show a great deal of improvement over Paper I, while the P waves, with the exception of $3P_2$, get worse but not alarmingly so. The best over-all fit requires $g_{\epsilon NN}{}^2/4\pi \approx 13.5$ with a cutoff at 0.4 F. Our conclusion is then, that the addition of the ϵ meson with a coupling constant $g_{\epsilon NN}{}^2/4\pi \simeq 13.5$ leads in general to improvements over our results of Paper I. A fitting of the experimental results using suitable minimization techniques could possibly make the agreement even better. Finally, a fit including a $T=1$ scalar meson could also be attempted, although it is likely that that would take us too far from the framework of Sudarshan's theory in which our calculation is based.

ACKNOWLEDGMENT

Thanks are due Professor E. C. G. Sudarshan for many helpful discussions.

PHYSICAL REVIEW D VOLUME 1, NUMBER 7 1 APRIL 1970

Consequences of Unitarity in Some Models of *CP* Violation*

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(Received 11 June 1969; revised manuscript received 10 December 1969)

Using the model-dependent assumption of 2π saturation of the unitarity sum, we find estimates for $|\eta_{00}/\eta_{+-}|$ and $\mathrm{Re}\epsilon/|\eta_{+-}|$ for a certain class of theories. Two models are tested, and $|\eta_{00}/\eta_{+-}|$ is found to be different from the originally estimated value, which was based on $\mathrm{Re}\epsilon$ as input.

THE directly measured CP-violation parameters in the $K_L{}^0 \to 2\pi$ and $K_L{}^0 \to \pi l\nu$ decay experiments are η_{+-}, η_{00}, and $\mathrm{Re}\epsilon$.[1] In most theories of CP violation which have been proposed to explain the $K_L{}^0$-decay experiments, the only quantity rigorously predicted by the theory is the order of magnitude of the symmetry-breaking effect. Only the so-called superweaklike models[2] find strict values for $|\eta_{00}/\eta_{+-}|$, ϕ_{+-}, ϕ_{00}, and $\mathrm{Re}\epsilon$. When dealing with other theories, it thus appears as if much freedom is allowed for the actual values of these parameters, and estimates are sometimes based on the use of the experimental value of one of them as input ($\mathrm{Re}\epsilon$ in general).

It is the purpose of this note to show that the unitarity condition[3] can be used to provide estimates for $|\eta_{00}/\eta_{+-}|$ as well as $\mathrm{Re}\epsilon/|\eta_{+-}|$ in the framework of a certain class of CP-nonconservation models. This, of course, is in conflict with the use of $\mathrm{Re}\epsilon$ as input and leads to different results for models where that was done originally.

Following the standard phenomenological analysis of the K^0-$\bar{K}^0$ system,[4] let us denote the matrix element of the decay of K^0 into a two-pion standing-wave state with isospin I by $|A_I|e^{i\phi_I}$. The short- and long-lived neutral-kaon states are written in terms of the eigenstates of hypercharge as

$$|K_{S,L}{}^0\rangle = [2(1+|\epsilon_0|^2)]^{-1/2}\{(1+\epsilon_0)|K^0\rangle \pm (1-\epsilon_0)|\bar{K}^0\rangle\}. \quad (1)$$

Unlike Wu and Yang,[4] who choose $\phi_0=0$, we use the phase convention in which ϵ_0 is real. This is the phase convention in which the CP-violation phases ϕ_I are measurable quantities.[5]

Using the approximate $|\Delta I|=\frac{1}{2}$ rule for $K_S{}^0\to 2\pi$ and the smallness of the observed CP-violation effect, one finds

$$\eta_{+-} \approx \epsilon_0 + i\phi_0 + (|A_2|/|A_0|\sqrt{2})\phi_2 e^{i\delta},$$
$$\eta_{00} \approx \epsilon_0 + i\phi_0 - (|A_2|\sqrt{2}/|A_0|)\phi_2 e^{i\delta}, \quad (2)$$

where $\delta = \frac{1}{2}\pi + \delta_2 - \delta_0$.

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* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Grant No. EOOAR-68-0010, through the European Office of Aerospace Research.

† Sponsored in part by the National Science Foundation University Science Development Project.

[1] A survey of the experimental situation relevant to CP violation, which contains the definition of the various parameters, has been given by J. Steinberger at the CERN Topical Conference on Weak Interactions, Geneva, 1969 (unpublished).

[2] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964). The superweaklike models are those theories which predict $|\epsilon'| \ll |\epsilon|$.

[3] J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965*, edited by M. Alston-Garnjost (University of California Press, Berkeley, Calif., 1967).

[4] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).

[5] G. Charpak and M. Gourdin, lectures delivered at the Matscience Institute, Madras, India, 1966 and 1967, p. 54 (unpublished).

The unitarity condition[3] can be written in the following form:

$$\epsilon_0(\Gamma_S+2i\Delta M)=\sum_n \langle n|T|K_S{}^0\rangle^*\langle n|T|K_L{}^0\rangle, \quad (3)$$

where the sum extends over all open channels (including phase space), $\Delta M=M_L-M_S$, and Γ_S is the total decay rate of $K_S{}^0$.

We now *assume* that the 2π contributions dominate the right-hand side of Eq. (3). This condition is obviously satisfied in any model in which CP is violated only (or predominantly) in the 2π decay mode of K^0. From this model-dependent assumption, one obtains

$$\epsilon_0\approx\phi_0+|A_2/A_0|^2\phi_2. \quad (4)$$

Denoting by r (y) the ratio between the $I=2$ and $I=0$ CP-conserving (violating) amplitudes, we obtain from Eqs. (2) and (4)

$$\begin{aligned}\eta_{+-}&\approx\phi_0(1+ry+i+2^{-1/2}ye^{i\delta}),\\ \eta_{00}&\approx\phi_0(1+ry+i-2^{1/2}ye^{i\delta}),\\ \epsilon_0&\approx\phi_0(1+ry).\end{aligned} \quad (5)$$

These relations provide estimates of the ratios $|\eta_{00}/\eta_{+-}|$ and $\epsilon_0/|\eta_{+-}|$ in terms of a single model-dependent parameter y. For the $\pi\pi$ phase shifts and the parameter r, which measures the validity of the $|\Delta I|=\frac{1}{2}$ rule in $K_S{}^0\to 2\pi$ decays, we may use the following values[6]:

$$\begin{aligned}\delta_2-\delta_0&=-(50\pm20)^\circ,\\ r&=0.06.\end{aligned} \quad (6)$$

Let us now examine two such models of CP violation which have been proposed recently.

In a mode presented by Delaney and Welling,[7] CP violation is introduced in the effective three-particle weak interaction of pseudoscalar mesons, namely, only in the 2π decay mode of K^0. In this model, the $|\Delta I|=\frac{1}{2}$ and $|\Delta I|>\frac{1}{2}$ CP-violating amplitudes are found to be of comparable strength:

$$y=1.2. \quad (7)$$

The authors, who seem to overlook the consequences of the unitarity sum rule, use the charge asymmetry parameter ϵ_0 as input,[8] and obtain

$$|\eta_{00}/\eta_{+-}|=1.5. \quad (8)$$

On the other hand, we obtain from Eqs. (5)–(7)

$$|\eta_{00}/\eta_{+-}|<0.3, \quad (9)$$

which seems to be inconsistent with the present experimental information.[1]

In a model suggested by Yun,[9] CP is explicitly violated only in the two-pion decay mode of the K meson, through C nonconservation in the electromagnetic interaction. The relative strength of the $|\Delta I|>\frac{1}{2}$ to $|\Delta I|=\frac{1}{2}$ $K\to2\pi$ amplitudes is smaller for the CP-violating transitions than for the CP-conserving ones:

$$y<r. \quad (10)$$

Thus, while Yun obtains—again taking ϵ_0 as input—

$$|\eta_{00}/\eta_{+-}|=1.23, \quad (11)$$

which is substantially distinguishable from the superweak prediction,[2] we would expect [from Eqs. (5), (6), and (10)] the equality $|\eta_{00}|=|\eta_{+-}|$ to hold up to 6% in this theory.

[6] J. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 281. The quantitative conclusions drawn from Eqs. (5) are insensitive to possible deviations from the above-quoted values.

[7] R. M. Delaney and D. J. Welling, Phys. Rev. **176**, 1841 (1968).

[8] The three measured values of the charge asymmetry (Ref. 1), although consistent with each other, still provide only a crude estimate of ϵ_0.

[9] S. K. Yun, Phys. Rev. **178**, 2439 (1969).