Test of Veneziano Model Using ρ-Meson Spin-Density Matrix Elements

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It is shown in conventional Regge theory that the spin-density matrix elements of ρ mesons produced by scattering pions off nucleons have the properties that $(\rho_{11}-\rho_{1,-1})/\rho_{00}$ and $2(|\rho_{10}|/\rho_{00})^2$ are equal and dependent only on pion exchange at very high energies. The ratio $\rho_{10}/\rho_{00}\simeq(\mathrm{Re}\rho_{10})/\rho_{00}$ in a generalized Veneziano model is then found to agree with data at 8-GeV/c pion laboratory momentum.

I. INTRODUCTION

In a recent article[1] it was shown that in a model[2,3] of Reggeization based on Feynman diagrams the spin-density matrix elements of ρ mesons produced by scattering pions off nucleons have the properties that the ratios $(\rho_{11}-\rho_{1,-1})/\rho_{00}$ and $2(|\rho_{10}|/\rho_{00})^2$ are equal and dependent only on pion exchange at very high energies. Unfortunately neither minimal nor maximal derivative coupling separately in the model led to agreement with data[4] at 8-GeV/c pion laboratory momentum.

The present work shows that the above conclusions[4a] regarding the density matrix elements hold generally in the usual Regge theory (simple poles in the angular momentum plane and factorization of residues), thus offering a new method of isolating the elusive pion trajectory and of testing future models of Regge couplings. In the latter context, the ratio $(\mathrm{Re}\rho_{10})/\rho_{00}$ is calculated in a generalized Veneziano model,[5] namely, that of Bardakci and Ruegg,[6] and is found to agree with the data[4] at 8 GeV/c.

II. CONVENTIONAL REGGE THEORY

Consider the process

$$\pi(q_1)+N(p_1)\rightarrow\rho(q_2)+N(p_2),$$

where the bracketed quantities are four-momenta in terms of which the independent kinematic invariants are $s=(q_1+p_1)^2$ and $t=(q_1-q_2)^2$. In calculating the spin-density matrix elements of the produced ρ meson, Gottfried and Jackson[7] have shown that they may be expressed particularly simply in terms of the t-channel helicity amplitudes[8] in a special reference frame. This special frame (the Gottfried-Jackson frame) is the one in which the ρ meson is at rest ($\mathbf{q}_2=0$), the z direction is in the direction of the pion's three-momentum $\mathbf{q}_1$, and the y direction is along the production plane normal $\mathbf{p}_1\times\mathbf{p}_2$. Then in terms of the t-channel helicity amplitudes the spin-density matrix elements are[7]

$$\rho_{mm'}=K\sum_{\bar\lambda,\lambda}f_{m;\bar\lambda\lambda}(t,s)f_{m';\bar\lambda\lambda}(t,s)^*,\tag{1}$$

where $\lambda,\bar\lambda$ are the helicities of $N,\bar N$ and m (and m') is the spin projection of the ρ meson along the z axis; K is chosen[9] to make $\sum_m\rho_{mm}=1$.

Reggeization is carried out in the usual manner. First a partial-wave decomposition is made in the t channel

$$f_{m;\bar\lambda\lambda}(t,s)=\sum_J(2J+1)\langle m|T^J(t)|\bar\lambda\lambda\rangle d_{\bar\lambda-\lambda,m}{}^J(\theta_t),\tag{2}$$

where θ_t is the t-channel center-of-mass scattering angle. Next, eigenstates of parity are introduced and the amplitudes $\langle m|T^J|\bar\lambda\lambda\rangle$ are expressed in terms of amplitudes for the two parity states. This calculation proceeds just as in Ref. 7, with the result[10]

$$\begin{aligned}
\langle 1|T^J|\tfrac12\,\tfrac12\rangle&=a_{10}{}^N+a_{10}{}^A,\\
\langle 1|T^J|-\tfrac12\,-\tfrac12\rangle&=a_{10}{}^N-a_{10}{}^A,\\
\langle 1|T^J|\tfrac12\,-\tfrac12\rangle&=a_{11}{}^N+a_{11}{}^A,\\
\langle 1|T^J|-\tfrac12\,\tfrac12\rangle&=a_{11}{}^N-a_{11}{}^A,\\
\langle 0|T^J|\tfrac12\,\tfrac12\rangle&=a_{00}{}^A,\\
\langle 0|T^J|\tfrac12\,-\tfrac12\rangle&=a_{01}{}^A,\\
\langle -1|T^J|\tfrac12\,\tfrac12\rangle&=-a_{10}{}^N+a_{10}{}^A,\\
\langle -1|T^J|-\tfrac12\,-\tfrac12\rangle&=-a_{10}{}^N-a_{10}{}^A,\\
\langle -1|T^J|\tfrac12\,-\tfrac12\rangle&=-a_{11}{}^N+a_{11}{}^A,\\
\langle -1|T^J|-\tfrac12\,\tfrac12\rangle&=-a_{11}{}^N-a_{11}{}^A,\\
\langle 0|T^J|-\tfrac12\,-\tfrac12\rangle&=-a_{00}{}^A,\\
\langle 0|T^J|-\tfrac12\,\tfrac12\rangle&=-a_{01}{}^A,
\end{aligned}\tag{3}$$

where $a_{10}{}^N$, $a_{11}{}^N$ are amplitudes for normal spin-parity states $[P=(-1)^J]$ and $a_{10}{}^A$, $a_{11}{}^A$, $a_{00}{}^A$, $a_{01}{}^A$ are amplitudes[11] for abnormal spin-parity states $[P=-(-1)^J]$.

[1] R. A. Morrow, Phys. Rev. **185**, 1764 (1969).
[2] L. Van Hove, Phys. Letters **24B**, 183 (1967); L. Durand III, Phys. Rev. **154**, 1537 (1967); **161**, 1610 (1967).
[3] R. Blankenbecler and R. L. Sugar, Phys. Rev. **168**, 1597 (1968).
[4] Aachen-Berlin-CERN Collaboration, Phys. Letters **22**, 533 (1966), for ρ^+ data; I. Derado, J. A. Poirier, N. N. Biswas, N. M. Cason, V. P. Kenney, and W. D. Shepard, *ibid.* **24B**, 112 (1967), for ρ^- data. In the present analysis this latter set of data is replaced by the more comprehensive data of S. J. Barish, W. Selove, N. N. Biswas, N. M. Cason, P. B. Johnson, V. P. Kenney, J. A. Poirier, W. D. Shepard, and H. Yuta, Phys. Rev. **184**, 1375 (1969); Phys. Rev. D 1, 375E (1970). A similar replacement in Ref. 1 would not alter the conclusions drawn there, however.
[4a] Some of these results have been expressed by G. U. Das and C. D. Frogatt, Nucl. Phys. **B8**, 661 (1968). See also J. P. Ader, M. Capdeville, G. Cohen-Tannoudji, and Ph. Salin, Nuovo Cimento **56A**, 952 (1968).
[5] G. Veneziano, Nuovo Cimento **57**, 190 (1968).
[6] K. Bardakci and H. Ruegg, Phys. Letters **28B**, 342 (1968).
[7] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).
[8] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959).
[9] In Eq. (5) of Ref. 1, the factor $(1/N)$ should be replaced by (N).
[10] The notation used in (3) for $\langle m|T^J|\bar\lambda\lambda\rangle$ is $a_{|m|\,|\bar\lambda-\lambda|}$.
[11] These amplitudes are simply u_J, v_J, U_J, V_J, W_J, and X_J, respectively, of Ref. 7.

Finally, a Sommerfeld-Watson transformation is made in (2) and, for large s, only the leading normal and abnormal spin-parity trajectories α_ω and α_π, respectively, are retained. In particular, the presence of Regge cuts is ignored. The result is (including signature)

$$f_{m;\bar{\lambda}\lambda}(t,s) \to \sum_{i=\pi,\omega} (-\pi)(2\alpha_i+1)\sigma_i\delta_i\beta_{|m||\bar{\lambda}-\lambda|}{}^i \times(\sin\pi\alpha_i)^{-1}[(-1)^{\bar{\lambda}-\lambda}d_{\bar{\lambda}-\lambda,-m}{}^{\alpha_i}(\pi-\theta_t)]|_{s\to\infty}, \quad (4)$$

where

$$\sigma_i=\tfrac{1}{2}(1+\tau_i e^{-i\pi\alpha_i}) \quad \text{with} \quad \tau_\pi=+1, \tau_\omega=-1$$

and[12]

$$\delta_i=(\delta_{i\pi}+m\delta_{i\omega})(2\bar{\lambda}\delta_{i\pi}+\delta_{i\omega}).$$

Next the asymptotic forms of the rotation matrix elements are found, the usual threshold factors are removed from the residues,[7] and the residues are factorized[13] with the result that (4) takes the form[14]

$$f_{m;\bar{\lambda}\lambda}(t,s) \to \sum_{i=\pi,\omega} g^i(t)\delta_i\gamma_{|m|}{}^i\chi_{|\bar{\lambda}-\lambda|}{}^i(s/s_0)^{\alpha_i}C_{\bar{\lambda}-\lambda,m}{}^i, \quad (5)$$

where $g^i(t)$ contains the factors $(\sin\pi\alpha_i)^{-1}$ and σ_i among others, $\gamma_{|m|}{}^i$, $\chi_{|\bar{\lambda}-\lambda|}{}^i$ are the helicity-dependent factors in the residues, and the $C_{\bar{\lambda}-\lambda,m}{}^i$ coming from the expansion of the rotation matrix elements are[15]

$$\begin{aligned} &C_{1,0}{}^i=C_{0,1}{}^i=-C_{-1,0}{}^i=-C_{0,1}{}^i=-i[\alpha_i/(\alpha_i+1)]^{1/2},\\ &C_{\pm1,\pm1}{}^i=-C_{\pm1,\mp1}{}^i=\alpha_i/(\alpha_i+1), \quad C_{0,0}{}^i=1. \end{aligned} \quad (6)$$

It then follows upon substitution of (5) and (6) into (1) that the $\rho_{mm'}$ split into two terms at high energy, one dependent only on pion exchange and the other only on ω exchange:

$$\rho_{mm'}/K \to \rho_{mm'}(\pi)/K+\rho_{mm'}(\omega)/K \quad \text{as } s\to\infty,$$

with

$$\begin{aligned} \rho_{11}(\pi)/K&=-\rho_{1,-1}(\pi)/K=2|g^\pi|^2|\gamma_1{}^\pi|^2[|\chi_0{}^\pi|^2 +|\chi_1{}^\pi|^2|\alpha_\pi/(\alpha_\pi+1)|]|\alpha_\pi/(\alpha_\pi+1)|(s/s_0)^{2\alpha_\pi},\\ \rho_{11}(\omega)/K&=\rho_{1,-1}(\omega)/K=2|g^\omega|^2|\gamma_1{}^\omega|^2[|\chi_0{}^\omega|^2 +|\chi_1{}^\omega|^2|\alpha_\omega/(\alpha_\omega+1)|]|\alpha_\omega/(\alpha_\omega+1)|(s/s_0)^{2\alpha_\omega},\\ \rho_{10}(\pi)/K&=i2|g^\pi|^2\gamma_1{}^\pi\gamma_0{}^{\pi*}[|\chi_0{}^\pi|^2 +|\chi_1{}^\pi|^2|\alpha_\pi/(\alpha_\pi+1)|][\alpha_\pi/(\alpha_\pi+1)]^{1/2}(s/s_0)^{2\alpha_\pi},\\ \rho_{00}(\pi)/K&=2|g^\pi|^2|\gamma_0{}^\pi|^2[|\chi_0{}^\pi|^2 +|\chi_1{}^\pi|^2|\alpha_\pi/(\alpha_\pi+1)|](s/s_0)^{2\alpha_\pi},\\ \rho_{10}(\omega)/K&=0=\rho_{00}(\omega)/K. \end{aligned} \quad (7)$$

It is straightforward to check that in arriving at (7), terms of order $s^{\alpha_\pi+\alpha_\omega-1}$ have been dropped from $\rho_{1,\pm1}/K$ and ρ_{10}/K, while terms of order $s^{2\alpha_\omega-2}$ have been dropped from $\rho_{1,\pm1}/K$, both in comparison to terms of order $s^{2\alpha_\pi}$, the justification being that $\alpha_\omega-1<\alpha_\pi$.

It follows from (7) that at high energy

$$(\rho_{11}-\rho_{1,-1})/\rho_{00}\simeq2|\rho_{10}|^2/\rho_{00}{}^2 \simeq \frac{2|\alpha_\pi/(\alpha_\pi+1)||\gamma_1{}^\pi|^2}{|\gamma_0{}^\pi|^2} \quad (8)$$

and is dependent only on the pion trajectory and how it couples to the external pion and ρ meson. This thus offers a means of studying the pion trajectory even though each $\rho_{mm'}$ depends also on the parameters of the ω Reggeon. (Note the presence of the factor K.) A model calculation exploiting this isolated dependence on the pion is described in Sec. III and is compared there with experimental data.[4] A justification for comparing this model calculation done at infinite energy with the data taken at 8 GeV/c is that the data do satisfy, within rather large uncertainties, *one* asymptotic relation, namely, the left equality of Eq. (8). This was shown in Fig. 3 of Ref. 1.

III. VENEZIANO MODEL

As remarked in the Introduction, neither minimal nor maximal derivative coupling, separately, in a model of Reggeization based on Feynman diagrams gave $\rho_{10}/\rho_{00}\simeq(\text{Re}\rho_{10})/\rho_{00}$ in agreement with the 8-GeV/c data.[4] Presumably, some unknown mixture of the two couplings is needed for agreement. A clearer-cut prediction is available in a generalized[6] Veneziano model, however. Since only ρ_{10}/ρ_{00} at high energy is wanted in the present study, this generalized Veneziano model with spinless nucleons and pion exchange is relevant as indicated by (8). That Eq. (8) holds for the model is apparent because the model has the required properties of conventional Regge theory, namely, Regge behavior at high energy and factorization of residues. The calculational details to follow are exactly similar to those of Jones and Wyld[16] in a related study.

Briefly, the calculation proceeds as follows. The Bardakci-Ruegg model[6] is applied to the reaction

$$\pi^++p \to p+\pi^0+\pi^+ \quad (9)$$

assuming spinless nucleons, the residue of the pole at $s_{\pi^0\pi^+}=m_\rho{}^2$ when $\alpha_{\pi^0\pi^+}$ is the ρ-meson trajectory is determined, the high-energy limit is found, the $l=1$ content of the residue is extracted, and the helicity amplitudes for the reaction $\pi^++p\to p+\rho^+$ are identified. These amplitudes are then used to find $(\text{Re}\rho_{10})/\rho_{00}$ which is compared with the data.[4]

To facilitate carrying out these calculations, the elegant analysis of Bialas and Pokorski[17] is appealed to. Considering the reaction (9) for definiteness, the identification of their labels with the particles of interest is $A\to p$, $B\to\pi^+$, $1\to\bar{p}$, $2\to\pi^0$, and $3\to\pi^-$ (recall that A, B, 1, 2, and 3 are all incoming in Ref. 17).

[12] The symbol δ_i enters through the realization that Eqs. (3) may be written as $\langle m|T^J|\bar{\lambda}\lambda\rangle=\sum_{i=A,N}\delta_i a_{|m||\bar{\lambda}-\lambda|}{}^i$, with $A\equiv\pi$, $N\equiv\omega$.

[13] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. Gribov and I. Pomeranchuk, *ibid.* **8**, 343 (1962).

[14] Consideration of kinematical singularities will alter the asymptotic expression for the smallest values of $|t|$. See L. Jones, Phys. Rev. **163**, 1523 (1967).

[15] In fact $C_{\bar{\lambda}-\lambda,m}{}^i\equiv[(-1)^{\bar{\lambda}-\lambda}d_{\bar{\lambda}-\lambda,-m}{}^{\alpha_i}(\pi-\theta_t) \times(-\frac{1}{2}\cos\theta_t)^{-\alpha_i}\Gamma(\alpha_i+1)^2/\Gamma(2\alpha_i+1)]_{s\to\infty}$.

[16] L. Jones and H. W. Wyld, Jr., Phys. Rev. Letters **23**, 814 (1969).

[17] A. Bialas and S. Pokorski, Nucl. Phys. **B10**, 399 (1969); hereafter referred to as BP.

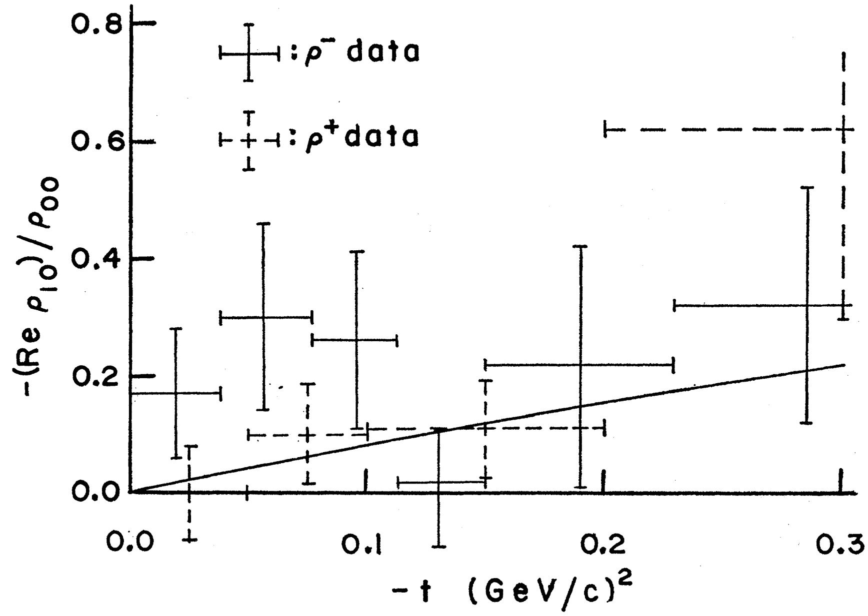

FIG. 1. A comparison of the prediction of the *asymptotic* Veneziano model with experimental data (Ref. 4) at 8-GeV/c pion laboratory momentum.

The amplitude for process (9) then consists of contributions from the 12 diagrams of Fig. 3 in BP. Since the trajectory α_{23}, the ρ trajectory, is to be set equal to 1, only those diagrams with particles 2 and 3 adjacent are important; this rules out half of the diagrams. Next, diagrams 4 and 6 of Fig. BP3 are important for *backward* scattering of the proton and may be neglected. This leaves only four diagrams, namely, 1, 2, 7, and 10 of Fig. BP3. It is possible to show,[18] however, that 7 (10) is handled by including signature in 1 (2) in the limit $s_{AB}\to\infty$. Finally, when the $l=1$ content of the residues of diagrams 1 and 2 at the pole $\alpha_{23}=1$ is extracted, it is found that the contributions are equal but opposite. Hence, attention will be restricted to $B_5(1)$, the amplitude of diagram 1 of Fig. BP3.

According to (BP12), the amplitude $B_5(1)$ in the limit $s_{AB}\to\infty$ with s_{23} fixed is

$$B_5(1)\to B_4(-\alpha_{AB},\,-\alpha_{A1})B_4(-\alpha_{23},\,-\alpha_{B3})\times F(-\alpha_{A1},\,-\alpha_{23};\,-\alpha_{B3}-\alpha_{23};\,\alpha_{13}/\alpha_{AB})\,.$$

Using (BP16) and the properties of the hypergeometric function,[19] the residue of this amplitude at $\alpha_{23}=1$ is

$$R(B_5(1))=\lim_{\alpha_{23}\to 1}\,(\alpha_{23}-1)B_5(1)\to -B_4(-\alpha_{AB},\,-\alpha_{A1})\times(1+\alpha_{B3}-\alpha_{A1}\alpha_{13}/\alpha_{AB})\,. \quad (10)$$

The next step is to insert the required trajectories and evaluate (10) in the Gottfried-Jackson frame, introducing simultaneously the spherical angles (θ,ϕ) of particle 3 in this frame. The necessary functions are

$$\alpha_{A1}=\alpha_\pi(s_{A1})=\alpha'(s_{A1}-m_\pi^2)\,,$$
$$\alpha_{B3}=\alpha_\rho(s_{B3})=\alpha'(s_{B3}-m_\rho^2)+1\,,$$
$$\alpha_{AB}\to\alpha' s_{AB}\,,$$
$$\alpha_{13}/\alpha_{AB}\to s_{13}/s_{AB}\to 1-s_{12}/s_{AB}\,,$$
$$s_{B3}=m_B^2+m_3^2+(2s_{23})^{-1}\times[(s_{23}-m_2^2+m_3^2)(s_{A1}-s_{23}-m_B^2)+\cos\theta\,\lambda(s_{23},m_2^2,m_3^2)^{1/2}\lambda(s_{A1},s_{23},m_B^2)^{1/2}]\,,$$
$$s_{12}\to s_{AB}\{1-(s_{23}-m_2^2+m_3^2)/(2s_{23})-[\cos\phi\,\sin\theta\,(-s_{A1}/s_{23})^{1/2}+\cos\theta\,(s_{A1}+s_{23}-m_B^2)/(2s_{23})]\times\lambda(s_{23},m_2^2,m_3^2)^{1/2}\lambda(s_{A1},s_{23},m_B^2)^{-1/2}\}\,,$$
$$\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2zx\,,$$

where we use a universal slope for the trajectories and where an arrow denotes the high-energy limit.

It is a straightforward matter[16] then to extract the t-channel helicity amplitudes for production of the ρ meson. Denoting them by f_m, the result is (with $s_{23}=m_\rho^2$ understood)

$$f_0\to Q[\lambda(s_{A1},s_{23},m_B^2)-(s_{A1}-m_\pi^2)(s_{A1}+s_{23}-m_B^2)]\,,$$
$$f_1=-f_{-1}\to Q(-2s_{A1}s_{23})^{1/2}(s_{A1}-m_\pi^2)\,,$$

with

$$Q=-B_4(-\alpha_{AB},\,-\alpha_{A1})(\tfrac{4}{3}\pi)^{1/2}\alpha'\lambda(s_{23},m_2^2,m_3^2)^{1/2}\times\lambda(s_{A1},s_{23},m_B^2)^{-1/2}(2s_{23})^{-1}.$$

Consequently, in the notation of Sec. II,

$$\frac{-\rho_{10}}{\rho_{00}}\to\frac{-(\mathrm{Re}\rho_{10})}{\rho_{00}}\to\frac{(-2t)^{1/2}(m_\pi^2-t)}{m_\rho(m_\rho^2-m_\pi^2-3t)}. \quad (11)$$

Comparison of this *asymptotic* expression with 8-GeV/c experimental data is made in Fig. 1. The reasonable location of the curve suggests that more ambitious calculations be pursued with the Veneziano model.

[18] W. J. Zakrzewski, Nucl. Phys. **B14**, 458 (1969).

[19] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, 4th ed. (Cambridge U. P., Cambridge, England, 1962), p. 288.