Pion-Pole Mechanism and the Application of the Veneziano Model to $K_2^0 \to \pi^+\pi^-\gamma$ Decay*

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The CP-conserving decay mode $\pi^+\pi^-\gamma$ of the K_2^0 is calculated in the pion-pole model (with η-π^0 mixing) with the strong-interaction part of the decay amplitude described by the Veneziano model. Because of sign ambiguities introduced by a model of physical PVV couplings (which fits both $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$ rates) and the unknown sign of η-π^0 mixing relative to the weak K-π mixing, values of the branching ratio $r((K_2^0 \to \pi^+\pi^-\gamma)/(K_2^0 \to \text{all modes}))$ are obtained ranging from 7.6×10^{-5} to 3.1×10^{-4}, but all are consistent with the present experimental upper limit of 4×10^{-4}.

1. INTRODUCTION

THERE has recently been a revival of interest[1–3] in the pion-pole model[4] of the weak and electromagnetic decays of the K and η mesons. Decay transitions in this model are viewed as going via a two-step process (and these steps may possibly be reversed in order): (1) The K (or η) meson converts to a pion *weakly* (or *electromagnetically*) through a phenomenological mixing of $K^\pm$, K_2^0 (or η) meson with the π meson; (2) the pion then converts to the final state via *strong* interactions. With no adjustable parameters, the model has been applied[2] to the decay spectra of $K^\pm \to 3\pi$, $K_2^0 \to 3\pi$, and $\eta \to 3\pi$, the K_1^0-K_2^0 mass difference, and the $\rho \to \pi\eta$ and $K_2^0 \to 2\gamma$ branching ratios, and the theoretical predictions have all been reported in good agreement with experiment.[2] It was subsequently noted,[3] after a reassessment of the experimental situation,[3,5] that this apparent success is somewhat spoiled in the last case ($K_2^0 \to 2\gamma$ branching ratio), the "naive" (pion-) pole model[3] accounting for only 40% of the (average) experimental value of the branching ratio, with important amplitude corrections coming from an omitted η pole (about 19% of the "naive" amplitude) and (probably sizable) nonpole contributions.[6] Moreover, the problem of reconciling theory and experiment is further complicated here by our ignorance of the signs[3,7] $\text{sgn}(G_\omega/G_\rho)$ and $\text{sgn}(\lambda_\pi/\gamma_\pi)$ (and hence the signs of the amplitude corrections). The question of $\text{sgn}(G_\omega/G_\rho)$ results from an indeterminacy[3,7] in the phenomenological model of PVV couplings introduced in Ref. 2 which fits both the $\pi^0 \to 2\gamma$ and the $\eta \to 2\gamma$ rates; in fitting[2] the couplings of the "effective" weak Hamiltonian

$$\mathcal{H}_w = G_F\lambda_\pi[(\varphi^{(4+i5)/\sqrt{2}}\varphi^{(1-i2)/\sqrt{2}} + \text{H.c.}) - \varphi^6\varphi^3] \quad (1.1)$$

and of the "effective" Hamiltonian describing η-π^0 mixing

$$\mathcal{H}_{\text{em}} = \alpha\gamma_\pi\varphi^8\varphi^3 \quad (1.2)$$

to the $K \to 3\pi$ and $\eta \to 3\pi$ total decay rates, respectively, the question of $\text{sgn}(\lambda_\pi/\gamma_\pi)$ does not arise.

In this paper we extend the pion-pole model (with η-π^0 mixing)[8] to the (CP-invariant)[9,10] $\pi^+\pi^-\gamma$ mode of K_2^0 decay. [The amplitude for the inner bremsstrahlung accompanying the CP-noninvariant decay $K_2^0 \to \pi^+\pi^-$ does not interfere[9] with the CP-invariant mode; for a calculation of its contribution to the calculated branching ratio[11] $r((K_2^0 \to \pi^+\pi^-\gamma)/(K_2^0 \to \text{all modes}))$ the reader should consult Ref. 9.] In order to shift the burden of uncertainty in such a calculation onto the pion-pole mechanism, we shall adopt the Veneziano model[12] for the "strong-interaction" part of the decay amplitude.[13] The improvement afforded by this model over simple pole-dominance models has been remarked on repeatedly in the very recent literature[14]; we take it seriously here as a good approximate theory of the strong interactions in the decay process under investigation. Some of the details of the calculation, a tabular comparison of the resulting (CP-conserving) branching ratios with those of Refs. 9 and 10, and plots of the photon energy distribution for the various cases are given in the next section. It should be noted that for the branching ratio $r((K_2^0 \to \pi^+\pi^-\gamma)/(K_2^0 \to \text{all}))$ there is only the experimental upper limit of 4×10^{-4} at

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[1] C. Lovelace, Phys. Letters **28B**, 265 (1968).

[2] R. Arnowitt, P. Nath, P. Pond, and M. H. Friedman (to be published).

[3] R. Rockmore, Phys. Rev. (to be published).

[4] See Ref. 2 for the appropriate historical footnote; however, in this paper our interest will be centered on a later contribution (cited below as Ref. 9) unfortunately not mentioned there.

[5] This analysis leaned heavily on a discussion by B. Aubert, in Proceedings of the CERN Topical Conference on Weak Interactions, Geneva, 1969 (unpublished).

[6] However, it should be stressed that this pole model *does* give the correct order of magnitude for $r((K_2^0 \to 2\gamma)/(K_2^0 \to \text{all modes}))$.

[7] R. Rockmore, Phys. Rev. (to be published).

[8] However, we do not consider the possibility of nonpole corrections as in Ref. 3.

[9] C. S. Lai and B. L. Young, Nuovo Cimento **52A**, 83 (1967).

[10] S. Oneda, Y. Kim, and D. Korff, Phys. Rev. **136**, B1064 (1964).

[11] We find it ranges from 6 to 21% of the branching ratio for CP-invariant decay for photon energies $k \geq 10$ MeV.

[12] G. Veneziano, Nuovo Cimento **57A**, 190 (1968); C. Lovelace (to be published).

[13] We make the usual assumption of the vector dominance of the electromagnetic current in the various crossed-photon off-shell pion photoproduction amplitudes which come into the calculation.

[14] See, e.g., Lovelace (Ref. 12), as well as R. G. Roberts and F. Wagner, in Proceedings of the CERN Topical Conference on Weak Interactions, Geneva, 1969 (unpublished).

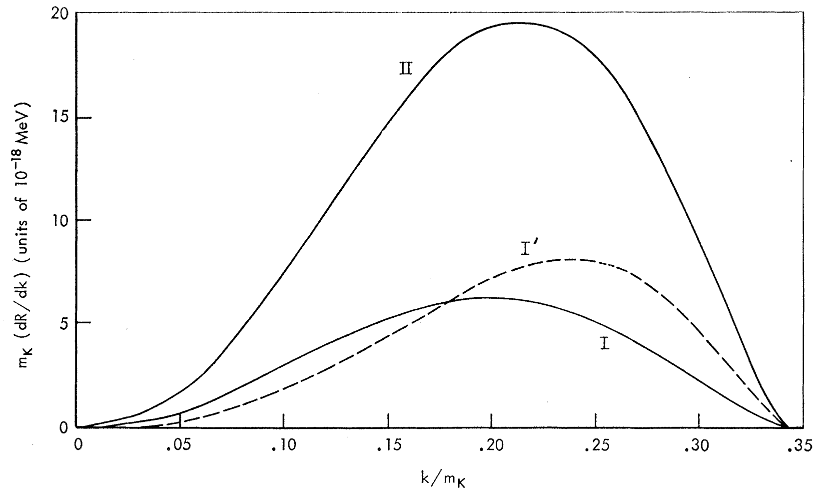

FIG. 1. Photon energy distribution for the (CP-invariant) decay $K_2{}^0 \to \pi^+\pi^-\gamma$ versus photon energy k in units of the K-meson mass. The dashed curve I′ is the energy distribution for a constant matrix element $M(k=0,\ \omega_+=\frac{1}{2}m_K)$, and may be directly compared with the distribution plotted in Fig. 1 of Ref. 9. I and II are the distributions for *no* η-π^0 mixing ($\gamma_\pi=0$), and correspond to the cases $\mathrm{sgn}(G_\omega/G_\rho)=+$ and $\mathrm{sgn}(G_\omega/G_\rho)=-$, respectively.

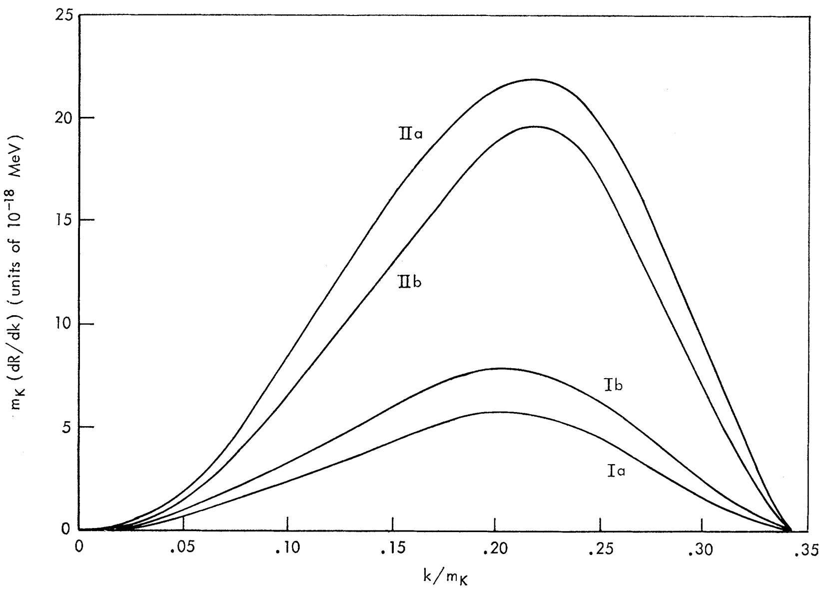

FIG. 2. Photon energy distribution for the (CP-invariant) decay $K_2{}^0 \to \pi^+\pi^-\gamma$ versus photon energy k in units of the K-meson mass. Ia and Ib are distributions *including* the effect of η-π^0 mixing and correspond to the cases $\mathrm{sgn}(G_\omega/G_\rho)=+$, $\mathrm{sgn}(\lambda_\pi/\gamma_\pi)=+$ and $\mathrm{sgn}(G_\omega/G_\rho)=+$, $\mathrm{sgn}(\lambda_\pi/\gamma_\pi)=-$, respectively; IIa and IIb correspond to the cases $\mathrm{sgn}(G_\omega/G_\rho)=-$, $\mathrm{sgn}(\lambda_\pi/\gamma_\pi)=+$ and $\mathrm{sgn}(G_\omega/G_\rho)=-$, $\mathrm{sgn}(\lambda_\pi/\gamma_\pi)=-$, respectively.

present.[15] Since all our calculated cases are consistent with this number (as indeed are the predictions of Refs. 9 and 10), a further refinement of this experimental limit is needed before we can safely discard some of these possible values and reach a conclusion regarding $\mathrm{sgn}(G_\omega/G_\rho)$ and $\mathrm{sgn}(\lambda_\pi/\gamma_\pi)$.

2. APPLICATION OF VENEZIANO MODEL TO CP-INVARIANT $K_2{}^0 \to \pi^+\pi^-\gamma$

The introduction of the Veneziano model into the calculation of the CP-invariant mode $\pi^+\pi^-\gamma$ of $K_2{}^0$ decay is simple once a vector-meson pole analysis in the s, t, and u channels[16] of the decay amplitude

$$\begin{aligned}\langle\pi^+(p_+)\pi^-(p_-)\gamma(k)\mathrm{out}|\mathcal{H}_w|K_2{}^0(p)\rangle &= \frac{G_F\lambda_\pi}{m_\pi{}^2-m_K{}^2}\langle\pi^+(p_+)\pi^-(p_-)\gamma(k)\mathrm{out}|j^3(0)|0\rangle \\ &+\frac{G_F\lambda_\pi}{m_K{}^2-m_\pi{}^2}\langle\pi^-(p_-)\gamma(k)\mathrm{out}|j^{(4-i5)/\sqrt{2}}(0)|K_2{}^0(p)\rangle \\ &-\frac{G_F\lambda_\pi}{m_K{}^2-m_\pi{}^2}\langle\pi^+(p_+)\gamma(k)\mathrm{out}|j^{(4+i5)/\sqrt{2}}(0)|K_2{}^0(p)\rangle \quad (2.1)\end{aligned}$$

[15] R. C. Thatcher, A. Abashian, R. J. Abrams, D. W. Carpenter, R. E. Mischke, B. M. K. Nefkens, J. H. Smith, L. J. Verhey, and A. Wattenberg, Phys. Rev. **174**, 1674 (1968).

[16] We define

$$s=-(p_++p_-)^2=-(p-k)^2=m_K{}^2-2m_Kk,$$
$$t=-(p_++k)^2=m_\pi{}^2-2kp_+\cos\theta_++2k\omega(p_+)=-m_K{}^2+m_\pi{}^2+2m_K[\omega(p_+)+k],$$

and

$$u=-(p_-+k)^2=m_K{}^2+m_\pi{}^2-2m_K\omega(p_+),$$

with $s+t+u=m_K{}^2+2m_\pi{}^2$; $(p_\pm)_\mu\equiv$ momentum of $\pi^\pm$, $k_\mu\equiv$photon momentum, $p_\mu\equiv K_2{}^0$ momentum.

TABLE I. Theoretical predictions regarding the CP-invariant decay $K_2{}^0 \to \pi^+\pi^-\gamma$.

$R(K_2{}^0 \to \pi^+\pi^-\gamma)$ (10^3 sec^{-1})	$r\left(\dfrac{K_2{}^0 \to \pi^+\pi^-\gamma}{K_2{}^0 \to \text{all modes}}\right)$	Remarks
1.7	9.14×10^{-5}	$(\gamma_\pi=0)$ $\operatorname{sgn}(G_\omega/G_\rho)=+$
5.24	2.82×10^{-4}	$(\gamma_\pi=0)$ $\operatorname{sgn}(G_\omega/G_\rho)=-$
1.42	7.63×10^{-5}	$\operatorname{sgn}(\lambda_\pi/\gamma_\pi)=+$ $\operatorname{sgn}(G_\omega/G_\rho)=+$
2.43	1.31×10^{-4}	$\operatorname{sgn}(\lambda_\pi/\gamma_\pi)=-$ $\operatorname{sgn}(G_\omega/G_\rho)=+$
5.78	3.11×10^{-4}	$\operatorname{sgn}(\lambda_\pi/\gamma_\pi)=+$ $\operatorname{sgn}(G_\omega/G_\rho)=-$
4.74	2.55×10^{-4}	$\operatorname{sgn}(\lambda_\pi/\gamma_\pi)=-$ $\operatorname{sgn}(G_\omega/G_\rho)=-$
1.7	1×10^{-4}	Ref. 9
2–3	$1.14\text{–}1.7\times10^{-4}$	Ref. 10

has been made. However, the details of such a calculation are too well known by now to bear any more than a brief repetition here of a few salient points. These are that we use the effective PPV and PPV couplings[3] given by

$$\mathcal{H}_{PPV}=G_{\rho\pi\pi}f_{ijk}\varphi_\lambda{}^i\varphi^j\partial_\lambda\varphi^k\,, \tag{2.2}$$

$$\mathcal{H}_{PVV}=(4\lambda/C_i)d_{ijk}\epsilon_{\mu\nu\alpha\beta}\varphi^i\partial_\mu\varphi_\nu{}^j\partial_\alpha\varphi_\beta{}^k\,, \tag{2.3}$$

with[17] $G_{\rho\pi\pi}{}^2=m_\rho{}^2/2C_\pi{}^2$ and[2] $\lambda=0.348$; the constants G_ρ and G_ω which appear in the resulting expression are defined by

$$\langle\gamma(k)\,|\,\varphi_\nu{}^3(0)\,|\,0\rangle=-e\epsilon_\nu{}^*(k)G_\rho/m_\rho{}^2\,, \tag{2.4}$$

$$\begin{aligned}\langle\gamma(k)\,|\,\varphi_\nu{}^8(0)\,|\,0\rangle&=\langle\gamma(k)\,|\,\omega_\nu(0)\,|\,0\rangle\\&=-e\epsilon_\nu{}^*(k)G_\omega/\sqrt{3}m_\omega{}^2.\end{aligned} \tag{2.5}$$

We find

$$\langle\pi^+(p_+)\pi^-(p_-)\gamma(k)\text{out}\,|\,\mathcal{H}_w\,|\,K_2{}^0(p)\rangle=\epsilon_{\mu\nu\alpha\beta}k_\mu\epsilon_\nu{}^*(k)p_{-\alpha}p_{+\beta}\frac{8eG_F\lambda_\pi\lambda G_{\rho\pi\pi}}{C_\pi(m_K{}^2-m_\pi{}^2)}M(s,t,u)\,, \tag{2.6}$$

with

$$\begin{aligned}M(s,t,u)=&\frac{\alpha' G_\omega}{3m_\omega{}^2}[B(1-\alpha(t),\,1-\alpha(s))+B(1-\alpha(u),\,1-\alpha(s))+B(1-\alpha(u),\,1-\alpha(t))]\\&+\frac{\alpha' G_\rho}{m_\rho{}^2}\frac{C_\pi}{C_K}[B(1-\alpha_{A_2}(t),\,1-\alpha_{K^*}(s))-B(1-\alpha_{A_2}(t),\,1-\alpha_{K^*}(u))]\\&-\frac{\alpha' G_\omega}{3m_\omega{}^2}\frac{C_\pi}{C_K}[B(1-\alpha(t),\,1-\alpha_{K^*}(s))+B(1-\alpha(t),\,1-\alpha_{K^*}(u))-2B(1-\alpha_{K^*}(s),\,1-\alpha_{K^*}(u))]\\&-\frac{\alpha\gamma_\pi}{m_\eta{}^2-m_K{}^2}\frac{C_\pi}{C_\eta}\frac{\alpha' G_\rho}{\sqrt{3}m_\rho{}^2}[B(1-\alpha_{A_2}(t),\,1-\alpha(s))+B(1-\alpha_{A_2}(u),\,1-\alpha(s))-B(1-\alpha_{A_2}(u),\,1-\alpha_{A_2}(t))]\,,\end{aligned} \tag{2.7}$$

where $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$, and where we take[18] $\alpha(t)=\alpha_{A_2}(t)\simeq0.5+0.9\times10^{-6}\ t$ MeV^{-2} and $\alpha_{K^*}(t)\simeq280.+0.9\times10^{-6}\ t$ MeV^{-2}. One finds for the CP-invariant rate,[19]

$$\begin{aligned}R(K_2{}^0\to\pi^+\pi^-\gamma)]_{(CP\text{-inv})}&=\int_0^{(1/2m_K)(m_K{}^2-4m_\pi{}^2)}dk(dR/dk)=\int dkd\omega_+d(\cos\theta_+)\delta\\&\times\left[\cos\theta_+-\frac{(k+\omega_+-m_K)^2-p_+{}^2-k^2-m_\pi{}^2}{2p_+k}\right]m_Kk^2p_+{}^2\sin^2\theta_+\left[\frac{4\alpha G_F{}^2\lambda_\pi{}^2\lambda^2G_{\rho\pi\pi}{}^2}{\pi^2C_\pi{}^2(m_K{}^2-m_\pi{}^2)^2}\right]|M(s,t,u)|^2\\&=\left[\frac{4\alpha G_F{}^2\lambda_\pi{}^2\lambda^2G_{\rho\pi\pi}{}^2m_K}{\pi^2C_\pi{}^2(m_K{}^2-m_\pi{}^2)^2}\right]\int_0^{(1/2m_K)(m_K{}^2-4m_\pi{}^2)}dk\int_{\frac12m_K-\frac12k-\frac12k[1-4m_\pi{}^2/(m_K{}^2-2m_Kk)]^{1/2}}^{\frac12m_K-\frac12k+\frac12k[1-4m_\pi{}^2/(m_K{}^2-2m_Kk)]^{1/2}}d\omega_+\\&\times\{k^2(\omega_+{}^2-m_\pi{}^2)-\tfrac14[(m_K-\omega_+-k)^2-\omega_+{}^2-k^2]^2\}\,|M(s,t,u)|^2.\end{aligned} \tag{2.8}$$

[17] C_π is 97 MeV; first-order symmetry breaking implies $3C_\eta=4C_K-C_\pi$, and we use $(C_K/C_\pi)^2=1.17$.

[18] See, e.g., H. Goldberg and Y. Srivastava, Phys. Rev. Letters **22**, 1340 (1969).

[19] The expression for the branching ratio $r((K_2{}^0\to\pi^+\pi^-\gamma)/(K_2{}^0\to2\gamma))$ given by Eq. (17) of Ref. 9 is in error, although the plotted curve (for the photon energy distribution) and branching ratio are correct. Equation (17) of Ref. 9 should read

$$r((K_2{}^0\to\pi^+\pi^-\gamma)/(K_2{}^0\to2\gamma))=\frac{m_\pi{}^8\lambda^2}{6\pi^3\alpha C_\pi{}^4m_K{}^2}\int_0^{(1/2m_K)(m_K{}^2-4m_\pi{}^2)}k^3dk\left(1-\frac{4m_\pi{}^2}{m_K{}^2-2m_Kk}\right)^{1/2}(m_K{}^2-2m_Kk-4m_\pi{}^2).$$

The predictions of expression (2.7) for all possible cases of interest are displayed in Table I along with the earlier calculations of Refs. 9 and 10 for comparison. As we noted earlier, these values are still consistent with the present experimental upper limit. The photon energy distributions for various cases are ploted in Figs. 1 and 2.

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Radiative Corrections to $K_{\mu 3}$ Decays*

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We have calculated the radiative corrections to the Dalitz plot for $K_{\mu 3}{}^{\pm}$ and $K_{\mu 3}{}^0$ decays, assuming a phenomenological weak K-π vertex and using perturbation theory. The answer depends logarithmically on a cutoff. We have evaluated all terms which contribute to order α analytically, without any approximations concerning the smallness of the muon mass or the "real inner bremsstrahlung." Thus, the dependence on the parameter ξ (the ratio of the form factor f_-/f_+) is exact. The radiative corrections to the $K_{\mu 3}{}^0$ Dalitz plot, muon spectrum, and lifetime average around 2% over most of their respective ranges and are not especially sensitive to the cutoff. The radiative corrections to $K_{\mu 3}{}^{\pm}$ decays is a fraction of a percent over most of the Dalitz plot and is sensitive to the cutoff. The radiative correction to the $\Delta I=\frac{1}{2}$ rule prediction for the ratio of the charged and neutral decay rates is approximately 2%. The final-state Coulomb correction accounts for most of this numerical result, the rest being model-dependent noise.

I. INTRODUCTION

In two previous papers,[1,2] we have derived expressions for the radiative corrections to the Dalitz plots in $K_{l3}{}^{\pm}$ and $K_{l3}{}^0$ decays using a phenomenological model for the weak interaction and perturbation theory. The processes considered are examples of strangeness-changing leptonic weak decays which can be analyzed experimentally in great detail. The numerous theoretical predictions for the form factors involved in these decays can, in principle, be tested by sufficiently fine measurements. Such measurements, of course, require an estimate of the radiative corrections for their interpretation. The numerical estimates which we have given were limited to the electron modes, where the approximation $m_e \to 0$ is valid. Unfortunately, in this limit the dependence on one of the form factors, f_-, is neglected, since these terms are proportional to $m_e{}^2$. In this paper we remove this restriction and present numerical estimates of the radiative corrections applicable to the muon modes. We have performed all the necessary integrations by analytical means, thus avoiding some lengthy numerical computations. Therefore, within the limitations of our model, the dependence on ξ, the ratio of the form factors f_-/f_+, is evaluated exactly.

Briefly, let us recall the assumptions underlying our previous calculations. First, we assume a phenomenological weak interaction for the hadrons using vector currents and characterized by the usual form factors f_+ and f_-. In momentum space, the Lagrangian takes the familiar form

$$\mathcal{L}\sim[(p_K+p_\pi)_\alpha f_+ + (p_K-p_\pi)_\alpha f_-]\bar{u}_\nu\gamma_\alpha\tfrac{1}{2}(1-i\gamma_5)v_l\,.$$

Our normalization is such that in the limit of unitary symmetry, the form of the weak K-π vertex is the same as the weak π-π vertex, the latter being given by the conserved vector current hypothesis.[3] Assuming also the octet hypothesis of Cabibbo,[4] we have $f_+ \to G_\beta{}^V \tan\theta$, $\xi = f_-/f_+ \to 0$, where $G_\beta{}^V$ is the weak-coupling constant determined from O^{14} decay and θ is the Cabibbo angle. Second, we calculate the radiative corrections to lowest order in α using perturbation theory and assuming minimal electromagnetic coupling. In particular, the gauge-invariant substitution $p_\alpha \to p_\alpha - eA_\alpha$ for the charged particles present gives rise to Feynman diagrams in which the weak and electromagnetic currents act at the same vertex.[5] Third, electromagnetic corrections to strong-interaction renormalization graphs are ignored; instead, we use phenomenological form factors and the physical masses of the particles involved. Finally, in calculating the radiative corrections, we shall neglect the momentum dependence of the form factors. If the form factors are expanded in the usual manner, $f_\pm(q^2)=f_\pm(0)(1+\lambda_\pm q^2/m_\pi)$, this amounts to neglecting terms of order $\alpha\lambda_\pm$, where $\lambda_\pm$ are small parameters characterizing the energy dependence of

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