Charge Quantization, Compactness of the Gauge Group, and Flux Quantization*

C. N. YANG
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 5 September 1969)

The relationship between charge quantization and the compactness of the gauge group is discussed. Also, remarks are made about charge quantization and the observation of flux quantization in superconductors.

I. INTRODUCTION

A QUESTION that has been frequently raised is the equality of the absolute charges of the proton and the electron. Experimentally[1] they are equal to a high degree of accuracy. To understand the equality, Dirac has made the ingenious theoretical[2] proposal of the existence of magnetic monopoles, which have not yet been detected.

In Sec. II of this paper, we discuss the logical relationship[3] between the quantization of the electric charge and the mathematical concept of the compactness of the gauge group. We shall see that they are intimately related. Let us here emphasize that in mathematics the concept of compactness for a group[4] is of primary importance. For Lie groups, compactness is a property of the global structure of the group, which has a determining influence on the nature of the representations of the group. It is, in fact, through its influence on the representations that the compactness of the gauge group has a bearing on the quantization of charges.

In Sec. III we remark on the unit of flux quantization and the question of charge quantization.

II. CHARGE QUANTIZATION AND COMPACTNESS OF GAUGE GROUP

We consider a space-time-independent gauge transformation on charged fields ψ_j of charge e_j:

$$\psi_j \to \psi_j' = \psi_j \exp(ie_j\alpha)\,. \tag{1}$$

In usual discussions one confines oneself to infinitesimal values of α. In the present discussion we consider instead *finite* values of α.

If the different e_j's $(=e_1, e_2, \ldots)$ of different fields are not commensurate with each other, the transformation (1) is different for all real values of α, and the gauge group must be defined so as to include all real values of α. Hence, the group is not compact.

If, on the other hand, all different e_j's are integral multiples of e, a universal unit of charge, then for *two values* of α different by an integral multiple of $2\pi/e$, the transformations (1) for any fields ψ_j are the *same*. In other words, two transformations (1) are indistinguishable if their α's are the same modulo $2\pi/e$. Hence the gauge group as defined by (1) is compact.

III. REMARKS ON FLUX QUANTIZATION AND CHARGE QUANTIZATION

In the experiment of flux quantization,[5] one finds that magnetic flux trapped in superconducting rings are in whole units of $2\pi ch/2e$. What should e be if the electron and the proton do not have the same charge? The answer is that e should be the electron charge, since electron pairs, not the protons, are the "basic group"[6] that possess off-diagonal long-range order in a superconductor. To illustrate this point further, let us assume that spin-up electrons and spin-down electrons have charges $-e$ and $-e'$, respectively, and that the basic group is a pair of electrons with opposite spins. The flux unit would then be $2\pi ch/(e+e')$. If, on the other hand, there are two kinds of spin-up electrons with charges e and e' which are incommensurate, and two kinds of spin-down electrons with similar charges, and, furthermore, if pairs of electrons $e-e$, $e-e'$, $e'-e$, and $e'-e'$ of opposite spins all have off-diagonal long-range order in a superconductor, then the flux unit is an integral multiple of $2\pi ch/2e$, $2\pi ch/(e+e')$, and $2\pi ch/2e'$. Hence it is ∞. To summarize, the existence of a finite flux quantization unit merely reflects on the quantized nature of the charge of the basic groups in a superconductor, and does not necessarily imply that electric charge is always quantized. But if the flux quantization unit were found to be ∞, one would have concluded that the electric charge is not quantized.

* Partially supported by U. S. Atomic Energy Commission Contract No. AT(30-1)-3668B.

[1] G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. (U. S.) **45**, 1301 (1959).

[2] P. A. M. Dirac, Proc. Roy. Soc. (London) **A133**, 60 (1931); Phys. Rev. **74**, 817 (1948).

[3] It has been brought to the author's attention that this relationship had been referred to in J. Schwinger, Phys. Rev. **125**, 1047 (1962); cf. also E. Progovecki, J. Math. Phys. **5**, 442 (1964); and S. Doplicher, R. Haag, and J. E. Roberts, Commun. Math. Phys. **13**, 1 (1969).

[4] See, e.g., H. Weyl, *The Classical Groups* (Princeton U. P., Princeton, N. J., 1946).

[5] B. S. Deaver and W. H. Fairbank, Phys. Rev. Letters **7**, 43 (1961); R. Doll and M. Näbauer, *ibid.* **7**, 51 (1961).

[6] C. N. Yang, Rev. Mod. Phys. **34**, 694 (1962).