead, in both experiments the observed deuterium vapor pressure did not correspond to the temperature of the jacket as measured by the hydrogen vapor pressure, if the Tapper[15] tabulation for deuterium is used. We believe that the temperature of the deuterium was equal to that of the jacket, and we therefore determined the deuterium density using the hydrogen temperature, as determined by its vapor pressure. The C-R group used instead the measured deuterium vapor pressure to obtain their density.[1] This difference in procedure resulted in a 0.7% higher cross section by the C-R group.

A new value for the pure isospin $I=0$ cross section σ_0 at 3.00 GeV/c can be computed by first scaling the $\sigma_T(p\text{-}p)$ and $\sigma_T(p\text{-}d)$ of Ref. 1 by fixed factors to be consistent with the data presented here and then using the standard procedure of Ref. 3. The value obtained for σ_0 is 38.57±0.23 mb, if one chooses for the average inverse square separation of the nucleon in the deuteron $\langle r^{-2}\rangle=0.0311$ mb^{-1} [1]; this value of σ_0 is to be compared with 40.75±0.15 mb from the C-R group.[16] The quoted errors are statistical only. By choosing $\langle r^{-2}\rangle=0.0327$ mb^{-1},[3] our best value for σ_0 becomes 39.01±0.23 mb. This variation in σ_0 with $\langle r^{-2}\rangle$ is somewhat larger than the statistical error. We wish to emphasize that the 1.17±0.09 mb difference in the deuteron total cross sections between our value and that of Ref. 9 has an important consequence. Namely, the cross section in the pure $I=0$ state which is deduced from $\sigma_T(p\text{-}p)$ and $\sigma_T(p\text{-}d)$ differs by an even larger fractional amount. The discrepancy in σ_0 at 3.00 GeV/c is 2.18±0.27 mb, i.e., 5.6%.

[15] R. J. Tapper, NIRL/R/95 (unpublished).

[16] A value of 41.88±0.15 mb for σ_0 was quoted in Ref. 1, where $\sigma_T(p\text{-}d)=83.45\pm0.05$ mb. The value of $\sigma_0=40.75\pm0.15$ mb given above was recomputed using $\sigma_T(p\text{-}d)=82.95\pm0.05$ mb from Ref. 9.

Using the values of $\sigma_1=\sigma_T(p\text{-}p)$ and σ_0 from the present experiment, we obtain a value of 41.67±0.09 mb for $\sigma_T(p\text{-}n)$. This is in agreement with the measurements of $\sigma_T(n\text{-}p)$ of Palevsky *et al.*,[17] who obtained 40.3±1.4 mb at 3.00 GeV/c. Values for σ_0 and σ_1 from Ref. 1 give 43.17±0.08 mb for $\sigma_T(p\text{-}n)$, which is somewhat in disagreement with Ref. 17. If one recomputes the value of σ_0 [16] using the value of $\sigma_T(p\text{-}d)=82.95\pm0.05$ mb from Riley[9] (which was obtained using the jacket temperature for determining the deuterium density), one then obtains $\sigma_T(p\text{-}n)=42.61\pm0.08$ mb, which is closer to the value of Ref. 17. Furthermore, the discrepancy in $\sigma_T(p\text{-}d)$ leads to different conclusions about the screening correction for the deuteron. Kreisler *et al.*,[18] using values for $\sigma_T(p\text{-}p)$ and $\sigma_T(p\text{-}d)$ from Ref. 1 together with $\sigma_T(n\text{-}p)$ from Ref. 17, obtained 1.3±1.4 mb for the screening correction. As a result, they concluded that the data are more consistent with a screening correction which rises with momentum in the region above 3 GeV/c. The screening correction at 3 GeV/c, corresponding to $\sigma_T(p\text{-}p)$, $\sigma_T(p\text{-}d)$, and $\sigma_T(p\text{-}n)$ from the present experiment, is 4.1±1.2 mb, where the quoted uncertainty is obtained from the estimated uncertainty in $\langle r^{-2}\rangle$.[3] This result would indicate instead that the screening correction is approximately constant from 3 to 20 GeV/c. The screening correction for π mesons has also been observed and is consistent with being constant in this energy region.[2,3]

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[17] H. Palevsky, J. L. Friedes, R. J. Sutter, R. E. Chrien, and H. R. Muether, *Proceedings of the International Conference on Nuclear Physics, Paris, 1964*, edited by P. Gugenberger (Editions du Centre National de la Recherche Scientifique, Paris, 1965), p. 162.

[18] M. N. Kreisler, L. W. Jones, M. J. Longo, and J. R. O'Fallon, Phys. Rev. Letters **20**, 468 (1968).