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Minimal Gauge-Invariant One-Vector-Meson-Exchange Model for Associated Δ and Charged ρ Photoproduction*

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A gauge-invariant one-vector-meson-exchange (OVE) model is constructed for the process $\gamma+N\rightarrow\rho^{\pm}+\Delta$. The form of the model is obtained by selecting the simplest extension of the OVE diagram consistent with gauge invariance. The $\Delta\rho N$-coupling strength is considered and discussed in terms of vector-meson dominance. It is found that the interference between the OVE and one-pion-exchange amplitudes vanishes.

I. INTRODUCTION

EVIDENCE for the reaction $\gamma+p\rightarrow\rho+\Delta$ has been found[1–6] in the process $\gamma+p\rightarrow p+\pi^{+}+\pi^{-}+\pi^{0}$. The one-pion-exchange (OPE) mechanism with absorption[2,3,7] has provided a qualitative description of the present experimental data for the unpolarized differential cross section, but fails to predict consistent decay correlation coefficients.[3]

We consider in this paper the formulation of a gauge-invariant model to study the contribution of the one-vector-meson-exchange (OVE) mechanism to the process. Although it is unlikely that vector-meson exchange would dominate the process, it can be expected to make an important contribution in the forward direction. The importance of the contribution might be enhanced by the weakness of the $\gamma\rho\pi$ coupling.

An OVE model also provides the attractive possibility of obtaining information regarding the electromagnetic moments and therefore the internal structure of the charged ρ mesons. The feasibility of determining the electromagnetic moments of the ρ through an OVE model has been discussed in an earlier paper[8] by the author for the process $\gamma+n\rightarrow\rho^{-}+p$.

In Sec. II we present a summary of the kinematics and our choice of conventions. In Sec. III we construct the OVE model and note that the interference between the OVE and OPE amplitudes vanishes, thereby allowing the incoherent addition of the OVE and OPE contributions.

II. KINEMATICS

The process $\gamma+N\rightarrow\rho^{\pm}+\Delta$ is represented diagrammatically and the corresponding momenta and helicities are labeled in Fig. 1. The notation for the polarization vectors, spinors, and other kinematical parameters is summarized in Table I. Our metric, normalization, and γ-matrix conventions are those of Bjorken and Drell.[9] The Rarita-Schwinger spinors[10] for the spin-$\frac{3}{2}$ Δ particles are characterized by the following normalization, completeness relation, and subsidiary conditions:

$$\bar{u}_{\sigma}(p\lambda)u^{\sigma}(p\lambda')=\delta_{\lambda\lambda'}\,, \tag{1}$$

$$\sum_{\lambda}u_{\sigma}(p\lambda)\bar{u}_{\phi}(p\lambda)=-\frac{\not{p}+M_{\Delta}}{2M_{\Delta}}\Gamma_{\sigma\phi}(p,M_{\Delta})\,, \tag{2}$$

where

$$\Gamma_{\sigma\phi}(p,M_{\Delta})=g_{\sigma\phi}-\tfrac{1}{3}\gamma_{\sigma}\gamma_{\phi}-\tfrac{2}{3}\frac{p_{\sigma}p_{\phi}}{M_{\Delta}^{2}}+\tfrac{1}{3}\frac{p_{\sigma}\gamma_{\phi}-\gamma_{\sigma}p_{\phi}}{M_{\Delta}}\,, \tag{3}$$

$$(\not{p}-M_{\Delta})u_{\sigma}(p,\lambda)=0\,, \tag{4}$$

$$u_{\sigma}(p\lambda)\gamma^{\sigma}=0\,, \tag{5}$$

$$u_{\sigma}(p\lambda)p^{\sigma}=0\,. \tag{6}$$

The invariants have the standard form

$$s=(p_i+k)^2=(p_f+q)^2\,, \tag{7}$$

$$t=(q-k)^2=(p_i-p_f)^2\,, \tag{8}$$

$$u=(p_i-q)^2=(p_f-k)^2\,. \tag{9}$$

The covariant transition matrix for the process can be written in the general form

$$\mathfrak{M}_{fi}=\zeta_{\nu}^{*}(q\lambda_q)\bar{u}_{\sigma}(p_f\lambda_f)I^{\sigma\nu\mu}u(p_i\lambda_i)\epsilon_{\mu}(k\lambda_k)\,. \tag{10}$$

The requirement of gauge invariance imposes a familiar condition on the form of Eq. (10):

$$0=\zeta_{\nu}^{*}(q\lambda_q)\bar{u}_{\sigma}(p_f\lambda_f)I^{\sigma\nu\mu}u(p_i\lambda_i)k_{\mu}\,. \tag{11}$$

TABLE I. Kinematical notation.

Particle	Four-momentum	Helicity	Polarization vector or spinor	Mass
Nucleon	p_i	λ_i	$u(p_i\lambda_i)$	M
Δ baryon	p_f	λ_f	$\bar{u}_{\sigma}(p_f\lambda_f)$	M_{Δ}
Photon	k	λ_k	$\epsilon_{\mu}(k\lambda_k)$	0
ρ meson	q	λ_q	$\zeta_{\nu}^{*}(q\lambda_q)$	m

* Supported in part by the National Science Foundation.
[1] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Nuovo Cimento **41**, 270 (1966) (phase-space plots).
[2] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Nuovo Cimento **48**, 26 (1967) (total cross section).
[3] Cambridge Bubble Chamber Group, Phys. Rev. **169**, 180 (1968) (relative yields and differential cross sections).
[4] Y. Eisenberg *et al.*, Phys. Rev. Letters **22**, 669 (1969).
[5] J. Ballam *et al.*, Bull. Am. Phys. Soc. **14**, 517 (1969).
[6] J. Ballam *et al.*, Phys. Rev. D **1**, 94 (1970).
[7] K. Schilling (private communication to the Cambridge and ABBHHM groups).
[8] R. B. Clark, Phys. Rev. **187**, 1993 (1969).
[9] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[10] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

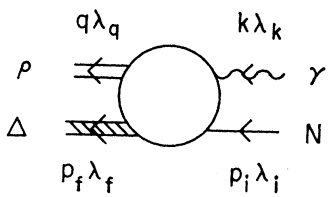

FIG. 1. Schematic diagram for the process $\gamma+N\rightarrow\rho+\Delta$.

The determination of the form of $I^{\sigma\nu\mu}$ from an OVE model which is consistent with Eq. (11) will be the object of the next section.

III. MODEL

The philosophy behind our construction of the OVE model for the general $\gamma+N\rightarrow\rho+\Delta$ process is to find the simplest extension of the amplitude representing the OVE diagram which is consistent with gauge invariance. The first step is to consider the amplitude representing the OVE diagram shown in Fig. 2(a). We then encounter the problem of finding the minimal set of additional terms necessary to ensure gauge invariance.

A. Vertex Expressions

The expressions corresponding to the $\rho\gamma\rho$ vertex[11] and the $\Delta\rho N$ vertex,[12] which are represented in Fig. 3,

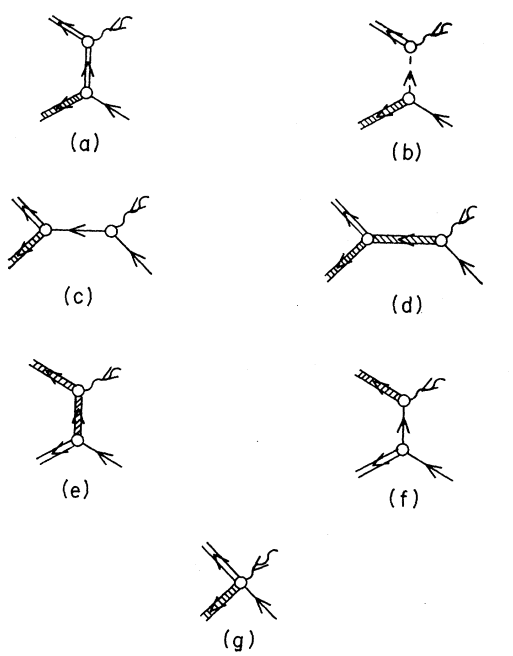

FIG. 2. Diagrammatic representation of the Born terms. (a) OVE; (b) OPE; (c) s-channel nucleon pole; (d) s-channel baryon pole; (e) u-channel baryon pole; (f) u-channel nucleon pole; (g) contact term.

[11] V. Glaser and B. Jaksic, Nuovo Cimento **5**, 1197 (1957); L. Durand III, Phys. Rev. **123**, 1393 (1961); W. K. Tung, *ibid.* **139**, B547 (1965).

[12] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963); A. J. Dufner and Y. S. Tsai, Phys. Rev. **168**, 1801 (1968).

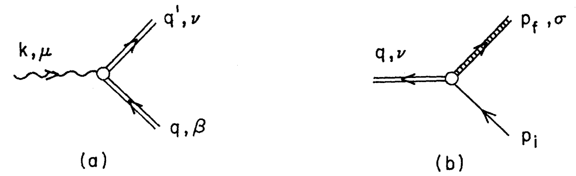

FIG. 3. Vertex diagrams for (a) the $\rho\gamma\rho$ vertex and (b) the $\Delta\rho N$ vertex.

can be written in the form

$$\mathcal{V}_{\rho\gamma\rho}=ie\epsilon_\mu(k\lambda_k)\zeta_\nu^*(q'\lambda')\mathcal{R}^{\nu\mu\beta}(q',k,q)\zeta_\beta(q\lambda)\,, \qquad (12)$$

where

$$\mathcal{R}^{\nu\mu\beta}(q',k,q)=\{\mathcal{K}_1[(q'+q)^\mu g^{\nu\beta}]+\mathcal{K}_2[k^\nu g^{\mu\beta}-k^\beta g^{\mu\nu}] +(\mathcal{K}_3/m^2)[(q'+q)^\mu k^\nu k^\beta]\}\,, \qquad (13)$$

and

$$\mathcal{V}_{\Delta\rho N}=-ig_{\Delta\rho N}\zeta_\nu^*(q\lambda_q)\bar{u}_\sigma(p_f\lambda_f)(i\gamma_5) \times\mathcal{C}^{\sigma\nu}(p_f,q,p_i)u(p_i\lambda_i)\,, \qquad (14)$$

where

$$\mathcal{C}^{\sigma\nu}(p_f,q,p_i)=[C_3(q^\sigma\gamma^\nu-g^{\sigma\nu}\boldsymbol{q})+C_4(q^\sigma p_f^\nu-g^{\sigma\nu}q\cdot p_f) +C_5(q^\sigma p_i^\nu-g^{\sigma\nu}q\cdot p_i)]. \qquad (15)$$

The connection between the $\mathcal{K}$'s and the electromagnetic moments of the ρ are given by the following expressions[11,8] in terms of e_ρ, the ρ-meson charge:

$$\mathcal{K}_1=e_\rho/e\,, \qquad (16)$$

$$\mathcal{K}_2=(e_\rho/e)(1+\mu_\rho^{\text{anomalous}})\,, \qquad (17)$$

$$\mathcal{K}_3=(e_\rho/e)\tfrac{1}{2}(Q_\rho+\mu_\rho^{\text{anomalous}})\,, \qquad (18)$$

where $1+\mu_\rho^{\text{anomalous}}$ gives the magnetic dipole moment in units $e_\rho/2m$ and Q_ρ gives the electric quadrupole moment in units of $1/m^2$.

The parameters in Eq. (15) are assumed to be constants and values for their magnitudes can be obtained from the experimental analysis of the $\Delta\gamma N$-vertex[12] if we assume the validity of vector-meson dominance.[13]

With the conventions prescribed in Eqs. (14) and (15), the assumption of vector-meson dominance[13] rerequires that $g_{\Delta^+\rho^0 p}\approx 2g_{p\rho^0 p}$ as can be seen from the following argument. Since the $\Delta^+\gamma p$ vertex is isovector, vector-meson dominance suggests that the $\Delta\gamma N$ coupling is mediated by a ρ^0 meson, which implies the approximate algebraic relation

$$G_i^{\Delta\gamma p}(k^2)\approx\frac{em^2}{f_\rho}\frac{1}{m^2-k^2}G_i^{\Delta\rho p}\,,$$

where $G_i^{\Delta\gamma p}(k^2)$ and $G_i^{\Delta\rho p}$ are the vertex couplings for the ith kinematical terms defined in Eqs. (14) and (15). The constant f_ρ has the usual property $f_\rho=f_{N\rho N}=2g_{p\rho^0 p}$,[13]

[13] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961); Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **8**, 79 (1962).

since the $\gamma\rho^0$ coupling is independent of the $\Delta\rho N$ vertex. The constancy of $G_i^{\Delta\rho p}$ is based on the assumption that strong-interaction couplings are constant and, therefore, momentum-transfer-independent. For real photons, we have $k^2=0$, and the relation becomes $G_i^{\Delta\gamma p}(0) \approx eG_i^{\Delta\rho p}/f_\rho$. If we adopt the usual convention for the $\Delta\gamma p$ coupling,[12] viz., $G_i^{\Delta\gamma N}(k^2)=eC_i(k^2)$, we obtain $G_i^{\Delta\rho p}\approx f_\rho C_i(0)$. With the definition of $g_{\Delta\rho N}$ given in Eqs. (14) and (15), we conclude that $g_{\Delta^+\rho^0 p}\approx 2g_{p\rho^0 p}$. From the isospin coupling at the $\Delta\rho N$ vertex, we obtain the relation

$$g_{p\rho^-\Delta^{++}}=(\tfrac{3}{2})^{1/2}g_{p\rho^0\Delta^+}=\sqrt{3}g_{p\rho^+\Delta^0}. \tag{19}$$

B. OVE Amplitude

The combination of the vertex expressions given in Eqs. (11) and (12) with vector-meson propagator yields the following expression for the amplitude representing the OVE diagram shown in Fig. 2(a):

$$\mathfrak{M}_{\text{OVE}}=\zeta_\nu^*(q\lambda_q)u_\sigma(p_f\lambda_f)I_{\text{OVE}}^{\sigma\nu\mu}u(p_i\lambda_i)\epsilon_\mu(k\lambda_k)\,, \tag{20}$$

where

$$\begin{aligned}I_{\text{OVE}}^{\sigma\nu\mu}=&[-ieg_{\Delta\rho N}(i\gamma_5)/(t-m^2)]\{\mathcal{H}_1 2q^\mu\mathcal{C}^{\sigma\nu}(p_f,\,q-k,\,p_i)\\&+\mathcal{H}_2[k^\nu\mathcal{C}^{\sigma\mu}(p_f,\,q-k,\,p_i)-g^{\nu\mu}k_\beta\mathcal{C}^{\sigma\beta}(p_f,\,q-k,\,p_i)]\\&+(\mathcal{H}_3/m^2)2q^\mu k^\nu k_\beta\mathcal{C}^{\sigma\beta}(p_f,\,q-k,\,p_i)\}\,.\end{aligned} \tag{21}$$

An inspection of Eq. (21) shows that the term linear in $\mathcal{H}_2$ is gauge-invariant, but that those linear in $\mathcal{H}_1$ and $\mathcal{H}_3$ are not. The $\mathcal{H}_3$ terms may be rendered gauge-invariant by an addition which results in the replacement of $2q^\mu k^\nu$ by $2(q^\mu k^\nu-g^{\mu\nu}k\cdot q)$. Since $2k\cdot q=-(t-m^2)$, the added term is pole-free and may therefore be attributed to the contact term represented in Fig. 3(g).

The $\mathcal{H}_1$ terms are more difficult and require the addition of terms from the electric monopole parts of the crossed-channel Born terms.[3] Prior to considering the crossed-channel Born terms, we perform a decomposition of the $\mathcal{H}_1$ term into a part which may be rendered independently gauge-invariant and the part which must appear in the crossed-channel combination. This decomposition is obtained by recognizing in Eq. (15) the linear nature of $\mathcal{C}^{\sigma\nu}(a,b,c)$ with respect to b which allows us to perform the separation

$$\mathcal{C}^{\sigma\nu}(p_f,\,q-k,\,p_i)=\mathcal{C}^{\sigma\nu}(p_f,\,q,\,p_i)-\mathcal{C}^{\sigma\nu}(p_f,\,k,\,p_i)\,. \tag{22}$$

Since $\mathcal{C}^{\sigma\nu}(p_f,\,k,\,p_i)$ is linear in k, we can express it in the contracted form

$$\mathcal{C}^{\sigma\nu}(p_f,k,p_i)=k_\beta\mathfrak{D}^{\sigma\nu\beta}(p_f,p_i)\,. \tag{23}$$

Gauge invariance of the k term of $\mathcal{C}$ may be obtained by the replacement of $2q^\mu\mathcal{C}^{\sigma\nu}(p_f,k,p_i)$ by $2[q^\mu\mathcal{C}^{\sigma\nu}(p_f,k,p_i)-k\cdot q\,\mathfrak{D}^{\sigma\nu\mu}(p_i,p_f)]$. Once again the factor $k\cdot q$ in the additional term removes the pole and allows the added term to be attributed to the contact term.

In the Appendix we show that the electric monopole[14] parts of the s- and u-channel Born terms contribute non-gauge-invariant terms which are linear in $\mathcal{C}^{\sigma\nu}(p_f,q,p_i)$. We can combine these terms and replace the non-gauge-invariant term in Eq. (21), $\mathcal{H}_1[2q^\mu/(t-m^2)]\mathcal{C}^{\sigma\nu}(p_f,q,p_i)$, with the gauge-invariant combination

$$\frac{\mathcal{E}^\mu\mathcal{C}^{\sigma\nu}(p_f,q,p_i)}{t-m^2}=\left(\mathcal{H}_1\frac{2q^\mu}{t-m^2}+F_1{}^N\frac{2p_i{}^\mu}{s-M^2}+F_1{}^\Delta\frac{2p_f{}^\mu}{u-M_\Delta{}^2}\right)\times\mathcal{C}^{\sigma\nu}(p_f,q,p_i)\,. \tag{24}$$

In Eq. (24), $F_1{}^N$ and $F_1{}^\Delta$ represent the electric monopole moments of the nucleon and the Δ. The requirement that the expression in Eq. (24) satisfy the gauge-invariance condition implies the relation

$$F_1{}^N=\mathcal{H}_1+F_1{}^\Delta\,. \tag{25}$$

Equation (25) obviously expresses the conservation of charge in the reaction $\gamma+N\to\rho+\Delta$.

With the additions described above, we may write the expression for $I_{\text{OVEC}}^{\sigma\nu\mu}$ in a form which will finally represent our minimal gauge-invariant OVE model:

$$\begin{aligned}I_{\text{OVEC}}^{\sigma\nu\mu}=&\frac{-eg_{\Delta\rho N}(i\gamma_5)}{t-m^2}\{\mathcal{E}^\mu\mathcal{C}^{\sigma\nu}(p_i,q,p_f)\\&+\mathcal{H}_1 2[q^\mu\mathcal{C}^{\sigma\nu}(p_i,k,p_f)-k\cdot q\mathfrak{D}^{\sigma\nu\mu}(p_i,p_f)]\\&+\mathcal{H}_2[k^\nu\mathcal{C}^{\sigma\mu}(p_i,\,q-k,\,p_f)-g^{\nu\mu}k_\beta\mathcal{C}^{\sigma\beta}(p_i,\,q-k,\,p_f)]\\&+(\mathcal{H}_3/m^2)2(q^\mu k^\nu-k\cdot qg^{\mu\nu})k_\beta\mathcal{C}^{\sigma\beta}(p_i,\,q-k,\,p_f)\}\,.\end{aligned} \tag{26}$$

Now that the form of $I_{\text{OVEC}}^{\sigma\nu\mu}$ has been established, we are able to compare the OVE and OPE amplitudes. The OPE amplitude which represents the diagram given in Fig. 3(b) can be expressed in the form

$$I_{\text{OPE}}^{\sigma\nu\mu}=[g_{\gamma\pi\rho}g_{\Delta\pi N}/(t-m_\pi{}^2)](q-k)^\sigma\epsilon^{\nu\mu\alpha\beta}k_\alpha q_\beta\,. \tag{27}$$

G parity gives us the useful relation for the $g_{\gamma\rho\pi}$ coupling constants:

$$g_{\rho^-\pi^-\gamma}=g_{\rho^+\pi^+\gamma}=g_{\rho^0\pi^0\gamma}\,. \tag{28}$$

Isospin coupling at the $\Delta\pi N$ vertex yields the relation analogous to Eq. (19):

$$g_{p\pi^-\Delta^{++}}=(\tfrac{3}{2})^{1/2}g_{p\pi^0\Delta^+}=\sqrt{3}g_{p\rho^+\Delta^0}\,. \tag{29}$$

It can be shown directly[15] that the interference term for the unpolarized cross section,

$$\begin{aligned}\sum_{\{\lambda\}}|\mathfrak{M}_{\text{int}}|^2=&(-g_{\mu\mu'})(-g_{\nu\nu'}+q_\nu q_{\nu'}/m^2)\,\text{Tr}\{(\not p_f+M_\Delta)\\&\times\Gamma_{\sigma\sigma'}(p_f,M_\Delta)I_{\text{OVEC}}^{\sigma\nu\mu}(\not p_i+M)I_{\text{OPE}}^{\sigma'\nu'\mu'}\}\,,\end{aligned} \tag{30}$$

vanishes identically.

[14] Our use of the term electric monopole moment for the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons refers to the Dirac moments which, strictly speaking, reduce to the electric monopole moments only in the static limit.

[15] R. H. Dalitz, in *Proceedings of the International School of Physics "Enrico Fermi," Course 33*, edited by L. W. Alvarez (Academic, New York, 1966), p. 171.

The contributions of the OPE and OVE terms may therefore be added incoherently in the cross section.

IV. CONCLUSIONS

In summary, we have constructed a minimal gauge-invariant OVE model for the process $\gamma+N\to\rho^{\pm}+\Delta$, which is the simplest extension of the OVE diagram and includes only those additional terms from the crossed-channel Born terms and contact terms which are necessary for gauge invariance. We have also discussed the $\Delta\rho N$-coupling strength in terms of vector-meson dominance and found that the interference between the OPE and OVE amplitudes vanishes. Detailed numerical calculations are presently in progress for the unpolarized differential cross sections and decay correlation coefficients for energies in the near-threshold region.

ACKNOWLEDGMENTS

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APPENDIX: s- AND u-CHANNEL BORN TERMS

The electric monopole part[14] of the s-channel nucleon pole term has the form

$$I_{Ns}{}^{\sigma\nu\mu}=-ieg_{N\rho\Delta}(i\gamma_5)\mathcal{C}^{\sigma\nu}(p_f,\,q,\,p_i+k)\\ \times[(\boldsymbol{p}_i+\boldsymbol{k}+M)/(s-M^2)]F_1{}^N\gamma^\mu. \quad (\mathrm{A1})$$

By the use of the Dirac equation, we may replace $(\boldsymbol{p}_i+\boldsymbol{k}+M)\gamma^\mu$ by $2p_i{}^\mu+\boldsymbol{k}\gamma^\mu$. Since we are explicitly searching for the terms which must be added to the OVE expression in order to ensure gauge invariance, we disregard the part of the expression linear in $\boldsymbol{k}\gamma^\mu$ because it is independently gauge-invariant.

In the expression which remains, we may exploit the linearity of $\mathcal{C}^{\sigma\nu}$ and perform the decomposition analogous to Eq. (15):

$$\mathcal{C}^{\sigma\nu}(p_f,\,q,\,p_i+k)=\mathcal{C}^{\sigma\nu}(p_f,q,p_i)+\mathcal{C}^{\sigma\nu}(p_f,q,k)\\ =\mathcal{C}^{\sigma}(p_f,q,p_i)+\mathfrak{D}_s{}^{\sigma\nu\beta}k_\beta. \quad (\mathrm{A2})$$

The $\mathfrak{D}_s$ term is also discarded since it may be rendered gauge-invariant independently by the replacement of $p_i{}^\mu\mathfrak{D}_s{}^{\sigma\nu\beta}k_\beta$ by $p_i{}^\mu\mathfrak{D}_s{}^{\sigma\nu\beta}k_\beta-k\cdot p_i\mathfrak{D}_s{}^{\sigma\nu\mu}$. The added term is seen to be pole-free from the relation $2k\cdot p_i=s-M^2$ and can therefore be attributed to the contact term.

The non-gauge-invariant remnant of the s-channel electric Born term which must be combined with the crossed-channel terms therefore has the form

$$I_{NsG}{}^{\sigma\nu\mu}=-ieg_{N\rho\Delta}(i\gamma_5)\mathcal{C}^{\sigma\nu}(p_f,q,p_i)\\ \times[2p_i{}^\mu/(s-M^2)]F_1{}^N. \quad (\mathrm{A3})$$

The electric monopole part of the u-channel baryon pole term has the form

$$I_{\Delta u}{}^{\sigma\nu\mu}=-ieg_{N\rho\Delta}F_1{}^\Delta g^{\sigma\sigma'}\gamma^\mu[(\boldsymbol{p}_f-\boldsymbol{k}+M)/(u-M_\Delta{}^2)]\\ \times\Gamma_{\sigma'\sigma''}(p_f-k,\,M_\Delta)(i\gamma_5)\mathcal{C}^{\sigma''\nu}(p_f-k,\,q,\,p_i). \quad (\mathrm{A4})$$

Once again we use the Dirac equation for the replacement of $\gamma^\mu(\boldsymbol{p}_f-\boldsymbol{k}+M_\Delta)$ by $2p_f{}^\mu-\gamma^\mu\boldsymbol{k}$, and disregard the $\gamma^\mu\boldsymbol{k}$ part. The $\mathcal{C}^{\sigma\nu}$ term may be treated as above by performing a linear decomposition and the addition of a pole-free contact term.

The propagator $\Gamma_{\sigma'\sigma''}(p_f-k,\,M_\Delta)$ has the form given in Eq. (3) and is reduced by the use of the subsidiary conditions [Eqs. (5) and (6)] to the form

$$\Gamma_{\sigma'\sigma''}=\left(g_{\sigma'\sigma''}+\frac{2}{3}\frac{k_{\sigma'}(p_f-k)_{\sigma''}}{M_\Delta{}^2}-\frac{1}{3M_\Delta}k_{\sigma'}\gamma_{\sigma''}\right). \quad (\mathrm{A5})$$

The terms arising from (A5) which are linear in $k_{\sigma'}$ may be rendered gauge-invariant independently by the replacement of $2p_f{}^\mu k^\sigma$ by $2(p_f{}^\mu k^\sigma-k\cdot p_f g^{\mu\sigma})$, which is attributable to the contact term.

The non-gauge-invariant remnant of the u-channel electric Born term is therefore of the form

$$I_{\Delta uG}{}^{\sigma\nu\mu}=-eg_{N\rho\Delta}F_1{}^\Delta[2p_f{}^\mu/(u-M_\Delta{}^2)]\\ \times(i\gamma_5)\mathcal{C}^{\sigma\nu}(p_f,q,p_i). \quad (\mathrm{A6})$$

The terms given in (A5) and (A6) must be combined with the corresponding part of the t-channel electric Born term in order to obtain a gauge-invariant expression.