while the extra factors in (4.37) are

$$\prod_{s>t}{}' \left\{1-\frac{(y_s-y_{i+j})(y_s-y_i)(\tilde{y}_t-y_{i+j})(\tilde{y}_t-y_i)}{(y_s-\tilde{y}_t)^2(y_{i+j}-y_i)}\right\}^{-p_sp_t}. \tag{4.39}$$

We could replace y_s and $\tilde{y}_t$ in (4.39) by $\tilde{y}_s$ and y_t, or we could take the square root of the product of the factors with and without this replacement.

ACKNOWLEDGMENT

I should like to thank Dr. John Schwarz for informing me of his and T. H. Burnett's results before publication.

PHYSICAL REVIEW D VOLUME 1, NUMBER 6 15 MARCH 1970

Relativistic Quark Model Based on the Veneziano Representation. II. General Trajectories*

STANLEY MANDELSTAM

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

(Received 26 September 1969)

The model previously proposed is extended to include multiquark trajectories. Once any trajectories with more than a single quark and antiquark are included, it is necessary to include trajectories where the number of quarks plus the number of antiquarks, which we call the total quark number, is arbitrarily large. The necessary factorization properties of the multiparticle Veneziano amplitudes will hold provided the intercept of the leading trajectory is a polynomial function of the total quark number, and the degeneracy of the levels on all but the leading trajectory will increase with the order of the polynomial. It is possible to construct two different models depending on whether one allows nonplanar duality diagrams. The model with nonplanar diagrams resembles more closely the nonrelativistic harmonic-oscillator quark model, and the nonplanar duality diagrams must be associated with the nonplanar Veneziano amplitudes discussed in a previous paper. One can introduce $SU(3)$ symmetry-breaking by making the intercept depend on the number of strange and nonstrange quarks separately, and one then obtains a modified Gell-Mann–Okubo mass formula.

1. INTRODUCTION

A RELATIVISTIC quark model has been proposed and applied to meson trajectories by Mandelstam[1] and by Bardakci and Halpern.[2] In the present paper we wish to extend the model to other trajectories. We shall discuss the general properties of the multiquark trajectories, as well as the symmetry properties of the three-quark states. The spin and unitary-spin degrees of freedom will only be mentioned insofar as they are connected with the symmetry properties. We hope to treat the more detailed spin properties of the baryon trajectories in a subsequent paper.

Within the framework of the model presented in I, it appeared that one need not introduce resonances consisting of more than two quarks. Once one requires the presence of three-quark states, however, it is necessary to introduce trajectories where the number of quarks and antiquarks is arbitrarily large. We shall examine such trajectories in Sec. 2. For baryon-antibaryon scattering, it has already been pointed out by Rosner[3] that exotic resonances with two quarks and two antiquarks must occur in the intermediate states, and one can apply similar reasoning to more complicated reactions. Following Delbourgo and Salam,[4] we shall refer to the number of quarks plus the number of antiquarks as the total quark number, and resonances with an arbitrarily large total quark number must be present. Our model in its present form does not appear to require trajectories with a net quark number greater than 3, though it can certainly accommodate such trajectories. Until we know how to extend our model beyond the narrow-response approximation, we cannot answer the question whether trajectories of baryon number greater than 1 occur in this approximation, or only in higher orders.

If our model is to be at all acceptable on experimental and theoretical grounds, it is necessary that the mass of the lightest particle with a given total quark number be an increasing function of the quark number. As long as the resonances with a total quark number of 4 or greater are sufficiently heavy, they will decay rapidly into resonances with smaller total quark numbers, and they will not appear experimentally as narrow resonances. We therefore have to inquire whether the model allows different trajectories to have different intercepts. The question to be investigated concerns the factorization properties of the multiparticle Veneziano

* This work was supported by the U. S. Atomic Energy Commission.

[1] S. Mandelstam, Phys. Rev. **184**, 1625 (1969); hereafter referred to as I.

[2] K. Bardakci and M. B. Halpern, Phys. Rev. **183**, 1456 (1969).

[3] J. Rosner, Phys. Rev. Letters **21**, 950 (1968).

[4] R. Delbourgo and A. Salam, Phys. Rev. **172**, 1727 (1968).

amplitude,[5,6] which lie at the basis of the relativistic quark model. The original treatment of such factorization properties assumed that all trajectories had the same intercept.

We shall show in Sec. 2 that the residues at the poles of the multiparticle Veneziano amplitude can still be expressed as a finite sum of factored terms, provided that the intercept of the leading trajectory is a polynomial function of the total quark number. The degeneracy of the resonances will depend on the order of the polynomial. If the intercept is a linear function of the total quark number, the degeneracy of the spectrum of ordinary mesons, i.e., of mesons with a total quark number of 2, will be the same as in a model without nucleons and exotic resonances. The degeneracy of the spectrum of other resonances will be an increasing function of the total quark number, as is to be expected on intuitive grounds. If the intercept is a higher polynomial function of the total quark number, the degeneracy of all ordinary or exotic resonances on nonleading trajectories will be increased, and even the degeneracy of the resonances on the first subsidiary trajectory will increase indefinitely with the order of the polynomial.

In order that the spectrum of resonances not be too complicated, we therefore have to postulate that the intercept of the leading trajectory be a polynomial of low degree in the total quark number. We might be tempted to assume that the intercept was a linear function of the total quark number. The masses of highly exotic resonances would then be proportional to the square root of the total quark number, however, and those of sufficiently high charge and hypercharge would be stable. It is not absolutely prohibited that such a model might be the appropriate narrow-resonances approximation to nature, since the range of validity of the model will probably not extend to resonances of very high mass. A model without the infinite system of stable exotic mesons is obviously to be preferred, and the simplest such model is one where the intercept of the leading trajectory is a quadratic function of the total quark number.

It is possible to construct two different models with all the features mentioned above, depending upon whether one allows nonplanar as well as planar duality diagrams. The models with only planar diagrams and with all diagrams will be discussed in Secs. 3 and 4, respectively. According to the general principles of the relativistic quark model, each duality diagram is to be associated with the topologically similar multiparticle Veneziano diagram. The nonplanar duality diagrams will therefore be associated with the nonplanar Veneziano amplitudes which we have discussed in a previous paper.[7]

Of the two models, that containing nonplanar as well as planar diagrams has the closer resemblance to the nonrelativistic harmonic-oscillator quark model. All baryon states on the leading trajectory of such a model are symmetric in the three quarks, and we shall hereafter refer to it as the symmetric quark model. The planar-diagram model, on the other hand, possesses some nonsymmetric states as well. The presence of one such state, the $l=0$ **70** has already been pointed out by Mandula, Rebbi, Slansky, Weyers, and Zweig.[8] While the symmetric quark model is preferable in this respect, the planar-diagram model is not in definite contradiction with experiment, since the $l=0$ **70** is only weakly coupled and may simply appear as a contribution to the continuum. We shall discuss such an interpretation in Sec. 3.

The planar-diagram model is simpler than the symmetric quark model in regard to the spectrum of resonances on the leading trajectory. As has been shown in Refs. 5 and 6, the leading trajectory in a planar Veneziano amplitude has no "orbital" degeneracy, though there will of course be a degeneracy associated with the spin and unitary-spin degrees of freedom. The extra degeneracy associated with the exotic resonances affects only the subsidiary trajectories, and no trajectory is infinitely degenerate. In nonplanar Veneziano amplitudes, the resonances on the leading baryon trajectory are degenerate, and the degeneracy increases with the angular momentum. The spectrum of resonances on the leading baryon trajectory of our symmetric quark model is identical to that of the nonrelativistic quark model, where the degeneracy also increases with the angular momentum.[9]

The meson-baryon coupling constants in the planar-diagram model are two-thirds as large as in the symmetric quark model. A convenient comparison of the $B\bar{B}M$ and MMM coupling constants may be made on the basis of the coupling with the neutral vector mesons. We showed in I that it was possible to adjust a single parameter in such a way that the coupling constant of the vector mesons to the other mesons was universal, and we can treat the $B\bar{B}M$ vertex in a similar manner. On comparing the $B\bar{B}M$ and MMM coupling constants, we then find that the symmetric quark model gives results in accord with vector-meson universality, while the couplings of the vector mesons to the baryons in the planar-diagram model are too small by a factor $\frac{2}{3}$.

The models constructed in I and in Secs. 2–4 of the present paper possess exact $SU(3)$ symmetry. A convenient method of introducing symmetry breaking is to assume that the intercept of the leading trajectory is a polynomial function, not only of the total quark number, but of the number of strange and nonstrange quarks separately. The resulting formula for the mass splitting resembles the Gell-Mann–Okubo formula

[5] K. Bardakci and S. Mandelstam, Phys. Rev. **184**, 1640 (1969).
[6] S. Fubini and G. Veneziano, Nuovo Cimento **64A**, 811 (1969).
[7] S. Mandelstam, preceding paper, Phys. Rev. D **1**, 1720 (1970).
[8] J. Mandula, C. Rebbi, R. C. Slansky, J. Weyers, and G. Zweig, Phys. Rev. Letters **22**, 1147 (1969).
[9] The degeneracy of the resonances on the leading trajectory of the nonrelativistic harmonic-oscillator quark model has been discussed by P. G. O. Freund and R. Waltz (unpublished).

without the term responsible for the $\Sigma\Lambda$ mass difference. If we assume that the intercept is a quadratic function of the quark models, our formula differs in detail from the Gell-Mann–Okubo formula. It contains an extra parameter, but we are able to fit the baryon and meson multiplets with a single formula, whereas the usual Gell-Mann–Okubo formula involves the masses of the baryon multiplets and the squares of the masses of the pseudoscalar octet. It is also worth pointing out that our present derivation does not depend on the smallness of the $SU(3)$ symmetry breaking.

2. EXOTIC RESONANCES

The general principles of the model are the same as those applied to meson trajectories in I. For meson-baryon scattering we shall have the processes represented by Fig. 1(a) and its crossed diagram and by Fig. 1(b), possibly together with further processes represented by nonplanar diagrams which we shall discuss in Sec. 4. In momentum space the diagrams correspond to multiparticle Veneziano amplitudes[10] with the external particles in the same cyclic order as in the diagrams. From such multiparticle amplitudes one can factor out meson-baryon amplitudes with external particles of any spin. The lines in Fig. 1 also represent δ functions in spin and unitary spin. The coupling therefore has the form originally proposed by Capps[11] and, in their internal degrees of freedom, the diagrams are the duality diagrams used by Harari,[12]

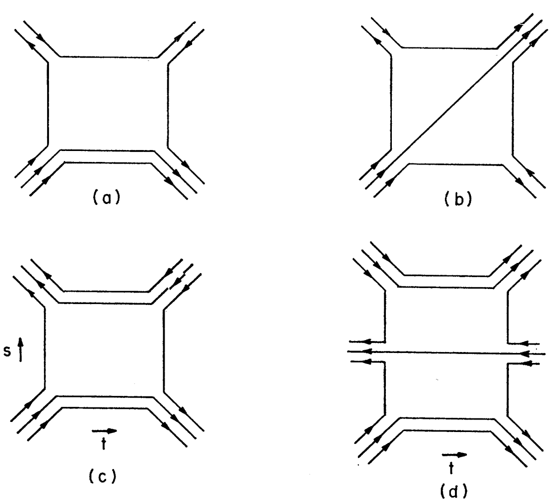

FIG. 1. Duality diagrams for various processes.

[10] K. Bardakci and H. Ruegg, Phys. Letters **28B**, 342 (1968); M. A. Virasoro, Phys. Rev. Letters **22**, 37 (1969); H. M. Chan and S. T. Tsou, Phys. Letters **28B**, 485 (1969); C. J. Goebel and B. Sakita, Phys. Rev. Letters **22**, 257 (1969); K. Bardakci and H. Ruegg, Phys. Rev. **181**, 1884 (1969).

[11] R. H. Capps, Phys. Rev. **168**, 1731 (1968).

[12] H. Harari, Phys. Rev. Letters **22**, 562 (1969).

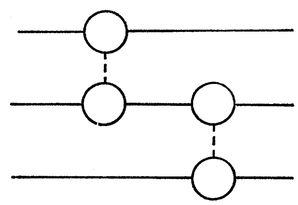

FIG. 2. A sequence of reactions contained in Fig. 1(d).

by Rosner,[13] by Matsuoka, Minomiya, and Sawada,[14] and, in a rather different form, by Neville.[15]

The diagram for baryon antibaryon scattering is shown in Fig. 1(c); there will be a similar diagram with the s and t channels interchanged. It will be noticed immediately that the resonances in the t channel are exotic mesons consisting of two quarks and two antiquarks. Rosner[3] was the first to draw attention to such exotic mesons, and their presence is a necessary feature of our model. By viewing Fig. 1(c) from the s channel, we see that the intermediate state is an ordinary meson, and the $B\bar{B}M$ coupling is required for consistency with the meson-baryon amplitude.

One can employ similar reasoning to prove the existence of exotic baryons consisting of four quarks and one antiquark. A diagram for the process $B\bar{B}B \to B\bar{B}B$ is shown in Fig. 1(d). Such a diagram must be present in order to represent the sequence of exchanges shown in Fig. 2. The resonances in the t channel of Fig. 1(d) are the exotic baryons under discussion. It is evident that exotic mesons and baryons with an arbitrarily large number of quarks and antiquarks are present in the model. This type of reasoning leaves open the question of the existence of resonances with a net quark number greater than 3.

We have pointed out that a model with an infinity of meson and baryon resonances would be completely unacceptable unless the masses of the resonances increases with the total quark number. We now wish to investigate the factorization properties of our amplitude when the intercepts of the trajectories depend on the total quark number, and to prove the results already quoted in the Introduction.

We begin by restricting ourselves to planar amplitudes. Figure 3 represents a general diagram for such an amplitude, and we are interested in the factorization properties when the diagram is divided by the dashed line. As in Ref. 5, we write the Veneziano integrand in the form

$$I=I_1I_2\prod_{i=1}^{j}\prod_{k=j+1}^{N}(1-wy_iy_k)^{-s_{ik}-s_{i+1,k-1}+s_{i,k-1}+s_{i+1,k}}, \tag{2.1}$$

[13] J. Rosner, Phys. Rev. Letters **22**, 689 (1969).

[14] T. Matsuoka, K. Minomiya, and S. Sawada. Progr. Theoret. Phys. (Kyoto) **42**, 56 (1969).

[15] D. Neville, Phys. Rev. Letters **22**, 494 (1969).

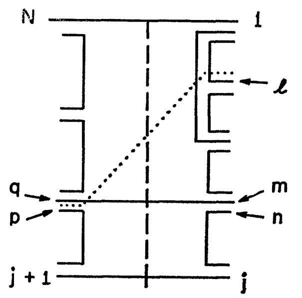

FIG. 3. A general n-point duality diagram.

where

$$w = u_{1j}, \tag{2.2a}$$

$$y_i = u_{1i} \cdots u_{1,j-1}, \quad i \leq 2 \leq j-1 \tag{2.2b}$$

$$y_j = 1, \quad y_1 = 0, \tag{2.2c}$$

$$y_k = u_{j+2,N} \cdots u_{k,N}, \quad j+2 \leq k \leq N-1 \tag{2.2d}$$

$$y_{j+1} = 1, \quad y_N = 0, \tag{2.2e}$$

$$S_{ik} = \alpha_{ik}(s_{ik}), \quad \text{if} \quad k \geq i+1 \tag{2.2f}$$

$$S_{jj} = S_{j+1,j+1} = 0, \tag{2.2g}$$

$$S_{j+1,j} = -1. \tag{2.2h}$$

We have used the notation of Chan and Tsou[10] for the u's. The subscripts ik refer to the channel with particles i to k in clockwise order round the diagram. I_1 and I_2 are the Veneziano integrands associated with the two halves of the diagram.

In units for which the slope of the trajectory is unity, we may write

$$S_{ik} = s_{ik} + b_{ik}, \quad k \geq i+1 \tag{2.3}$$

where b_{ik} is the intercept of the leading trajectory in the channel ik. We supplement (2.3) with the further definitions:

$$b_{j,j} = b_{j+1,j+1} = -\mu^2, \tag{2.4a}$$

$$b_{j+1} = 1. \tag{2.4b}$$

The quanity μ^2 in (2.4a) is the mass of the quark, so that $b_{j,j}$ and $b_{j+1,j+1}$ are defined as the intercept which a trajectory would have it the mass of its lowest member were equal to the quark mass, We may then write (2.4a) as

$$I = I_1 I_2 \prod_{i=1}^{j} \prod_{k=j+1}^{N} (1 - w y_i y_k)^{-2p_i p_k + \epsilon_{ik}}, \tag{2.5a}$$

where

$$\epsilon_{ik} = -b_{ik} - b_{i+1,k-1} + b_{i,k-1} + b_{i+1,k}. \tag{2.5b}$$

If all the intercepts are the same, so that all the quantities b_{ik} are zero except $b_{j+1,j}$, it follows from (2.5b) that all ϵ's are zero except $\epsilon_{j,j+1}$. The expression (2.5a) thus has the factorization properties described in Refs. 5 and 6. If the ϵ's are not zero the factorization properties become more complicated and, unless restrictions are imposed on the b's, the residue at a pole cannot be written as a finite sum of factored terms at all.

We now assume that the intercept b_{ik} is a polynomial function of the total quark number and, to begin, we shall suppose that it depends linearly on the quark number. Most of the ϵ's are then equal to zero. Let us examine the channel lp (Fig. 3). If we denote the total quark number by ν, we may write

$$\nu_{lp} = 3, \quad \nu_{l+1,p-1} = 3, \quad \nu_{l,p-1} = 4, \quad \nu_{l+1,p} = 2. \tag{2.6}$$

As long as b_{ik} depends linearly on ν_{ik}, it follows from (2.5b) and (2.6) that $\epsilon_{lp} = 0$. We may treat the majority of the variables ϵ_{ik} in a similar way. The points i and k will be on the right and the left of the diagram, respectively, and let us imagine a dotted line drawn just above these points. The quantity $\nu_{ik} - \nu_{i+1,k}$ will be equal to ± 1 according to whether the quark line drawn to the point i crosses this dotted line or not. Exactly the same rule serves to determine the value of $\nu_{i,k-1} - \nu_{i+1,k-1}$, so that ϵ_{ik}, given by (2.5b), is zero.

The only ϵ's for which this reasoning falls are the quantities such as ϵ_{mq} (Fig. 3), where the points m and q are jointed by a quark line. In that case

$$\nu_{mq} = 0, \quad \nu_{m+1,q-1} = 0, \quad \nu_{m,q-1} = 1, \quad \nu_{m+1,q} = 1, \tag{2.7}$$

and ϵ_{mq} is not zero. Thus, in Fig. 3, only two of the ϵ's are nonzero, $\epsilon_{j,j+1}$ and ϵ_{mq}. These two ϵ's correspond to the two quark lines which cross the diagram from left to right. No ϵ is associated with the line $1N$, since the factor $(1 - w y_1 y_N)^{-2p_1 p_N + \epsilon_{1N}}$ is absent by virtue of (2.2c) and (2.2e). The quantity $\epsilon_{j,j+1}$ is nonzero even if the intercept of all trajectories is the same, but the other nonzero ϵ's such as ϵ_{mq} are new features of our present model. The number of nonzero ϵ's is equal to the number of quark lines crossing the diagram besides the top line or, in other words, to the total quark number of the channel minus 1.

We observed in Ref. 5 that the term $\epsilon_{j,j+1}$ in the exponent increased the degeneracy of the resonances, and any other nonzero ϵ will have a similar effect. As long as the number of nonzero ϵ's is finite and independent of the number of external lines, we may easily repeat the reasoning of Refs. 5 and 6 to show that the residue at each pole is equal to the sum of a finite number of factorizable terms. The resonances on the leading trajectory will be nondegenerate, but the degeneracy of all other resonances will increase with the number of nonzero ϵ's. In ordinary-meson channels, where the total quark number is 2, the only nonzero ϵ is $\epsilon_{j,j+1}$, so that the degeneracy of the resonances is exactly the same as in a model where all leading trajectories have the same intercept. The degeneracy of the resonances in other channels will be an increasing function of the total quark number.

We turn next to the case where the intercept b_{ik} is a quadratic function of the total quark number. Thus

$$-b_{ik} = (\alpha \nu_{ik})^2 + \beta \nu_{ik} + \gamma. \tag{2.8}$$

For later reference we may generalize our formula to systems where we have several different quarks, e.g., strange and nonstrange quarks in a model with broken $SU(3)$. Equation (2.8) then becomes

$$-b_{ik}=(\sum_r \alpha_r\nu_{r,ik})^2+\sum_r \beta_r\nu_{r,ik}+\gamma\,, \qquad (2.9)$$

where $\nu_{r,ik}$ is the number of quarks of type r in the channel ik.[16]

For simplicity we shall limit our investigation to the spectrum of ordinary mesons. The only lines which pass from the left to the right of Fig. 3 are then the top and the bottom lines. The channels ik may be divided into four classes as shown in Figs. 4(a)–4(d); the number of vertical quark lines is arbitrary. In the first case, Fig. 4(a),

$$\begin{aligned}\epsilon_{ik}=(\sum_r \alpha_r\nu_{r,ik})^2+\sum_r \beta_r\nu_{r,ik}+\gamma+(\sum_r \alpha_r\nu_{r,ik}-\alpha_i-\alpha_k)^2\\ +\sum_r \beta_r\nu_{r,ik}-\beta_i-\beta_k+\gamma-(\sum_r \alpha_r\nu_{r,ik}-\alpha_i)^2\\ -\sum_r \beta_r\nu_{r,ik}+\beta_i-\gamma-(\sum_r \alpha_r\nu_{r,ik}-\alpha_k)^2\\ -\sum_r \beta_r\nu_{r,ik}+\beta_k-\gamma\,,\end{aligned}$$

i.e.,

$$\epsilon_{ik}=2\alpha_i\alpha_k\,. \qquad (2.10)$$

The parameters α_i, α_k, β_i, and β_k represent α_r and β_r for the quarks in the external lines i and k. With the configuration shown in Fig. 4(b), one again obtains the result $\epsilon_{ik}=2\alpha_i\alpha_k$. With Figs. 4(c) and 4(d), $\epsilon_{ik}=-2\alpha_i\alpha_k$. We may therefore write the general equation.

$$\epsilon_{ik}=2\epsilon_i\alpha_i\epsilon_k\alpha_k \quad \text{unless} \quad i=j \text{ and } k=j+1\,, \qquad (2.11)$$

where ϵ_i is equal to $+1$ if the quark leading from the external line i goes towards the top of the diagram, and equal to -1 if the quark goes towards the bottom of the diagram.

As in all previous examples, the value of ϵ_{ik} when $i=j$ and $k=j+1$ will not be given by the equation valid for other values of i and k.

If we substitute (2.11) in (2.5a), we obtain the equation

$$I=I_1I_2\prod_{i=1}^{j}\prod_{k=j+1}^{N}(1-wy_iy_k)^{-2p_ip_k+2\epsilon_i\alpha_i\epsilon_k\alpha_k}(1-w)^{\epsilon'}, \qquad (2.12a)$$

where

$$\epsilon'=\epsilon_{j,j+1}-2\epsilon_j\alpha_j\epsilon_{j+1}\alpha_{j+1}\,. \qquad (2.12b)$$

We may rewrite (2.12a) as the exponential of a logarithm, so that it becomes

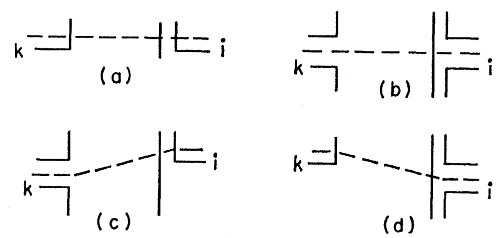

FIG. 4. Various channels ik in Fig. 3.

$$I=I_1I_2\exp\left\{2\sum_{r=1}^{\infty}\frac{w^r}{r}\Big[(\sum_{i=1}^{j}p_iy_i^r)(\sum_{k=j+1}^{n}p_ky_k^r) -(\sum_{i=1}^{j}\epsilon_i\alpha_iy_i^r)(\sum_{k=j+1}^{n}\epsilon_k\alpha_ky_k^r)-\epsilon'\Big]\right\}. \qquad (2.13)$$

On expanding the exponential in (2.13), we notice that the coefficient of w^r is indeed equal to the sum of a finite number of factored terms. As the second term within the square bracket does not involve the scalar product p_ip_j, it does not contribute to the leading trajectory, which remains nondegenerate. The term in question does contribute to the subsidiary trajectories, so that the degeneracy of the spectrum is greater than in a model where all trajectories have the same intercept.

One may treat more complicated cases in the same way. If the intercept is a polynomial function of the total quark number, the degeneracy of a particular resonance will increase with the order of the polynomial and with the total quark number of the resonance. All resonances are finitely degenerate, but the degeneracy of even the first subsidiary ordinary-meson trajectory increases indefinitely with the order of the polynomial. One would therefore expect the intercept to be a polynomial of low degree in the total quark number and, as we explained in the Introduction, the optimal choice is a quadratic function if we demand that there be no infinite family of highly exotic stable particles.

We may easily generalize our results to the two-quark channels of a nonplanar amplitude. It has been shown in Ref. 7 that the factorization properties in such channels are the same as those of planar amplitudes. We had assumed that all trajectories had the same intercept, but we can easily prove a similar theorem in a model where the intercept depends upon the total quark number. All results obtained in this section for the factorization properties of two-quark-channel amplitudes are therefore true in models with planar and nonplanar diagrams.

If the intercept of the leading trajectory is a quadratic or higher polynomial in the total quark number, we cannot predict the masses of the exotic mesons. The masses of the ordinary mesons and baryons only give us two points with which to determine the coefficients of the polynomial. If the quadratic term were absent,

[16] One could also write down a more general formula in which the first term was an arbitrary quadratic function of the variables $\nu_{r,ik}$. The degeneracy of the spectrum would be greater than that corresponding to (2.9).

we would obtain a mass of 1.6–1.8 BeV for the exotic mesons with a total quark number of 4. The quadratic term will increase this value, since it must be negative in order that the mass of the highly exotic resonances be an increasing function of the total quark number.

With a sufficiently large quadratic term, the exotic mesons will have a considerable Q value for S-wave decay into ordinary mesons. They may therefore be too broad to appear experimentally as resonances. In our present model there is no coupling between one exotic meson and two ordinary mesons, but the linear-trajectory dynamical scheme is only to be regarded as a weak-coupling approximation to nature, and the coupling in question will occur when we improve on the approximation.

3. PLANAR-DIAGRAM MODEL

In this and the following section we are interested in the symmetry properties of multiquark states and, in particular, of three-quark states. One may construct two models with different symmetry properties. In the present section, we shall examine a model with only planar duality diagrams.

The scattering of ordinary mesons and baryons will involve the duality diagrams of Figs. 1(a) and 1(b), together with the same two diagrams drawn upside down. If s and u are the meson-baryon channels and t is the meson-meson channel, the st and tu terms will correspond to Fig. 1(a) and the su term to Fig. 1(b). The even orbital-angular-momentum states in the s channel will be given by the sum of Figs. 1(a) and 1(b), the odd orbital-angular-momentum states by their difference.

We observe that Fig. 1(b) may be obtained from Fig. 1(a) by intechanging the top and bottom quarks on the right-hand side, followed by twisting the entire right-hand side through 180°. The last operation does not affect the meaning of the diagram in any way, so that the two diagrams are related to one another by interchanging a pair of quarks in the final state. It follows that the purely antisymmetrical state and the purely symmetrical state will be absent from the even and odd orbital-angular-momentum trajectories, respectively. The even orbital-angular-momentum trajectories will possess symmetrical multiplets and multiplets of mixed symmetry or, in $SU(6)$, the **56** and **70** representation.[17] Odd angular-momentum trajectories will possess multiplets of mixed symmetry and antisymmetrical multiplets or in $SU(6)$, the **70** and **20** representations. The **20** will of course not contribute to meson-baryon scattering. These results have been obtained by Mandula, Rebbi, Slansky, Weyers, and Zewig.[8]

[17] For simplicity we shall express our results in the framework of $SU(6)$. If we adopt the interpretation given in I, the narrow resonances do form representations of $SU(6)$, though the trajectories form representations of $SU(6,6)$ or $SU(12)$.

A more detailed analysis shows that the amplitudes represented by Figs. 1(a) and 1(b) involve the **56** and the **70** in the following proportions:

$$\text{Fig. 1(a)}:\quad 15(\mathbf{56})+16(\mathbf{70})\,, \tag{3.1a}$$

$$\text{Fig. 1(b)}:\quad 15(\mathbf{56})-\ 8(\mathbf{70})\,. \tag{3.1b}$$

We confirm that the amplitude for states of odd angular momenta, which is obtained by subtracting (3.1b) from (3.1a), is a pure **70**. States of even angular momenta involve the **56** and the **70** in the ratio 15/4.

The existence of the **70** multiplet in trajectories of even orbital angular momentum represents a difference between the planar-diagram model and the nonrelativistic symmetric quark model. The spectrum of our present model is not restricted to states which are symmetric in the spin, unitary spin, and orbital degrees of freedom of the quarks. No **70** multiplets with even angular momentum have been observed in pion-nucleon phase-shift analyses. Before we dismiss the planar-diagram model out of hand, however, we should investigate possible changes which may occur when we improve on the narrow-resonance approximation.

The linear Regge trajectories will acquire a curvature as the coupling is turned on. If the forces are attractive, the curves of Reα against s will move upward from their positions in the Born approximation, just as in potential theory. Since the weak-coupling linear trajectories are inclined at 45° to the horizontal, this upward movement will be accompanied by a movement to the left, and the Q value for a resonance with a given angular moment will be decreased. The most strongly bound particles or resonances will therefore lie in the channels with the strongest attractive forces. We may go further and make the interpretation that the resonances on a trajectory which is not moved a substantial distance to the left by the coupling will be so wide that they appear experimentally as part of the continuum. The dynamical scheme will now have the property, expected in a bootstrap model, that particles or narrow resonances exist only in those channels where the attractive forces are sufficiently strong. We actually used such an interpretation in I, where we assumed that the V' and Π' did not correspond to particles or narrow resonances, and that no narrow resonances existed on repulsive trajectories.

We have seen that the attractive forces in the **70** even-l channels are indeed much weaker than in the **56** even-l channels; the squares of the coupling constants are in the ratio 4/15. We may therefore assume that the forces in the **70** even-l channels are not sufficient to produce narrow resonances. The S-wave states in such channels would correspond to the 11, 13, and 31 states of the pion-nucleon system. The latter two have not been observed at an energy below 1.8 BeV. The prominent 11 Roper resonance is usually assigned to the second trajectory, and we are not proposing to change that assignment. If the resonances in question had approximately the same energy as the $l=1$ resonances, they could easily escape detection. Their coupling to the πN system would be about one-third as great as that of the $l=1$ resonances. On the other hand, if their partial width for decay into the $\pi\pi N$ mode is estimated from that of the Roper resonance with the appropriate phase-space correction, it is found that their total width would be roughly comparable to that of the $l=1$ resonances.

A second difference between the planar-diagram model and the nonrelativistic quark model lies in the nature of the spectrum of resonances with higher angular momentum. In our present model, all resonances on the leading trajectory are nondegenerate in their orbital degrees of freedom. The degeneracy of resonances on nonleading trajectories depends on the total quark number, but no trajectory is infinitely degenerate. In the nonrelativistic harmonic-oscillator quark model the degeneracy of the resonances will again depend on the total quark number, but now the resonances on the leading trajectory will be degenerate if the total quark number is greater than 2; the degeneracy will be roughly proportional to the $(\nu-2)$ power of the angular momentum.

From the duality diagrams of Fig. 1 one may immediately construct diagrams for the vertices. Thus, from Figs. 1(a) and 1(b), one can infer that the $B\bar{B}M$ vertex is given by the sum of Figs. 5(a) and 5(b). From Figs. 1(c) one infers that the vertex which couples an exotic meson to a baryon-antibaryon pair is given by the sum of Figs. 5(c) and 5(d). In general, by referring to Fig. 1(d) and more complicated diagrams, one may conclude that any exotic baryon or meson is represented by a diagram such as Fig. 6, where the top two quarks have their arrows pointing in the same direction, following which all arrows alternate until the bottom two quarks again have their arrows pointing in the same direction. The vertex between any three particles can now be drawn as in Figs. 5(a)–5(d), with no lines crossing. Each vertex will consist of the sum of two terms with different cyclic ordering of the three particles. Note that it is impossible to draw a vertex such as Fig. 5(e); at least one quark line must pass between any pair of particles.

For processes where the number of baryons plus antibaryons exceeds four, half of the diagrams will

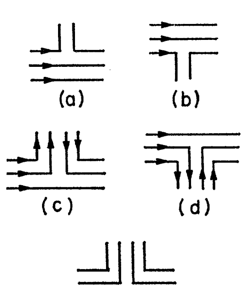

FIG. 5. Vertex functions in the planar-diagram model.

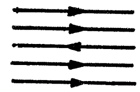

FIG. 6. Quarks in an exotic resonance.

occur with minus sign owing to the Fermi statistics of the baryons. A particular diagram can be chosen and given a plus sign; any diagram obtained from the selected diagram by interchanging an odd number of baryon pairs or antibaryon pairs is then given a minus sign.

One may construct more complicated models which possess the exotic mesons and baryons just discussed, together with exotic mesons and baryons where more than two adjacent quark lines have their arrows in the same direction. For instance, one can have a model where states of baryon number two exists in the narrow-resonance approximation. The method of constructing the diagrams is straightforward.

An important feature of the planar-diagram model is that it does not possess vector-meson universality. The ratio between the different MMV vertices and between the different $B\bar{B}V$ vertices is in accord with vector-meson universality, provided one fixes certain mixing parameters as has been explained in I. However, the ratio between the $B\bar{B}V$ vertex and the MMV vertex is two-thirds of that predicted by vector-meson universality, since the meson in Figs. 5(a) and 5(b) cannot interact with the middle quark of the baryon.

In summary, we may mention the following three features of the planar-diagram model:

(i) The baryon trajectories of even orbital angular momentum possess a **70** multiplet as well as a **56**.

(ii) The resonances on the leading trajectory have no orbital degeneracy, as opposed to the states of the harmonic-oscillator quark model.

(iii) The model does not possess vector-meson universality, since the meson can only interact with the two outer quarks of a multiquark particle.

Though point (i) may be regarded as a drawback of the model, we have seen that it does not necessarily imply that the model should be rejected. With regard to point (ii), the degeneracy of the higher resonances on the leading trajectory has thus far received no experimental support, and we may therefore prefer our present model to the harmonic-oscillator model on the grounds of simplicity of the spectrum. Point (iii), like point (i), may be regarded as a drawback of the model, since vector-meson universality is an appealing feature which may possibly be helpful in constructing a representation of current algebra. However, since our model in its present form cannot make exact quantitative predictions of coupling constants, we should not regard the violation of vector-meson universality as strong experimental evidence against the model.

By including nonplanar as well as planar diagrams, we can construct a model which possesses only symmetrical three-quark states on the leading trajectory. It therefore agrees with the symmetric harmonic-oscillator model in points (i) and (ii), and it does possess vector-meson universality. We turn now to a description of this model.

4. SYMMETRIC QUARK MODEL

If all the baryon states are to be symmetric in the three quarks, it is necessary that a quark in an intermediate state of a scattering process should be able to go into any quark in the initial or final states. For meson-baryon scattering one would therefore expect diagrams such as Figs. 7(a) and 7(b) as well as Figs. 1(a) and 1(b). There are 54 diagrams in all, since each of the three quarks of the initial and final nucleon may annihilate with the antiquark of the meson and, in addition, the three quarks which pass from the initial to the final state may do so in six possible ways.

According to the principles of the relativistic quark model, each duality diagram is associated with a topologically similar multiparticle Veneziano diagram. Corresponding to Figs. 7(a) and 7(b) we therefore have the diagrams of Figs. 8(a) and 8(b), where we have drawn dashed lines across the meson and nucleon. These diagrams represent the scattering of ten external quarks, and they are to be interpreted in the sense of Ref. 7; we construct multiparticle Veneziano amplitudes with responances in those channels for which Figs. 8(a) and 8(b) possess intermediate states. By factorizing the meson-baryon amplitude from the ten-point Veneziano amplitude, we obtain the amplitude for the scattering of mesons and baryons of arbitrary spin.

Of the 54 duality diagrams, the 18 where the quark of the incoming meson passes into the quark of the outgoing meson [Figs. 1(a) and 7(b)] have intermediate states in the s and t channels. They therefore correspond to an st term in meson-baryon scattering. The remaining 36, such as Figs. 1(b) and 7(a), correspond to an su term in meson-baryon scattering.

In duality diagrams for processes where the total number of baryons and antibaryons exceeds four, some of the diagrams will again occur with minus signs owing to the Fermi statistics of the baryons or the para-Fermi statistics of the quarks. Each quark will be given a three-valued degree of freedom in addition to its spin, unitary spin, and orbital degrees of freedom. The three quarks in a single baryon have different values for this degree of freedom. We insert an extra

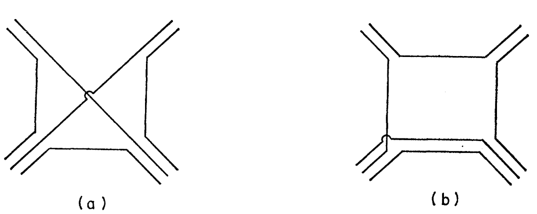

FIG. 7. Nonplanar duality diagrams.

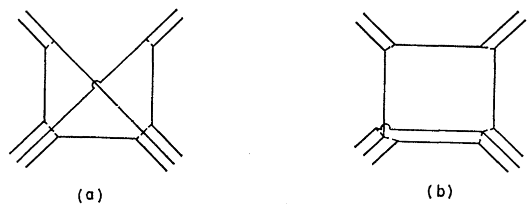

FIG. 8. Nonplanar Veneziano amplitudes.

minus sign for every pair of crossed lines between two quarks with the same value of the new degree of freedom.

The vertex diagrams are obtained by combining the quarks in all possible ways. The meson-baryon vertex will consist of the sum of 18 diagrams, since the quark and the antiquark lines of the meson may pass into any of the three quark lines of the nucleon, and the remaining two nucleon lines may pass into one another in two possible ways. Vertices such as Fig. 5(e) are excluded. In order that the vertices combine consistently in both the s and t channels to give the duality diagrams such as Figs. 1(a) and 7(b), it is necessary to multiply all $B\bar{B}M$ vertices and all meson-baryon amplitudes by a factor of $\frac{1}{6}$.

Our present model does possess vector-meson universality, because any of the baryon quarks in the $B\bar{B}M$ vertex may pass into the meson quarks. The BBM coupling constant is $\frac{3}{2}$ as large as in the planar-diagram model, there being 18 vertex diagrams and an over-all factor of $\frac{1}{6}$, as opposed to the two diagrams in the latter model.

We next examine the factorization properties of our amplitude, and we shall show that the spectrum of resonances on the leading trajectory is indeed the same as in the nonrelativistic harmonic-oscillator quark model. In the analysis of nonplanar diagrams given in Ref. 7, we had shown that the most direct formula had to be modified in order to obtain the amplitude with the simplest spectrum of intermediate states. We shall repeat some of the formulas here in order to exhibit the relationship between planar and nonplanar diagrams.

The general nonplanar diagram has been represented in Fig. 9(a). All the solid and dashed lines in Figs. 8(a) and 8(b) have here been represented by solid lines, and we are interested in the spectrum in the channel cut by the dashed line. The factorization properties of the

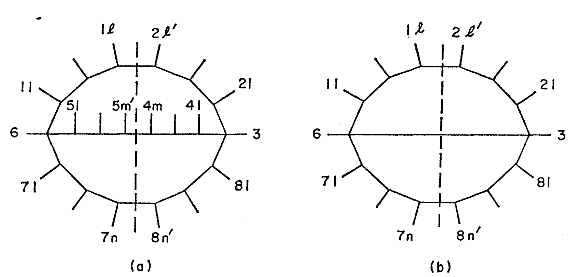

FIG. 9. Nonplanar and planar multiparticle Veneziano amplitudes.

amplitude can be obtained from an expansion of the Veneziano integrand in powers of w, the integration variable corresponding to the channel in question. If we are only interested in the leading trajectory, we may drop all but the highest power of the angular momentum for a given power of w. The expansion for the general unmodified nonplanar amplitude has the form

$$I_1I_2\sum_{n=0}^{\infty}(n!)^{-1}(2P_1P_2+2P_5P_4+2P_7P_8-2P_1P_4-2P_1P_8 \\ -2P_5P_2-2P_5P_8-2P_7P_2-2P_7P_4)^n w^{n-\alpha-1}, \quad (4.1)$$

where I_1 and I_2 are the Veneziano integrands for the two halves of the diagram, and α is the trajectory function for the channel of interest. The P's are defined by the equations

$$P_\alpha=\sum_\beta p_{\alpha\beta}y_{\alpha\beta}, \quad (4.2)$$

where $y_{\alpha\beta}$ is the product of the u's for all left-hand subchannels which include the particles $\alpha\beta$ and 6 if $\alpha\beta$ is on the left of the diagram, or the product of the u's for all right-hand subchannels which include the particles $\alpha\beta$ and 3 if $\alpha\beta$ is on the right of the diagram.

Let us compare this formula with the corresponding formula for a planar diagram. Figure 9(b) represents a general such diagram, the middle solid line corresponding to the middle quark line of Figs. 2(a) and 2(b). The expansion of the amplitude in powers of w has been given in Refs. 5 and 6. With neglect of lowest powers of angular momentum, it has the form

$$I_1I_2\sum_{n=0}^{\infty}(n!)^{-1}\{2(\sum_\beta p_{1\beta}\tilde{y}_{1\beta}+p_6\tilde{y}_6+\sum_\beta p_{7\beta}\tilde{y}_{7\beta}) \\ \times(\sum_\beta p_{2\beta}\tilde{y}_{2\beta}+p_3\tilde{y}_3+\sum_\beta p_{8\beta}\tilde{y}_{8\beta})\}^n w^{n-\alpha-1}. \quad (4.3)$$

The variables $\tilde{y}$ have been defined in Ref. 5, as well as in Sec. 2 of the present paper. We may eliminate the momenta p_6 and p_3 by using the equations

$$\sum_\beta p_{1\beta}+p_6+\sum_\beta p_{7\beta}=\sum_\beta p_{2\beta}+p_3+\sum_\beta p_{8\beta}\approx 0,$$

to leading order in the angular momentum. On doing so and making use of the relations between the u's, we may rewrite (4.3) as follows:

$$I_1I_2\sum_{n=0}^{\infty}(n!)^{-1}[2(-P_1+P_7)(-P_2+P_8)]^n w^{n-\alpha-1} \\ =I_1I_2\sum_{n=0}^{\infty}(n!)^{-1}(2P_1P_2+2P_7P_8 \\ -2P_1P_8-2P_7P_2)^n w^{n-\alpha-1}, \quad (4.4)$$

where the P's are defined as in (4.2). The variables P_4 and P_5 are zero for the diagram under consideration, as there are no terms in the summation (4.2) when r is equal to 4 or 5. We thus observe that the same formula (4.1) holds for planar and nonplanar diagrams.

In Ref. 7 we showed that one can obtain a simpler spectrum of intermediate states by multiplying the Veneziano integrand by the function

$$\prod_{r>s}(1-z_{rs})^{-p_rp_s}. \quad (4.5)$$

The product is over pairs of momenta from different groups $11\cdots21$, $51\cdots41$, and $71\cdots81$ of Fig. 9(a); the variable z_{rs} is defined as the product of the u's for all channels which include one of the particles r and s and one of the particles 3 and 6. In the modified formula, (4.1) becomes replaced by the expression

$$I_1I_2\sum_{k=0}^{\infty}(n!)^{-1}(2P_1P_2+2P_5P_4+2P_7P_8-P_1P_4-P_1P_8 \\ -P_5P_2-P_5P_8-P_7P_2-P_7P_4)^n w^{n-\alpha-1} \\ =I_1I_2\sum_{n=0}^{\infty}(n!)^{-1}[\tfrac{1}{3}(2P_1-P_5-P_7) \\ \times(2P_2-P_4-P_8)+\tfrac{1}{3}(-P_1+2P_5-P_7) \\ \times(-P_2+2P_4-P_8)+\tfrac{1}{3}(-P_1-P_5+2P_7) \\ \times(-P_2-P_4+2P_2)]w^{n-\alpha-1}. \quad (4.6)$$

We may also modify the planar amplitudes by multiplying the Veneziano integrand by (4.5). The effect is again to replace (4.1) by (4.6). As in (4.4), at least two of the P's will be zero for a planar diagram. In a model with only planar diagrams we would not modify the formula in this way, since it results in a complication of the spectrum; the leading trajectory, which previously had no orbital degeneracy, now becomes infinitely degenerate. In our present model, however, we already have an infinitely degenerate leading trajectory due to the nonplanar diagrams, and we shall now prove that the effect of modifying the planar amplitudes is to remove the nonsymmetrical quark states.

In Fig. 9(a), the three solid lines cut by the dashed line represent the transfer of three quarks from the left to the right of the diagram. Let us denote the quarks on the left of the diagram by the indices 1, 5, and 7, those on the right by the indices 2, 4, and 8. The tensor $I_1(2P_1-P_5-P_7)^{n_1}(-P_1+2P_5-P_7)^{n_2}(-P_1-P_5+2P_7)^{n_3}$ corresponds to an initial state where the three quarks are in an n_1th, n_2th, and n_3th level of the harmonic-oscillator spectrum. Spurious states are eliminated by the relation

$$(2P_1-P_5-P_7)+(-P_1+2P_5-P_7) \\ +(-P_1-P_5+2P_7)=0.$$

The curly brackets of (4.6) may then be represented symbolically by the expression

$$\delta_{12}\delta_{54}\delta_{78}\,, \tag{4.7}$$

the subscripts on the Kronecker δ referring to states of the harmonic-oscillator spectrum. Since the lines going across the diagram also represent quark lines in a duality diagram, we may interpret the subscripts as referring to spin and isotopic-spin indices as well as to states of the harmonic-oscillator spectrum.

In addition to Fig. 9(a), there will be five further diagrams where the groups 1, 5, and 7 are joined to the groups 2, 4, and 8 in all possible ways. When we take the sum of all six diagrams, (4.7) is replaced by

$$\delta_{12}\delta_{54}\delta_{78}+\delta_{14}\delta_{58}\delta_{72}+\delta_{18}\delta_{52}\delta_{74}+\delta_{14}\delta_{52}\nu_{78} +\delta_{12}\delta_{58}\delta_{74}+\delta_{18}\delta_{54}\delta_{72}\,. \tag{4.8}$$

It is evident from (4.8) that the only intermediate states are those which are symmetric in the space, spin, and isotopic-spin degree of freedom of the quarks taken together.

The factor (4.5) will of course modify the meson-baryon amplitudes which have been factored out of the general amplitude. We discussed this point in Ref. 7 and, although we confined our attention in that paper to nonplanar diagrams, the same reasoning applied to planar diagrams. The *st* and *tu* terms are unaffected by the modification, and they will be given by an ordinary β function. The *su* term, on the other hand, is given by the formula

$$\int_0^1 dx\, x^{-\alpha(s)-1}(1-x)^{-\alpha(u)-1} \times[1-x(1-x)]^{\frac{1}{2}(s+u-M^2-\mu^2)}\,. \tag{4.9}$$

Let us verify that the $l=0$ and $l=1$ states consist of a pure **56** and a pure **70**, respectively, as implied by the symmetry of the quark states. The *st* and *su* terms will again involve the **56** and **70** in the proportions given by (3.1a) and (3.1b). In the present model there are twice as many *su* diagrams as *st* diagrams, so that the sum of the diagrams will give a pure **56**. For the $l=1$ resonance it is not difficult to show that the last factor of (4.9) reduces the residue in the *su* term by a factor of $\frac{1}{2}$. The multiplet constitution is therefore obtained simply by subtracting (3.1b) from (3.1a), and the resonance is a pure **70**.

We conclude with the remark that the duality diagrams which we have drawn in Figs. 7 and 8 are the simplest possible nonplanar diagrams. In general, we could construct nonplanar diagrams with any number of crossed lines, and it is necessary that all diagrams be present in a consistent bootstrap scheme. In Ref. 7 we confined our attention to diagrams with only one pair of crossed lines, but it would be surprising if the methods could not be extended to the general case. We must assume that such an extension is possible if we are to complete our scheme in the narrow-resonance approximation.

5. $SU(3)$ SYMMETRY BREAKING

In the $SU(3)$-symmetric model which we have examined thus far, the mass of the lowest resonance on the leading trajectory depends only on the total quark number. An obvious method of introducing $SU(3)$-symmetry breaking is to make the mass depend on the number of strange and nonstrange quarks separately. The octet character of the mass formula then arises naturally out of the theory. One does not obtain a sufficiently general mass formula by this method, as the formula for the mass splitting of the nucleon octet contains only one term, the Σ and Λ having the same mass. It may be necessary to introduce another type of $SU(3)$-symmetry breaking or, alternatively, the term responsible for the $\Sigma\Lambda$ mass difference may appear as a higher-order effect in the width of the resonances.

Let us assume that the intercept depends quadratically on the total quark number; we have shown in Sec. 2 that this is the simplest possible assumption if we are to avoid stable particles of high charge and strangeness. From Eq. (2.4), we find that the mass of the lowest resonance on the leading trajectory is given by the formula

$$\mu^2=(\alpha_n\nu_n+\alpha_s\nu_s)^2+\beta_n\nu_n+\beta_s\nu_s+\gamma\,, \tag{5.1}$$

where ν_s and ν_n are the number of strange and nonstrange quarks. We may rewrite (5.1) in the form

$$\mu^2=[\alpha_n\nu+(\alpha_s-\alpha_n)\nu_s]^2+\beta_n\nu+(\beta_s-\beta_n)\nu_s+\gamma\,, \tag{5.2}$$

where ν is the total quark number. Hence

$$\mu^2=A\nu_s^2+B\nu_s+C\,, \tag{5.3a}$$

where

$$A=(\alpha_s-\alpha_n)^2\,, \tag{5.3b}$$

$$B=2\alpha_n\nu(\alpha_s-\alpha_n)+\beta_s-\beta_n\,, \tag{5.3c}$$

$$C=\alpha_n^2\nu^2+\beta_n\nu+\gamma\,. \tag{5.3d}$$

We notice that the constant A is independent of the multiplet, while B is a function of the total quark number. If the symmetry-breaking parameters are small, A will be a small quantity of second order.

Equation (5.3a) provides us with a mass-splitting formula. We may compare it directly with experiment for the spin-$\frac{3}{2}$ decuplet and for the vector-meson nonet, where each particle contains a definite number of strange quarks. We shall not compare the formula with experiment for the nucleon octet, as the $\Sigma\Lambda$ term is absent. The mass splitting of the pseudoscalar octet cannot be treated rigorously until we have a model where the **35** is not degenerate with the η'. In such a model the η would not contain a definite number of strange quarks. We shall make the assumption, justified by the success of the Gell-Mann–Okubo mass formula, that one can simply put ν_s for the η equal to its average value of $\frac{4}{3}$.

We then find that the masses of the three multiplets are reasonably well represented by the following formulas:

Spin-$\frac{3}{2}$ decuplet: $\mu^2=0.02\nu_s^2+0.17\nu_s+1.56$;

Vector-meson nonet: $\mu^2=0.02\nu_s^2+0.18\nu_s+0.59$;

Pseudoscalar octet: $\mu^2=0.02\nu_s^2+0.20\nu_s+0.02$.

The formula for the pseudoscalar octet is the least accurate, since it would give a mass of 575 MeV for the η, as opposed to the experimental mass of 550 MeV. However, our present formula is not appreciably worse than the ordinary Gell-Mann–Okubo formula.

If the constant A in (5.3a) were zero, we would obtain a Gell-Mann–Okubo formula in the squares of the masses, without the Σ-Λ term. If $A=B^2/4C$, we obtain a Gell-Mann–Okubo formula in the masses. By leaving A arbitrary, we have one more parameter than the ordinary Gell-Mann–Okubo formula, but we can fit the three multiplets under consideration with a mass-splitting formula of the same type. The term $A\nu_s^2$ is of second order in the symmetry breaking, and in any case it represents a fairly small effect, but the fit with such a term is definitely better than that with a simple Gell-Mann–Okubo formula in the squares of the masses.

Another point worth mentioning is that our formula is not a perturbation formula in the symmetry breaking, at least not for the spin-$\frac{3}{2}$ decuplet and the vector-meson nonet. We nowhere assume that the $SU(3)$-symmetry-breaking terms are small.

6. CONCLUDING REMARKS

At first sight our model appears to differ from the usual quark models by the presence of exotic mesons of arbitrarily high total quark number. The distinction between ordinary and exotic resonances is that the latter have a higher Q value for decay across a given centrifugal barrier. It is to be expected of any reasonable model that poles should occur in all channels of the S matrix. The channels would differ from one another according to the distance of the poles from the real axis. Our present model may be favored from the point of view of nucleon democracy, since it admits of no difference in principle between two- and three-quark channels on the one hand, and multiquark channels on the other.

Since the total quark number in our model can assume any positive integral value, it may be of interest to attempt to Reggeize the quark number in the manner proposed by Delbourgo and Salam.[4] We shall not investigate this point in the present paper, however.

We have left open the question of choosing between the planar-diagram model and the symmetric quark model. The two models do not differ drastically from one another in their experimental predictions and, as neither is meant to be an exact representation of nature, it may be difficult to make such a choice. The difference between the coupling constants provides one obvious possible method of distinguishing between the models. Using the fact that vector-meson universality is fairly well satisfied in nature, we might decide in favor of the symmetric quark model. We should bear in mind the limitations in accuracy of our model, however, and, in particular, the model predicts a value for the ratio $g_{\pi\rho\omega}/g_{\pi\pi\rho}$ which is somewhat smaller than the experimental value. If one compares the meson-baryon coupling constants with the constant $g_{\pi\rho\omega}$ instead of with $g_{\pi\pi\rho}$, one obtains roughly equal good results with the two models. The "experimental" determination of the coupling constant $g_{\pi\rho\omega}$ is of course subject to some uncertainty.

If the higher resonances on the leading baryon trajectory turn out to be complex, we would have strong evidence in favor of the symmetric quark model. It would be difficult to draw any conclusion from a failure to resolve the resonances. The degenerate trajectories may be so close together that the spacing between the resonances is small compared to their width, in which case they would appear experimentally as a single resonance. On the other hand, the degeneracy may be broken to a considerable extent, so that the relationship between the degenerate resonances is not evident. The resonances which are increased in mass by the breaking of the degeneracy may not appear experimentally as narrow resonances.

It is quite possible that the two models are equally good representations of nature in the narrow-resonance approximation, and that the differences between them are of the order of magnitude of the effects due to finite widths of the resonances. Each model possesses trajectories which do not appear in the other, and the symmetric quark model has a greater degree of degeneracy. The extra trajectories, and the breaking of the degeneracy, could appear as higher-order effects. It may even be that the two models are different starting-points which eventually lead to the same dynamical scheme. We cannot discuss such questions until we are able to improve on the narrow-resonance approximation, however.

ACKNOWLEDGMENTS

I should like to acknowledge helpful discussions with K. Bardakci, G. F. Chew, M. B. Halpern, and J. Shapiro.