Experimental Search for CP-Violating Decay $K_S \to \pi^+\pi^-\pi^0$†

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We have studied the time distribution of the neutral K decays in 53 hydrogen bubble-chamber events of the type $K^-p \to \bar{K}^0 n$, neutral $K \to \pi^+\pi^-\pi^0$. In 20 of these events, the neutral K decay occurs less than four K_S mean lives after production. We find no evidence for a CP-violating amplitude. We point out that a combination of experiments of our type, using an initial $\bar{K}^0$ state, with experiments using an initial K^0 will provide much more information than either type of experiment alone. This is demonstrated when we combine our results with those of an earlier experiment based on 18 K^0 events.

I. INTRODUCTION

As an experimental test of CP conservation in the process neutral $K \to \pi^+\pi^-\pi^0$, we have analyzed the time distribution of 53 $\pi^+\pi^-\pi^0$ decays of neutral K mesons produced in the reaction $K^-p \to \bar{K}^0 n$. In this section we introduce the phenomenological parameters of the time distribution, and state the predictions of various selection rules regarding their values.

We assume that the neutral K meson does not decay into predominantly nonsymmetric three-pion final states; that is, we assume the $I=0$, nonsymmetric $I=1$, and $I=2$ final states are not strongly favored. Then the total rate of decay into three pions is dominated by the contributions of the symmetric $I=1$ and $I=3$ final states, and, considering only these contributions, we may define decay amplitudes $a(K_S \to \pi^+\pi^-\pi^0)$ and $a(K_L \to \pi^+\pi^-\pi^0)$, averaged over the final-state phase space, in such a way that the three-pion decay rate at proper time t of an initial $\bar{K}_0$ state is[1]

$$\Gamma(t;\,x,y) = \tfrac{1}{2}\Gamma(K_L \to \pi^+\pi^-\pi^0)\left[(x^2+y^2)e^{-\lambda_S t} + e^{-\lambda_L t} - 2(x\cos\delta t + y\sin\delta t)e^{-(\lambda_S+\lambda_L)t/2}\right], \quad (1)$$

where

$$a(K_S \to \pi^+\pi^-\pi^0)/a(K_L \to \pi^+\pi^-\pi^0) = x + iy\,,$$

λ_S^{-1} and λ_L^{-1} are the K_S and K_L mean lifetimes, and δ is the mass difference $m(K_S)-m(K_L)$, in units such that $\hbar = c = 1$.[2]

Then CPT invariance and no $I=3$ state imply $x=0$,[3] while CP conservation implies both $x=0$ and $y=0$.

II. SCANNING AND SELECTION OF CANDIDATES

We obtained $\bar{K}^0$ mesons by exposing the 25-in. Lawrence Radiation Laboratory hydrogen bubble chamber to a K^- beam with momenta 310–430 MeV/c. In a total of 1.3 million pictures, we found about 18 000 reactions of the type $K^-p \to \bar{K}^0 n$ followed by a visible decay of the neutral K meson. Most of these were $\pi^+\pi^-$ decays, but we have found 53 events in which there is a $\pi^+\pi^-\pi^0$ decay.

Our method for finding three-pion decays was basically similar to that used for the leptonic neutral K decays in the same bubble-chamber exposure.[4] The pictures were scanned for V's; if both a V and a 0-prong were found, they were measured, and four-constraint (4C) fits to $\bar{K}^0$ and Λ production and two-body decay were attempted. If no satisfactory fits were obtained, fits to all three-body K^0 decay hypotheses were attempted. Events with a satisfactory 1C three-body decay fit were remeasured, and those with a confidence level for the three-pion fit greater than 0.02, and more than 100 times greater than that for any other decay fit, were called three-pion decay candidates. We rejected from our sample of candidates all those in pictures containing extra 0-prongs not clearly associated with other events. In this way we obtained those decays which are associated with a unique production vertex, but fitted only the 1C hypothesis of $\bar{K}^0$ production and three-pion decay.

In order to ensure that our scanning and measuring efficiencies are independent of the decay time of the neutral K, we have applied three geometrical cuts to the three-pion decay candidates. First, the V must lie inside a fiducial volume that is everywhere at least 5 cm from the limits of the visible region of the bubble chamber. Second, the projected distance between the 0-prong and the V must be great enough for the event not to be mistakenly called a 2-prong event. On the basis of a study of the observed distribution of 5000 $K_S \to \pi^+\pi^-$ decays, which are subject to the same scanning bias, we excluded a region that forms a rectangle 6 mm long and 3.5 mm wide around the 0-prong, when projected on the xy plane of the bubble-chamber coordinate system. This plane represents a rough average of the three camera views in the 25-in. chamber.

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[1] In writing Eq. (1), we make approximations of the form $\langle x^2\rangle = \langle x\rangle^2 = x^2$, $\langle y^2\rangle = \langle y\rangle^2 = y^2$, where the angular bracket indicates an average over phase space. Such approximations are expected to be good when the decay is not dominated by nonsymmetric amplitudes. Regeneration in liquid hydrogen, and the fact that the K_S and K_L are not precisely eigenstates of CP, give negligible corrections to Eq. (1).

[2] We use the values $\lambda_S^{-1} = 0.862\times10^{-10}$ sec, $\lambda_L^{-1} = 5.38\times10^{-8}$ sec, $\delta = -0.544\times10^{10}$ sec^{-1}, from the compilation of the Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[3] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **14**, 835 (1965).

[4] B. R. Webber, F. T. Solmitz, F. S. Crawford, Jr., and M. Alston-Garnjost, Phys. Rev. Letters **21**, 498 (1968); **21**, 715(E) (1968); B. R. Webber (Ph.D. thesis), UCRL Report No. UCRL-19226, 1969 (unpublished).

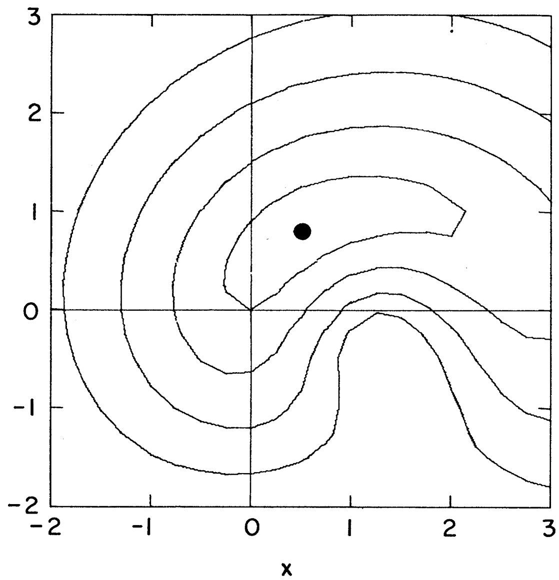

FIG. 1. Contours of equal likelihood in the xy plane, for this experiment. The solid circle marks the likelihood peak and contours indicate relative likelihood $e^{-(1/2)n^2}$, where $n=1, 2, 3, 4$.

Finally, we rejected events in which either decay track had a dip angle greater than 60° or less than −60°.

III. ELIMINATION OF BACKGROUND

Before examining the remaining three-pion candidates, we made a cut to remove a substantial background of electron pairs, due to Dalitz decays of π^0's from $\Lambda \rightarrow n\pi^0$ and $K_S \rightarrow \pi^0\pi^0$ decays. In this cut we interpreted the V tracks as electrons, and rejected the event if the pair mass was less than 50 MeV.[5] After this cut, and those described in the previous section, we were left with 61 events, which were carefully inspected on a scanning table.

The three-pion mode is kinematically quite distinct from the other decay modes of the neutral K, so the background due to these other modes was already negligible at this stage of the analysis. Poorly measured Λ decays show some tendency to fit the three-pion neutral-K-decay hypothesis, and this background was eliminated by inspection of the ionization of the positive decay track, which, for all visible Λ decay modes, is that of a proton. A more troublesome contamination consisted of "events" in which the V was not in fact the decay of a neutral particle, but rather the scattering or decay of an incoming charged pion or muon, which just happened to occur in the same picture as a 0-prong event. If the direction of motion of the particles forming the V could not be determined by study of ionization, δ rays, and energy loss, we calculated the missing mass at the vertex with the appropriate track reversed, and rejected the event if the missing mass squared lay within 4 standard deviations of a possible value [0 for pion decay, 0 to (105 MeV)^2 for muon decay, and (938 MeV)^2 for pion-proton elastic scattering]. In the case of a possible pion-proton scattering, we also required the recoiling proton to be invisible (missing momentum within 4 standard deviations of the range 0–80 MeV/c) before rejecting the event.

Of the 61 candidates that were inspected, five were rejected as Λ decays and three as possible decays or scatterings of incoming charged particles. Thus 53 events remained after our inspection.

We used the UCRL program PHONY[6] to generate simulated events in which the three-pion decays were distributed according to the spectrum and time distribution for K_L decay. We found that about 45% of these simulated events failed to satisfy selection criteria similar to those described above, but the remaining 55% showed no biases in their decay-time distribution. We also generated samples of simulated leptonic, two-pion, and radiative two-pion neutral K decays and verified that no events of these types were able to satisfy our selection criteria.

A potential source of background that would be very difficult to eliminate involves the coincidence of a three-body wall V with an unassociated 0-prong event in the same picture. Such a combination might give a spurious fit to the three-pion decay hypothesis. To examine this possibility, we searched a sample of 1.6×10^5 pictures and found 29 wall V's inside our decay fiducial volume. For each wall V, we simulated an unassociated 0-prong event by measuring a beam track associated with a real event in the same picture. None of the 0-prong plus V combinations satisfied our selection criteria for three-pion decays. Since only 9% of our pictures contain unassociated 0-prong events, this source of background is therefore negligible in our experiment.[7]

We concluded that the 53 events that satisfied our selection criteria were three-pion neutral K decays, with a background contamination of less than one event.

IV. MAXIMUM-LIKELIHOOD ANALYSIS

We have used the maximum-likelihood method to estimate the parameters x and y in Eq. (1). For each of our 53 events, we calculate three quantities: t^i, the proper time after production at which the neutral K decayed, $t_{\min}{}^i$, the time at which it left the excluded region around the 0-prong, and $t_{\max}{}^i$, the time at which its extended line of flight would have left the decay fiducial volume. Then, since we expect our detection efficiency to be constant in the interval from

[5] We were prepared to reject events in which the pair mass was between 50 and 135 MeV, if the track densities were consistent with the electron-pair hypothesis. However, we found that none of our candidates was rejected on this basis.

[6] E. R. Burns, Jr. (of UCRL), and Y. Oren and D. Drijard (private communication).

[7] If we had found n 0-prong plus V combinations that satisfied our selection criteria, we would expect a background of about $0.09n(1.3\times10^6)/(1.6\times10^5)=0.7n$ events.

$t_{\min}{}^i$ to $t_{\max}{}^i$, the likelihood function is

$$\mathcal{L}(x,y)=\prod_{i=1}^{53}\Gamma(t^i;\,x,y)\bigg/\int_{t_{\min}{}^i}^{t_{\max}{}^i}\Gamma(t;\,x,y)dt, \qquad (2)$$

where the distribution function $\Gamma(t;x,y)$ is given in Eq. (1).

Contours of equal likelihood in the xy plane are shown in Fig. 1. The likelihood maximum occurs at the point

$$x=0.5,\quad y=0.8, \qquad (3)$$

and, since the contour at $e^{-0.5}$ relative likelihood (1 standard deviation) encloses the point $x=y=0$, we may say that this result is in good agreement with CP conservation. If we assume CPT invariance and no $I=3$ final state, then x is constrained to be zero and we find

$$y=0.45\pm0.45. \qquad (4)$$

The decay-time distribution of our 53 events is shown in Fig. 2. In this figure we also show the predicted distributions for $x=y=0$ (solid curve) and for our most likely values, $x=0.5$, $y=0.8$ (broken curve), both curves being normalized to a total of 53 events. These predictions were calculated in the following way. For given values of x and y, the predicted time distribution is clearly

$$n(t;\,x,y)=\sum_{\text{all }\bar{K}^0\text{'s}}\epsilon_{3\pi}\Gamma(t;\,x,y)\eta_i(t), \qquad (5)$$

where $\epsilon_{3\pi}$ is the probability that a $\pi^+\pi^-\pi^0$ decay in the interval from $t_{\min}{}^i$ to $t_{\max}{}^i$ will survive our cuts, $\Gamma(t;\,x,y)$ is given by Eq. (1), and

$$\begin{aligned}\eta_i(t)&=1 \quad \text{if} \quad t_{\min}{}^i<t<t_{\max}{}^i\\ &=0 \quad \text{otherwise}.\end{aligned} \qquad (6)$$

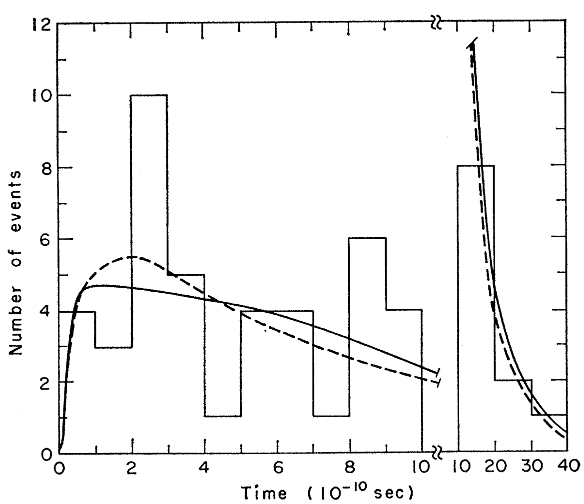

FIG. 2. Time distribution of the 53 events. The distributions predicted by CP conservation ($x=y=0$) and our best fit ($x=0.5$, $y=0.8$) are shown by the solid and broken curves, respectively.

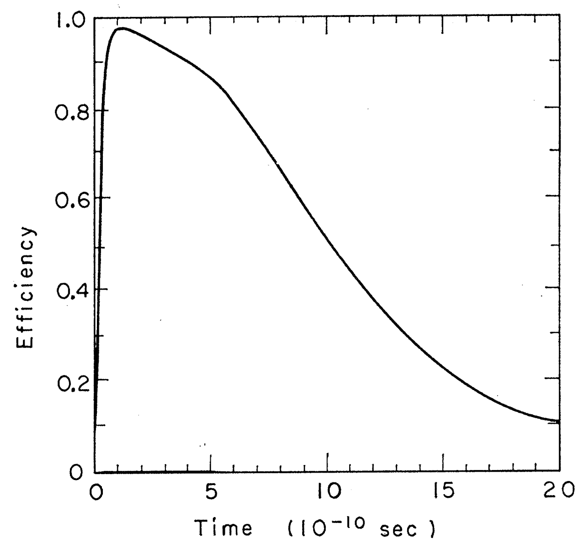

FIG. 3. Geometrical efficiency function.

We cannot directly evaluate the sum in Eq. (5) because we do not, in fact, observe all $\bar{K}^0$'s, but only those that have a visible decay. However, we can replace the sum over all $\bar{K}^0$'s by a weighted sum over observed $\pi^+\pi^-$ decays, where the weight for each event is the inverse of the probability that a $\bar{K}^0$ produced with that momentum and direction would give an observed $\pi^+\pi^-$ decay. This probability is

$$p_i=\tfrac{1}{2}\epsilon_{2\pi}\Gamma(K_S\to\pi^+\pi^-)\int_{t_{\min}{}^i}^{t_{\max}{}^i}e^{-\lambda_S t}dt, \qquad (7)$$

where $\epsilon_{2\pi}$ is the probability that a $\pi^+\pi^-$ decay in the interval from $t_{\min}{}^i$ to $t_{\max}{}^i$ will be observed. Thus,

$$n(t;\,x,y)=\epsilon_{3\pi}\Gamma(t;\,x,y)\sum_{\pi^+\pi^-}\eta_i(t)p_i^{-1}. \qquad (8)$$

Equation (8) may also be written in the form

$$n(t;\,x,y)=N\epsilon_{3\pi}\Gamma(t;\,x,y)\epsilon(t), \qquad (9)$$

where N is the total number of $\bar{K}^0$'s,

$$N=\sum_{\pi^+\pi^-}p_i^{-1}, \qquad (10)$$

and $\epsilon(t)$ is the geometrical efficiency function, determined from the distribution of $\pi^+\pi^-$ decays,

$$\epsilon(t)=\sum_{\pi^+\pi^-}\eta_i(t)p_i^{-1}\Big/\sum_{\pi^+\pi^-}p_i^{-1}. \qquad (11)$$

This function is shown in Fig. 3. Multiplying it by $\Gamma(t;\,x,y)$, with the appropriate choices of x and y, and adjusting the normalization to give a total of 53 events, we obtain the curves in Fig. 2. Since neither curve shows any serious systematic disagreement with the data, we may say that the parametrization of the distribution in terms of x and y, according to Eq. (1),

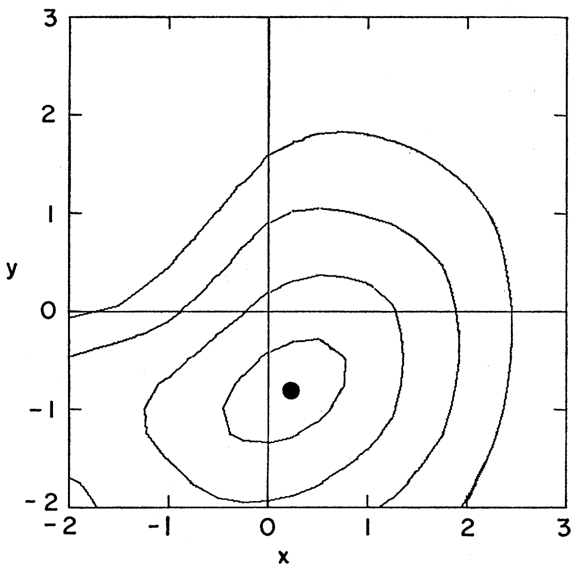

FIG. 4. Likelihood contours for the 18 K^0 events of Anderson *et al.* The contours have the same meaning as those in Fig. 1.

seems to be appropriate, and the results of our likelihood analysis are in qualitative agreement with the data.

It was important to check the normalization of the distribution of three-pion decays, in addition to its form, in order to search for unexplained losses of events, which might produce time-dependent biases. We made this check by using some of our events to measure the absolute rate of the decay $K_L \to \pi^+\pi^-\pi^0$. By integrating Eq. (8), we see that the predicted total number of events is

$$n=\int_0^\infty n(t;\,x,y)dt=\epsilon_{3\pi}\sum_{\pi^+\pi^-}p_i^{-1}\times\int_{t_{\min}^i}^{t_{\max}^i}\Gamma(t;\,x,y)dt. \quad (12)$$

Thus

$$n=\frac{\epsilon_{3\pi}\Gamma(K_L\to\pi^+\pi^-\pi^0)}{\epsilon_{2\pi}\Gamma(K_S\to\pi^+\pi^-)}Q(x,y), \quad (13)$$

where

$$Q(x,y)=\sum_{\pi^+\pi^-}\int_{t_{\min}^i}^{t_{\max}^i}[(x^2+y^2)e^{-\lambda_S t}+e^{-\lambda_L t}-2(x\cos\delta t+y\sin\delta t)e^{-(\lambda_S+\lambda_L)t/2}]dt \Big/ \int_{t_{\min}^i}^{t_{\max}^i}e^{-\lambda_S t}dt \quad (14)$$

$$=2.53\times10^5[1-0.122x+0.144y+0.057(x^2+y^2)], \quad (15)$$

by explicit evaluation. In making this evaluation, one must apply the same geometrical cuts to the $\pi^+\pi^-$ events as were used for the $\pi^+\pi^-\pi^0$ events. We also had to apply some kinematical cuts to eliminate background from the $\pi^+\pi^-$ sample. From Monte Carlo simulation, we found that these cuts gave $\epsilon_{2\pi}=0.75$. This value is due mainly to the cut at 60° on the dip angles of the decay tracks, which alone led to the rejection of 22% of the $\pi^+\pi^-$ decays.

In order to calculate $\epsilon_{3\pi}$ by Monte Carlo simulation, we had to modify some of the selection criteria described in Sec. III. For example, according to those criteria, we rejected an event in which the V fitted the decay of an incoming charged pion only when visual examination showed the V tracks to be consistent with this hypothesis. Since we could not simulate such an examination, in calculating $\epsilon_{3\pi}$ we simply rejected all events with V fits to the scattering or decay of incoming charged pions or muons. Because of this and similar changes in our criteria, we were left with 43 events for the measurement of the absolute rate of $K_L\to\pi^+\pi^-\pi^0$, and the detection efficiency was found to be $\epsilon_{3\pi}=0.46$. Equations (13) and (15) therefore give[8]

$$\Gamma(K_L\to\pi^+\pi^-\pi^0)=2.20\times10^6[1-0.122x+0.144y+0.057(x^2+y^2)]^{-1}\ \text{sec}^{-1}. \quad (16)$$

Our result for the $K_L\to\pi^+\pi^-\pi^0$ absolute rate depends on the values of x and y, because the fraction of our 43 events that is due to K_L decay, rather than K_S decay, depends on the values of x and y. For $x=y=0$, we have

$$\Gamma(K_L\to\pi^+\pi^-\pi^0)=(2.20\pm0.35)\times10^6\ \text{sec}^{-1}, \quad (17)$$

and for our most likely values, $x=0.5$ and $y=0.8$,

$$\Gamma(K_L\to\pi^+\pi^-\pi^0)=(1.98\pm0.32)\times10^6\ \text{sec}^{-1}. \quad (18)$$

Both these values are in good agreement with the current world average,[9] $\Gamma(K_L\to\pi^+\pi^-\pi^0)=(2.36\pm0.10)\times10^6$ sec^{-1}. We therefore find no evidence of unexplained losses of events in our experiment.

V. CONCLUSIONS

We conclude that the values we obtain for x and y are in good agreement with the CP-conserving values $x=0$ and $y=0$. However, the error for our value of x is very large, as shown by the likelihood plot in Fig. 1. The rather weak assumptions of CPT invariance and no $I=3$ final state constrain x to be zero, and, making these assumptions, we obtain the result (4) for y, which corresponds to the limit (at the 2-standard-deviation level)

$$\Gamma(K_S\to\pi^+\pi^-\pi^0)/\Gamma(K_L\to\pi^+\pi^-\pi^0)<2.0, \quad (19)$$

that is,

$$\Gamma(K_S\to\pi^+\pi^-\pi^0)/\Gamma(K_S\to\text{all})<4\times10^{-4}. \quad (20)$$

[8] We use the current world-average value $\Gamma(K_S\to\pi^+\pi^-)=0.794\times10^{10}$ sec^{-1}, from the Particle Data Group compilation (Ref. 2).

[9] Particle Data Group, Ref. 2.

These results are in good agreement with those of earlier experiments,[10,11] in which an initial K^0 state, rather than a $\bar{K}^0$ state, was used. Because we have a $\bar{K}^0$ initial state, we are most sensitive to the combination of parameters

$$\Gamma(\bar{K}^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0) = \tfrac{1}{2}[(1-x)^2+y^2], \quad (21)$$

where $\Gamma(\bar{K}^0 \to \pi^+\pi^-\pi^0)$ is the initial decay rate. For this ratio we obtain the result (at the 1-standard-deviation level)

$$0.2 < \Gamma(\bar{K}^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0) < 1.0\,. \quad (22)$$

The experiments with an initial K^0, on the other hand, are most sensitive to

$$\Gamma(K^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0) = \tfrac{1}{2}[(1+x)^2+y^2]. \quad (23)$$

Combining (21) and (23), we see that

$$x = \tfrac{1}{2}[\Gamma(K^0 \to \pi^+\pi^-\pi^0) - \Gamma(\bar{K}^0 \to \pi^+\pi^-\pi^0)]/\Gamma(K_L \to \pi^+\pi^-\pi^0)\,. \quad (24)$$

Thus, a combination of experiments with initial states K^0 and $\bar{K}^0$ provides a test for CPT violation or $I=3$ final states, through a measurement of x, that is much more precise than would be possible in either type of experiment alone.

Although Eq. (24) provides a simple way of measuring x, the greatest amount of information on both x and y will be obtained by multiplying together the likelihood functions for K^0 and $\bar{K}^0$ experiments. We have used this method to combine our results with those of Anderson *et al.*[10] Figure 4 shows the likelihood contour plot for that experiment, in which 18 events of the type $\pi^- p \to \Lambda K^0$, neutral $K \to \pi^+\pi^-\pi^0$, were observed. Figure 5 shows the product of the likelihood functions in Figs. 1 and 4, that is, the combined result of the two experiments. The 1-standard-deviation contour in

[10] J. A. Anderson, F. S. Crawford, Jr., R. L. Golden, D. Stern, T. O. Binford, and V. Gordon Lind, Phys. Rev. Letters **14**, 475 (1965); **15**, 645 (1965).

[11] L. Behr, V. Brisson, P. Petiau, E. Bellotti, A. Pullia, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, H. Huzita, A. Sconza, B. Aubert, L. M. Chounet, J. P. Lowys, and C. Pascaud, Phys. Letters **22**, 540 (1966).

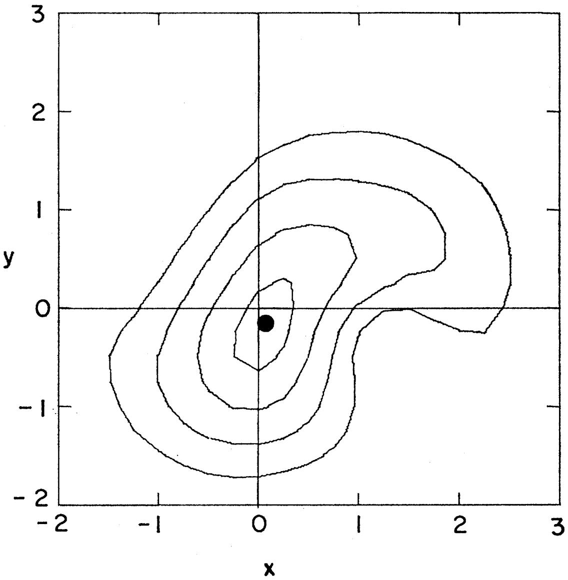

FIG. 5. Combined likelihood contours for this experiment and that of Anderson *et al.* The contours have the same meaning as those in Fig. 1.

Fig. 5 gives

$$x = 0.05 \pm 0.30\,, \quad y = -0.15 \pm 0.45\,, \quad (25)$$

in excellent agreement with CP conservation. We see in Eq. (25) the expected vast improvement in the precision of the determination of x.

If we constrain x to be zero, then the combined experiments give a result corresponding to (4):

$$y = -0.25 \pm 0.40\,, \quad (26)$$

which in turn gives the combined result (at the 2-standard-deviation level)

$$\Gamma(K_S \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0) < 1.1\,, \quad (27)$$

that is,

$$\Gamma(K_S \to \pi^+\pi^-\pi^0)/\Gamma(K_S \to \text{all}) < 2.2 \times 10^{-4}. \quad (28)$$

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