Regge Analysis of π^0 and π^+ Photoproduction at Backward Angles*

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By correlating recent experimental results on π^0 and π^+ photoproduction at backward angles with results on $\pi^- p \to p\,\rho^-$ at the same angles, it is found that any explanation of the three processes in terms of Regge poles must involve at least two isospin-$\frac{1}{2}$ trajectories. Furthermore, the two trajectories must be almost degenerate. Such a solution makes definite predictions about other processes which can be checked by experiments.

I. INTRODUCTION

THE first data[1] on the photoproduction of π^+ mesons at backward angles imposed restrictions on the contribution of various trajectories.[2] These restrictions become more limiting now that there are accurate data[3–5] available for the processes $\gamma p \to n\pi^+$, $\gamma p \to p\pi^0$, and $\pi^- p \to p\rho^-$. In fact, by correlating these processes one arrives at the conclusion that an analysis of the data in terms of traditional Regge poles is nontrivial and it requires the contribution of at least three trajectories. Furthermore, the contribution of the $I=\frac{3}{2}$ trajectories to photoproduction has an upper bound and the $I=\frac{1}{2}$ trajectories must be almost degenerate.

In view of the above results, we feel it is justified to neglect for the moment the contribution of the absorption cuts,[6] which has not yet been formulated completely, and deal with a three-pole model which leads to definite conclusions. These conclusions can easily be checked experimentally. In case they are violated we feel that a simple pole model is inadequate and impractical and that the contribution of cuts is essential.

In Sec. II we give the basic Regge formalism and relate the residues to the coupling constants of the Born diagrams determined in the isobar model.[7] In Sec. III we summarize the experimental situation that requires us to introduce the N_γ trajectory, obtain from vector-meson dominance an upper bound for the contribution of the $I=\frac{3}{2}$ trajectories to photoproduction, and give the parametrization of the amplitudes. Section IV is a discussion of the main results of the model and a proposal of experimental checks.

II. FORMALISM

The kinematics of photoproduction have been discussed in several places. We adopt the CGLN notation.[8] The CGLN amplitudes have kinematic singularities, which become apparent when one writes them in terms of the Ball[9] amplitudes which satisfy the Mandelstam representation:

$$\mathfrak{F}_1(W)=\frac{W-M}{8\pi W}(E_1+M)^{1/2}(E_2+M)^{1/2}\left[B_1-\tfrac{1}{2}(W-M)B_4+\frac{t-\mu^2}{2(W-M)}(B_3-\tfrac{1}{2}B_4)\right], \tag{2.1a}$$

$$\mathfrak{F}_2(W)=\frac{W-M}{8\pi W}q\left(\frac{E_1+M}{E_2+M}\right)^{1/2}\left[-B_1-\tfrac{1}{2}(W+M)B_4+\frac{t-\mu^2}{2(W+M)}(B_3-\tfrac{1}{2}B_4)\right], \tag{2.1b}$$

$$\mathfrak{F}_3(W)=\frac{W-M}{8\pi W}q(E_1+M)^{1/2}(E_2+M)^{1/2}\left[\frac{2(W-M)}{t-\mu^2}B_2-B_3+\tfrac{1}{2}B_4\right], \tag{2.1c}$$

$$\mathfrak{F}_4(W)=\frac{W-M}{8\pi W}q^2\left(\frac{E_1+M}{E_2+M}\right)^{1/2}\left[-\frac{2(W+M)}{t-\mu^2}B_2-B_3+\tfrac{1}{2}B_4\right], \tag{2.1d}$$

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[1] R. Anderson *et al.*, Phys. Rev. Letters **21**, 479 (1968).
[2] E. A. Paschos, Phys. Rev. Letters **21**, 1855 (1968). The proposal of this paper is the simplest choice in the absence of the nucleon dip. The new data on $\pi^- p\to p\rho^-$ do not allow such a large $I=\frac{3}{2}$ contribution. We take this opportunity to correct two misprints which occurred in the final printing of this paper: (i) In Eq. (1), the Γ function $\Gamma(\alpha-\frac{1}{2})$ should be replaced by $\Gamma(\alpha+\frac{3}{2})$; (ii) in Eq. (11), $A_{1/2,1/2}$ should read like Eq. (2.5b) in this text.
[3] D. Tompkins *et al.*, Phys. Rev. Letters **23**, 725 (1969).
[4] R. L. Anderson *et al.*, Phys. Rev. Letters **23**, 721 (1969).
[5] E. W. Anderson *et al.*, Phys. Rev. Letters **22**, 102 (1969).
[6] J. V. Beaupre, SLAC Report No. SLAC-107, UC-34(TH) (unpublished).
[7] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963); Ph. Salin, *ibid.* **28**, 1294 (1963).
[8] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957); referred to as CGLN.
[9] J. S. Ball, Phys. Rev. **124**, 2014 (1961); G. Zweig, Nuovo Cimento **32**, 689 (1964).

where

$$W=\sqrt{u}\,,\quad k=(u-M^2)/2W\,,\quad E_1\pm M=(W\pm M)^2/2W\,,\quad E_2\pm M=[(W\pm M)^2-\mu^2]/2W\,,$$
$$q=[(W+M)^2-\mu^2]^{1/2}[(W-M)^2-\mu^2]^{1/2}/2W\,.$$

The cosine of the u-channel scattering angle is given by

$$z_u=\cos\theta_u=-\frac{2su-2M^2u+(u-M^2)(u+M^2-\mu^2)}{(u-M^2)[u-(M+\mu)^2]^{1/2}[u-(M-\mu)^2]^{1/2}}. \tag{2.2}$$

The phase conventions for the several square roots appearing above are these: All expressions of the form $(W-a)^{1/2}$ have a branch point at $W=a$, and the cut lies along the negative real $(W-a)$ axis. Slightly above the cut, the phase of $(W-a)^{1/2}$ is $\frac{1}{2}\pi$. Products like $(W^2-a^2)^{1/2}$ have cuts between $+a$ and $-a$, and they satisfy

$$(W^2-a^2)^{1/2}\xrightarrow[|W|\to\infty]{}W.$$

Thus in the backward direction $(z_s=-1)$ one has $z_u=-1$, and at $u=0$, $z_u=+1$.

The fact that z_u is not large for backward angles is not very alarming since a sequence of daughter trajectories can restore the Regge asymptotic behavior.

A. McDowell Symmetry

Since the $B_i(s,u)$ are even functions of W, we can derive relations between $\mathfrak{F}_i(W)$ and $\mathfrak{F}_i(-W)$. With the phase convention adopted in the previous section, we obtain

$$\mathfrak{F}_1(W)=\mathfrak{F}_2(-W)\,, \tag{2.3a}$$

$$\mathfrak{F}_3(W)=\mathfrak{F}_4(-W)\,. \tag{2.3b}$$

B. Singularities at $u=0$

Since the line $u=0$ is within the physical region, special care must be taken so that the combinations of sines and cosines of $\frac{1}{2}\theta_u$, together with the linear combinations of the $\mathfrak{F}_i(W)$ which appear in the helicity amplitudes, are smooth around the line $u=0$. Assuming that the B_i amplitudes approach constants as $W\to 0$, we find that the most singular behavior of the sums and differences is given by

$$\mathfrak{F}_1(W)+\mathfrak{F}_2(W)\to C_1/W^2\,, \tag{2.4a}$$

$$\mathfrak{F}_1(W)-\mathfrak{F}_2(W)\to C_2/W\,, \tag{2.4b}$$

$$\mathfrak{F}_3(W)+\mathfrak{F}_4(W)\to C_3/W^2\,, \tag{2.4c}$$

$$\mathfrak{F}_3(W)-\mathfrak{F}_4(W)\to C_4/W^3\,. \tag{2.4d}$$

The trigonometric functions of $\frac{1}{2}\theta_u$ behave as

$$\sin\tfrac{1}{2}\theta_u\propto W\,,\quad \cos\tfrac{1}{2}\theta_u\to 1\,.$$

Therefore, the following u-channel helicity amplitudes vary smoothly over the line $u=0$:

$$A_{1/2,3/2}=-\sqrt{2}\pi W(\mathfrak{F}_3+\mathfrak{F}_4)\sin\theta_u\cos\tfrac{1}{2}\theta_u\,, \tag{2.5a}$$

$$A_{1/2,1/2}=\sqrt{2}\pi W[2(\mathfrak{F}_2-\mathfrak{F}_1)\cos\tfrac{1}{2}\theta_u+(\mathfrak{F}_3-\mathfrak{F}_4)\sin\theta_u\sin\tfrac{1}{2}\theta_u]\,, \tag{2.5b}$$

$$A_{-1/2,3/2}=\sqrt{2}\pi W(\mathfrak{F}_3-\mathfrak{F}_4)\sin\theta_u\sin\tfrac{1}{2}\theta_u\,, \tag{2.5c}$$

$$A_{-1/2,1/2}=\sqrt{2}\pi W[2(\mathfrak{F}_2+\mathfrak{F}_1)\sin\tfrac{1}{2}\theta_u+(\mathfrak{F}_3+\mathfrak{F}_4)\sin\theta_u\cos\tfrac{1}{2}\theta_u]\,. \tag{2.5d}$$

C. Normalization

The differential cross section is given by

$$\frac{d\sigma}{du}=\frac{1}{4\pi s k_s^2}\sum_{\lambda\mu}|A_{\lambda\mu}|^2\,, \tag{2.6}$$

where k_s is the center-of-mass momentum in the s channel, and $A_{\lambda\mu}$ represents the u-channel amplitudes, $\lambda(\mu)$ being the final (initial) center-of-mass helicity.

At 180°, where $z_s=z_u=-1$, only the $A_{-(1/2,1/2)}$ amplitude is nonvanishing. In the case that the contribution of the other amplitudes is non-negligible, we expect a minimum at 180°.

D. Reggeization

The Reggeization of the amplitudes incorporates all of the above properties. Because of the McDowell symmetry, we Reggeize only $\mathfrak{F}_2$ and $\mathfrak{F}_4$; $\mathfrak{F}_1$ and $\mathfrak{F}_3$ are obtained by reflection in W. In terms of the electric and magnetic multipoles, $M_{l\pm}(W)$ and $E_{l\pm}(W)$, with $j=l\pm\frac{1}{2}$, the partial-wave expansions are given by

$$\mathfrak{F}_2(W)=\sum_{l=1}[(l+1)M_{l+}(W)+lM_{l-}(W)]P_l'(z_u)\,, \tag{2.7a}$$

$$\mathfrak{F}_4(W)=\sum_{l=1}[M_{l+}(W)-E_{l+}(W)-M_{l-}(W)-E_{l-}(W)]P_l''(z_u)\,. \tag{2.7b}$$

The combinations lM_{l-}, lE_{l-}, $(l+1)E_{l+}$, and $(l+1)M_{l+}$ can have dynamic poles in the complex j plane. For a given Regge pole at $j=\alpha$, the leading contributions come from M_{l-} and E_{l-}. Thus we keep only the lM_{l-} in $\mathfrak{F}_2$. Assuming

$$M_{l-}(W)=\frac{\beta_2(W)}{j-\alpha}\,, \tag{2.8}$$

we obtain the Reggeization

$$\mathfrak{F}_2(W)=(\sqrt{\pi})\beta_2(W)\frac{1+\eta e^{-i\pi(\alpha-1/2)}}{\sin\pi(\alpha-\frac{1}{2})\Gamma(\alpha+\frac{1}{2})}\left(\frac{s}{kq}\right)^{\alpha-1/2}. \tag{2.9}$$

For $\mathfrak{F}_4$ we assume

$$E_{l-}(W)+M_{l-}(W)=\frac{\beta_4(W)}{j-\alpha} \quad (2.10)$$

and obtain

$$\mathfrak{F}_4(W)=-2(\sqrt{\pi})\beta_4(W) \times\frac{1+\eta e^{-i\pi(\alpha-1/2)}}{\sin\pi(\alpha-\frac{1}{2})\Gamma(\alpha-\frac{1}{2})}\left(\frac{s}{kq}\right)^{\alpha-3/2}. \quad (2.11)$$

Introducing the scaling parameter s_0 and extracting the kinematic singularities, we obtain a set of residues $\gamma_i(W)$ which contain only dynamic information:

$$\beta_2(W)=[(W-M)^2-\mu^2]^{1/2}(1-M^2/u) \times(kq/s_0)^{\alpha-1/2}\gamma_2(W), \quad (2.12a)$$

$$\beta_4(W)=\tfrac{1}{2}q[(W-M)^2-\mu^2]^{1/2}(1-M^2/u) \times(kq/s_0)^{\alpha-3/2}\gamma_4(W). \quad (2.12b)$$

The resulting forms of the Reggeized amplitudes are

$$\mathfrak{F}_2(W)=\frac{u-M^2}{(\sqrt{\pi})u}[(W-M)^2-u^2]^{1/2}\sum_\alpha \gamma_2{}^\alpha \times(1+\eta e^{-i\pi(\alpha-1/2)})\Gamma(\tfrac{1}{2}-\alpha)\left(\frac{s}{s_0}\right)^{\alpha-1/2}, \quad (2.13a)$$

$$\mathfrak{F}_4(W)=\frac{u-M^2}{(\sqrt{\pi})u}q[(W-M)^2-u^2]^{1/2}\sum_\alpha \gamma_4{}^\alpha \times(1+\eta e^{-i\pi(\alpha-1/2)})\Gamma(\tfrac{3}{2}-\alpha)\left(\frac{s}{s_0}\right)^{\alpha-3/2}. \quad (2.13b)$$

E. Born Diagrams

We evaluate the residues of the three leading trajectories at the poles by using Born diagrams. The nucleon residue at the pole is obtained by using Feynman rules and then using Eqs. (2.8) and (2.12a) to obtain

$$\gamma_2{}^E(N_\alpha,W)=\frac{eG}{32\pi}\frac{d\alpha}{du}, \quad (2.14a)$$

$$\gamma_2{}^M(N_\alpha,W)=\frac{\mu_i G}{32\pi}(W+M)\frac{d\alpha}{du}, \quad (2.14b)$$

where $G^2/4\pi=14.5$, $e^2/4\pi=1/137$, $\mu_p=1.78e/2M$, and $\mu_n=-1.91e/2M$.

In estimating the residues of the N_γ and the Δ trajectories, we use the isobar model of Gourdin and Salin.[7] The contribution of the N_γ resonance to the $M_{2-}(W)$ partial wave is

$$M_{2-}(W)=\tfrac{1}{6}L_2[(E_1+M)(E_2+M)]^{1/2}/8\pi W, \quad (2.15)$$

where

$$L_2=0.1[e/\sqrt{(4\pi)}][G/\sqrt{(4\pi)}]4\pi. \quad (2.16)$$

In (2.15), we have corrected for the normalization differences between Refs. 7 and 8, but we have not included any isotopic-spin factors. Using (2.15), (2.8), and (2.12a), we can obtain $\gamma_2(N_\gamma)$. In a similar way, we can estimate the Δ residue. The numerical results, using the coupling constants of Gourdin and Salin,[7] are

$$\gamma_2(N_\alpha)/\gamma_2(N_\gamma)\approx 1.0 \quad\text{and}\quad \gamma_2(N_\gamma)/\gamma_2(\Delta)\approx 0.1.$$

III. EXPERIMENTAL SUMMARY AND PARAMETRIZATION

The π^+ and π^0 photoproduction data[3,4] can be combined in such a way that one can make rather strong arguments about the contributions of the allowed baryon exchanges. On the other hand, application of vector-meson dominance imposes additional constraints on the contributions of the trajectories. There are three rather important features of the data, and we discuss them in detail:

(a) The traditional nucleon trajectory has a nonsense zero at $u\simeq-0.15$ (BeV/c)^2, which appears in all four helicity amplitudes. The absence of a dip in this u region in both the π^0 and π^+ photoproduction eliminates the dominance of the N_α trajectory. Presumably this argument is weakened if the nucleon contribution is composed of the usual Regge contribution and an N_α-Pomeranchukon cut[6]; or if there is a fixed pole in the N_α residues at $\alpha(N_\alpha)=-\frac{1}{2}$. Since the understanding of cuts and fixed poles is rather limited, we restrict our discussion to conventional Regge poles.

(b) If one makes a conventional Regge model containing only the N_α and the Δ poles, only the Δ contribution survives at $u\simeq-0.15$ (BeV/c)^2. Since the pure $\Delta(I=\frac{3}{2})$ contribution involves only the isovector part of the photon, the vanishing of the $I=\frac{1}{2}$ contribution(s) implies

$$\left(\frac{d\sigma}{du}\right)_{\gamma p\to p\pi^0}=2\left(\frac{d\sigma}{du}\right)_{\gamma p\to n\pi^+}. \quad (3.1)$$

The recent experimental data show that these two cross sections are equal for $-0.3<u<0.0$. Therefore, there must exist a nonvanishing $I=\frac{1}{2}$ contribution at $u\simeq-0.15$. Since we are forced to consider another $I=\frac{1}{2}$ trajectory, we may ask whether the $\Delta(I=\frac{3}{2})$ contribution is completely negligible. The present data are consistent with an $I=\frac{1}{2}$ contribution alone, since one can vary the isoscalar and isovector ratios as a function of u to account for the observed cross sections.

(c) The above argument does *not*, however, eliminate the $I=\frac{3}{2}$ exchange contributions. Assuming vector-meson dominance, there is an experimental way to estimate the $I=\frac{3}{2}$ contribution to $\gamma p\to n\pi^+$ and $\gamma p\to p\pi^0$. This method relates the photoproduction contributions to the pure Δ exchange process $\pi^-p\to p\rho^-$ by

$$\left[\Delta\text{ contribution to }\frac{d\sigma}{du}(\pi^-p\to n\rho^0)\right] =\frac{2}{9}\frac{d\sigma}{du}(\pi^-p\to p\rho^-), \quad (3.2a)$$

TABLE I. Comparisons of upper bounds for $I=\frac{3}{2}$ exchanges with the data and calculated cross sections. The second and third columns give the upper bounds for the contribution of $I=\frac{3}{2}$ exchanges to the photoproduction cross sections. Columns four and five give the interpolated π^0 and π^+ experimental cross sections. Columns six and seven give the percent contribution of the Δ to the calculated cross section.

u (BeV^2)	Upper bound for π^0 (nb/BeV^2)	Upper bound for π^+ (nb/BeV^2)	$d\sigma/du(\pi^0)$ (nb/BeV^2)	$d\sigma/du(\pi^+)$ (nb/BeV^2)	Calculated % Δ to π^0	Calculated % Δ to π^+
$E=$ 8 BeV						
−0.02	5.48	2.74	6.7	5.6	35	18
−0.06	4.56	2.28	6.2	6.3	...	...
−0.13	3.36	1.68	5.0	6.1	69	28
−0.18	2.04	1.02	3.8	5.8	110[a]	31[a]
−0.24	1.40	0.70	2.7	4.5	...	32[a]
−0.32	0.96	0.48	2.2	3.0	126[a]	32[a]
$E=$16 BeV						
−0.10	0.56	0.28	0.87	...	110[a]	33
−0.14	0.60	0.30	0.79	0.78	...	41
−0.20	0.48	0.24	0.59	...	154[a]	43
−0.26	0.36	0.18	0.40	0.61	...	43
−0.33	0.24	0.12	0.27	0.57	184[a]	44[a]
−0.43	0.16	0.08	0.22	...	169[a]	46[a]
−0.57	0.096	0.048	0.13	...	132[a]	58[a]

[a] The calculated cross section exceeds the bound for these points.

$$\left[\Delta\text{ contribution to }\frac{d\sigma}{du}(\gamma p-n\pi^+)\right]=\frac{1}{4}\frac{e^2}{\gamma_\rho^2}\left(\frac{p_\pi}{p_\rho}\right)^2\rho_{11}(u)$$
$$\times\left[\Delta\text{ contribution to }\frac{d\sigma}{du}(\pi^-p\to n\rho^0)\right].\quad(3.2\text{b})$$

Accurate knowledge of the helicity density-matrix element $\rho_{11}(u)$ and of the γ-ρ coupling constant together with the $\pi^-p\to p\rho^-$ data[5] can determine accurately the Δ contribution to photoproduction cross sections. The knowledge of $\rho_{11}(u)$ is rather limited, but one can still obtain an upper bound on the Δ contribution. Table I shows the upper bounds for π^+ and π^0 photoproduction obtained for $\rho_{11}(u)=\frac{1}{2}$, $\gamma_\rho^2/4\pi=\frac{1}{2}$. Comparison of the upper bounds and the experimental photoproduction cross sections show that the $I=\frac{3}{2}$ trajectories could be a significant part of the photoproduction cross sections. In particular, Δ exchange could account for most of the π^0 photoproduction cross section and about $\frac{1}{4}$ to $\frac{1}{2}$ of the π^+ cross sections.

In summary, the arguments (a)–(c) imply that an $I=\frac{1}{2}$ contribution, different from the traditional nucleon, is needed. Argument (c) does not imply that the Δ is necessarily negligible, as was emphasized by Kane.[10]

We therefore use not only the N_α and Δ trajectories but also the N_γ trajectory in our parametrization of the amplitudes. The N_γ trajectory is the needed $I=\frac{1}{2}$ exchange suggested by arguments (a)–(c).

The trajectories are obtained as follows: The N_α and Δ trajectories are taken from the previous parametrizations of Refs. 11 and 12 to the reactions $\pi^+p\to p\pi^+$, $\pi^-p\to p\pi^-$, and $\pi^-p\to p\rho^-$. The nucleon trajectory that we use is

$$\alpha=-0.34+0.093W+1.15W^2\quad(W\text{ in BeV})\quad(3.3)$$

taken from the work of Chiu and Stack.[11] For the Δ trajectory, we added a term linear in W to Shih's determination,[12] so that the trajectory passes through the Δ (1238) mass, to obtain

$$\alpha=0.05+0.25W+0.75W^2.\quad(3.4)$$

The N_γ trajectory is constrained to pass through the first two N resonances [$N_\gamma(1520)\frac{3}{2}^-$, $N_\gamma(2190)\frac{7}{2}^-$] and also contains a term linear in W. The trajectory intercept was chosen as a free paramter:

$$\alpha=x-(0.385+1.105x)W+(0.88+0.278x)W^2.\quad(3.5)$$

The residues $\gamma_i^\alpha(W)$ have the following form:

$$\gamma_2=I_\alpha a_\alpha(1+b_\alpha W),\quad(3.6\text{a})$$
$$\gamma_4=I_\alpha c_\alpha(1+d_\alpha W)e^{hu},\quad(3.6\text{b})$$

where I_α is an isospin factor which depends on the coupling of the various trajectories to the u-channel vertices, and is given in Table II. The parameter S/V (see Table II) occurs for the nucleon trajectories because the nucleons can couple to both the isovector and isoscalar components of the photon. We take the ratio to be a constant and the same for all residues and $I=\frac{1}{2}$ trajectories.

TABLE II. The isotopic-spin dependence of the amplitudes. The parameter S/V is the isoscalar-to-isovector coupling ratio for the $I=\frac{1}{2}$ exchanges, and is determined by the fit to the data.

Reaction	I_α N_α, N_γ	Δ
$\gamma p\to n\pi^+$	$-\sqrt{2}(1+S/V)$	1
$\gamma p\to p\pi^0$	$(1-S/V)$	$\sqrt{2}$
$\gamma n\to p\pi^-$	$-\sqrt{2}(1-S/V)$	1

[10] G. Kane, talk presented at the informal meeting on Processes at Backward Angles, SLAC, 1969 (unpublished).
[11] C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).
[12] C. C. Shih, Phys. Rev. Letters **22**, 105 (1969).

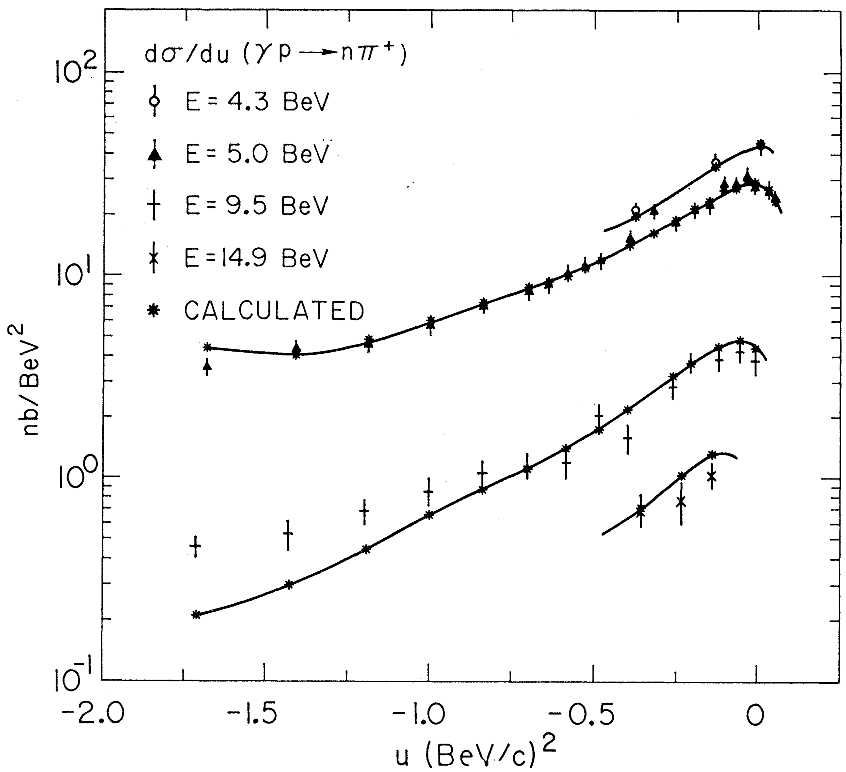

FIG. 1. Fit to π^+ photoproduction. The photon momentum varies somewhat about the indicated average. The theoretical points are calculated using the photon momentum of each point. The data are from Ref. 3.

As can be seen in Figs. 1 and 2, we obtained an excellent fit to the photoproduction data. The χ^2 was found to be 92.5 for 61 degrees of freedom. The quality of the fit did not change appreciably when a different nucleon trajectory,[13]

$$\alpha = -0.38 + 0.91W^2, \tag{3.7}$$

was substituted for the one given in Eq. (3.3).

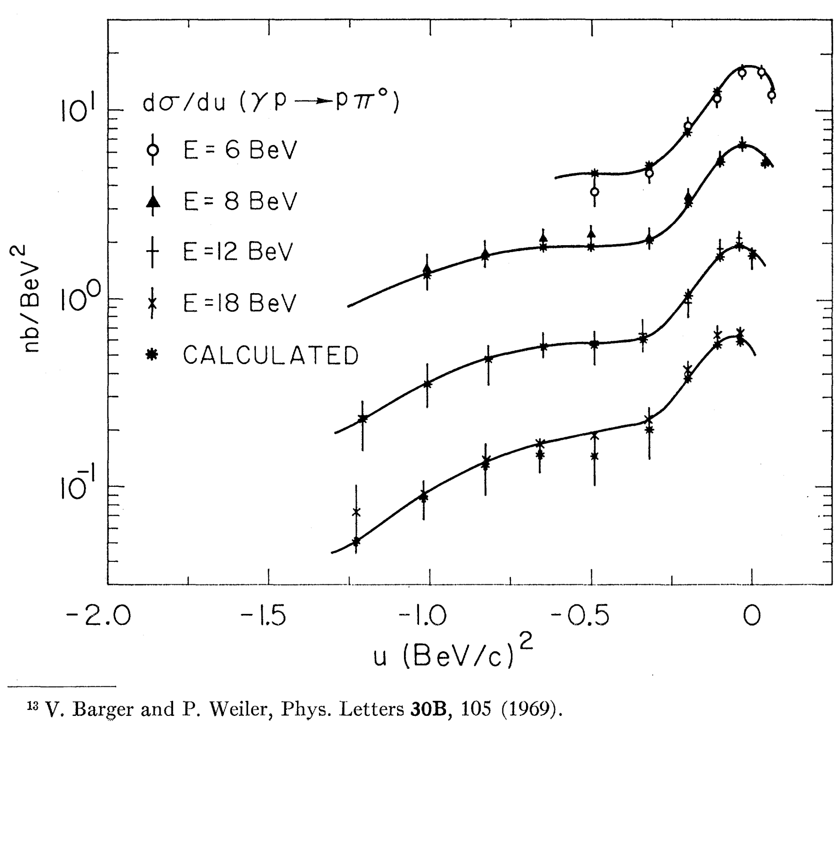

FIG. 2. Fit to π^0 photoproduction data of Ref. 4.

[13] V. Barger and P. Weiler, Phys. Letters **30B**, 105 (1969).

The rest of the parameters are given by

$$N_\alpha:\ a=-1.41\times10^{-3}/\text{BeV}^2,\quad b=8.43/\text{BeV},$$
$$c=1.10\times10^{-2}/\text{BeV}^3,\quad d=-22.9/\text{BeV};$$
$$\Delta:\ a=-3.66\times10^{-3}/\text{BeV}^2,\quad b=7.78/\text{BeV},$$
$$c=5.25\times10^{-2}/\text{BeV}^3,\quad d=1.97/\text{BeV};$$
$$N_\gamma:\ a=6.45\times10^{-2}/\text{BeV}^2,\quad b=0.031/\text{BeV},$$
$$c=-8.20\times10^{-2}/\text{BeV}^3,\quad d=-0.778/\text{BeV},$$
$$s_0=3.83\ \text{BeV}^2,\quad S/V=-0.376,$$
$$h=4.19/\text{BeV}^2,\quad x=\alpha(N_\gamma,0)=-0.546,$$

giving the N_γ trajectory

$$\alpha=-0.546+0.22W+0.73W^2.$$

The large u dependence of the residue γ_4 is unsatisfactory. We tried to eliminate this dependence by imposing several constraints, but the only solutions that we found have a χ^2 larger than 250. These searches lead us to believe that extrapolations of the residues to the poles can be unstable. Therefore, estimates for the ratios of the residues using the isobar model are at best suggestive.

IV. CONCLUSIONS AND EXPERIMENTAL CHECKS

An interpretation of the data in terms of three trajectories demands a delicate balance among them. In fact, this analysis relies heavily on the degeneracy between the N_α and N_γ trajectories, as well as the dominance of the Δ trajectory in some regions. We summarize below those conclusions of the model that can be checked experimentally.

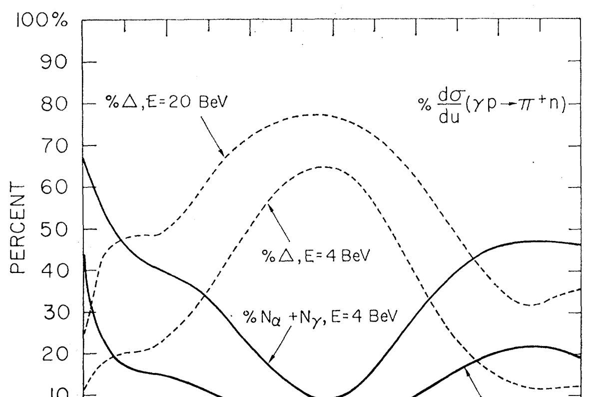

FIG. 3. Percentage contributions of $I=\frac{1}{2}$ $(N_\alpha+N_\gamma)$ and $I=\frac{3}{2}$ (Δ) exchanges to the calculated $\gamma p\to n\pi^+$ cross section. The N_γ signature minimum is quite evident near $u=-1.2$ BeV^2.

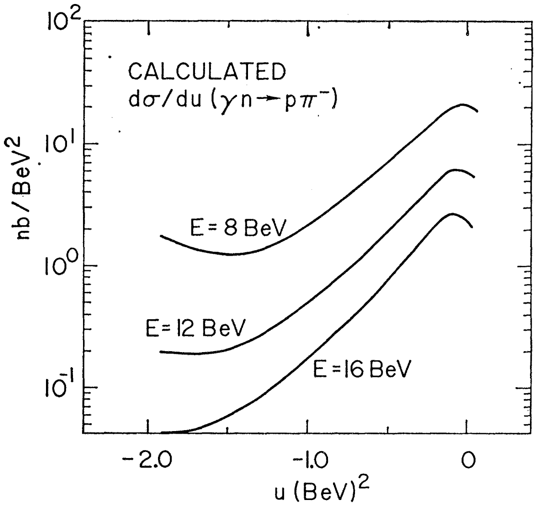

FIG. 4. Predicted cross section for $\gamma n\to p\pi^-$ at incident photon energies of 8, 12, and 16 BeV.

A. N_α-N_γ Degeneracy

In the region $-1\lesssim u\lesssim 0.0$, the N_α and N_γ trajectories are almost degenerate. Such a degeneracy has already been proposed in the literature:

(a) Several theoretical models predict an $N_\alpha-N_\gamma$ exchange degeneracy.[14]

(b) Both the absence of resonances in the pp system and the absence of the N_α dip in $pp\to d\pi^+$ can be explained in terms of degenerate N_α and N_γ trajectories.[15] This degeneracy can be checked at places where the Δ contribution is small by comparing $\gamma p\to n\pi^+$ to the crossing symmetric process $\bar{p}n\to\pi^-\gamma$. Furthermore, a better check of the exchange degeneracy can be obtained by comparing the crossing-symmetric processes $pp\to d\pi^+$ and $\pi^-p\to\bar{p}d$.[15]

B. Δ Contribution

We find that the Δ contribution is large and in some places dominant. The percentages of the $I=\frac{1}{2}$ and Δ contributions to π^+ production at photon energies of 4 and 20 BeV are shown in Fig. 3. The upper bounds of the Δ contribution, given in Table I, are exceeded by our model for $|u|\gtrsim0.3$ BeV^2, for the π^+ data, and most of the π^0 data. In the large-u region, where the N_γ has a wrong-signature minimum, the Δ trajectory is dominant. It would be very interesting to extend the $\pi^-p\to p\rho^-$ data to this region to determine whether or not the Δ contribution of the model drastically violates the upper bounds. A large violation for a simple Regge model would imply that the role of cuts is very important and cannot be ignored in photoproduction.

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Another motivation for extending the $\pi^- p \to p\rho^-$ data to larger values of $|u|$ comes from the following observation. When we separate the isovector contribution of the photon, by setting S/V equal to zero in our solution, we can relate it to $\rho_{11} d\sigma/du(\pi^- p \to n\rho^0)$ using the vector-meson dominance model. Such a comparison has been done by Guiragossian[16] who concludes that the agreement is satisfactory at $E_{\rm lab}=4$ BeV and $-1.0 \lesssim u < 0.0$, but it is not very satisfactory in the larger $|u|$ region.

C. Photoproduction of π^- at Backward Angles

The $\gamma n \to p\pi^-$ cross section can be predicted within this model. Figure 4 gives the calculated cross section at 8, 12, and 16 BeV. The $\gamma n \to p\pi^-$ cross section is about two to three times the $\gamma p \to n\pi^+$ cross section. This enhancement is related to the S/V parameter through the equation

$$\frac{d\sigma}{du}(\pi^\pm) = |N|^2\left(1 \pm \frac{S}{V}\right)^2 + \left(\frac{d\sigma}{du}\right)_{\rm int}\left(1 \pm \frac{S}{V}\right) + |\Delta|^2,$$

where $|N|^2$, $(d\sigma/du)_{\rm int}$, and $|\Delta|^2$ are the contributions to the cross sections from the nucleon, nucleon-Δ interference, and Δ, respectively. Since $S/V = -0.376$ from our solution and since the $(d\sigma/du)_{\rm int}$ is positive as it follows from Fig. 3, we expect the π^- cross section to be larger.

At places where the N_γ trajectory is dominant, the prediction that

$$\frac{d\sigma}{du}(\gamma n \to p\pi^-) > \frac{d\sigma}{du}(\gamma p \to n\pi^+)$$

is rather general[13] and it should be checked experimentally.

Our solutions at 180° extrapolated to low energies pass through the mean of the different cross sections.

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Broken Chiral Symmetry. I. Continuous Transitions between Subgroups*

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We investigate the general properties of the Gell-Mann model for chiral $U(3)\otimes U(3)$ symmetry breaking. From a study of the two-point functions, we find that the symmetry-breaking parameters cannot assume arbitrary values, but must be confined in specified domains. The boundaries of these domains are related to several interesting subgroup symmetries. We present arguments to show that one must have essential singularities at those values of the symmetry-breaking parameter which correspond to subgroup symmetries realized via the emergence of zero-mass bosons. In a suitable singularity-free range of physical interest, we next discuss the possibility of continuous transitions between different symmetry subgroups, and show how, with the use of a variational principle, one can obtain some mass formulas and relations between other physically relevant quantities in a nonperturbative manner. In particular, the relation obtained by Gell-Mann, Oakes, and Renner for the symmetry-breaking parameter is obtained naturally in this manner. Also, it is shown that this formalism requires the existence of scalar mesons.

1. INTRODUCTION

The chiral $SW(2) \equiv SU^{(+)}(2)\otimes SU^{(-)}(2)$ and $SW(3) \equiv SU^{(+)}(3)\otimes SU^{(-)}(3)$ groups have been introduced and studied by many authors.[1–7] Most of these approaches can be classified as dynamical or kinematical. In the dynamical method, one assumes explicit forms for the Lagrangian possessing an approximate $SW(2)$ or $SW(3)$ group symmetry, while in the kinematical approach one employs more general principles such as the algebra[5] of currents and the transformation properties of the symmetry-violating interactions. Actually, one can further categorize the dynamical method. One approach is based on a linear realization[1,3–6] of the chiral group with the Lagrangian ex-

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