Spin-$\frac{3}{2}$ Polarization in N^* Photoproduction†

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The explicit expressions for spin-$\frac{3}{2}$ density-matrix elements are obtained, and a straightforward way to get the decay distribution of a spin-$\frac{3}{2}$ particle is given. The photoproduction $\gamma+p \rightarrow N^{*++}+\pi^-$ and the decay $N^{*++} \rightarrow p+\pi^+$ are considered on the assumption of one axial-vector-meson exchange. The density-matrix elements of N^{*++} contain one parameter which is the ratio of two form factors, and they are compared with the recent experimental data from DESY. A special case of the linearly polarized photon is also considered, and the decay distribution of N^* turns out to be close to the Sakurai-Stodolsky distribution near the threshold. Several other models are also considered and compared with the axial-vector-meson-exchange model.

I. INTRODUCTION

THE helicity formalism of Jacob and Wick[1,2] is known to be very useful in determining the decay distribution of high-spin particles. In this formalism the decay distribution of a high-spin particle is expressed in terms of the spin-density matrix elements and the polar angles of one of the decay products. It contains the information on the production process through the density-matrix elements.

Shay, Song, and Good[3] have obtained the spin-density matrix for a spin-$\frac{3}{2}$ particle in a covariant form and also in terms of spin-$\frac{3}{2}$ matrices. We[4] have shown how the polarization differential cross section and the density matrix for a spin-$\frac{3}{2}$ particle can be obtained in the production of N^* by neutrinos. Florescu and Minnaert[5] have obtained the density matrix and the polarization correlation function of a baryonic resonance of arbitrary half-integer spin in high-energy neutrino reactions. King[6] has considered the process $\pi+N\rightarrow N^*+\pi$ and obtained the density matrix and the decay distribution of N^*. Sakurai and Stodolsky[7,8] have considered the process $\gamma+N\rightarrow N^*\rightarrow N+\pi$ and obtained the decay distribution of N^*. Jackson and Pilkuhn[9] have discussed the one-pion- and one-vector-meson-exchange models in several cases including the production and decay of a spin-$\frac{3}{2}$ particle. However, the methods used by Sakurai and Stodolsky and by Jackson and Pilkuhn are different from ours. Recently Lüke, Scheunert, and Stichel[10] have considered the process $\gamma+p\rightarrow N^{*++}+\pi^-$ and obtained the density-matrix elements of N^{*++} using a model developed by Stichel and Scholz[11] in which the one-pion-exchange (OPE) model has been modified to achieve gauge invariance by including additional diagrams.

In the recent experiments of N^* photoproduction by the Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration[12,13] and by the Cambridge Bubble Chamber Group,[14] the data have been compared with the OPE model and with a model developed by Stichel and Scholz. The Cambridge Bubble Chamber Group has also considered a resonance model.

The purpose of this paper is to present the explicit values of the density-matrix elements for a spin-$\frac{3}{2}$ particle, to give a straightforward way to get its decay distribution, and to discuss the photoproduction of N^*. The transition amplitude of N^* photoproduction is obtained on the assumption of one-axial-vector-meson exchange, and it turns out to be the same as the transition amplitude obtained by Narayanaswamy and Renner[15] in the limit they used. The differential cross sections, the density matrix of N^* in its production process, and the decay distribution of N^* are obtained. The density-matrix elements of N^* contain one parameter, and they are compared with recent experimental data from DESY. The linear polarized photons are also considered, and it is found that the decay distribution becomes the Sakurai-Stodolsky distribution near threshold when the momentum of the N^* is assumed to be small in the laboratory system. Also, the other models considered by Narayanaswamy and Renner,[15] Gourdin and Salin,[16] and Sakurai and Stodolsky,[7] and the one-vector-meson-exchange model discussed by Jackson and Pilkuhn,[9] are compared with the one-axial-vector-meson-exchange model.

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Finally, some useful formulas in calculating the absolute square of the transition amplitude in N^* production processes are listed in the Appendix.

II. SPIN-$\frac{3}{2}$ DENSITY MATRIX

The spin-density matrix for a spin-$\frac{3}{2}$ particle given in Refs. 3 and 4 is of the following form in its rest frame:

$$\rho=\tfrac{1}{4}(1+2B_i\mathbf{S}_i/A+C_{ij}\mathbf{S}_{ij}/2A+D_{ijk}\mathbf{S}_{ijk}/6A)\,. \quad (1)$$

The coefficients in this expression can be chosen differently, and here the expression in Ref. 4 is used. Since the C_{ij} and D_{ijk} are made of vector components, they can be written as

$$C_{ij}=C_i{}^AC_j{}^B\,,\quad D_{ijk}=D_i{}^AD_j{}^BD_k{}^C\,. \quad (2)$$

Then the density-matrix elements are as follows:

$$\begin{aligned}
4A\rho_{\frac{3}{2},\frac{3}{2}(-\frac{3}{2},-\frac{3}{2})}&=A\pm3B_3+\tfrac{1}{2}(3C_3{}^AC_3{}^B-\mathbf{C}^A\cdot\mathbf{C}^B)\\
&\quad\pm(3/20)(5D_3{}^AD_3{}^BD_3{}^C-D_3{}^A\mathbf{D}^B\cdot\mathbf{D}^C\\
&\qquad-D_3{}^B\mathbf{D}^C\cdot\mathbf{D}^A-D_3{}^C\mathbf{D}^A\cdot\mathbf{D}^B)\,,\\
4A\rho_{\frac{1}{2},\frac{1}{2}(-\frac{1}{2},-\frac{1}{2})}&=A\pm B_3-\tfrac{1}{2}(3C_3{}^AC_3{}^B-\mathbf{C}^A\cdot\mathbf{C}^B)\\
&\quad\mp(9/20)(5D_3{}^AD_3{}^BD_3{}^C-D_3{}^A\mathbf{D}^B\cdot\mathbf{D}^C\\
&\qquad-D_3{}^B\mathbf{D}^C\cdot\mathbf{D}^A-D_3{}^C\mathbf{D}^A\cdot\mathbf{D}^B)\,,\\
4A\rho_{\frac{3}{2},\frac{1}{2}(-\frac{1}{2},-\frac{3}{2})}&=\sqrt{3}\{B_-\pm\tfrac{1}{2}(C_3{}^AC_-{}^B+C_3{}^BC_-{}^A)\\
&\quad+(1/20)[D_-{}^A(5D_3{}^BD_3{}^C-\mathbf{D}^B\cdot\mathbf{D}^C)\\
&\quad+D_-{}^B(5D_3{}^CD_3{}^A-\mathbf{D}^C\cdot\mathbf{D}^A)\\
&\qquad+D_-{}^C(5D_3{}^AD_3{}^B-\mathbf{D}^A\cdot\mathbf{D}^B)]\}\,,\\
4A\rho_{\frac{3}{2},-\frac{1}{2}(\frac{1}{2},-\frac{3}{2})}&=\sqrt{3}[\tfrac{1}{2}C_-{}^AC_-{}^B\pm\tfrac{1}{4}(D_3{}^AD_-{}^BD_-{}^C\\
&\qquad+D_3{}^BD_-{}^CD_-{}^A+D_3{}^CD_-{}^AD_-{}^B)]\,,\\
4A\rho_{\frac{1}{2},-\frac{1}{2}}&=2B_--(3/20)[D_-{}^A(5D_3{}^BD_3{}^C-\mathbf{D}^B\cdot\mathbf{D}^C)\\
&\quad+D_-{}^B(5D_3{}^CD_3{}^A-\mathbf{D}^C\cdot\mathbf{D}^A)\\
&\qquad+D_-{}^C(5D_3{}^AD_3{}^B-\mathbf{D}^A\cdot\mathbf{D}^B)]\,,\\
4A\rho_{\frac{3}{2},-\frac{3}{2}}&=\tfrac{3}{4}D_-{}^AD_-{}^BD_-{}^C\,,
\end{aligned} \quad (3)$$

where $B_-=B_1-iB_2$, etc., and the lower signs of $\pm$ and $\mp$ correspond to the values of ρ with the subscripts in parentheses. The other matrix elements can be obtained from the Hermitian property of the density matrix. In order to get the decay distribution, one does not actually need all of these matrix elements; it has a much simpler form. For example, let a coordinate system be chosen such that the z axis is perpendicular to the production plane and the x and y axes are in the production plane with the y axis along the direction of the momentum of one of the incoming particles. Then the decay distribution[6,17] is

$$\begin{aligned}
W(\theta,\phi)=(1/8\pi)[&(\rho_{\frac{1}{2},\frac{1}{2}}+\rho_{-\frac{1}{2},-\frac{1}{2}})(1+3\cos^2\theta)\\
&+(\rho_{\frac{3}{2},\frac{3}{2}}+\rho_{-\frac{3}{2},-\frac{3}{2}})3\sin^2\theta\\
&-\mathrm{Re}(\rho_{\frac{3}{2},-\frac{1}{2}}+\rho_{\frac{1}{2},-\frac{3}{2}})2\sqrt{3}\sin^2\theta\cos2\phi\\
&+\mathrm{Im}(\rho_{\frac{3}{2},-\frac{1}{2}}+\rho_{\frac{1}{2},-\frac{3}{2}})2\sqrt{3}\sin^2\theta\sin2\phi]\,,
\end{aligned} \quad (4)$$

where θ and ϕ are the polar angles of the nucleon momentum produced in the N^* decay process. From Eq. (3) one gets

$$\begin{aligned}
\rho_{\frac{1}{2},\frac{1}{2}}+\rho_{-\frac{1}{2},-\frac{1}{2}}&=(2A+\mathbf{C}^A\cdot\mathbf{C}^B-3C_3{}^AC_3{}^B)/4A\,,\\
\rho_{\frac{3}{2},\frac{3}{2}}+\rho_{-\frac{3}{2},-\frac{3}{2}}&=(2A-\mathbf{C}^A\cdot\mathbf{C}^B+3C_3{}^AC_3{}^B)/4A\,,\\
\rho_{\frac{3}{2},-\frac{1}{2}}+\rho_{\frac{1}{2},-\frac{3}{2}}&=\sqrt{3}[2C_-{}^AC_-{}^B+(D_3{}^AD_-{}^BD_-{}^C\\
&\quad+D_3{}^BD_-{}^CD_-{}^A+D_3{}^CD_-{}^AD_-{}^B\\
&\qquad+3D_-{}^AD_-{}^BD_-{}^C)]/16A\,,\\
\rho_{\frac{3}{2},-\frac{1}{2}}+\rho_{\frac{1}{2},-\frac{3}{2}}&=\sqrt{3}C_-{}^AC_-{}^B/4A\,.
\end{aligned} \quad (5)$$

Gottfried and Jackson[2] have used a different coordinate system in which the z axis is along the direction of the momentum of the incoming nucleon and the y axis is normal to the production plane.

III. PHOTOPRODUCTION OF N^*

Sakurai and Stodolsky[7] have considered the pure $M1$ transition in N^* photoproduction, and Stodolsky[8] has obtained the transition amplitude for the reaction $\gamma+N\to N^*$ in the form

$$G\bar{u}_\mu A_\mu u\,, \quad (6)$$

where G is a coupling constant, u_μ and u are the N^* spinor and N spinor, respectively, and $A_\mu=\epsilon_{\mu\nu\rho\sigma}N_\nu{}^*k_\rho\varepsilon_\sigma$. From now on N_μ, $N_\nu{}^*$, q_ρ, k_τ, and ε_σ will denote the momenta of the nucleon, N^*, pion, photon, and the polarization of photon, respectively.

When the direct interaction is considered in the reaction $\gamma+p\to N^{*++}+\pi^-$, the transition amplitude considered by Stichel and Scholz[11] is

$$F\bar{u}_\mu\varepsilon_\mu u\,. \quad (7)$$

This equation is not gauge invariant. However, if ε_μ is replaced by $\varepsilon_\mu-k_\mu(\varepsilon q)/kq$, it is gauge invariant. [Here (kq) implies the scalar product of four-vectors, $k\cdot q$.] This equation can be considered as a special case of Eq. (11).

Another possibility in the reaction $\gamma+p\to N^{*++}+\pi^-$ is the one-particle-exchange model. Especially in the photoproduction of charged pions, the exchange of π, ρ, A_1, and A_2 are all allowed.[18]

For one-pion exchange, the transition amplitude becomes of the form[11]

$$H(\varepsilon q)\bar{u}_\mu Q_\mu u/(m_\pi{}^2-t)\,, \quad (8)$$

where Q_μ is the momentum transfer four-vector, $t=-Q^2$, and H is a form factor. If one uses the formulas in the Appendix and Eq. (5), one obtains immediately the density-matrix elements in the Jackson-Gottfried system:

$$\rho_{\frac{1}{2},\frac{1}{2}}=\tfrac{1}{2}\,,\quad \rho_{\frac{3}{2},\frac{3}{2}}=\rho_{\frac{3}{2},\frac{1}{2}}=\rho_{\frac{3}{2},-\frac{1}{2}}\,, \quad (9)$$

and this unmodified OPE model does not agree with experimental data.[10,18,19] Stichel and Scholz[11] modified the OPE model to achieve gauge invariance by including additional diagrams.

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Jackson and Pilkuhn[9] have extensively considered several cases of the one-vector-meson-exchange model. According to their treatment, the transition amplitude in the process $\gamma+p\to N^{*++}+\pi^-$ can be written in the following form, when a vector meson is exchanged:

$$M=\frac{f_{\gamma\pi V}F_V}{m_V^2-t}\bar{u}_\mu\left(G_1P_\mu-i\frac{G_2}{\Delta}Q_\mu\gamma_\nu P_\nu\right)\gamma_5 u\,, \tag{10}$$

where $P_\mu=i\epsilon_{\mu\nu\rho\sigma}k_\nu Q_\rho\varepsilon_\sigma$ and $\Delta=m_N+m_{N^*}$. $f_{\gamma\pi V}$, F_V, G_1, and G_2 are form factors, and here higher-derivative terms are neglected. Hereafter, the subscripts γ, N, N^*, π, V, and A denote the photon, nucleon, N^*, pion, vector meson, and axial-vector meson, respectively.

Next we suppose that the exchange particle in the reactions $\gamma+p\to N^{*++}+\pi^-$ is an axial-vector meson, e.g., an A_1 particle, instead of a vector meson. The vertex factor $\gamma A_1\pi$ for the transition amplitude can be obtained from the amplitude of the A_1-π-γ reaction considered by Riazuddin and Fayyazuddin,[20] and the vertex factor NA_1N^* has been considered by Jackson and Pilkuhn.[9] In the latter case the transition amplitude becomes

$$M=\frac{f_{\gamma\pi A}F_A}{m_A^2-t}\bar{u}_\mu\left(F_1B_\mu-i\frac{F_2}{\Delta}Q_\mu\gamma_\nu B_\nu\right)u\,, \tag{11}$$

where $B_\mu=\varepsilon_\mu-k_\mu(\varepsilon q)/kq$ and the form factor $f_{\gamma\pi A}$ has been calculated by Riazuddin and Fayyazuddin, but F_A, F_1, and F_2 are undetermined. The density-matrix elements and differential cross sections obtained from Eq. (11) may give suitable values for the form factors when they are compared with experimental data. In the limit in which the nucleon momentum of the incoming nucleon is very small in the N^* rest frame, the second term in Eq. (11) can be neglected.

The absolute square of the transition amplitude of Eq. (11) can be obtained immediately using the formulas given in the Appendix. In the rest frame of N^* one obtains

$$|M|^2=\frac{f_{\gamma\pi A}{}^2m_{N^*}{}^2}{12E_N}\left|\frac{F_A}{m_A^2-t}\right|^2\times\left[A+\frac{K_1K_2+K_2K_3+K_3K_1}{4|K|}C_{ij}s_{ij}+\frac{K_1K_2K_3}{4|K|}D_{ijk}s_{ijk}\right], \tag{12}$$

and the differential cross sections become

$$\left(\frac{d\sigma}{dt}\right)_{\text{unp}}=\frac{m_{N^*}f_{\gamma\pi A}{}^2}{6\pi(s-m_N{}^2)^2}\left|\frac{F_A}{m_A^2-t}\right|^2A\,, \tag{13}$$

$$\left(\frac{d\sigma}{dt}\right)_{\text{pol}}=\frac{1}{4}\left(\frac{d\sigma}{dt}\right)_{\text{unp}}\times\left[1+\frac{K_1K_2+K_2K_3+K_3K_1}{4|K|\mathbf{A}}C_{ij}s_{ij}+\frac{K_1K_2K_3}{4|K|\mathbf{A}}D_{ijk}s_{ijk}\right], \tag{14}$$

where $s=-(k+N)^2$. The density matrix becomes

$$\rho=\tfrac{1}{4}(1+C_{ij}\mathbf{S}_{ij}/2A+D_{ijk}\mathbf{S}_{ijk}/6A)\,, \tag{15}$$

and A, C_{ij}, and D_{ijk} have different values according to the polarization states of the incident photon. Since time-reversal invariance holds in the present process, the form factors can be assumed to be real and then the D_{ijk} do not appear.

If the incident photon is assumed to be unpolarized, one obtains

$$\begin{aligned}A={}&F_1{}^2(E_N+m_N)\left(1-\frac{2E_\gamma E_\pi}{kq}-\frac{m_\pi{}^2E_\gamma{}^2}{(kq)^2}\right)-2F_2{}^2\Delta^{-2}\mathbf{N}^2\left(m_N-\frac{E_\gamma(qN)+E_\pi(kN)}{kq}-\frac{m_\pi{}^2E_\gamma(kN)}{(kq)^2}\right)\\&-2F_1F_2\Delta^{-1}m_N(E_N-m_{N^*})\left(1+\frac{2E_\gamma E_\pi}{kq}+\frac{m_\pi{}^2E_\gamma{}^2}{(kq)^2}\right)\\&+F_1F_2\Delta^{-1}\left[(QN+QN^*)\left(1+\frac{2E_\gamma E_\pi}{kq}+\frac{m_\pi{}^2E_\gamma{}^2}{(kq)^2}\right)-2\mathbf{N}^2\right],\\C_{ij}={}&F_1{}^2(E_N+m_N)(kq)^{-1}\left(2k_iq_j+\frac{m_\pi{}^2\mathbf{k}_i\mathbf{k}_j}{(kq)^2}\right)+2F_2{}^2\Delta^{-2}\mathbf{N}_i\mathbf{N}_j\left(m_N-\frac{E_\gamma(qN)+E_\pi(kN)}{kq}-\frac{m_\pi{}^2E_\gamma(kN)}{(kq)^2}\right)\\&+2F_1F_2\Delta^{-1}(kq)^{-1}m_N\left(E_\gamma\mathbf{N}_i\mathbf{q}_j+E_\pi\mathbf{N}_i\mathbf{k}_j+\frac{m_\pi{}^2E_\gamma\mathbf{N}_i\mathbf{k}_j}{kq}\right)\\&+2F_1F_2\Delta^{-1}(kq)^{-1}\left[-\mathbf{N}^2\mathbf{k}_i\mathbf{k}_j\left(2+\frac{m_\pi{}^2}{kq}\right)+\mathbf{N}_i\mathbf{N}_j\left((kq)+E_\gamma(2E_N-2E_\pi-m_{N^*})-\frac{m_\pi{}^2E_\gamma{}^2}{kq}\right)\right.\\&\left.+\mathbf{N}_i\mathbf{k}_j\left((E_\gamma+E_\pi)(2E_N-m_{N^*})-2\mathbf{N}^2+\frac{m_\pi{}^2E_\gamma(2E_N-m_{N^*})}{kq}\right)\right],\end{aligned} \tag{16}$$

[20] Riazuddin and Fayyazuddin, Phys. Rev. **171**, 1428 (1968).

and one can also get $\rho_{\frac{3}{2},\frac{3}{2}}$, $\rho_{\frac{3}{2},\frac{1}{2}}$, and $\rho_{\frac{3}{2},-\frac{1}{2}}$ using Eqs. (3) and (16). In the Gottfried-Jackson system, they become

$$\rho_{\frac{3}{2},\frac{3}{2}}=\tfrac{1}{4}+(8A)^{-1}\left\{\frac{4(E_N+m_N)(\mathbf{k}\cdot\mathbf{N})}{kq}+\frac{(E_N+m_N)(3k_3{}^2-\mathbf{k}^2)[2(kq)+m_\pi{}^2]}{(kq)^2}\right.$$
$$+4\left(\frac{RN}{\Delta}\right)^2\left(m_N-\frac{E_\gamma(qN)+E_\pi(kN)}{kq}-\frac{m_\pi{}^2E_\gamma(kN)}{(kq)^2}\right)+4\frac{R}{\Delta}m_N\frac{E_\gamma\mathbf{N}^2+(E_\gamma+E_\pi)(\mathbf{N}\cdot\mathbf{k})+m_\pi{}^2E_\gamma(\mathbf{N}\cdot\mathbf{k})/kq}{kq}$$
$$+2\frac{R}{\Delta}\left[-\frac{(3k_3{}^2-\mathbf{k}^2)\mathbf{N}^2[2(kq)+m_\pi{}^2]}{(kq)^2}+2\mathbf{N}^2\left(1+\frac{E_\gamma(E_N-E_\gamma-E_\pi)}{kq}-\frac{m_\pi{}^2E_\gamma{}^2}{(kq)^2}\right)\right.$$
$$\left.\left.+2(kq)^{-1}(\mathbf{N}\cdot\mathbf{k})\left((E_\gamma+E_\pi)(2E_N-m_{N^*})-2\mathbf{N}^2-\frac{m_\pi{}^2E_\gamma(2E_N-m_{N^*})}{kq}\right)\right]\right\}, \quad (17)$$

$$\rho_{\frac{3}{2},\frac{1}{2}}=\left(\frac{\sqrt{3}k_-}{4A(kq)^2}\right)\left((E_N+m_N)\{N(kq)+k_3[m_\pi{}^2+2(kq)]\}+\frac{R}{\Delta}Nm_N[(kq)(E_\gamma+E_\pi)+m_\pi{}^2E_\gamma]\right.$$
$$\left.+\frac{R}{\Delta}N\{-2k_3N[m_\pi{}^2+2(kq)]+(E_\gamma+E_\pi)(2E_N-m_{N^*})(kq)-2\mathbf{N}^2(kq)+m_\pi{}^2E_\gamma(2E_N-m_{N^*})\}\right),$$

$$\rho_{\frac{3}{2},-\frac{1}{2}}=\frac{\sqrt{3}}{8A}k_-{}^2\left((E_N+m_N)-\frac{2R\mathbf{N}^2}{\Delta}\right)\frac{m_\pi{}^2+2(kq)}{(kq)^2},$$

where $R=F_1/F_2$ and $N=|\mathbf{N}|$. Comparison of these density-matrix elements with the recent experimental data[19] is shown in Fig. 1 for various E_γ intervals. Here the central value of each E_γ interval is used and N^* is treated as a particle with sharp mass (1236 MeV). The form factors are assumed to be constant and four values of R are considered.

If experimental data on the process $\gamma+p\rightarrow N^{*++}+\pi^-$ with linearly polarized photons are available, the ratio R of two form factors can be cross checked, and in this case the theoretical results for the density-matrix elements are simpler in the axial-vector-meson-exchange model. Since the polarization ε_μ of the incident photon with $\varepsilon_4=0$ is perpendicular to k_μ and N_μ in the laboratory system, it can be assumed to be perpendicular to the production plane, especially in the neighborhood of threshold. Then Eq. (11) becomes

$$M=\frac{f_{\gamma\pi A}F_A}{m_A{}^2-t}\bar{u}_\mu\left(F_1\varepsilon_\mu-i\frac{F_2}{\Delta}Q_\mu\gamma_\nu\varepsilon_\nu\right)u. \quad (18)$$

Again, if the formulas given in the Appendix are used, one can obtain from Eq. (18)

$$A=F_1{}^2(E_N+m_N)+\frac{F_2{}^2(E_N-m_N)\mathbf{N}^2}{\Delta^2}-\frac{F_1F_2\mathbf{N}^2}{\Delta},$$
$$C_{ij}=-F_1{}^2(E_N+m_N)\varepsilon_i\varepsilon_j-\frac{F_2{}^2(E_N-m_N)\mathbf{N}_i\mathbf{N}_j}{\Delta^2}+\frac{2F_1F_2(\mathbf{N}^2\varepsilon_i\varepsilon_j+\mathbf{N}_i\mathbf{N}_j)}{\Delta}. \quad (19)$$

Now adopting the coordinate system specified in Sec. II, where the z axis is perpendicular to the production plane, and using Eqs. (4), (5), and (19), the decay distribution can be written in the form

$$W(\theta,\phi)=\frac{1}{8\pi}\left[\left(1-\frac{3}{4A\Delta^2}F_2{}^2(E_N-m_N)\mathbf{N}^2\right)(1+3\cos^2\theta)+\frac{9F_2{}^2(E_N-m_N)\mathbf{N}^2\sin^2\theta}{4A\Delta^2}+\frac{3[2\Delta F_1F_2-F_2{}^2(E_N-m_N)]\mathbf{N}^2\sin^2\theta\cos2\phi}{4A\Delta}\right]. \quad (20)$$

Therefore, if the momentum $\mathbf{N}$ is negligibly small, the decay distribution becomes just the Sakurai-Stodolsky distribution

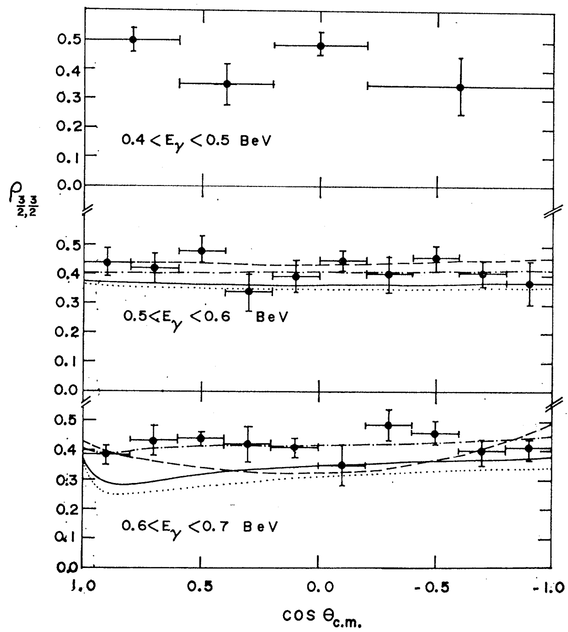
$$1+3\cos^2\theta. \quad (21)$$

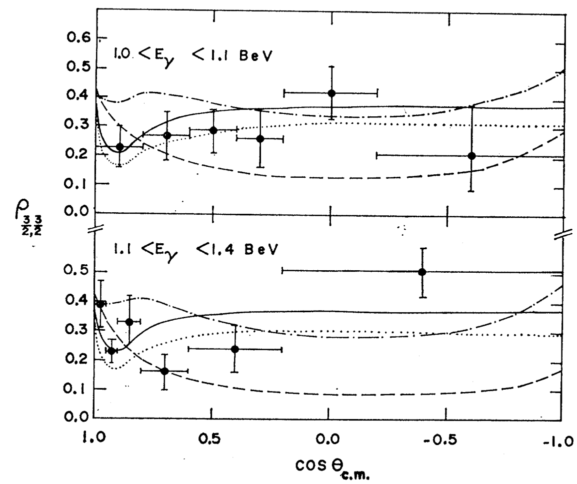
This is the case when the N^* momentum is small in the laboratory system.

When another system considered by Gottfried and Jackson is used, the decay distribution is

$$W(\theta,\phi)=(1/4\pi)[3\rho_{\frac{3}{2},\frac{3}{2}}\sin^2\theta+\rho_{\frac{1}{2},\frac{1}{2}}(1+3\cos^2\theta)-2\sqrt{3}\,\mathrm{Re}(\rho_{\frac{3}{2},-\frac{1}{2}})\sin^2\theta\cos2\phi-2\sqrt{3}\,\mathrm{Re}(\rho_{\frac{3}{2},\frac{1}{2}})\sin2\theta\cos\phi]. \quad (22)$$

Here θ is the angle between the incoming and decay nucleons in the N^* rest frame and ϕ is the azimuthal angle of the decay nucleon measured from the production plane. The explicit values of the spin-density





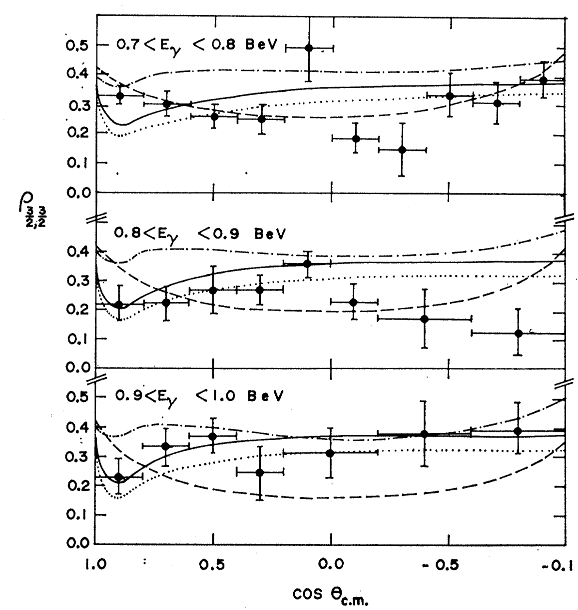


FIG. 1. Reaction $\gamma+p\rightarrow N^{*++}+\pi^-$. Density-matrix elements of N^{*++} in the Gottfried-Jackson system for various intervals of photon energy E_γ and for various values of R, the ratio of two form factors. The full curve is for $R=0$, the dashed curve for $R=7$, the dot-dashed curve for $R=3$, and the dotted curve for $R=-1$. $\theta_{\rm c.m.}$ is the angle between the incoming proton and N^{*++} in the c.m. system. (a) $\rho_{\frac{3}{2},\frac{3}{2}}$.

matrix elements in this case are

$$\begin{aligned}\rho_{\frac{3}{2},\frac{3}{2}}&=\tfrac{3}{8}(1+\alpha)^{-1},\\ \rho_{\frac{1}{2},\frac{1}{2}}&=\tfrac{1}{2}-\rho_{\frac{3}{2},\frac{3}{2}},\\ \rho_{\frac{3}{2},-\frac{1}{2}}&=\frac{\sqrt{3}}{8(1+\beta)},\\ \rho_{\frac{3}{2},\frac{1}{2}}&=0,\end{aligned}\tag{23}$$

where α and β are defined as

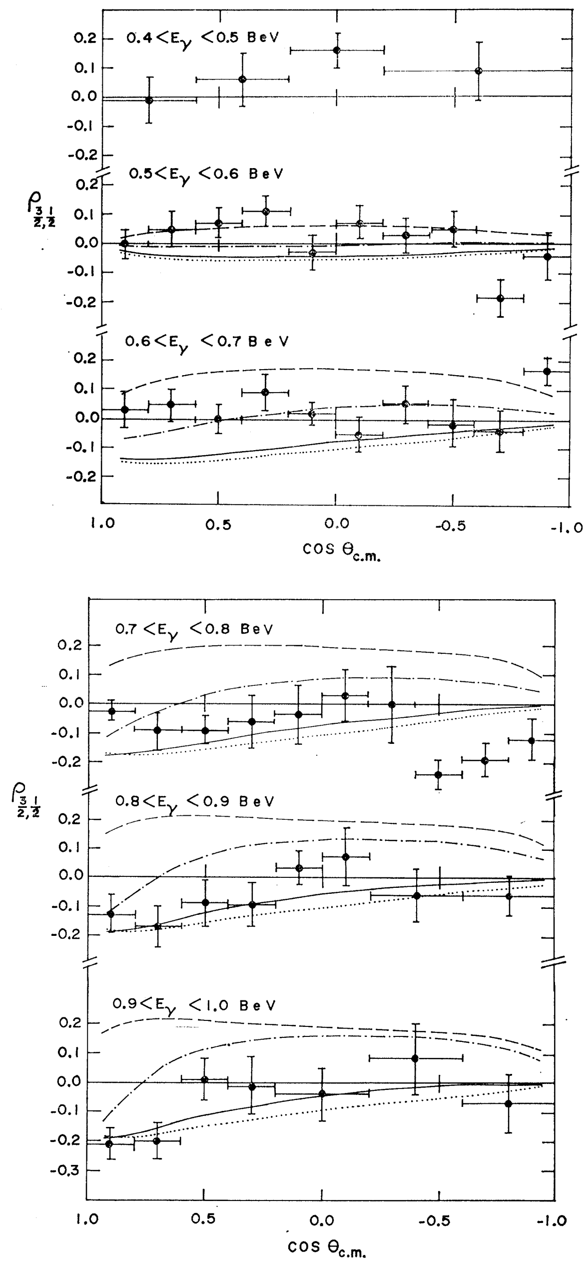
$$\begin{aligned}\alpha&=[(E_N-m_N)R/\Delta]^2-(E_N-m_N)R/\Delta,\\ \beta&=\frac{[(E_N-m_N)R/\Delta]^2+(E_N-m_N)R/\Delta}{1-2(E_N-m_N)R/\Delta}.\end{aligned}\tag{24}$$

When the decay distribution of Eq. (22) is averaged over angles, it becomes

$$\begin{aligned}W(\theta)&=\tfrac{1}{4}(1+3\cos^2\theta)+\rho_{\frac{3}{2},\frac{3}{2}}(1-3\cos^2\theta),\\ W(\phi)&=(1/2\pi)[1-(4/\sqrt{3})\,\mathrm{Re}(\rho_{\frac{3}{2},-\frac{1}{2}})\cos 2\phi],\end{aligned}\tag{25}$$

and in terms of α and β one obtains

$$\begin{aligned}W(\theta)&=\frac{(5+2\alpha)-(3-6\alpha)\cos^2\theta}{8(1+\alpha)},\\ W(\phi)&=\frac{3+2\beta-2\cos^2\phi}{4\pi(1+\beta)}.\end{aligned}\tag{26}$$



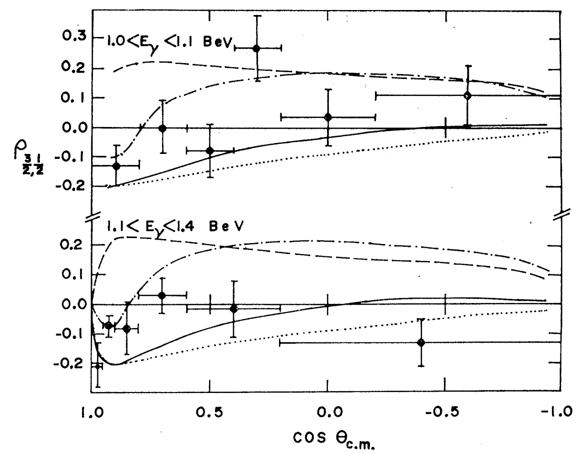


FIG. 1 (*Continued*). (b) $\rho_{\frac{3}{2},\frac{1}{2}}$.

Since $W(\theta)$ and $W(\phi)$ must be positive semidefinite, there are restrictions on possible α and β values. From Eq. (26) these restrictions become

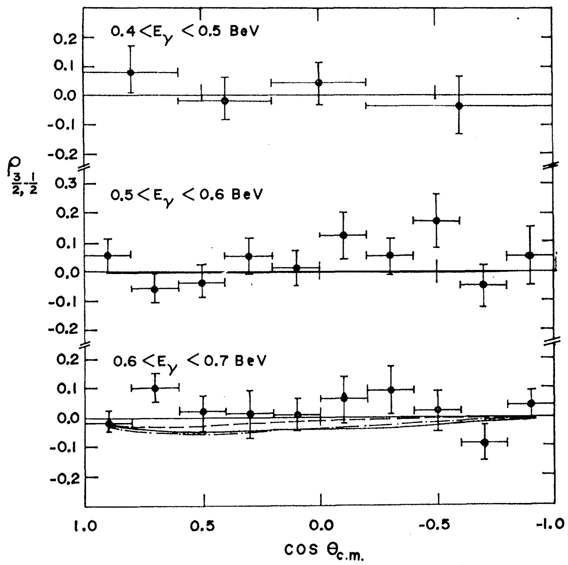
$$\alpha > -\tfrac{1}{4} \quad \text{or} \quad \alpha < -\tfrac{5}{2}, \qquad \beta > -\tfrac{1}{2} \quad \text{or} \quad \beta < -\tfrac{3}{2}. \tag{27}$$

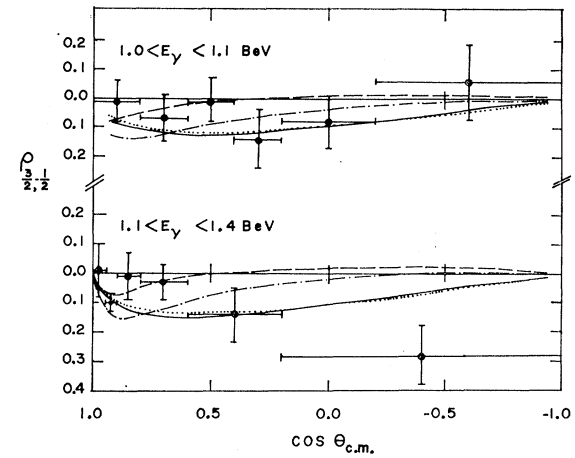
If **N** is negligibly small in the N^* rest frame, the decay distributions become

$$W(\theta) = (5-3\cos^2\theta)/8, \quad W(\phi) = (3-2\cos^2\phi)/4\pi. \tag{28}$$

The first term in Eq. (18) gives the same decay distribution as Eq. (6) does, because A_μ in Eq. (6) and ε_μ in Eq. (18) have the same property in the sense that they are orthogonal to the momenta of the particles. In the limit in which the momenta of the incoming nucleon is very small in the N^* rest frame, the second term in Eq. (18) can be neglected and it reproduces the $M1$ coupling results for $\gamma + N \rightarrow N^*$. Stodolsky has used a different method and obtained the same result, i.e., Eq. (21) or Eq. (28).

Narayanaswamy and Renner have considered another approach and, in the limit where the four-momentum of the outgoing pion in the production process of N^* is zero, they have obtained a transition amplitude similar to that in Eq. (18). Q_μ in Eq. (18) is replaced by k_μ in





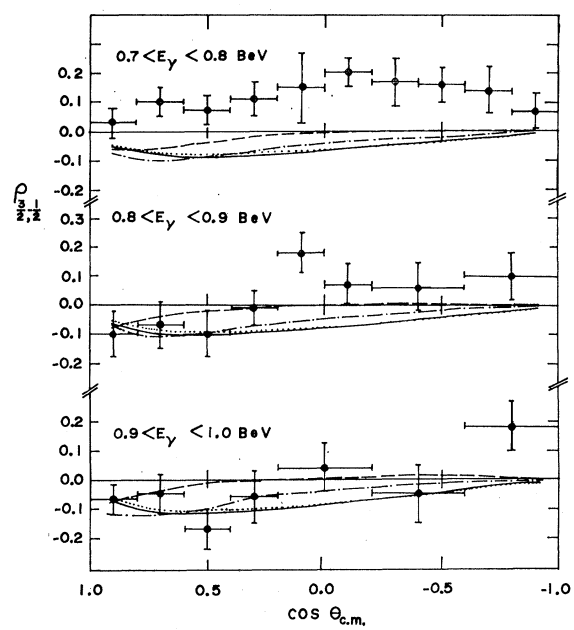


FIG. 1 (*Continued*). (c) $\rho_{\frac{3}{2},-\frac{1}{2}}$.

the limit $q_\mu \to 0$ and the form factors are defined differently. The decay distributions are then the same and $\mathbf{N}$ can be replaced by $\mathbf{k}$ and $\mathbf{N}^2$ by the square of photon energy E_γ^2. They have also considered the threshold limit in which the center-of-mass system is the common rest frame of N^* and π. Then the Adair angle and the angle θ in Eqs. (25) and (28) are the same. The result of Narayanaswamy and Renner can also be compared with the photoproduction of pions considered by Gourdin and Salin where γ_5 is put in. In this case the decay distribution has the same form as Eq. (20), but the signs of the m_N's are changed.

When the one-vector-meson-exchange model is considered, one can obtain the absolute square of the transition amplitude given by Eq. (10) immediately by using the formulas given in the Appendix. If the incident photon polarizations are summed, density-matrix elements can be obtained from Eq. (17) by changing the signs of the m_N's except for those in Δ. However, when the incident photon is linearly polarized, the decay distribution is not similar to the Sakurai-Stodolsky distribution except when the static limit[9] ($m_N \simeq m_{N^*}$, $\mathbf{N}^2 = -t \ll 4m_N^2$), the dipole model ($G_1 = G_2$), and $|P_4^2| \ll |P^2|$ ($|\mathbf{k}\times\mathbf{Q}| \ll |(kQ)|$) in the N^* rest frame are assumed. Also, in the vector-meson-exchange model, one cannot consider the limit in which the four-momentum of the pion produced with N^* is zero as Narayanaswamy and Renner have done. In this limit the transition amplitude obtained by Stichel and Scholz becomes Eq. (7).

IV. DISCUSSION

In Fig. 1 the density-matrix elements contain one free parameter R which is the ratio of two form factors. The first term in Eq. (11) seems to give the general trend of the density matrix, while the second term with various values of R gives the corrections. Here we assumed that the form factors are constant, but in general they are not. $R=7$ seems to given the better fit with the experimental data at low energies, but as the incident photon energy increases, smaller values of R are preferable. Extensive investigation of the density matrix together with differential cross sections will determine the exact form of the form factors.

Any justification from which Eq. (11) can be obtained gives the same decay distribution. There is no special reason why we use the axial-vector-meson-exchange model; this model is simply used as one of the other possibilities describing N^* photoproduction. In order to confirm the validity of one-axial-vector-meson exchange, one should also obtain the correct differential cross sections. However, there are three form factors unspecified, and the investigation of density-matrix elements seems to be the first step to undertake because we do not have to know the explicit value of each form factor in this case.

The comparison of our results with experimental data is not confined to the low energies. It is known at present that at high energies ($E_\gamma \geq 2$ BeV), background problems and low statistics make it difficult to study the density matrix as a function of momentum transfer, and no results are available from the present DESY experiments; however, high statistics may probably be expected in the near future.

The first term of Eq. (10) for the vector-meson-exchange model also gives the same result for the density-matrix elements as in the case $R=0$ when the polarization states of the incident photons are summed, but in general Eq. (10) gives different results.

Finally, if one wants to fit the cross section with experimental data, one should probably use a Regge-pole exchange. Cooper[21] has considered the reaction $\gamma+p\rightarrow N^{*+}+\pi^0$, and the same method may be used for the present case. However, the results of any other model or differential cross sections will be investigated further.

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APPENDIX

The following formulas are useful in calculating the absolute square of the transition amplitude where a spin-$\frac{3}{2}$ particle spinor is included and is polarized:

$$(\bar{u}_\mu u)(\bar{u}u_\nu)=\frac{m_N m_{N^*}-NN^*}{48|K|E_N E_{N^*}}[3(N_\mu{}^*N_\nu{}^*+m_{N^*}{}^2\delta_{\mu\nu})+(K_1K_2+K_2K_3+K_3K_1)(2N_\mu{}^*N_\nu{}^*+2m_{N^*}{}^2\delta_{\mu\nu}-3m_{N^*}{}^2\eta_\mu\eta_\nu)]$$
$$+\frac{m_{N^*}}{96|K|E_N E_{N^*}}\{\epsilon_{\mu\nu\lambda\tau}(K_1+K_2+K_3)(4(NN^*)N_\lambda{}^*\eta_\tau-m_{N^*}{}^2N_\lambda\eta_\tau-5m_{N^*}m_N N_\lambda{}^*\eta_\tau)$$
$$+3K_1K_2K_3(2(NN^*)N_\lambda{}^*\eta_\tau+m_{N^*}{}^2N_\lambda\eta_\tau-m_{N^*}m_N N_\lambda{}^*\eta_\tau)$$
$$+(N_\mu{}^*\epsilon_{\nu\lambda\tau\rho}-N_\nu{}^*\epsilon_{\mu\lambda\tau\rho})[-(K_1+K_2+K_3)N_\lambda N_\tau{}^*\eta_\rho+3K_1K_2K_3N_\lambda N_\tau{}^*\eta_\rho]\}. \quad \text{(A1)}$$

A special case of the above equation is

$$|\bar{u}_\mu A_\mu u|^2=\frac{m_N m_{N^*}-NN^*}{48|K|E_N E_{N^*}}\{3[(AN^*)^2+m_{N^*}{}^2A^2]$$
$$+(K_1K_2+K_2K_3+K_3K_1)[2(AN^*)^2+2m_{N^*}{}^2A^2-3m_{N^*}{}^2(A\eta)^2]\}, \quad \text{(A2)}$$

$$|\bar{u}_\mu A_\mu\gamma_\nu B_\nu u|^2=\frac{2(BN)(BN^*)-B^2(NN^*)-B^2m_N m_{N^*}}{48|K|E_N E_{N^*}}\{3[(AN^*)^2+m_{N^*}{}^2A^2]$$
$$+(K_1K_2+K_2K_3+K_3K_1)[2(AN^*)^2+2m_{N^*}{}^2A^2-3m_{N^*}{}^2(A\eta)^2]\}, \quad \text{(A3)}$$

[21] F. Cooper, Phys. Rev. 167, 1314 (1968).

$$(\bar{u}_\mu A_\mu \gamma_\nu B_\nu u)^\dagger (\bar{u}_\lambda C_\lambda u)$$

$$\begin{aligned}
&= \mp \frac{i m_N}{96|K|E_N E_{N^*}} \{-6(BN^*)[(AN^*)(CN^*)+m_{N^*}{}^2(AC)] \\
&\quad \pm (K_1+K_2+K_3)m_{N^*}[6(BN^*)\langle AC\eta N^*\rangle + m_{N^*}{}^2\langle ABC\eta\rangle - (AN^*)\langle BC\eta N^*\rangle - (CN^*)\langle AB\eta N^*\rangle] \\
&\quad +2(K_1K_2+K_2K_3+K_3K_1)[3m_{N^*}{}^2(BN^*)(A\eta)(C\eta) - 2m_{N^*}{}^2(AC)(BN^*) - 2(AN^*)(BN^*)(CN^*)] \\
&\quad \pm 3K_1K_2K_3 m_{N^*}[(AN^*)\langle BC\eta N^*\rangle + (CN^*)\langle AB\eta N^*\rangle - m_{N^*}{}^2\langle ABC\eta\rangle]\} \\
&\mp \frac{i}{96|K|E_N E_{N^*}} \{-3m_{N^*}[2(BN)((AN^*)(CN^*)+m_{N^*}{}^2(AC)) + m_{N^*}{}^2((AN)(BC)-(AB)(CN)) \\
&\quad +(AN^*)((BC)(NN^*)-(BN^*)(CN)) + (CN^*)((AN)(BN^*)-(AB)(NN^*))] \\
&\quad \pm (K_1+K_2+K_3)[4(AN^*)(CN^*)\langle BN\eta N^*\rangle + 4m_{N^*}{}^2(AC)\langle BN\eta N^*\rangle + 3m_{N^*}{}^2(CN)\langle AB\eta N^*\rangle \\
&\quad +3m_{N^*}{}^2(AN)\langle BC\eta N^*\rangle + 3m_{N^*}{}^2(NN^*)\langle ABC\eta\rangle - 3m_{N^*}{}^2(N\eta)\langle ABCN^*\rangle \\
&\quad +2m_{N^*}{}^2(BN)\langle AC\eta N^*\rangle + m_{N^*}{}^2(A\eta)\langle BCNN^*\rangle - m_{N^*}{}^2(C\eta)\langle ABNN^*\rangle] \\
&\quad +(K_1K_2+K_2K_3+K_3K_1)m_{N^*}[2(BN)(3m_{N^*}{}^2(A\eta)(C\eta) - 2m_{N^*}{}^2(AC) - 2(AN^*)(CN^*)) \\
&\quad -5(AN^*)((BC)(NN^*)-(BN^*)(CN)) - 5(CN^*)((AN)(BN^*)-(AB)(NN^*)) \\
&\quad -6((C\eta)(AN^*)-(A\eta)(CN^*))((BN^*)(N\eta)-(B\eta)(NN^*)) + 6m_{N^*}{}^2(C\eta)((AN)(B\eta)-(AB)(N\eta)) \\
&\quad +6m_{N^*}{}^2(A\eta)((BC)(N\eta)-(CN)(B\eta)) - 5m_{N^*}{}^2((AN)(BC)-(AB)(CN))] \\
&\quad \pm 3K_1K_2K_3[2\langle BN\eta N^*\rangle((AN^*)(CN^*)+m_{N^*}{}^2(AC) - 3m_{N^*}{}^2(A\eta)(C\eta)) \\
&\quad +m_{N^*}{}^2(-(A\eta)\langle BCNN^*\rangle + (C\eta)\langle ABNN^*\rangle - (N\eta)\langle ABCN^*\rangle + (AN)\langle BC\eta N^*\rangle \\
&\quad +(CN)\langle AB\eta N^*\rangle + (NN^*)\langle ABC\eta\rangle)]\}, \qquad \text{(A4)}
\end{aligned}$$

where the lower signs in $\pm$ and $\mp$ indicate the value for $(\bar{u}_\mu A_\mu \gamma_\nu B_\nu u)(\bar{u}_\lambda C_\lambda u)^\dagger$, and the notation given in Ref. 4 is used. If γ_5 is put in between $\bar{u}_\mu$ and u in Eqs. (A1), (A3), and (A4), the signs of the m_N's are changed.

If the spinors normalized by Jackson and Pilkuhn are used, the above equations should be multiplied by a factor $4E_N E_{N^*}/m_{N^*}{}^2$. When one sums over the polarization states of N^*, the terms containing the K_i's vanish and the rest are multiplied by a factor $16|K|/3$.