Constraints on $|\eta_{00}|$ from Electromagnetic C Violation*

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Assuming that the hadronic electromagnetic current contains an isoscalar C-even part, we use the soft-pion technique and pole dominance to determine the isospin of the final state in the CP-violating part of the $K^0 \to 2\pi$ amplitude. If the usual dimensional arguments for the order of magnitude of the amplitude hold, as they appear to do for the observed value of η_{+-}, then values of η_{00} close to η_{+-} are excluded.

I. INTRODUCTION

THE suggestion[1] that CP-violating effects in weak decays may be radiative corrections due to a C-violating electromagnetic interaction is appealing by reason of elegance and economy, and because it affords an explanation of the order of magnitude of the effect. Although many tests of this hypothesis have been proposed, and many experiments have been performed, unequivocal confirmation or refutation is still lacking. The observation, at the 3-standard-deviation level, of asymmetries in various η decays[2] and the possible observation[3] of a breakdown of detailed balance in the reaction $\gamma+d \to p+n$ may be taken as evidence of the correctness of the hypothesis; the extremely low upper limit on the electric dipole moment of the neutron,[4] on the other hand, is somewhat disturbing.

In this paper we assume that the hadronic electromagnetic current has a C-even part, which we take to be an isoscalar and to commute with the axial generators of $SU(2)\otimes SU(2)$. These properties are natural if the current in question has a nonvanishing charge. However, the asymmetry observed in $\eta \to 3\pi$ appears to be mainly in the $I=2$ final state. This necessitates the presence of an isovector component of the C-even electromagnetic current, and furthermore indicates that the $\Delta I=0$ C-violating transition is somehow suppressed. The question of the effect of an isovector C-even current is one of the generalizations of this paper that should be investigated.

Because the electromagnetic interaction does not conserve isospin, the hypothesis of electromagnetic C breaking allows for an appreciable $\Delta I=\frac{3}{2}$ component in CP-violating processes. A reliable estimate of this effect allows one to confront the model with experiment. As we shall see, the experimental situation is at present so unclear, particularly in regard to the value of $|\eta_{00}|$, that it is impossible to tell whether the theory fares well or badly in the confrontation. We do find, however, that one of the presently observed values for $|\eta_{00}|$ (3.6×10^{-3}) is preferred over the other (2.2×10^{-3}).

As is well known,[5] the soft-pion technique predicts the $\Delta I=\frac{1}{2}$ rule in nonleptonic, CP-conserving, weak decays. Since the rule is experimentally well satisfied, we are encouraged to believe in the validity of the soft-pion approximation, and propose to use the technique to study CP-violating decays. This has been done by several authors[6] in the context of the Glashow model. For ordinary weak interactions, the validity of the $\Delta I=\frac{1}{2}$ rule, in those transitions for which it is predicted by the soft-pion calculation, is a straightforward consequence of the $SU(2)\otimes SU(2)$ symmetry of the theory in the limit of zero pion mass. The electromagnetic interaction breaks both $SU(2)$ and $SU(2)\otimes SU(2)$ symmetry, and both properties are required if it is to modify the $\Delta I=\frac{1}{2}$ rule in the soft-pion limit. The degree to which it does force a breaking of the $\Delta I=\frac{1}{2}$ rule is, therefore, closely related to its breaking of chiral invariance, which is a dynamical question and cannot be determined on the basis of symmetry considerations alone.

We investigate the degree of chiral symmetry breaking of the amplitudes by inserting single-particle or resonant states between a photon loop and the weak Hamiltonian. Most of these states either do not contribute as a consequence of some selection rule, or are decoupled in the soft-pion limit. This simplifies the analysis, and makes possible an estimate of the exact amount of breaking of the $\Delta I=\frac{1}{2}$ rule.

In Sec. II we set up the formalism and display the results of reducing out the two pions. We do this in more generality than is strictly required, in order to bring out the underlying mathematical structure. In Sec. III we derive the consequence of the pole approximation. In Sec. IV we apply this result to derive certain limits on the observed CP-violating parameters. With certain assumptions concerning the order of magnitude of radiative corrections, we find that $\eta_{00}\gtrsim 2.4\times10^{-3}$. Because of the degree of uncertainty concerning the magnitude of the relevant radiative corrections, we cannot make a stronger statement than that the range

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[1] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[2] M. Gormley, E. Hyman, W. Lee, T. Nash, J. Peoples, C. Schultz, and S. Stein [Phys. Rev. Letters **21**, 402 (1968)] see an asymmetry in $\eta \to 3\pi$ of $(1.5\pm0.5)\%$ in the $I=2$ final state.

[3] K. Goulianos, Bull. Am. Phys. Soc. **14**, 598 (1969); M. Longo, *ibid.* **14**, 598 (1969).

[4] J. K. Baird, P. D. Miller, W. B. Dress, and N. F. Ramsey, Phys. Rev. **179**, 1285 (1969).

[5] M. Suzuki, Phys. Rev. **144**, 1154 (1966).

[6] B. R. Holstein, Phys. Rev. **171**, 1668 (1968); Y. T. Chiu and J. Schechter, *ibid.* **167**, 1345 (1968).

of values for η_{00} near 3.6×10^{-3} is favored by this model over those for which $\eta_{00}\approx\eta_{+-}$. Section V summarizes our assumptions and results. The effects of nonresonant contributions to the amplitudes are studied in the Appendix. Assuming that these contributions exhibit approximate $SU(2)\otimes SU(2)$ invariance, we find that they contribute at most a 30% correction.

II. REDUCTION OF PIONS

We define the operators

$$T_V{}^{abc}\equiv\int d^4x\int d^4y\, e^{iq\cdot y}\times T(J_\mu{}^a(x)J_\mu{}^b(x)V_\nu{}^c(y)K_\lambda(0))D^{\nu\lambda}(y)\,,$$
$$T_A{}^{abc}\equiv\int d^4x\int d^4y\, e^{iq\cdot y}\times T(J_\mu{}^a(x)J_\mu{}^b(x)A_\nu{}^c(y)K_\lambda(0))D^{\nu\lambda}(y)\,,\qquad(2.1)$$

where a, b, and c are $SU(3)$ indices, K_λ is the C-even isoscalar electromagnetic current, and $J_\mu{}^a\equiv V_\mu{}^a+A_\mu{}^a$. We shall be interested in relating the matrix elements $\langle(2\pi)_I|T_{V,A}{}^{abc}|K^0\rangle$, where $(2\pi)_I$ denotes a standing-wave state of two pions with total isospin I, to $\langle 0|T_{V,A}{}^{abc}|K^0\rangle$. The relationship becomes exact only in the limit of zero four-momentum of each pion.

It is convenient to replace the T^{abc} by operators of definite total isospin I_T. We form these operators by combining the two J currents to form a weak-Hamiltonian operator of definite isospin I_W, and then adding the third current, which we shall take to be an isovector. (Experimentally,[2] the isoscalar ordinary electromagnetic current couples only very weakly to the isoscalar C-even current, and we shall ignore it here.) Adopting the notation $M_{V,A}{}^{(2I_W)(2I_T)}$ according to whether the third current is a vector or an axial vector, we define

$$M_V{}^{11}=-T_V{}^{\pi^+K^-\pi^0}+T_V{}^{\pi^0K^-\pi^+}+0+(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^0\bar{K}^0\pi^0}+\sqrt{2}T_V{}^{\pi^-\bar{K}^0\pi^+},\qquad(2.2)$$

$$M_V{}^{31}=T_V{}^{\pi^+K^-\pi^0}-T_V{}^{\pi^0K^-\pi^+}+3(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^+\bar{K}^0\pi^-}+\sqrt{2}T_V{}^{\pi^0\bar{K}^0\pi^0}+(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^-\bar{K}^0\pi^+},\qquad(2.3)$$

$$M_V{}^{13}=T_V{}^{\pi^+K^-\pi^0}+\tfrac{1}{2}T_V{}^{\pi^0K^-\pi^+}+0-(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^0\bar{K}^0\pi^0}+(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^-\bar{K}^0\pi^+},\qquad(2.4)$$

$$M_V{}^{33}=\tfrac{1}{2}T_V{}^{\pi^+K^-\pi^0}-2T_V{}^{\pi^0K^-\pi^+}-3(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^+\bar{K}^0\pi^-}+(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^0\bar{K}^0\pi^0}+\sqrt{2}T_V{}^{\pi^-\bar{K}^0\pi^+},\qquad(2.5)$$

$$M_V{}^{35}=T_V{}^{\pi^+K^-\pi^0}+T_V{}^{\pi^0K^-\pi^+}-(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^+\bar{K}^0\pi^-}+\sqrt{2}T_V{}^{\pi^0\bar{K}^0\pi^0}-(\sqrt{\tfrac{1}{2}})T_V{}^{\pi^-\bar{K}^0\pi^+},\qquad(2.6)$$

and similarly for $V\to A$. If we define the two five-component vectors $B^\pm=(M_{V\pm A}{}^{11},\ M_{V\pm A}{}^{13},\ M_{V\pm A}{}^{31},\ M_{V\pm A}{}^{33},\ M_{V\pm A}{}^{35})$, we may denote the results of the reduction of the pions economically by means of four 5×5 matrices, $N_I{}^\pm$, for $I=0,\,2$:

$$(2\pi)_I|B_i{}^\pm|K^0\rangle\to\sum_{j=1}^{5}(N_I{}^\pm)_{ij}\,0|B_j{}^\pm|K^0\rangle\,,\qquad(2.7)$$

in which the arrow denotes the zero-momentum limit for both pions, and we ignore a common proportionality constant.

With the conventional normalization

$$(\pi\pi)_0\equiv(\sqrt{\tfrac{1}{3}})(\pi^+\pi^-+\pi^0\pi^0+\pi^-\pi^+)\,,$$
$$(\pi\pi)_2\equiv(\sqrt{\tfrac{1}{6}})(\pi^+\pi^--2\pi^0\pi^0+\pi^-\pi^+)\,,\qquad(2.8)$$

the matrices N_I are[7]

$$N_0{}^+=(\sqrt{\tfrac{1}{3}})\begin{pmatrix}\tfrac{3}{4}&0&0&0&0\\0&3\tfrac{3}{4}&0&0&0\\0&0&\tfrac{3}{4}&0&0\\0&0&0&3\tfrac{3}{4}&0\\0&0&0&0&8\tfrac{3}{4}\end{pmatrix},\qquad(2.9)$$

$$N_0{}^-=(\sqrt{\tfrac{1}{3}})\begin{pmatrix}4\tfrac{3}{4}&0&0&0&0\\0&1\tfrac{3}{4}&0&0&0\\0&0&10\tfrac{3}{4}&0&0\\0&0&0&7\tfrac{3}{4}&0\\0&0&0&0&2\tfrac{3}{4}\end{pmatrix},\qquad(2.10)$$

$$N_2{}^+=(\sqrt{\tfrac{1}{6}})\begin{pmatrix}0&0&0&0&0\\0&3&0&0&0\\0&0&0&0&0\\0&0&0&3&0\\0&0&0&0&8\end{pmatrix},\qquad(2.11)$$

$$N_2{}^-=(\sqrt{\tfrac{1}{6}})\begin{pmatrix}0&-4&0&0&0\\-2&-1&0&0&0\\0&0&0&5\tfrac{3}{5}&7\tfrac{1}{5}\\0&0&7&-3\tfrac{2}{5}&-1\tfrac{4}{5}\\0&0&4&-\tfrac{4}{5}&-1\tfrac{3}{5}\end{pmatrix}.\qquad(2.12)$$

We note that (a) all the N's but $N_2{}^-$ are diagonal; (b) $N_0{}^+$ is proportional to $I_T(I_T+1)$; (c) $N_2{}^+$ has zeros for elements corresponding to $I_T=\tfrac{1}{2}$; and (d) $N_2{}^-$ is diagonal in I_W. All of these properties are, of course, consequences of the fact that the reduction of the pions does not destroy the $SU(2)\otimes SU(2)$ character of the operator.

We have treated this problem in greater generality than we really need in order to bring out the underlying mathematical simplicity of the result. For what follows, we shall be interested in the reduction of $T_V{}^{\pi^+K^-\pi^0}$. Inverting Eqs. (2.2)–(2.6), and using (2.9)–(2.12), we find

$$\langle(2\pi)_0|T_V{}^{\pi^+K^-\pi^0}|K^0\rangle\to(\sqrt{\tfrac{1}{3}})\langle 0|[-(11/4)M_V{}^{11}+2M_A{}^{11}+(23/4)M_V{}^{31}-5M_A{}^{31}]|K^0\rangle,\qquad(2.13)$$

$$\langle(2\pi)_2|T_V{}^{\pi^+K^-\pi^0}|K^0\rangle\to(\sqrt{\tfrac{1}{6}})\langle 0|[-2M_V{}^{11}+2M_A{}^{11}+5M_V{}^{31}-5M_A{}^{31}]|K^0\rangle.\qquad(2.14)$$

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[7] For a detailed discussion of the reduction process applied to the uncorrected weak Hamiltonian, see B. R. Holstein, Ref. 6. No essential complications arise here.

We point out that if $M_V^{11}=M_A^{11}$ and $M_V^{31}=M_A^{31}$, (2.14) gives $\langle(2\pi)_2|T_V^{\pi^+K^-\pi^0}|K^0\rangle=0$, the chiral-symmetry limiting value. Furthermore, if $M_V^{11}=M_V^{31}=0$, we have a ratio of $\sqrt{2}$ between the $I=0$ and $I=2$ amplitudes. This ratio corresponds to

$$\langle\pi^0\pi^0|T_V^{\pi^+K^-\pi^0}|K^0\rangle=0\,,$$

and results from the fact that the reduction of *neutral* pions does not generate a T_A^{abc}.

III. POLE APPROXIMATION

We have learned as much as we can from symmetry considerations alone. Further progress requires some dynamics. As a first approximation, we shall assume that the amplitudes T^{abc} are dominated by single-particle intermediate states. This is similar in spirit to the vector-meson-dominance model[8] of ordinary nonleptonic weak interactions, and to the calculation of Yun.[9] We consider here all allowed single-particle intermediate states, rather than restricting our attention to an arbitrary subset of them. We assume that high-spin states are dynamically suppressed by angular momentum barriers (presence of momenta in the amplitudes) and therefore concentrate on the states of lowest spin.

$J=0$ states: There are no known scalar states with the appropriate quantum numbers ($S=0, C=-1$, or $S=1$). As for the pseudoscalar states, the K does not contribute because the electromagnetic photon loop, being odd under T, has no diagonal matrix elements between single-particle states of zero spin; the η and η' do not contribute because the reduction of a single pion will not produce a $\Delta Q=0$, $\Delta P=-1$ photon-loop operator; the pion contributes a term that is linear in the momentum of the intermediate state. (This can be seen because the contribution vanishes in the soft-pion limit.) Furthermore, the momentum of the intermediate state, and of any state that occurs with a parity-violating photon-loop operator, is on the pion mass shell. Therefore in the soft-pion limit this state is decoupled.

$J=1$ states: The ρ, ω, and ϕ do not contribute at all because they are coupled to a conserved current, and $[J^\mu,K^\mu]=0$. The A_1 and K^* are decoupled in the soft-pion limit. (They are at least linear in one power of a momentum which goes to zero.) The $K_A(1230)$ and $K_A(1320)$, however, occur with a parity-conserving photon loop and are thus on the K mass shell. This contribution remains in the soft-pion limit. They contribute only to M_V^{11}.

Thus, for $J<2$, the only amplitude that receives any resonant contributions is M_V^{11}. For higher spins, it will be generally true that the contribution to M_A^{ij} will vanish in the soft-pion limit. $I=1$, $S=0$ states [e.g., the $A_2(1315)$ with $J^P=2^+$] can contribute both to M_V^{11} and M_V^{31}.

However, the latter contribution, which involves a $\Delta I=\frac{3}{2}$ weak transition, is expected to be dynamically suppressed. Therefore, as far as resonant contributions are concerned, M_V^{11} dominates in Eqs. (2.13) and (2.14). We shall neglect the other amplitudes in what follows, reserving a discussion of the effects of their inclusion for the Appendix. This predicts a ratio of

$$\langle(2\pi)_2|T_V^{\pi^+K^-\pi^0}|K^0\rangle/\langle(2\pi)_0|T_V^{\pi^+K^-\pi^0}|K^0\rangle =4\sqrt{2}/11\cong\tfrac{1}{2}\,. \quad (3.1)$$

IV. EFFECTS OF *CP* MIXING IN EIGENSTATES

The comparison of the result of Sec. III to experiment is complicated by the fact[10] that the decaying state is not an eigenstate of CP. The phenomenological description of this state of affairs has been complicated by the fact that the relative phase of the K^0 and $\bar{K}^0$ states is unobservable, and must be chosen by convention. This corresponds to a basic ambiguity in the assigning of the observed CP-violating effects to dynamics, on the one hand, and to state mixing, on the other. Traditionally,[11] one chooses to describe CP violation in the $I=0$ decay mode as due entirely to state mixing. This has the disadvantage that it gives the "CP phase factor" α, defined by

$$CP|K^0\rangle=e^{i\alpha}|\bar{K}^0\rangle\equiv e^{i\alpha}CPT|K^0\rangle\,, \quad (4.1)$$

a nontrivial (and unknown) value, determined by the relative strength of the CP-conserving amplitudes in the $I=0$ 2π channel. For the moment we shall not specify the value of α. In terms of the 2×2 mass matrix $M-i\Gamma$, defined by

$$(M-i\Gamma)\begin{pmatrix}K^0\\ \bar{K}^0\end{pmatrix}=i\frac{d}{dt}\begin{pmatrix}K^0\\ \bar{K}^0\end{pmatrix}, \quad (4.2)$$

we may write the physical eigenstates K_L and K_S in terms of the CP eigenstates K_+ and K_- defined by $CP|K_\pm\rangle=\pm|K_\pm\rangle$. We find

$$K_L=\frac{1}{\sqrt{2}}[(\epsilon\cos\tfrac{1}{2}\alpha+i\sin\tfrac{1}{2}\alpha)K_+ +(\cos\tfrac{1}{2}\alpha+i\epsilon\sin\tfrac{1}{2}\alpha)K_-]\,, \quad (4.3a)$$

$$K_S=\frac{1}{\sqrt{2}}[(\cos\tfrac{1}{2}\alpha+i\epsilon\sin\tfrac{1}{2}\alpha)K_+ +(\epsilon\cos\tfrac{1}{2}\alpha+i\sin\tfrac{1}{2}\alpha)K_-]\,, \quad (4.3b)$$

where

$$\epsilon=\frac{1}{2(|M_{12}|^2+|\Gamma_{12}|^2)}[(\mathrm{Re}M_{12}\,\mathrm{Im}\Gamma_{12}-\mathrm{Re}\Gamma_{12}\,\mathrm{Im}M_{12}) +i(\mathrm{Re}\Gamma_{12}\,\mathrm{Im}\Gamma_{12}+\mathrm{Re}M_{12}\,\mathrm{Im}M_{12})] \quad (4.4)$$

and we have neglected terms of order ϵ^2. We note that

[8] J. J. Sakurai, Phys. Rev. **156**, 1508 (1967); L. J. Clavelli, *ibid.* **154**, 1509 (1967).

[9] Suk Koo Yun, Phys. Rev. **178**, 2439 (1969).

[10] Since $\mathrm{Re}\,\epsilon\neq0$. See Ref. 13.

[11] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 180 (1964).

if ϵ is pure imaginary, K_L and K_S may be chosen to be eigenstates of CP, by a suitable choice of α—in other words, in this case all of the observed effect may be imputed to the dynamics. This corresponds to the fact [easily verified from Eq. (4.4)] that, to order ϵ, Imϵ may be transformed away by a suitable choice of α, while Reϵ (which is proportional to the nonorthogonality of the physical eigenstates, and therefore observable) is not affected by this choice.

If $\alpha \ll 1$, K_L (K_S) is mostly CP odd (even) and therefore corresponds to the long- (short-) lived eigenstate. If $\alpha \approx \pi$, the roles of L and S are reversed and they lose their mnemonic value. (For intermediate values, $\alpha \approx 1$, we no longer have $|\epsilon| \ll 1$, and the above analysis breaks down.) If we choose $\alpha \ll 1$, we have

$$\eta_{+-}=\epsilon+\epsilon'+i\,\mathrm{Im}A_0/\mathrm{Re}A_0\,, \qquad (4.5\mathrm{a})$$

$$\eta_{00}=\epsilon-2\epsilon'+i\,\mathrm{Im}A_0/\mathrm{Re}A_0\,, \qquad (4.5\mathrm{b})$$

with

$$\epsilon'=\frac{i}{\sqrt{2}}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)}\,, \qquad (4.6)$$

where

$$A_I\equiv\langle(2\pi)_I|H_w|K^0\rangle\,, \qquad (4.7)$$

and δ_I is the $(\pi\pi)_I$ scattering phase shift. ImA_I is proportional to the dynamical CP breaking in the $(2\pi)_I$ mode for $\alpha=0$. Since the CP-violating weak Hamiltonian, to second order in the electromagnetic interaction, is just $T_V{}^{\pi^+K^-\pi^0}$, Eq. (3.1) may be written

$$|\mathrm{Im}A_2/\mathrm{Im}A_0|\approx\tfrac{1}{2} \quad \text{for} \quad \alpha=0\,. \qquad (4.8)$$

In order to compare this result with experiment, we would need to calculate Reϵ. This quantity is proportional to the CP-violating radiative corrections to a second-order weak nonleptonic matrix element, and a precise calculation of it is not possible. However, the observed value of Reϵ allows us at least an order of magnitude estimate of some of the quantities which determine ϵ'. In what follows, we shall, for simplicity, adopt the phase convention $\alpha=0$, i.e., $CP|K^0\rangle=|\bar{K}^0\rangle$. Re$\epsilon$ is independent of this choice, and everything could be done without it, but the resulting discussion would be more complicated.

Expanding the T product in a sum over states,

$$M_{12}=\sum_n P[\langle K^0|H|n\rangle\langle n|H|\bar{K}^0\rangle/(M_{K^0}-M_n)]\,, \qquad (4.9\mathrm{a})$$

$$\Gamma_{12}=\pi\sum_f \rho_f\langle K^0|H|f\rangle\langle f|H|K^0\rangle\,. \qquad (4.9\mathrm{b})$$

Here the states n (of unperturbed energy M_n) form a complete set, P denotes a principal-value integral over the pole, and ρ_f is the density of the final states f into which a real decay process is possible. H is the weak Hamiltonian corrected to all orders in the electromagnetic interaction. With our phase convention, the imaginary parts of M_{12} and Γ_{12} are proportional to CP-violating radiative corrections to the weak Hamiltonian, which are contained in H. From the observed decay ratio, it is clear[12] that Γ_{12} is dominated by the $(2\pi)_0$ state. We shall assume that this holds also for ImΓ_{12}. [This amounts to assuming that radiative corrections to the $(2\pi)_0$ state are not significantly different in magnitude from those to other states. This assumption is made plausible by the observed value Re$\epsilon\approx 10^{-3}$, which is what one expects for radiative corrections.] The eigenvalue equation (4.2) leads to the identification $2\,\mathrm{Re}M_{12}=M_S-M_L$, and $2\,\mathrm{Re}\Gamma_{12}=\frac{1}{2}(\gamma_S-\gamma_L)$, so from experiment, Re$M_{12}\approx -Re\Gamma_{12}$. Equation (4.4) gives

$$\mathrm{Re}\epsilon=-(1/4\,\mathrm{Re}\Gamma_{12})(\mathrm{Im}\Gamma_{12}+\mathrm{Im}M_{12})\,. \qquad (4.10)$$

Inserting the $(2\pi)_0$ state in (4.9b), we obtain the estimate ImΓ_{12}/Re$\Gamma_{12}\approx -2\,$ImA_0/ReA_0, which gives

$$\mathrm{Re}\epsilon=R(1+\xi)\,, \qquad (4.11)$$

where we have defined

$$R\equiv\tfrac{1}{2}\,\mathrm{Im}A_0/\mathrm{Re}A_0\,, \quad \xi\equiv\mathrm{Im}M_{12}/\mathrm{Im}\Gamma_{12}\,.$$

In the same way, we find

$$\mathrm{Im}\epsilon=R(-1+\xi)\,. \qquad (4.12)$$

We emphasize that the form of (4.11) and (4.12) is dependent on the phase convention we have adopted ($CP|K^0\rangle=\bar{K}^0\rangle$), though the numerical value of Reϵ is independent of this choice.

The observables η_{+-} and η_{00} depend on the difference in phase shifts $\phi\equiv\delta_0-\delta_2$, for which only a very approximate value is available. However, it turns out that $|\eta_{+-}|$ and $|\eta_{00}|$ depend on ϕ only through the function $\sin\phi+\cos\phi$, which is very slowly varying over the accepted range[13] $\phi=50°\pm20°$. Within 10%, then, we find

$$|\eta_{+-}|=\sqrt{2}|R(\xi+\tfrac{3}{2})|\,, \qquad (4.13)$$

$$|\eta_{00}|=\sqrt{2}|R\xi|\,, \qquad (4.14)$$

where we have used the ratio, which is the main result of this paper, ImA_2/Im$A_0=\frac{1}{2}$.

Experimentally,[13] $|\eta_{+-}|=2\times10^{-3}$, Re$\epsilon$ is a bit uncertain but probably lies in the range $10^{-3}\leqslant\mathrm{Re}\epsilon\leqslant 2\times10^{-3}$, and the latest results for $|\eta_{00}|$ divide equally between $(2.2\pm0.4)\times10^{-3}$ and $(3.6\pm0.6)\times10^{-3}$. We propose to use the firm value for η_{+-}, together with the possible range for Reϵ and some intrinsic notions of the size of radiative corrections, to derive a possible range of values for η_{00}. We do not include ϕ_{+-} and ϕ_{00} in our analysis because they are rapidly varying functions of ϕ in the relevant range.

The family of solutions with $|\eta_{+-}|=-\sqrt{2}R(\xi+\frac{3}{2})$, together with Re$\epsilon\geqslant10^{-3}$, is incompatible with $|\eta_{00}|<8$

[12] L. Wolfenstein, Nuovo Cimento **62A**, 17 (1966).

[13] For these and other experimental data, see J. W. Cronin, rapporteur talk, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 292ff.

$\times 10^{-3}$, and so we reject it. Taking the other branch of the cut, we find solutions corresponding to $|\eta_{+-}| = +\sqrt{2}R(\xi+\frac{3}{2})$, with

$$-(1.17\times 10^{-3}) \leq R \leq 0.83\times 10^{-3}, \quad |\eta_{00}| = |\eta_{+-}| - (3/\sqrt{2})R. \tag{4.15}$$

In principle, this allows for $|\eta_{00}| = |\eta_{+-}|$. However, R ($\equiv \frac{1}{2}\,\mathrm{Im}A_0/\mathrm{Re}A_0$) is a typical second-order radiative correction, and as such is expected to be of order 10^{-3}, not zero. If R were to differ markedly from this value, the near equality $|\eta_{+-}| \simeq \alpha/\pi$ would have to be ascribed to a remarkable coincidence, and the model would lose one of its most appealing features.

As long as $R \neq 0$, this model is not compatible with exact equality between $|\eta_{+-}|$ and $|\eta_{00}|$. How close they may come to one another may best be investigated by writing (4.15) as

$$\frac{|\eta_{00}| - |\eta_{+-}|}{|\eta_{+-}|} \approx \frac{|R|}{R_0}, \tag{4.16}$$

where $R_0 = 10^{-3}$, the expected value for R, based on $\arg(A_0) \approx \alpha/\pi$. If one requires, e.g., that R be within a factor of 5 of R_0 for the believability of the model, then (4.16) places a lower limit of 2.4×10^{-3} on $|\eta_{00}|$. Of the two experimental values in favor at present, 2.2×10^{-3} gives $R/R_0 = 0.09$, while 3.6×10^{-3} gives $R/R_0 = 0.75$. The latter value does less violence to our intuitive notion of the size of radiative corrections, and is consistent with the dimensional estimate of $|\eta_{+-}|$. It is "preferred by the model" in this sense.

V. CONCLUSION

Assuming that the soft-pion limit is permissible for the $K^0 \to 2\pi$ decay amplitudes, and using the experimentally observed suppression of the $\Delta I = 0$ CP-violating electromagnetic interaction, we have related the extent of violation of the $\Delta I = \frac{1}{2}$ rule in CP-violating amplitudes to the breakdown of chirality of the interaction. We have estimated this effect by summing over single-particle intermediate states and low-lying resonances. This predicts a large chiral asymmetry, and gives a $\Delta I = \frac{3}{2}$ amplitude of about half the strength of the $\Delta I = \frac{1}{2}$ amplitude.

The effect of this breaking of the $\Delta I = \frac{1}{2}$ rule is not directly observable because the amplitudes for which it was calculated do not involve eigenstates of the full Hamiltonian. If the physical eigenstates were states of definite CP, this would pose no problem, since in that case the 2π decay of the longlived K^0 would proceed entirely through the CP-violating interaction that we have investigated. The isospin properties of this interaction would then be exactly mirrored in the observed decay. There is definite evidence, however, from semileptonic processes, that the K_L^0 is not an eigenstate of CP, and therefore its 2π decay mode is not mediated entirely by a CP-violating interaction. This complicates the theoretical problem of predicting the isospin of the final state, because it is difficult to obtain an estimate of the relative importance of the CP-violating and CP-conserving parts of the transition matrix.

In getting around this difficulty, we have been forced to assume the approximate validity of dimensional arguments for the relative size of second-order radiative corrections, which predict $|\mathrm{Im}A_0/\mathrm{Re}A_0| \approx \alpha/\pi$. If we write $|\mathrm{Im}A_0/\mathrm{Re}A_0| = C\alpha/\pi$, $C=1$ gives $|\eta_{00}| \approx 4\times 10^{-3}$ and $C = \frac{1}{10}$ gives $|\eta_{00}| \approx 2.2\times 10^{-3}$. The conclusion of this paper is that if the radiative correction to A_0 is adequately estimated by dimensional arguments (as the observed magnitude of η_{+-} apparently is), then values of η_{00} near η_{+-} are ruled out. The value $|\eta_{00}| = 3.6\times 10^{-3}$ is, on the contrary, well within the predicted range.

The effects of an isovector C-even current have not been considered. Barring accidental cancellations, however, it is expected that inclusion of such a current would break the $\Delta I = \frac{1}{2}$ rule even more badly. This would reinforce the conclusion found here. This point is under investigation.

APPENDIX: ESTIMATE OF CORRECTIONS TO POLE APPROXIMATION

In this paper we have estimated the breaking of chiral symmetry of the interaction by including in the sum over intermediate states only the low-mass low-spin particles and resonances. This is inherently a low-energy approach, and it is not surprising that it gives a maximal violation of chiral symmetry. It is expected[14] that at higher energies $SU(2)\otimes SU(2)$ may become asymptotically good. This implies that the states we have left out (multiparticle states and nonresonant channels), insofar as they represent higher-energy contributions, may shift our calculation in the direction of closer adherence to the $\Delta I = \frac{1}{2}$ rule. This, in turn, would weaken our result $\eta_{00} \neq \eta_{+-}$. Thus, the magnitude of the effect of nonpole contributions is a matter of some interest. We show here that these contributions, if they are approximately $SU(2)\otimes SU(2)$-invariant, actually contribute very little to the amplitude of interest because of a near cancellation.

We divide up the M_V^{11} amplitude into $(M_V^{11})_{\mathrm{pole}}$, which is what we have considered up to now, and

$$(M_V^{11})_{\mathrm{nonpole}} \equiv \eta (M_V^{11})_{\mathrm{pole}}.$$

We expect $\eta \lesssim 1$, and also

$$(M_V^{11})_{\mathrm{nonpole}} \approx (M_A^{11})_{\mathrm{nonpole}}.$$

Then, defining

$$\alpha \equiv (M_V^{31})_{\mathrm{nonpole}} / (M_V^{11})_{\mathrm{nonpole}},$$

[14] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967).

we have

$$\langle(2\pi)_0|T_V{}^{\pi^+K^-\pi^0}|K^0\rangle = (\sqrt{\tfrac{1}{3}})\langle 0|M_V{}^{11}|K^0\rangle(-11/4) \times[1+(3/11)\eta(1-\alpha)], \quad \text{(A1)}$$

$$\langle(2\pi)_2|T_V{}^{\pi^+K^-\pi^0}|K^0\rangle = (\sqrt{\tfrac{1}{6}})\langle 0|M_V{}^{11}|K^0\rangle(-2). \quad \text{(A2)}$$

Note that for $\alpha=1$ the nonpole contribution to (A1) vanishes, while even for the extreme case $\alpha\ll\eta\approx 1$, our conclusions are altered only by $\sim 30\%$. The coefficient 3/11 in (A1), which makes our result so insensitive to the nonpole terms, results from a near cancellation of much larger coefficients in (2.13). This cancellation could not have been predicted from symmetry considerations alone. It should be borne in mind, however, that in deriving this result we have used the assumption of approximate $SU(2)\otimes SU(2)$ symmetry for the nonpole part.

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Deviations from Charge Independence in Nuclear Forces*

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A simple mass-averaging device is described for the investigation of the effect of the pion mass difference on the nucleon-nucleon interaction. While the mass-difference correction in the one-pion-exchange contribution is known to be significant, we find that the corresponding correction plays a less significant role in the two-pion-exchange contribution. The effect of difference in the pion coupling constants is also discussed, and found to be equally significant in the one- and two-pion-exchange contributions. The applicability of our approach to the ρ-exchange contribution is pointed out.

1. INTRODUCTION

THE differences in the masses and coupling constants of charged and neutral pions lead to some deviations from charge independence in the pion theory of nuclear forces. The resulting correction for the one-pion-exchange (OPE) nucleon-nucleon interaction is well known,[1–3] but no reliable calculation of this correction for the two-pion-exchange (TPE) interaction is available. We shall, therefore, investigate deviations from charge independence by extending our earlier work[4,5] on the relativistic nucleon-nucleon interaction. In order to simplify the calculations, especially for the TPE interaction, a convenient method will be described for taking into account the effect of the pion mass difference by means of a mass-averaging device, which can be applied to any nucleon-nucleon scattering diagram. This method is also immediately applicable to the contribution of the ρ mesons.

It was shown by Breit *et al.*[6] that the addition of the TPE to the OPE nucleon-nucleon interaction leads to an improved agreement with the phenomenological phase parameters in higher partial waves, and similar results have been obtained more recently by Haracz and Sharma[7] and by Wortman.[8] Our calculation of deviations from charge independence in the TPE interaction can be used for a further refinement of these investigations.

We shall generally follow the same notation as in the earlier papers,[4,5] but we shall take $c=\hbar=1$. The nucleon mass will again be denoted by M, while the masses of the charged and neutral pions will be denoted by m_c and m_0, respectively.

It will be useful to keep in mind the following eigenvalues of the isospin operators for the two-nucleon system: $\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}$ takes the values 1 and -3 for the isotriplet and isosinglet states, respectively; $\tau_3{}^{(1)}\tau_3{}^{(2)}$ takes the value 1 for the p-p or n-n states and the value -1 for the n-p states; $\frac{1}{4}(1+\tau_3{}^{(1)})(1+\tau_3{}^{(2)})$ takes the value 1 for the p-p states and vanishes for the n-n or n-p states; $\frac{1}{4}(1-\tau_3{}^{(1)})(1-\tau_3{}^{(2)})$ takes the value 1

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