Interstellar Absorption of Cosmic X Rays

ROBERT L. BROWN
National Radio Astronomy Observatory, Green Bank, West Virginia 22901*

AND

ROBERT J. GOULD
University of California, San Diego, San Diego, California
(Received 15 December 1969)

Improved results on the total photo-ionization cross section of atomic helium are given. The total cross section includes contributions from simultaneous ionization and excitation of He^+ and double ionization; the combined effect of these two processes adds about 10% to the normal photo-ionization process where He^+ is left in the ground state. A lower abundance (10.92 based on 12.00 for hydrogen) is adopted for helium, based on recent radio determinations. In calculating the opacity due to K-shell photo-ionization of heavy elements, a lower abundance (8.00) is also adopted for neon. Brief mention is made of the effects of irregularities in the density distribution of the interstellar gas on the problems of both x-ray and radio-wave absorption.

I. INTRODUCTION

THE relevant atomic parameters for x-ray absorption by the interstellar gas have been computed by a number of authors in recent years. The first paper on this subject that was cited extensively was that of Strom and Strom.[1] Felten and Gould,[2] using more up-to-date (at that time) abundance determinations, gave a somewhat more accurate treatment of the problem, which was subsequently improved further by Bell and Kingston.[3] The principal contribution of the latter authors was the use of a more accurate cross section for the photo-ionization of atomic helium at x-ray energies.[4] In the present paper we outline the results of a still further improvement of the calculation of σ_{He}; these calculations are described in Sec. II. The other significant difference between our calculation of the general x-ray opacity of the interstellar gas and that in previous papers lies in the abundances we adopt for helium and neon. We believe that in the past few years the observational situation on this question has undergone some change in that a lower helium abundance and a considerably lower abundance of neon seems to be indicated at present. Since neon was previously thought to be the main contributor to the absorption above 0.9 keV, a lower assumed abundance will affect the calculated opacity proportionally. Also, below 0.53 keV, helium is the main contributor to absorption, so again a lower assumed abundance will be very significant. A more detailed discussion of these points and other questions bearing on the general results will be given in Secs. IV and V.

* Operated by Associated Universities, Inc., under contract with the National Science Foundation.

[1] S. E. Strom and K. M. Strom, Publ. Astron. Soc. Pacific **73**, 43 (1961).

[2] J. E. Felten and R. J. Gould, Phys. Rev. Letters **17**, 401 (1966).

[3] K. L. Bell and A. E. Kingston, Monthly Notices Roy. Astron. Soc. **136B**, 241 (1967).

[4] K. L. Bell and A. E. Kingston, Proc. Phys. Soc. (London) **90A**, 31 (1967).

II. HELIUM PHOTO-IONIZATION CROSS SECTION

Applicability of the Born Approximation

While detailed calculations of the helium photo-ionization cross section have been available for many years,[5] such results have not been widely applied to problems in x-ray astronomy because it was thought that in the energy domain of interest, $h\nu \sim 1$ keV, the Born approximation provided an adequate representation of the cross section in a concise form.[1,2] However, this impression has been shown to introduce substantial error by Salpeter and Zaidi (SZ),[6] who noted that the exact expression for the cross section is

$$\sigma_{\rm exact} = \sigma_{\rm Born} f_{\rm corr}, \tag{1}$$

where the correction factor is

$$f_{\rm corr} = 1 - 2\pi/E^{1/2} + C_2/E + \dots, \tag{2}$$

where E is in rydbergs. By comparison with the accurate expression for the cross section evaluated using a Coulomb wave function for the outgoing electron, they also obtained the coefficient C_2 of the $1/E$ term in the Born series. This second Born coefficient was also found to be quite large (~ 24) but is probably not very accurate.

The main point to be made here is that not only is the Born series only in inverse powers of the square root of the energy, but the coefficients in the expansion are large. Thus, surprisingly, at the relevant energies (~ 1 keV) for x-ray-absorption Born-type formulas are not very accurate. This is unfortunate because SZ developed methods for computing very simply the Born-limit photo-ionization cross section summed over the atomic states (including the continuum) of the residual species He^+. Since we are mainly interested in the helium cross section for photon energies around a

[5] S. S. Huang, Astrophys. J. **108**, 354 (1948).

[6] E. E. Salpeter and M. H. Zaidi, Phys. Rev. **125**, 248 (1962).

few tenths of a keV where the Born series converges very slowly, we have not used the SZ results. Instead, we have computed all cross sections using Coulomb wave functions for the outgoing electron.

Excited-State and Double-Ionization Contribution

We shall designate by σ_0 the cross section for photo-ionization of He with the residual ion left in the ground $1s$ state. Many calculations of σ_0 have been made, the most recent of which is that of Bell and Kingston,[4] who used a 20-parameter Hylleraas wave function for the ground state of He and a polarized-orbital continuum wave function for the system $He^+ - e$. However, at x-ray energies their results differ very little from that obtained using a six-parameter Hylleraas wave function for He and a shielded Coulomb wave function for the ejected electron if the momentum matrix element is employed. Polarization effects are more important near threshold where the electron is ejected with a low velocity.[7] At "moderate" energies like a few tenths of a keV the much simpler effects of the Coulomb field felt by the ejected electron are much more important than the polarization effects; we have already emphasized the slow convergence of the Born series due to these Coulomb effects.

A significant correction to the total absorption cross section results from the inclusion of the contribution from photo-ionization wherein He^+ is left in excited states or the continuum (double ionization). This effect, which was not considered by Bell and Kingston, adds about 10% to the cross section σ_0. The contribution from such "double jumps" is especially important for an atom like He where the "Hartree field" is very different before and after one electron makes a transition. The threshold for these effects is, of course, well below x-ray energies. While the correction ($\sim$10%) is perhaps less than the uncertainty in the helium abundance, errors in abundance determinations have been decreasing and a total helium absorption cross section with an error of no more than a few percent seems to be called for at the present time.

We present here the results of calculations of these additional contributions to the total helium photo-ionization cross section: full details may be found elsewhere.[8,9] We designate the total cross section for photo-ionizing He with He^+ left in an excited state by $\sigma_{\text{ion+ex}}$ and the cross section for double ionization by $\sigma_{2-\text{ion}}$. Double photo-ionization of He has previously been computed by Byron and Joachain,[10] who used a special convenient wave function for the initial ground state and unshielded ($Z=2$) Coulomb wave functions for the ejected electrons. Their calculations have been repeated using, instead, a more accurate six-parameter Hylleraas wave function for He and the result is very similar.[8] It is presented in Fig. 1 as a percentage of the total cross section. It should be mentioned that these calculations agree to within the experimental uncertainty with the results of Carlson.[11]

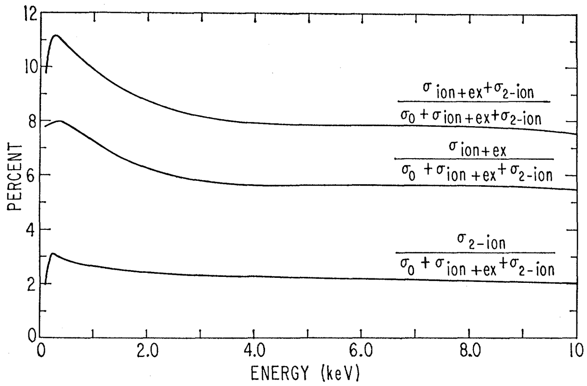

FIG. 1. Cross sections for simultaneous ionization and excitation (ion+ex) and double ionization (2-ion) of atomic helium in terms of the total photo-ionization cross section.

Simultaneous photo-ionization and excitation of the $2s$ state of He^+ has been computed by Dalgarno and Stewart[12] and Salpeter and Zaidi.[6] These calculations have been repeated and extended[9] to include other ns as well as np states of He^+; for these calculations a Byron-Joachain wave function[10] was used for He and a screened ($Z=1$) Coulomb wave function for the ejected electron. Transitions $(1s^2)$-$(ns,\epsilon p)$ and $(1s^2)$-$(np,\epsilon s)$ with $n=2$ to 10 were included and the total is plotted in Fig. 1. The main contribution here comes from $(1s^2)$-$(2s,\epsilon p)$ and the result is about 5% smaller than Salpeter and Zaidi's. The contribution from transitions to $(np,\epsilon s)$ is only about 1% of that to $(ns,\epsilon p)$ for $n=2$ to 10. Transitions to higher n states ($n=3$ to 10) and continuum p states contribute only about 20% of the total from $n=2$ to 10. The results for $(1s^2)$-$(2s,\epsilon p)$ agree well with experiments by Samson[13] and Carlson[11] as demonstrated elsewhere.[9]

In Fig. 1 the total contribution from simultaneous ionization and excitation and from double ionization is plotted. This represents the relevant correction to the σ_0 cross section and can be regarded to be fairly well established by experiments.

Total Cross Section

Experimental results on the total photo-ionization for helium ($\sigma_0+\sigma_{\text{ion+ex}}+\sigma_{2-\text{ion}}$) were summarized by

[7] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic, New York, 1957).

[8] R. L. Brown, Phys. Rev. A **1**, 586 (1970).

[9] R. L. Brown, Phys. Rev. A **1**, 341 (1970).

[10] F. W. Byron, Jr., and D. J. Joachain, Phys. Rev. **164**, 1 (1967). Coulomb wave functions with $Z=2$ are appropriate since predominantly in this process one of the electrons goes off at low energy while the other electron is ejected with a high energy (for which Coulomb effects are at least not *very* large). For the same reason, correlation effects are unimportant for the outgoing electrons, as Byron and Joachain show.

[11] A. T. Carlson, Phys. Rev. **156**, 142 (1966). See also Ref. 9.

[12] A. Dalgarno and A. L. Stewart, Proc. Phys. Soc. (London) **76**, 49 (1960).

[13] J. A. R. Samson, Phys. Rev. Letters **22**, 693 (1969).

Bell and Kingston.[4] The addition of $\sigma_{\text{ion+ex}}+\sigma_{2-\text{ion}}$ certainly improves the agreement with theory[9]; in fact, the old measurement by Dershem and Schein[14] at 44 Å agrees very well with the total theoretical cross section. The measurements by Lukirskii, Brytov, and Zimkina[15] indicate a higher value for the total cross section at this energy, however, but their data seem to have a lot of scatter. Measurements by Bearden[16] in the keV range are not very significant because of large experimental errors due to contaminants.

Helium Abundance

We adopt a helium logarithmic abundance of 10.92 based on the very recent work of Palmer *et al.*[17] This is a determination from rf recombination lines (high n-value transitions following recombination) from gaseous nebulae, and is probably the best measurement to date of the helium abundance in the interstellar gas. For a discussion of other observational determinations of the helium abundance the reader is referred to the paper by Palmer *et al.*[17] This abundance is about half as large as that assumed by Felten and Gould[2] and Bell and Kingston.[3] Thus our results for the relative contribution of helium are quite different.

In Fig. 2 we plot the total helium photo-ionization cross section together with that for hydrogen and He^+ (the latter may be calculated exactly[7]) as a function of the energy of the incoming photon. To facilitate interpolation from this figure we have actually plotted $[E/(1\text{ keV})]^3\sigma$, which is a more slowly varying quantity than σ, multiplied in the case of helium by the additional factor of the abundance ratio N_{He}/N_H so that the cross section is *per hydrogen atom*. We adopt a logarithmic helium abundance of 10.92 based on 12.00 for hydrogen (see Sec. II).

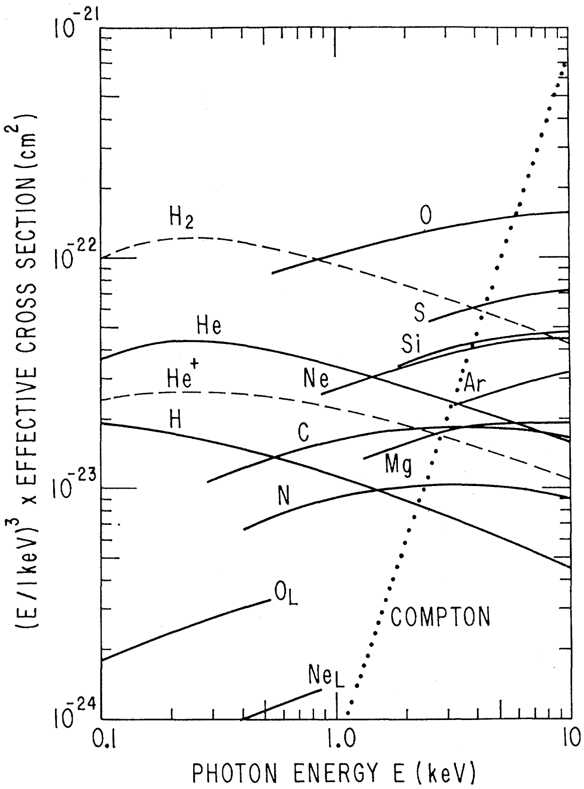

FIG. 2. Effective photo-ionization cross sections per hydrogen atom ($\sigma\times$abundance relative to hydrogen) multiplied by $[E/(1\text{ keV})]^3$ as a function of incident photon energy. The cross section for the case where helium is singly ionized is indicated as a dashed line. The molecular hydrogen cross section (per hydrogen *atom*) is also plotted as a dashed line. Of the heavy elements, below the K edge only the L-shell contributions of oxygen and neon are plotted. For all the heavy elements, however, above the K edge, the cross section plotted represents the total from the $K+L+M$ shells. Abundances employed are given in the text in Sec. IV. The effective cross section per hydrogen atom for Compton scattering is indicated by a dotted line and represents the total contribution from hydrogen and helium.

III. MOLECULAR HYDROGEN

While for the general interstellar gas it now appears that only a very small fraction of the hydrogen is in molecular form, in some dense regions hydrogen may be predominantly molecular. Therefore some instances may arise where it would be necessary to consider x-ray attenuation by the molecular species. Calculations on molecules are much more difficult than those for atoms and as such are subject to greater uncertainty, but the cross section for photo-ionization of the hydrogen molecule from the ground electronic and vibrational state has been well determined.[18] Several simplifying assumptions were employed to facilitate calculation; in particular, the ground $({}^1\Sigma_g{}^+)$ state of H_2 was represented by the Weinbaum approximation[19] and the final state of the system, taken to be that of a free electron and an $H_2{}^+$ ion in the $1s\sigma_g$ ground electronic state, was represented by the wave function tabulated by Bates, Ledsham, and Stewart.[20] From this and similar calculations[21] one obtains a value $\sigma(H_2)/\sigma(He)\sim 0.45$ in the energy domain of current interest. We have assumed that this ratio is independent of energy, that is, that the relative magnitude of the Born corrections for H_2 are the same as for He, and have plotted the result in Fig. 2. We plot the cross section *per hydrogen atom* or half the molecule cross section. There seems to have been some confusion in the past[3] on this point with respect to the interpretation of the data of Bearden.[16]

[14] E. Dersham and M. Schein, Phys. Rev. **37**, 1283 (1931).
[15] A. P. Lukirskii, I. A. Boytov, and T. M. Zimkina, Opt. i Spectroskopiya **17**, 438 (1964) [Opt. Spectry. (USSR) **17**, 234 (1964)].
[16] A. J. Bearden, J. Appl. Phys. **37**, 1681 (1966).
[17] P. Palmer, B. Zuckerman, H. Penfield, A. E. Lilley, and P. G. Mezger, Astrophys. J. **156**, 887 (1969).
[18] M. R. Flannery and U. Öpik, Proc. Phys. Soc. (London) **86**, 491 (1965).
[19] J. Weinbaum, J. Chem. Phys. **1**, 593 (1933).
[20] D. R. Bates, D. Ledsham, and A. L. Stewart, Phil. Trans. Roy. Soc. London **A246**, 215 (1953).
[21] M. Shimizu, J. Phys. Soc. Japan **15**, 1440 (1960).

In any case Bearden's data on H_2 and He cannot be relied on, especially at high energy, due to effects of contaminants and contributions from Compton scattering. However, this results at photon energies of 2.984, 3.444, and 4.510 keV are at least consistent with the above cross-section ratio.

IV. HEAVY ELEMENTS

By "heavy elements" we mean especially species like oxygen which is the most abundant element other than hydrogen and helium. For these species the main contribution to the opacity comes from K-shell photo-ionization; for oxygen, L-shell photo-ionization accounts for less than 10% of the atom's total cross section above the K-shell threshold. As in the work of Felten and Gould[2] we rely heavily on experimental results for cross sections and include the elements C, N, O, Ne, Mg, Si, S, and Ar. Recent calculations generally agree with the data to within the limits set by experimental uncertainty at these x-ray energies.[13] However, instead of assuming an $E^{-8/3}$ energy dependence for the cross section and fitting to the best experimental point above threshold, we have fitted all the data on each element to a least-squares adjusted polynomial. This results in a more realistic energy dependence for the cross section, steepening from the $E^{-8/3}$ dependence just above threshold toward the $E^{-7/2}$ Born-limit dependence. For C, N, O, Ne, and Ar, our sources of data are the works of Bearden,[16] Samson,[13] Wuilleumier,[22] and Henke, White, and Lundberg.[23] For Mg, Si, and S we have made use of the work of Guttman and Wagonfeld[24] whereby the cross section for the element is interpolated from values for other elements of adjacent Z. We consider the contribution from the heavy elements only above the K threshold. Below threshold we neglect the L-shell contribution except for O and Ne for which the data are taken from Ref. 23.

The main problem with the heavy elements lies in the adoption of abundances for the species. We have assumed the following logarithmic abundances (based on 12.00 for hydrogen): Element $(\log_{10}N) = \mathrm{H}(12.00)$, He(10.92), C(8.60), N(8.05), O(8.95), Ne(8.00), Mg(7.40), Si(7.50), S(7.35), Ar(6.88). With the exception of helium and neon these abundances are the same as those employed by Felten and Gould and Bell and Kingston. The old value (8.70) taken for neon now seems much too high. Recent observational work, especially that of Peimbert and Costero[25] on the Orion nebula, seems to point to a lower value for the interstellar gas and we have adopted the value 8.00, realizing that it could easily be off by ± 0.5 (in the logarithm). Neon is a difficult element for abundance determinations. One of the problems is that there are no collisionally excited optical lines from singly-ionized neon, and Ne^+ should be one of the dominant stages of ionization of the element in gaseous nebulae. Here the recent observations by Gillett and Stein[26] of the 12.8-μ line from the planetary nebula IC 418 are very relevant. This line, which originates from a fine-structure transition in Ne^+, was detected at only about one-tenth the strength predicted[27] on the assumption of the higher abundance 8.70. However, planetary nebulae may not be representative of the interstellar gas, so it is not clear how much weight to give to this result in relation to the problem at hand. Determinations of the neon abundance in stellar atmospheres[28] give a high value; it is found that $\mathrm{Ne/O} \approx 0.9$. Here again there is a large uncertainty, since a fairly detailed stellar-atmosphere model must be employed in the analysis. The value for cosmic-ray primaries is[29] $\mathrm{Ne/O} \approx 0.3$, but this number is very susceptible to revision. The general situation on the neon abundance is really not clear. Most astronomers tend to put more weight to determinations from gaseous nebulae and would probably favor the lower abundance (8.00) we adopt. Hopefully, future observations of the infrared 12.8-μ line will clarify the problem.

Cross sections for the heavy elements are given in Fig. 2. Actually the "effective" cross section

$$\sigma_e = (N_i/N_{\mathrm{H}})[E/(1\ \mathrm{keV})]^3 \sigma_i \qquad (3)$$

is plotted where N_i/N_{H} is the abundance with respect to hydrogen and the $[E/(1\ \mathrm{keV})]^3$ factor is inserted just to flatten the curves.

V. GENERAL RESULTS AND DISCUSSION

Some mention should be made here of the effects of Compton scattering which dominate photoelectric absorption above about 8 keV. In Fig. 2 we have plotted as a dotted line the total effective Compton cross section for hydrogen plus helium (He adds about 15% to the H contribution). The simple Thomson cross section was used per atomic electron; effects of binding have been ignored.[30] As can be seen in the figure, compared with photoelectric absorption Compton scattering contributes very little (except at high energies); in fact, the attenuation along the path from the other side of the galaxy in the plane would amount to only a few percent. This we shall ignore its effects.

Excluding Compton scattering, we have plotted, now on a semilog graph, the total effective cross section $(\times E^3)$ in Fig. 3, showing the jumps due to the various K edges. By far the biggest jump is the oxygen K edge

[22] F. Wuilleumier, J. Phys. (Paris) **26**, 776 (1965).
[23] B. L. Henke, R. White, and B. J. Lundberg, J. Appl. Phys. **28**, 98 (1957).
[24] A. J. Guttman and H. Wagonfeld, Acta Cryst. **22**, 335 (1967).
[25] M. Peimbert and R. Costero, Tonanzintla Bull. No. 31, 1969 (unpublished).
[26] F. C. Gillett and W. A. Stein, Astrophys. J. **155**, 197 (1969).
[27] T. N. Delmer, R. J. Gould, and W. Ramsay, Astrophys. J. **149**, 495 (1967).
[28] L. H. Aller, *Abundance of the Elements* (Interscience, New York, 1961).
[29] W. R. Webber, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1967), Vol. 46, Chap. 2.
[30] A brief discussion of this point may be found in Ref. 6.

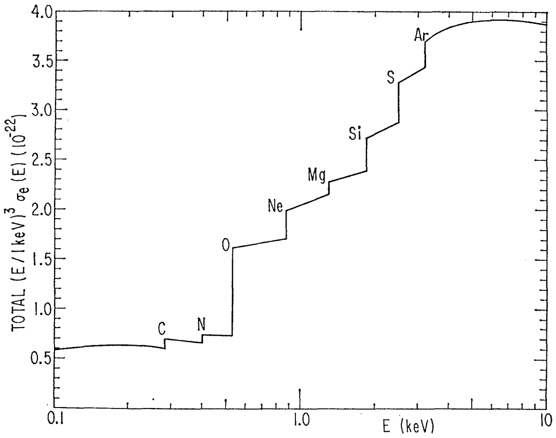

FIG. 3. Total photo-ionization cross section per hydrogen atom $[\times(E/1\text{ keV})^3$ in units $10^{-22}\text{ cm}^2]$ as a function of incident photon energy. The elements responsible for the jumps due to their respective K edges are indicated. The energies of these K thresholds have been tabulated in Ref. 2.

at 0.53 keV at which the effective cross section increases by about a factor of 2. The jump at the neon K edge at 0.87 keV is only about 10% as are the edges due to silicon and sulfur.

For the attenuation of x rays through the interstellar gas we could perhaps take the model in which the spatial dependence of the gas density is only in the z direction (normal to the galactic plane). With our position approximately in the plane, the absorption optical depth to a source at galactic latitude b_s and distance above the plane z_s would then be

$$\tau_s(E)=\sigma_e(\mathrm{H})n_H(0)(\csc b_s)\int_0^{z_s} a(z)dz, \qquad (4)$$

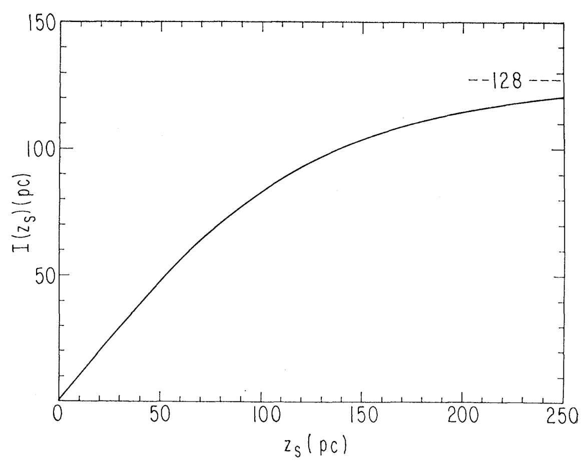

FIG. 4. The integral $I(z_s)=\int_0^{z_s} a(z)dz$ introduced in Eq. (4). (pc=parsec.)

where $\sigma_e(E)$ is the total effective cross section per hydrogen atom (from Fig. 3), $n_{\mathrm{H}}(0)$ is the total density of hydrogen (ionized or not)[31] in the plane, and $a(z)$ gives the z dependence of the gas density [normalized to $a(0)=1$]. The integral in Eq. (4) has been computed before from observational data in connection with work on radio-wave absorption[32]; it is reproduced here as Fig. 4. The value of $n_{\mathrm{H}}(0)$ is, including about 0.1 cm^{-3} from ionized matter, approximately 0.8 cm^{-3}.

Effects of a deviation from a smooth spatial gas density may be large, however. Bowyer, Field, and Mack,[33] from observations of the galactic latitude dependence of the x-ray background at 0.25 keV, concluded that the attenuation is only about one-third as large as it should be.[34] This effect has been interpreted by Bowyer and Field[35] to result from the discrete cloud structure of the interstellar gas. The idea here is that with a detector of finite angular resolution radiation is predominantly received from directions within the reception cone where the absorption optical depth is a minimum. Essentially the same type of effect has been suggested to explain radio observations.[32] Radio absorption takes place in the ionized interstellar gas which should have a more irregular spatial distribution than the neutral gas, so the magnitude of the effect for radio absorption should be larger than for x-ray absorption if the receiving beamwidths are the same.[36] There seems to be some weak evidence of the existence of such an effect for radio absorption[32]; however, some doubt has been cast concerning the effect for radio absorption.[37]

ACKNOWLEDGMENTS

We should like to thank, in particular, J. E. Felten and G. B. Field for reading an early draft of this paper and making some helpful comments. We have also benefited from conversations with J. C. Y. Chen and J. S. Miller. This research was supported by NASA (Grant NSG-357). R. J. G. thanks the Alfred P. Sloan Foundation for fellowship support during the later stages of this work.

[31] In considerations of x-ray absorption at energies above the oxygen K edge (0.53 keV) the total hydrogen density is the relevant parameter. However, at lower energies where H and He are the principal absorbers the ionization conditions do affect the opacity.

[32] R. J. Gould, Australian J. Phys. **22**, 189 (1969).

[33] C. S. Bowyer, G. B. Field, and J. E. Mack, Nature **217**, 32 (1968).

[34] This was on the basis of a larger helium abundance ($\log_{10} N_{\mathrm{He}}=11$), however.

[35] C. S. Bowyer and G. B. Field, Nature **223**, 573 (1969).

[36] If, however, the radio absorption takes place in the cool partially ionized HI clouds, the magnitude of the effect would be about the same for radio and x-ray absorption. For a detailed discussion of this possibility the reader is referred to Ref. 32.

[37] A. H. Bridle, Nature **221**, 648 (1969).