Chiral-Symmetry Model*†

E. KYRIAKOPOULOS
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 11 August 1969)

We have constructed a chiral-symmetry model in which the vector mesons (which are not introduced by means of the Yang-Mills technique) are described in terms of an antisymmetric second-rank tensor $T_{\mu\nu}$. As a result, the interaction Hamiltonian in the interaction representation contains additional contact-type terms. Besides the pseudoscalar and vector-meson nonets, the model contains two nonets with $J^{PC}=0^{--}$ and 1^{-+}. Their fields appear in even numbers in all terms in the expansion of our Lagrangians. A mass Lagrangian is introduced such that the mesons get their physical masses. This gives generalized partially conserved axial-vector current if higher-order contributions are neglected. A $U(3)_{\text{left}}\otimes U(3)_{\text{right}}$-invariant Lagrangian is found which gives the well-known PPV coupling and also a PVV coupling. A nonet of baryons is introduced, with the $Y_0^*(1405)$ as ninth baryon. Several chiral-invariant meson-baryon Lagrangians are considered, such that the nonderivative and derivative couplings of the pseudoscalar mesons to baryons, and the vector and tensor couplings of the vector mesons to baryons, are obtained. Also, two symmetry-breaking meson-baryon Lagrangians are needed to give the baryons their physical masses. The coupling constant of the tensor coupling of the ρ mesons to nucleons is correctly predicted. The width of the $Y_0^*(1405)$ and the ratio $|G_{NKY}|/|G_{\Sigma\pi Y}|$ are predicted to be 62 MeV and 2, respectively, which are close to the experimental values. The decay widths $\Gamma[\eta'(958)\rightarrow\eta+2\pi]$, $\Gamma(K^*\rightarrow K\pi)$, and $\Gamma(\varphi\rightarrow K^+K^-)$ are in reasonable agreement with experiment.

I. INTRODUCTION

THE success of the combination of current algebra[1] and partially conserved axial-vector current (PCAC)[2,3] has indicated an underlying $U(3)\otimes U(3)$ chiral symmetry. Weinberg[4] has suggested that an equivalent approach would be to use an effective Lagrangian which satisfies PCAC and proper current commutation relations. The method of phenomenological Lagrangians, which is less rigorous than the algebra of currents (or the algebra of fields[5]) and whose main advantage is that it requires much less elaborate techniques, has been used extensively in the last few years.[6–15]

To construct a phenomenological Lagrangian model, we consider a 12×12-dimensional meson matrix Φ. This dimensionality is expected, if the mesons are made out of a quark triplet and an antiquark triplet, and each quark and antiquark is a four-dimensional Dirac spinor. The mesons enter only through the matrix $M(\Phi)$, which is a nonlinear function of Φ. The quark-meson Lagrangian, and one term of the kinetic-energy meson Lagrangian, is invariant under the group $GL(6,c)_{\text{left}}\otimes GL(6,c)_{\text{right}}$, while the other Lagrangians are invariant under subgroups of $GL(6,c)_L\otimes GL(6,c)_R$. Also, all our Lagrangians are invariant under parity and charge conjugation, if the four meson nonets of our model have $J^{PC}=0^{-\pm}$, $1^{-\pm}$, and under time reversal.

In Sec. III the meson Lagrangian is constructed in such a way that to lowest order a free-meson Lagrangian is obtained, which leads to a positive-definite Hamiltonian. The mesons have their physical masses, and octet-singlet mixing is allowed in all nonets. A Lagrangian invariant under the group $U(3)_L\otimes U(3)_R$ is given which allows a PPV coupling, and also a PVV coupling.

In Sec. IV the vector and axial-vector currents coming from the meson Lagrangian are constructed, and it is shown that the axial-vector current satisfies generalized PCAC, if three-meson coupling terms are neglected.

In Sec. V we consider a nonet of baryons, where the resonance $Y_0^*(1405)$ is the ninth baryon, which transforms according to the representation $(3,3^*)$ and $(3^*,3)$ of $U(3)_L\otimes U(3)_R$. From the baryon kinetic-energy Lagrangian, two $U(3)_L\otimes U(3)_R$-invariant meson-baryon Lagrangians, and two symmetry-breaking meson-baryon Lagrangians, we obtain to lowest order the free Lagrangian of a nonet of baryons. All baryons have their physical masses. To next order, we get the nonderivative coupling of the pseudoscalar mesons to baryons and the tensor coupling of the vector mesons to baryons. Two more $U(3)_L\otimes U(3)_R$-invariant Lagrangians are introduced, which give the vector coupling of the vector mesons to baryons. This coupling is pure F type.

The physical vector mesons are described in terms of an antisymmetric second-rank tensor. This description

* Supported in part by the U. S. Atomic Energy Commission.
† Submitted to the Department of Physics, The University of Chicago, Chicago, Ill., in partial fulfillment of the requirements for the Ph.D. degree.
[1] M. Gell-Mann, (a) Phys. Rev. 125, 1067 (1962); (b) Physics 1, 63 (1964).
[2] Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
[3] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[4] S. Weinberg, Phys. Rev. Letters 18, 188 (1967).
[5] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).
[6] J. A. Cronin, Phys. Rev. 161, 1483 (1967).
[7] J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).
[8] L. S. Brown, Phys. Rev. 163, 1802 (1967).
[9] J. Schwinger, Phys. Letters 24B, 473 (1967).
[10] P. Chang and F. Gürsey, Phys. Rev. 164, 1752 (1967).
[11] S. Weinberg, Phys. Rev. 166, 1568 (1968).
[12] T. Shiozaki, Progr. Theoret. Phys. (Kyoto) 39, 195 (1968).
[13] B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1968).
[14] Y. Ohnuki and Y. Yamaguchi, Institute for Nuclear Study (Tokyo) Report, 1967 (unpublished).
[15] J. Schwinger, Phys. Rev. 167, 1432 (1968).

leads to an effective interaction Hamiltonian of the form $-\mathcal{L}_{\rm Int}+$(additional contact-type terms).[16] We have calculated in Ref. 16 the interaction Hamiltonian coming from a simple vector-meson–baryon interaction Lagrangian, and we have shown that the S matrix is covariant to any order in perturbation theory. In Sec. VI we start from a general interaction Lagrangian, calculate the interaction Hamiltonian, and prove that the S matrix we obtain is covariant to second order in perturbation theory. The value of the $G_{NK\Sigma}^2/4\pi$ which is calculated from the $\mathcal{H}_{\rm Int}(\bar{B}BP)$ is in reasonable agreement with experiment, while the value of $G_{NK\Lambda}^2/4\pi$ is larger than the experimental value. The calculated values of the decay width $\Gamma(Y_0^*\to\Sigma\pi)$ and the ratio $|G_{NKY}|/|G_{\Sigma\pi Y}|$ are close to the experimental values.

In Sec. VII the s-wave $\pi\pi$, πK, and KK scattering lengths are calculated. In these calculations the mass parameters and the mixing angle of the S^i fields enter. Finally, in Sec. VIII the decay rates of the strong decays $\eta'(958)\to\eta(495)+2\pi$, $K^*\to K\pi$, and $\varphi\to K^+K^-$ are calculated. The results are in reasonable agreement with experiment.

II. MESON MATRIX

Consider a triplet of quarks

$$q=\begin{bmatrix} q_1\\ q_2\\ q_3\end{bmatrix}, \tag{2.1}$$

each one of which is a four-dimensional Dirac spinor. Let M be a 12×12 matrix representing the mesons. As in the Nishijima-Gürsey-type model,[17–19,6] we write the following interaction Lagrangian:

$$\mathcal{L}_0=g[q_L^\dagger(\gamma_4M)q_R+q_R^\dagger(M^\dagger\gamma_4)q_L], \tag{2.2}$$

where[20]

$$q_{L,R}=\begin{bmatrix} \frac{1}{2}(1\pm\gamma_5)q_1\\ \frac{1}{2}(1\pm\gamma_5)q_2\\ \frac{1}{2}(1\pm\gamma_5)q_3\end{bmatrix}. \tag{2.3}$$

So the fields q_L and q_R have the left-handed (upper sign) and right-handed (lower sign) components, respectively. Consider the group $GL(6,c)_{\rm left}\otimes GL(6,c)_{\rm right}$ whose generators are

$$G_{\alpha,\mu\pm}=\tfrac{1}{2}\lambda_\alpha\otimes\Sigma_\mu\tfrac{1}{2}(1\pm\gamma_5)\equiv G_{\alpha,\mu}\tfrac{1}{2}(1\pm\gamma_5), \tag{2.4}$$

where $\alpha=0,\ldots,8$, $\mu=1,\ldots,4$, and the operators with the plus sign are generators of the group $GL(6,c)_L$, while the operators with the minus sign are generators of the group $GL(6,c)_R$. The operators Σ_μ can be divided into a Hermitian set $\{\Sigma^H\}$ which contains

$$\Sigma_j{}^H=\tfrac{1}{4}\epsilon_{jkl}\sigma_{kl},\quad j=1,2,3$$
$$\Sigma_T{}^H=1, \tag{2.5}$$

and an anti-Hermitian set $\{\Sigma^A\}$ which contains

$$\Sigma_j{}^A=\tfrac{1}{4}\epsilon_{jkl}\sigma_{kl}\gamma_5=\tfrac{1}{2}i\sigma_{4j},\quad j=1,2,3$$
$$\Sigma_4{}^A=i\gamma_5. \tag{2.6}$$

The operators

$$\Sigma_j'=\{\Sigma_j{}^H,\Sigma_j{}^A\},\quad j=1,2,3 \tag{2.7}$$

are generators of an $SL(2,c)$ group which will be identified with the Lorentz group.

The spinors q_L and q_R are taken to transform as follows:

$$\begin{aligned} q_L &\xrightarrow{GL(6,c)_L} e^{i\epsilon_{\alpha\mu}G_{\alpha\mu+}}q_L=e^{i\epsilon_{\alpha\mu}G_{\alpha\mu}}q_L,\\ q_R &\xrightarrow{GL(6,c)_L} q_R,\\ q_L &\xrightarrow{GL(6,c)_R} q_L,\\ q_R &\xrightarrow{GL(6,c)_R} e^{i\zeta_{\alpha\mu}G_{\alpha\mu-}}q_R=e^{i\zeta_{\alpha\mu}G_{\alpha\mu}}q_R,\end{aligned} \tag{2.8}$$

where $\epsilon_{\alpha\mu}$ and $\zeta_{\alpha\mu}$ are real constants. Equations (2.8) imply that the Lagrangian of Eq. (2.2) is invariant under the group $GL(6,c)_L\otimes GL(6,c)_R$ if the meson matrix transforms as follows:

$$\begin{aligned} M &\xrightarrow{GL(6,c)_L} M'=e^{i\epsilon_{\alpha\mu}G_{\alpha\mu}}M,\\ \gamma_4M^\dagger\gamma_4 &\xrightarrow{GL(6,c)_L} \gamma_4M'^\dagger\gamma_4=\gamma_4M^\dagger\gamma_4e^{-i\epsilon_{\alpha\mu}G_{\alpha\mu}},\\ M &\xrightarrow{GL(6,c)_R} M'=Me^{-i\zeta_{\alpha\mu}G_{\alpha\mu}},\\ \gamma_4M^\dagger\gamma_4 &\xrightarrow{GL(6,c)_R} \gamma_4M'^\dagger\gamma_4=e^{i\zeta_{\alpha\mu}G_{\alpha\mu}}\gamma_4M^\dagger\gamma_4.\end{aligned} \tag{2.9}$$

From Eqs. (2.9) we get under $GL(6,c)_L$ transformations

$$M'\gamma_4M'^\dagger\gamma_4=e^{i\epsilon_{\alpha\mu}G_{\alpha\mu}}M\gamma_4M^\dagger\gamma_4e^{-i\epsilon_{\alpha\mu}G_{\alpha\mu}},$$
$$\gamma_4M'^\dagger\gamma_4M'=\gamma_4M^\dagger\gamma_4M, \tag{2.10}$$

and under $GL(6,c)_R$ transformations

$$M'\gamma_4M'^\dagger\gamma_4=M\gamma_4M^\dagger\gamma_4,$$
$$\gamma_4M'^\dagger\gamma_4M'=e^{i\zeta_{\alpha\mu}G_{\alpha\mu}}\gamma_4M^\dagger\gamma_4Me^{-i\zeta_{\alpha\mu}G_{\alpha\mu}}. \tag{2.11}$$

A choice of M which satisfies Eqs. (2.10) and (2.11) is

$$\gamma_4M^\dagger\gamma_4=M^{-1}. \tag{2.12}$$

We shall make this choice.

Let Φ be a 12×12 meson matrix which is a direct product of a 4×4 Dirac space and a 3×3 unitary space.

[16] E. Kyriakopoulos, Phys. Rev. **183**, 1318 (1969).
[17] K. Nishijima, Nuovo Cimento **11**, 698 (1959).
[18] F. Gürsey, Nuovo Cimento **16**, 230 (1960); Ann. Phys. (N. Y.) **12**, 91 (1961).
[19] Y. Nambu (unpublished).
[20] We shall use Hermitian γ matrices in the Weyl representation in which

$$\gamma_j=\begin{pmatrix}0 & i\sigma_j\\ -i\sigma_j & 0\end{pmatrix},\quad \gamma_4=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$
$$\gamma_5=\begin{pmatrix}-1&0\\0&1\end{pmatrix},\quad \sigma_{\mu\nu}=\frac{1}{2i}[\gamma_\mu,\gamma_\nu].$$

Such a matrix can be written as follows:

$$\Phi=\gamma_5(1\otimes S+\gamma_\mu\otimes V_\mu'+\tfrac{1}{2}\sigma_{\mu\nu}\otimes T_{\mu\nu}+\gamma_\mu\gamma_5\otimes A_\mu+i\gamma_5\otimes P)\,, \quad (2.13)$$

where S, V_μ', $T_{\mu\nu}$ A_μ, and P represent 3×3 matrices of meson fields. If we take these fields to be anti-Hermitian only for $\mu=4$, and otherwise Hermitian, we find that the matrix Φ satisfies the relation

$$\gamma_4\Phi^\dagger\gamma_4=-\Phi. \quad (2.14)$$

Using the above equation, we find that the following forms of M satisfy Eq. (2.12):

$$M=e^{f\Phi}\,, \quad (2.15)$$

$$M=(1+f\Phi)/(1-f\Phi)\,, \quad (2.16)$$

where f is a real constant. Another form of M is given by Eq. (4.13) below.

The Lagrangian of Eq. (2.2) must be Lorentz-invariant. In addition, we want it to be invariant under parity, charge conjugation, and time reversal. Such a requirement will specify the P, C, and T of the mesons involved, while their spin will be determined from the free Lagrangians of each specific field, which are given in Sec. III.

A. Lorentz Transformations

Under the Lorentz transformation $\mathcal{U}(\Lambda^{-1})$ the spinors transform in the well-known fashion,

$$q_L(x)\rightarrow\mathcal{U}(\Lambda^{-1})q_L(x)\mathcal{U}(\Lambda^{-1})^{-1}=[S(\Lambda)\otimes1]q_L(\Lambda^{-1}x)\,, \quad (2.17)$$

$$q_R(x)\rightarrow\mathcal{U}(\Lambda^{-1})q_R(x)\mathcal{U}(\Lambda^{-1})^{-1}=[S(\Lambda)\otimes1]q_R(\Lambda^{-1}x)\,,$$

where the Lorentz-transformation matrix $S(\Lambda)$ is given by

$$S(\Lambda)e^{i\epsilon_j\Sigma_j^A}. \quad (2.18)$$

The operators Σ_j^A of the above equation are given by Eqs. (2.6), and the parameters ϵ_j are real constants. Also, if $\Phi(x)$ is given by Eq. (2.13), and the meson fields have the usual Lorentz-transformation properties, we find, since $[\gamma_5,S(\Lambda)]=[\gamma_5,S(\Lambda)^{-1}]=0$, that under Lorentz transformations

$$\Phi(x)\rightarrow\mathcal{U}(\Lambda^{-1})\Phi(x)\mathcal{U}(\Lambda^{-1})^{-1}=[S(\Lambda)\otimes1]\Phi(\Lambda^{-1}x)[S(\Lambda)^{-1}\otimes1]. \quad (2.19)$$

Equation (2.19) implies that

$$M(x)\rightarrow\mathcal{U}(\Lambda^{-1})M(x)\mathcal{U}(\Lambda^{-1})^{-1}=[S(\Lambda)\otimes1]M(\Lambda^{-1}x)[S(\Lambda)^{-1}\otimes1]. \quad (2.20)$$

We easily see from Eqs. (2.17) and (2.20) that the Lagrangian of Eq. (2.2) is invariant under Lorentz transformations.

B. Parity

Under the parity operation $\mathcal{P}$, we get

$$\mathcal{P}q_L(\mathbf{x},t)\mathcal{P}^{-1}=\eta_P\gamma_4q_R(-\mathbf{x},t)\,, \quad (2.21)$$

$$\mathcal{P}q_R(\mathbf{x},t)\mathcal{P}^{-1}=\eta_P\gamma_4q_L(-\mathbf{x},t)\,,$$

where η_P is a phase factor. So the Lagrangian $\mathcal{L}_0$ is invariant under parity if

$$\mathcal{P}M(x)\mathcal{P}^{-1}=M^\dagger(-\mathbf{x},t)\,. \quad (2.22)$$

Then Eq. (2.14) with (2.15) or (2.16) tells us that the above equation is satisfied if

$$\mathcal{P}\Phi(\mathbf{x},t)\mathcal{P}^{-1}=\Phi^\dagger(-\mathbf{x},t)=-\gamma_4\Phi(-\mathbf{x},t)\,. \quad (2.23)$$

Equation (2.23) implies that the fields S, V_μ', $T_{\mu\nu}$, A_μ, and P behave under parity as scalar, vector, tensor, pseudovector, and pseudoscalar, respectively.

C. Charge Conjugation

The Lagrangian $\mathcal{L}_0$ is invariant under the charge conjugation operation $\mathcal{C}$ if

$$\mathcal{C}M\mathcal{C}^{-1}=CM^TC^{-1}\,, \quad (2.24)$$

where M^T mean the transpose matrix of M and $C=\gamma_2\gamma_4$. Equation (2.24) is satisfied if

$$\mathcal{C}\Phi\mathcal{C}^{-1}=C\Phi^TC^{-1}\,, \quad (2.25)$$

which implies

$$C_S=C_{V'}=C_P=1,\quad C_T=C_A=-1. \quad (2.26)$$

The field V_μ' cannot be identified with the physical vector-meson field because it has the wrong charge conjugation.

D. Time Reversal

The Lagrangian $\mathcal{L}_0$ is invariant under time reversal $\mathcal{T}$ if

$$\mathcal{T}M(\mathbf{x},t)\mathcal{T}^{-1}=BM(\mathbf{x},-t)B^{-1}\,, \quad (2.27)$$

where $B=\gamma_1\gamma_3$. The above equation is satisfied if

$$\mathcal{T}\Phi(\mathbf{x},t)\mathcal{T}^{-1}=\gamma_1\gamma_3\Phi(\mathbf{x},-t)\gamma_3\gamma_1\,, \quad (2.28)$$

which implies

$$T_S=T_{V'}=T_A=1,\quad T_P=T_T=-1. \quad (2.29)$$

III. MESON LAGRANGIAN

In the following we shall write in the meson Lagrangian M^{-1} instead of $\gamma_4M^\dagger\gamma_4$ according to Eq. (2.12). We consider the meson Lagrangian

$$\mathcal{L}'=\mathcal{L}_M{}^1+\mathcal{L}_M{}^2+\mathcal{L}_M{}^m\,, \quad (3.1)$$

where

$$\mathcal{L}_M{}^1=-(1/8f^2)\,\mathrm{Tr}[(\partial_\mu M)(\partial_\mu M^{-1})]\,, \quad (3.2)$$

$$\begin{aligned}\mathcal{L}_M{}^2=&-(1/32f^2)\,\mathrm{Tr}[(\partial_\mu M)\gamma_\mu(\partial_\nu M^{-1})\gamma_\nu]\\&-(1/32f^2)\,\mathrm{Tr}[\gamma_\mu(\partial_\mu M)\gamma_\nu(\partial_\nu M^{-1})]\\&+(1/32f^2)\,\mathrm{Tr}[(\partial_\mu M)\gamma_\mu\gamma_\nu(\partial_\nu M^{-1})]\\&+(1/32f^2)\,\mathrm{Tr}[\gamma_\mu(\partial_\mu M)(\partial_\nu M^{-1})\gamma_\nu]\,,\end{aligned} \quad (3.3)$$

and $\mathcal{L}_M{}^m$ is the meson mass Lagrangian. For degenerate meson masses we may take

$$\mathcal{L}_M{}^m=(m^2/8f^2)\,\mathrm{Tr}(M+M^{-1})\,. \tag{3.4}$$

The Lagrangian $\mathcal{L}_M{}^1$ is invariant under the transformations of Eqs. (2.9), the Lagrangian $\mathcal{L}_M{}^2$ is invariant under the group $SL(2,c)\otimes[U(3)_L\otimes U(3)_R]$, where $SL(2,c)$ is the Lorentz group, while the Lagrangian $\mathcal{L}_M{}^m$ is invariant under the group $SL(2,c)\otimes U(3)$. In addition, the Lagrangians $\mathcal{L}_M{}^1$, $\mathcal{L}_M{}^2$, and $\mathcal{L}_M{}^m$ are invariant under parity, charge conjugation, and time reversal. We constructed the meson Lagrangian in such a way that to zero order in f, we obtain the sum of the free Lagrangians of the mesons of our model.

From Eqs. (3.1)–(3.4), we get

$$\begin{aligned}\mathcal{L}'=\tfrac{1}{2}\,\mathrm{Tr}\{&-(\partial_\mu V_\nu')(\partial_\mu V_\nu')+(\partial_\mu V_\mu')(\partial_\nu V_\nu')\\&+(\partial_\mu T_{\nu\lambda})(\partial_\nu T_{\mu\lambda})+(\partial_\mu A_\nu)(\partial_\nu A_\mu)-(\partial_\mu P)(\partial_\mu P)\}\\&+\tfrac{1}{2}m^2\,\mathrm{Tr}(SS-V_\mu'V_\mu'+\tfrac{1}{2}T_{\mu\nu}T_{\mu\nu}+A_\mu A_\mu-PP)\\&+3m^2/f^2+O(f^2)+\cdots.\end{aligned} \tag{3.5}$$

The above Lagrangian leads to a positive-definite Hamiltonian. From its free-meson part, we obtain the conditions

$$S=0,\quad \partial_\mu V_\mu'=0,\quad \partial_\mu A_\nu-\partial_\nu A_\mu=0\,, \tag{3.6}$$

while from the antisymmetry of the $T_{\mu\nu}$ tensor we get (details for the $T_{\mu\nu}$ field are given in Ref. 16)

$$\partial_\mu\partial_\nu T_{\mu\nu}=0\,. \tag{3.7}$$

So from Eqs. (3.5)–(3.7), together with the results of Sec. II on parity and charge conjugation of the mesons fields, we find that the model contains a nonet with $J^{PC}=1^{-+}$ described by the field V_μ', a nonet with $J^{PC}=1^{--}$ described by the field $\partial_\mu T_{\mu\nu}$, a nonet with $J^{PC}=0^{--}$ described by the field $\partial_\mu A_\mu$, and finally a nonet with $J^{PC}=0^{-+}$ described by the field P. So we get two nonets of vector mesons of opposite charge conjugation, and two nonets of pseudoscalar mesons which also have opposite charge conjugation. The nonets $\partial_\mu T_{\mu\nu}$ and P will be identified with the physical vector-meson and pseudoscalar-meson nonets, respectively.

If in a term in the expansion of the total Lagrangian, consisting of the sum of the meson Lagrangian of Eq. (3.27) and the baryon Lagrangian of Eq. (5.11), given later, the field V' appears m times and the field A n times, we find that the sum $m+n$ is always an even number. This means that in the framework of our Lagrangian the particles V' and A have strong interactions among themselves and with the other hadrons, are always produced in even number, and cannot decay to pseudoscalar mesons, vector mesons, and baryons via strong interactions. We shall assume that they decay by electromagnetic and weak interactions, which implies that their decay width is narrow.[21] Nonets with $J^{PC}=0^{--}$ and 1^{-+} are also predicted by the "new" quark model of Gell-Mann and Zweig.[22]

To introduce phenomenologically the physical masses of the mesons into the scheme, we consider in addition to $\mathcal{L}_M{}^m$ of Eq. (3.4) several other mass Lagrangian terms. In doing that we try to achieve the following things: (a) to separate the (common) mass of a nonet from the (common) mass of the other nonets; (b) to assign to the $I=1$ and $I=\frac{1}{2}$ members of the nonets their physical masses (apart from electromagnetic mass splitting, which we shall ignore); and (c) to allow within each nonet a mixing of the $SU(3)$ singlet with the $I=0$ member of the $SU(3)$ octet with adjustable masses and mixing angles.

Generalizing Eq. (3.4) according to the requirements just stated, we consider the following mass Lagrangian:

$$\begin{aligned}\mathcal{L}_M{}^m=&(1/f^2)\,\mathrm{Tr}[(M+M^{-1})(\alpha+\beta\lambda_8)]+(\gamma/f^2)\,\mathrm{Tr}[(\gamma_\mu M\gamma_\mu)(\gamma_\nu M^{-1}\gamma_\nu)]\\&+(\delta/f^2)\,\mathrm{Tr}[(\gamma_\mu M\gamma_\mu)\{\gamma_\nu M^{-1}\gamma_\nu,\lambda_8\}_+]+(\epsilon/f^2)\,\mathrm{Tr}[(\sigma_{\mu\nu}M\sigma_{\mu\nu})(\sigma_{\rho\sigma}M^{-1}\sigma_{\rho\sigma})]\\&+(\zeta/f^2)\,\mathrm{Tr}[(\sigma_{\mu\nu}M\sigma_{\mu\nu})\{\sigma_{\rho\sigma}M^{-1}\sigma_{\rho\sigma},\lambda_8\}_+]+(\xi/f^2)\,\mathrm{Tr}[(\gamma_\mu M\gamma_\mu)M^{-1}]+(\chi/f^2)\,\mathrm{Tr}[(\gamma_\mu M\gamma_\mu)\{M^{-1},\lambda_8\}_+]\\&+(1/f^2)\sum_Q\{\eta_Q\,\mathrm{Tr}[\Gamma_Q(M-M^{-1})]\,\mathrm{Tr}[\Gamma_Q(M-M^{-1})]+\theta_Q\,\mathrm{Tr}[\Gamma_Q(M-M^{-1})]\,\mathrm{Tr}[\Gamma_Q\lambda_8(M-M^{-1})]\\&+\kappa_Q\,\mathrm{Tr}[\Gamma_Q\lambda_8(M-M^{-1})]\,\mathrm{Tr}[\Gamma_Q\lambda_8(M-M^{-1})]\}\,,\end{aligned} \tag{3.8}$$

where, in the summation, Q takes the values S, V', T, A, and P and we have

$$\Gamma_S=\gamma_5,\quad \Gamma_{V'}=\gamma_5\gamma_\mu,\quad \Gamma_T=\gamma_5\sigma_{\mu\nu},\quad \Gamma_A=\gamma_\mu,\quad \Gamma_P=1\,. \tag{3.9}$$

Symmetry breaking has been introduced by means of the matrix λ_8. The terms with the coefficients γ, ϵ, and ξ are introduced to satisfy requirement (a), while the terms with the coefficients β δ ζ, and χ are introduced to satisfy

[21] It is well known that the strange particles behave in an analogous fashion. The intermediate bosons also have similar properties, if we assume that they belong to a unitary triplet [C. Ryan, S. Okubo, and R. E. Marshak, Nuovo Cimento **34**, 753 (1964); S. V. Pepper, C. Ryan, S. Okubo, and R. E. Marshak, Phys. Rev. **137**, B1259 (1965)]. Triality conservation implies that such particles have quadratic strong interactions with the nucleons. Such a theory explains the large mass (>1.5 BeV) of the conjectured intermediate bosons and also a 2% difference between the vector coupling constants G_β and G_μ characterizing β decay and μ decay, respectively.

[22] H. Harari, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 195. The "new" quark model predicts a 0^{--} nonet around the B-meson mass. It also predicts a 1^{-+} nonet, the daughter of the A_2 nonet. Finally, a second 1^{-+} nonet is predicted, which is the parity doublet of the A_1 nonet.

requirement (b). The terms in the summation $\sum_Q$ allow for the mixing of requirement (c). The $\mathcal{L}_M{}^m$ of Eq. (3.8) is invariant under parity, charge conjugation, and time reversal, and under the group $SL(2,c)\otimes U(2)$.

From Eqs. (3.8) and (3.9) we get[23] (omitting an unimportant constant term)

$$\mathcal{L}_M{}^m=-\tfrac{1}{2}\,\mathrm{Tr}[SS(\alpha_S+\beta_S\lambda_8)]-\tfrac{1}{2}\,\mathrm{Tr}[V_\mu'V_\mu'(\alpha_{V'}+\beta_{V'}\lambda_8)]+\tfrac{1}{4}\,\mathrm{Tr}[T_{\mu\nu}T_{\mu\nu}(\alpha_T+\beta_T\lambda_8)]$$
$$+\tfrac{1}{2}\,\mathrm{Tr}[A_\mu A_\mu(\alpha_A+\beta_A\lambda_8)]-\tfrac{1}{2}\,\mathrm{Tr}[PP(\alpha_P+\beta_P\lambda_8)]+\sum_Q\{\eta_Q'Q_0Q_0+\theta_Q'Q_0Q_8+\kappa_Q'Q_8Q_8\}+\cdots,\quad(3.10)$$

where Q_0 means the $SU(3)$ singlet part of the $U(3)$ nonet Q, while Q_8 means the $I=0$ member of the $SU(3)$ octet. Let us introduce the following notation for the members of the nonets:

$$\begin{aligned}P&=(\pi,K,\eta,\eta'),\\ T&=(\rho,K^*,\varphi,\omega),\\ S&=(\pi',K',\zeta',\xi'),\\ V'&=(\rho',K'^*,\varphi',\omega'),\\ A&=(\pi'',K'',\zeta'',\xi'').\end{aligned}\quad(3.11)$$

To satisfy requirements (a) and (b), we choose the coefficients α_Q and β_Q of Eq. (3.10) as follows:

$$\begin{aligned}\alpha_Q&=\tfrac{1}{3}(m_Q{}^2+2m_{Q'}{}^2),\\ \beta_Q&=(2/\sqrt{3})(m_Q{}^2-m_{Q'}{}^2),\end{aligned}\quad(3.12)$$

where

$$\begin{aligned}m_Q&=(m_\pi,m_\rho,m_{\pi'},m_{\rho'},m_{\pi''}),\\ m_{Q'}&=(m_K,m_{K^*},m_{K'},m_{K'^*},m_{K''}).\end{aligned}\quad(3.13)$$

We get Eqs. (3.12) if we make the following choice for the coefficients α, β, ξ, χ, γ, δ, ϵ, and ζ:

$$\begin{aligned}\alpha&=(1/24)(m_\pi{}^2+2m_K{}^2),\\ \beta&=(1/4\sqrt{3})(m_\pi{}^2-m_K{}^2),\\ \xi&=-(1/192)(m_{\pi'}{}^2+2m_{K'}{}^2+m_\pi{}^2+2m_K{}^2),\\ \chi&=(1/64\sqrt{3})(-m_{\pi'}{}^2+m_{K'}{}^2-m_\pi{}^2+m_K{}^2),\\ \gamma&=(1/1152)[9(m_\rho{}^2+2m_{K^*}{}^2)-8(m_{\rho'}{}^2+2m_{K'^*}{}^2)\\ &\qquad+\tfrac{3}{2}(m_\pi{}^2+2m_K{}^2)+\tfrac{5}{2}(m_{\pi'}{}^2+2m_{K'}{}^2)],\\ \delta&=(1/384\sqrt{3})[9(m_\rho{}^2-m_{K^*}{}^2)-8(m_{\rho'}{}^2-m_{K'^*}{}^2)\\ &\qquad+\tfrac{3}{2}(m_\pi{}^2-m_K{}^2)+\tfrac{5}{2}(m_{\pi'}{}^2-m_{K'}{}^2)],\\ \epsilon&=(1/4608)[4(m_{\rho'}{}^2+2m_{K'^*}{}^2)-3(m_\rho{}^2+2m_{K^*}{}^2)\\ &\qquad-\tfrac{3}{2}(m_\pi{}^2+2m_K{}^2)-\tfrac{1}{2}(m_{\pi'}{}^2+2m_{K'}{}^2)],\\ \zeta&=(1/1536\sqrt{3})[4(m_{\rho'}{}^2-m_{K'^*}{}^2)-3(m_\rho{}^2-m_{K^*}{}^2)\\ &\qquad-\tfrac{3}{2}(m_\pi{}^2-m_K{}^2)-\tfrac{1}{2}(m_{\pi'}{}^2-m_{K'}{}^2)].\end{aligned}\quad(3.14)$$

If the $\mathcal{L}_M{}^m$ is chosen as above, all the masses are arbitrary parameters except the masses $m_{\pi''}$ and $m_{K''}$, which are given by

$$\begin{aligned}m_{\pi''}{}^2&=m_{\rho'}{}^2-\tfrac{1}{2}(m_{\pi'}{}^2+m_\pi{}^2),\\ m_{K''}{}^2&=m_{K'^*}{}^2-\tfrac{1}{2}(m_{K'}{}^2+m_K{}^2).\end{aligned}$$

To satisfy requirement (c) we choose the coefficients η_Q, θ_Q, and κ_Q as follows:

$$\begin{aligned}\eta_Q&=C_Q[-\tfrac{1}{3}(m_Q{}^2+2m_{Q'}{}^2)\\ &\qquad+m_{Q''}{}^2\sin^2\lambda_Q+m_{Q'''}{}^2\cos^2\lambda_Q],\\ \theta_Q&=D_Q[-\tfrac{4}{3}\sqrt{2}(m_Q{}^2-m_{Q'}{}^2)\\ &\qquad+2(m_{Q''}{}^2-m_{Q'''}{}^2)\cos\lambda_Q\sin\lambda_Q],\\ \kappa_Q&=F_Q[\tfrac{1}{3}(m_Q{}^2-4m_{Q'}{}^2)\\ &\qquad+m_{Q''}{}^2\cos^2\lambda_Q+m_{Q'''}{}^2\sin^2\lambda_Q],\end{aligned}\quad(3.15)$$

where for $Q=(P,T,S,V',A)$ we have, respectively,

$$\begin{aligned}C_Q&=(1/384)(1,\ \tfrac{1}{2},\ 1,\ -1,\ 1),\\ D_Q&=(1/128\sqrt{6})(1,\ -\tfrac{1}{2},\ 1,\ -1,\ 1),\\ F_Q&=(1/256)(1,\ \tfrac{1}{2},\ 1,\ -1,\ 1).\end{aligned}\quad(3.16)$$

The masses m_Q and $m_{Q'}$ are given by expressions (3.13), while $m_{Q''}$ and $m_{Q'''}$ are given by

$$\begin{aligned}m_{Q''}&=(m_\eta,m_\varphi,m_{\zeta'},m_{\varphi'},m_{\zeta''}),\\ m_{Q'''}&=(m_{\eta'},m_\omega,m_{\xi'},m_{\omega'},m_{\xi''}).\end{aligned}\quad(3.17)$$

In the present case the masses and the mixing angles for all nonets can be fixed independently. The mixing angles λ_Q are defined by

$$\begin{aligned}Q''&=\cos\lambda_Q\,Q_8+\sin\lambda_Q\,Q_0,\\ Q'''&=-\sin\lambda_Q\,Q_8+\cos\lambda_Q\,Q_0.\end{aligned}\quad(3.18)$$

The Lagrangian $\mathcal{L}'(M)$ of Eq. (3.1) gives no coupling involving three meson fields. It does, however, contain in its expansion terms involving four mesons. We shall be interested only in terms which contain four pseudoscalar meson fields since we shall consider processes of the type $PP\to PP$. Such terms come from the Lagrangians $\mathcal{L}_M{}^1$ and $\mathcal{L}_M{}^m$ only. We find

$$\begin{aligned}\mathcal{L}_M{}^1&\to-(f^2/24)\,\mathrm{Tr}\{(P\overleftrightarrow{\partial}_\mu P)(P\overleftrightarrow{\partial}_\mu P)\},\\ \mathcal{L}_M{}^m&\to\tfrac{1}{3}f^2\,\mathrm{Tr}[P^4(\alpha+\beta\lambda_8)]+\mathcal{L}_M{}^{0,8}\\ &=\tfrac{1}{12}f^2\,\mathrm{Tr}\{P^4[\tfrac{1}{6}(m_\pi{}^2+2m_K{}^2)\\ &\qquad+(1/\sqrt{3})(m_\pi{}^2-m_K{}^2)\lambda_8]\}+\mathcal{L}_M{}^{0,8},\end{aligned}\quad(3.19)$$

where at least one of the four fields appearing in the Lagrangian $\mathcal{L}_M{}^{0,8}$ is the $SU(3)$ singlet or the $I=0$ mem-

[23] From the first of Eqs. (3.6), we see that the fields S vanish as free fields. However, they do not vanish in the presence of interactions, i.e., if we consider higher-order terms in the expansion of the Lagrangian $\mathcal{L}$, as we see from the equations of motion. Something similar happens in the $U(6,6)$ theory. In fact, A. Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee [Proc. Roy. Soc. (London) **285A**, 312 (1965)] argue that such fields can be treated as spurions and provide a mechanism for breaking the symmetry.

ber of the $SU(3)$ octet. The Lagrangian $\mathfrak{L}_M{}^{0,8}$ is given by

$$\mathfrak{L}_M{}^{0,8}=(32f^2/3)[2\eta_P \operatorname{Tr}(P)\operatorname{Tr}(P^3)+\theta_P \operatorname{Tr}(P)\operatorname{Tr}(P^3\lambda_8)+\theta_P \operatorname{Tr}(P\lambda_8)\operatorname{Tr}(P^3)+2\kappa_P \operatorname{Tr}(P\lambda_8)\operatorname{Tr}(P^3\lambda_8)]. \quad (3.20)$$

To complete the meson Lagrangian, we must introduce another one which will give three-meson coupling, and particularly which will couple the physical vector-meson nonet to two physical pseudoscalar-meson nonets. We want this Lagrangian to be invariant under the group $U(3)_L \otimes U(3)_R$ and of course invariant under parity, charge conjugation, time reversal, and Lorentz transformations. The three-meson coupling can be introduced by the following Lagrangian, which has the required symmetry properties:

$$\begin{aligned}\mathfrak{L}_M{}^3=-\frac{f_{\rho\pi\pi}}{32\sqrt{2}f^3 m_\rho}&\\ +\operatorname{Tr}[&(\partial_\mu M)\gamma_\mu(\partial_\nu M^{-1})\gamma_\nu(M\gamma_\rho M^{-1}\gamma_\rho-4)\gamma_5\\ &-(\partial_\mu M^{-1})\gamma_\mu(\partial_\nu M)\gamma_\nu(M^{-1}\gamma_\rho M\gamma_\rho-4)\gamma_5\\ &+\gamma_5(\gamma_\rho M^{-1}\gamma_\rho M-4)\gamma_\nu(\partial_\nu M^{-1})\gamma_\mu(\partial_\mu M)\\ &-\gamma_5(\gamma_\rho M\gamma_\rho M^{-1}-4)\gamma_\nu(\partial_\nu M)\gamma_\mu(\partial_\mu M^{-1})]. \quad (3.21)\end{aligned}$$

The presence of the coefficient $-f_{\rho\pi\pi}/32\sqrt{2}f^3 m_\rho$ is justified in Sec. VI. The above Lagrangian gives no coupling of two mesons and also no coupling of four mesons. We get

$$\mathfrak{L}_M{}^3=-\frac{\sqrt{2}if_{\rho\pi\pi}}{m_\rho}\operatorname{Tr}\{T_{\mu\nu}(\partial_\mu P)(\partial_\nu P)\}-\frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\epsilon_{\kappa\theta\nu\sigma}\operatorname{Tr}[\{\partial_\nu P,\partial_\mu T_{\mu\sigma}\}_+T_{\kappa\theta}]+\cdots, \quad (3.22)$$

where only the terms which couple a vector meson to two pseudoscalar mesons and a pseudoscalar meson to two vector mesons are shown.

We have proved[16] that the field V_μ defined by

$$V_\mu=\partial_\rho T_{\rho\mu}/m \quad (3.23)$$

represents a vector-meson field. From the free Lagrangian of the $T_{\mu\nu}$ field given in Eq. (3.5), we obtain the following equation of motion:

$$\partial_\mu\partial_\rho T_{\rho\nu}-\partial_\nu\partial_\rho T_{\rho\mu}-m^2 T_{\mu\nu}=0, \quad (3.24)$$

from which we get, using Eq. (3.23),

$$T_{\mu\nu}=\partial_\mu(V_\nu/m)-\partial_\nu(V_\mu/m). \quad (3.25)$$

Equations (3.23) and (3.25) will be used frequently. We get

$$\operatorname{Tr}[T_{\mu\nu}\partial_\mu P\partial_\nu P]=-\tfrac{1}{2}\operatorname{Tr}[\partial_\mu T_{\mu\nu}(P\overleftrightarrow{\partial}_\nu P)]+\tfrac{1}{2}\partial_\mu\operatorname{Tr}[T_{\mu\nu}(P\overleftrightarrow{\partial}_\nu P)]. \quad (3.26)$$

Using Eq. (3.23), we see that the first term on the right-hand side of the above equation is the well-known VPP coupling, apart from mass factors.

Our complete meson Lagrangian $\mathfrak{L}$ is given by

$$\mathfrak{L}=\mathfrak{L}_M{}^1+\mathfrak{L}_M{}^2+\mathfrak{L}_M{}^3+\mathfrak{L}_M{}^m, \quad (3.27)$$

where $\mathfrak{L}_M{}^m$ is given by Eq. (3.8).

IV. VECTOR AND AXIAL-VECTOR CURRENTS

From the complete meson Lagrangian $\mathfrak{L}$ of Eq. (3.27), we shall calculate the vector and axial-vector currents by applying the method of Gell-Mann and Lévy.[3] Consider the Lagrangian $\mathfrak{L}(q_\tau)$ and the transformation

$$q_\tau \to q_\tau+\epsilon(x)F_\tau(x), \quad (4.1)$$

where $\epsilon(x)$ is an infinitesimal space-time function. Using the Euler-Lagrange equation, we obtain the Gell-Mann–Lévy equation

$$\partial_\mu\frac{\delta\mathfrak{L}}{\delta[\partial_\mu\epsilon(x)]}=\frac{\delta\mathfrak{L}}{\delta\epsilon}. \quad (4.2)$$

In our case we consider the transformation

$$M\to e^{i\epsilon_\alpha\lambda_\alpha/2}Me^{-i\zeta_\alpha\lambda_\alpha/2}. \quad (4.3)$$

A. Vector Current

From the above transformation with $\zeta_\alpha=\epsilon_\alpha$ applied to the Lagrangian $\mathfrak{L}'$, we obtain the following vector current:

$$\begin{aligned}J_\mu{}'^\alpha\equiv&-\frac{\partial\mathfrak{L}'}{\partial(2\partial_\mu\epsilon_\alpha)}\\ =&-\frac{i}{16f^2}\operatorname{Tr}\{(\partial_\mu M)[M^{-1},\lambda_\alpha]+[M,\lambda_\alpha](\partial_\mu M^{-1})\}\\ &-\frac{i}{64f^2}\operatorname{Tr}\{[\partial_\nu M,\gamma_\nu][[M^{-1},\lambda_\alpha],\gamma_\mu]\\ &+[[M,\lambda_\alpha],\gamma_\mu][\partial_\nu M^{-1},\gamma_\nu]\}\\ =&\tfrac{1}{8}i\operatorname{Tr}(\partial_\mu\Phi[\Phi,\lambda_\alpha])+\tfrac{1}{32}i\operatorname{Tr}([\partial_\nu\Phi,\gamma_\nu]\\ &\times[[\Phi,\lambda_\alpha],\gamma_\mu])+\cdots. \quad (4.4)\end{aligned}$$

To the above current, we must add the current coming from the Lagrangian $\mathfrak{L}_M{}^3$. We have omitted this current because its exact expression is complicated, while when it is expanded it gives no contribution to zero order in f. Also, since $\mathfrak{L}_M{}^3$ is invariant under the group $U(3)_L \otimes U(3)_R$, it does not contribute to the divergence of the vector (as well as the axial-vector) current. To zero order in f, we get from Eq. (4.4) (now $J_\mu{}^\alpha$ is the current coming from the Lagrangian $\mathfrak{L}$)

$$\begin{aligned}J_\mu{}^\alpha=\tfrac{1}{2}i\operatorname{Tr}[\lambda_\alpha\{&(P\overleftrightarrow{\partial}_\mu P)-\tfrac{1}{2}([\partial_\nu(V_\sigma/m)\\ &-\partial_\sigma(V_\nu/m)]\overleftrightarrow{\partial}_\mu[\partial_\nu(V_\sigma/m)-\partial_\sigma(V_\nu/m)])\\ &+(V_\nu'\overleftrightarrow{\partial}_\mu V_\nu')-(A_\nu\overleftrightarrow{\partial}_\mu A_\nu)\}]+\cdots. \quad (4.5)\end{aligned}$$

To obtain the above expression, we have used the relation

$$\operatorname{Tr}(\partial_\nu T_{\mu\sigma}[T_{\sigma\nu},\lambda_\alpha]) = \tfrac{1}{2}\operatorname{Tr}(\lambda_\alpha\{[\partial_\nu(V_\sigma/m)-\partial_\sigma(V_\nu/m)]\overleftrightarrow{\partial}_\mu[\partial_\nu(V_\sigma/m) - \partial_\sigma(V_\nu/m)]\}), \quad (4.6)$$

which is true if we write $T_{\mu\nu}=\partial_\mu(V_\nu/m)-\partial_\nu(V_\mu/m)$, i.e., if we omit higher-order terms which appear in the expression for $T_{\mu\nu}$ according to the equations of motion. Also, the second and the third of Eqs. (3.6) have been used. The $J_\mu{}^\alpha$ of Eq. (4.5) has been normalized in such a way that its pseudoscalar-meson part is the isospin current of the pseudoscalar mesons.

The mass Lagrangian does not contribute to the vector current. It contributes, however, to its divergence. If we choose the mass Lagrangian as in Eq. (3.8), we find that the components $\alpha=1$, 2, 3, 8 of the vector current are conserved, while with the choice of Eq. (3.4) all components are conserved.

B. Axial-Vector Current

From the transformation of Eq. (4.3) with $\zeta_\alpha=-\epsilon_\alpha$ applied to the Lagrangian $\mathcal{L}$ of Eq. (3.27), we obtain the axial-vector current $J_{5\mu}{}^\alpha$:

$$\begin{aligned}J_{5\mu}{}^\alpha &\equiv -\frac{\partial\mathcal{L}}{\partial(2\partial_\mu\epsilon_\alpha)} = -\frac{i}{32f^2}\operatorname{Tr}[(\partial_\mu M)\{M^{-1},\lambda_\alpha\}_+ - \{M,\lambda_\alpha\}_+(\partial_\mu M^{-1})] - (i/128f^2)\operatorname{Tr}\{[\gamma_\nu,\partial_\nu M][\gamma_\mu,\{M^{-1},\lambda_\alpha\}_+] \\ &\quad + [\{M,\lambda_\alpha\}_+,\gamma_\mu][\gamma_\nu,\partial_\nu M^{-1}]\} + (\text{terms coming from } \mathcal{L}_M{}^3) \\ &= (1/\sqrt{2}f)\partial_\mu\varphi^\alpha + \tfrac{1}{16}if\operatorname{Tr}\{\lambda_\alpha[\Phi(\partial_\mu\Phi)\Phi - 2c\partial_\mu\Phi^3]\} + (if/128)\operatorname{Tr}([\gamma_\nu,\partial_\nu\Phi^2][\gamma_\mu,\{\Phi,\lambda_\alpha\}_+] \\ &\quad + [\{\Phi^2,\lambda_\alpha\}_+,\gamma_\mu][\gamma_\nu,\partial_\nu\Phi]) + \cdots + (\text{terms coming from } \mathcal{L}_M{}^3). \quad (4.7)\end{aligned}$$

In the above expression we have not explicitly indicated the terms which come from the Lagrangian $\mathcal{L}_M{}^3$ because they are complicated. Also, we have expressed the pseudoscalar-meson octet matrix P' as follows:

$$P' = (1/\sqrt{2})\lambda_\alpha\varphi^\alpha, \quad (4.8)$$

and we have assumed that the meson matrix M has the following expansion:

$$M = 1 + f\Phi + \tfrac{1}{2}f^2\Phi^2 + cf^3\Phi^3 + \cdots. \quad (4.9)$$

From Eq. (4.7), we find

$$\begin{aligned}J_{5\mu}{}^\alpha &= (1/\sqrt{2}f)\partial_\mu\varphi^\alpha \\ &\quad + \tfrac{1}{4}f\operatorname{Tr}\{\lambda_\alpha[P(\partial_\mu P)P - 2c\partial_\mu P^3]\} + \cdots \\ &\quad + (\sqrt{2}f_{\rho\pi\pi}/fm_\rho)f_{\alpha\beta\gamma}\partial_\nu\varphi^\beta \\ &\qquad \times[\partial_\nu(V_\mu{}^\gamma/m) - \partial_\mu(V_\nu{}^\gamma/m)] + \cdots. \quad (4.10)\end{aligned}$$

The first two terms of Eq. (4.10) are obtained from the Lagrangian $\mathcal{L}'$. From the terms which couple three mesons we have kept only those which contain three pseudoscalar-meson fields. The lowest-order contribution to the current $J_{5\mu}{}^\alpha$ coming from the Lagrangian $\mathcal{L}_M{}^3$ involves two meson fields, but both cannot be pseudoscalar-meson fields. If we keep the terms containing one pseudoscalar- and one vector-meson field, we get the last term of Eq. (4.10). The next-order contribution involves four mesons.

If the mass Lagrangian is given by Eq. (3.8), we find the following expression for the divergence of the axial-vector current:

$$\begin{aligned}\partial_\mu J_{5\mu}{}^\alpha &= m_\alpha{}^2[(1/\sqrt{2}f)\varphi^\alpha - \tfrac{1}{8}icf\operatorname{Tr}(\Phi^3\lambda_\alpha) + O(f^3) + \cdots] \\ &\quad + [\text{contribution of } \textstyle\sum_Q \text{ terms of Eq. (3.8)}], \quad (4.11)\end{aligned}$$

where

$$\begin{aligned}m_\alpha &= m_\pi \quad \text{for} \quad \alpha=1,2,3 \\ &= m_K \quad \text{for} \quad \alpha=4,5,6,7. \quad (4.12)\end{aligned}$$

As indicated in Eq. (4.11), the mass Lagrangian terms in the summation $\sum_Q$ contribute to $\partial_\mu J_{5\mu}{}^\alpha$. Their contribution is at least third-order[24] in the meson fields, and in the third-order case at least one of the meson fields is a Q_0 or Q_8 field. So if we neglect the terms which are third- or higher-order in the meson fields, the $J_{5\mu}{}^\alpha$ satisfies generalized PCAC. Also, we see from Eq. (4.11) that if we neglect the contribution coming from the Lagranian terms in the summation $\sum_Q$ and choose M such that $c=0$, the terms which will violate PCAC are fifth-order in the meson fields. The following choice of M has the expansion of Eq. (4.9) with $c=0$:

$$M = e^{f'\Phi}\frac{1+f'\Phi}{1-f'\Phi} = 1 + f\Phi + \tfrac{1}{2}f^2\Phi^2 + \tfrac{1}{8}f^4\Phi^4 + \cdots, \quad (4.13)$$

where

$$f' = f/(2\alpha+1), \quad \alpha = \tfrac{1}{6}(4^{1/3} - 2^{1/3} - 2). \quad (4.14)$$

The M of Eq. (4.13) satisfies Eq. (2.12).

In our model we have π-meson PCAC which is accurate to roughly 10%; K-meson PCAC which appears uncertain, and it is not clear what sort of accuracy can be expected from it even though there is no definite evidence against it; but not η-meson PCAC. It is argued[25] that, as in the π- and K-meson cases, η-meson PCAC is to be expected at least as long as η-η' mixing is neglected. In our approach we shall not neglect η-η' mixing.

V. MESON-BARYON LAGRANGIAN

We want to introduce the baryons into the model, i.e., to construct a Lagrangian which will contain the baryons and which will be to zero order in f the free

[24] Their next-order contribution is fifth-order in the meson fields.

[25] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

Lagrangian of the baryons. We assign the baryons to the representation (3*,3) and (3,3*) of $U(3)_L\otimes U(3)_R$ which was originally suggested by Gell-Mann[1(b)] because it gives a D-type axial-vector current, while the representation (8,1) and (1,8) gives pure F-type vector and axial-vector currents. The pure F-type axial-vector current seem to be ruled out experimentally.[26] The second important question is whether we shall use 2-component Weyl fields or 4-component Dirac fields. We choose the first possibility.[27,6] So if $B_\alpha{}^\beta$ is the baryon field, we write

$$B_\alpha{}^\beta=\begin{pmatrix}(B_R)_\alpha{}^{\beta'}\\ (B_L)_{\alpha'}{}^\beta\end{pmatrix}, \tag{5.1}$$

and we take

$$B_L=\tfrac{1}{2}(1+\gamma_5)B\sim(3_L{}^*,3_R), \qquad B_R=\tfrac{1}{2}(1-\gamma_5)B\sim(3_L,3_R{}^*). \tag{5.2}$$

From Eqs. (2.9) we find that the matrix M transforms as follows under the group $U(3)_L\otimes U(3)_R$:

$$M\sim(3_L,3_R{}^*). \tag{5.3}$$

We shall consider several meson-baryon Lagrangians. All of them will be invariant under parity, charge conjugation, time reversal, and Lorentz transformations. The following Lagrangians are in addition invariant under the group $U(3)_L\otimes U(3)_R$:

$$\begin{aligned}\mathfrak{L}_{MB}{}^1&=-\tfrac{1}{2}(B_L{}^\dagger)_\alpha{}^\beta\gamma_4\gamma_\mu\overleftrightarrow{\partial}_\mu(B_L)_\beta{}^\alpha-\tfrac{1}{2}(B_R{}^\dagger)_\alpha{}^\beta\gamma_4\gamma_\mu\overleftrightarrow{\partial}_\mu(B_R)_\beta{}^\alpha\\&=-\tfrac{1}{2}\mathrm{Tr}(\bar{B}\gamma_\mu\overleftrightarrow{\partial}_\mu B),\end{aligned} \tag{5.4}$$

$$\begin{aligned}\mathfrak{L}_{MB}{}^2&=\epsilon_{\alpha\gamma\delta}\epsilon^{\beta\zeta\eta}(B_L{}^\dagger)_\zeta{}^\gamma\gamma_4(M)_\beta{}^\alpha(B_R)_\eta{}^\delta+\epsilon_{\alpha\gamma\delta}\epsilon^{\beta\zeta\eta}(B_R{}^\dagger)_\zeta{}^\gamma(M^\dagger)_\beta{}^\alpha\gamma_4(B_L)_\eta{}^\delta\\&=-\Big(\sum_{i=1}^{8}\bar{B}_iB_i-2\bar{B}_0B_0\Big)-f\,\mathrm{Tr}[(\bar{B}\{B,S\}_+)+\tfrac{1}{2}(\bar{B}\sigma_{\mu\nu}\{B,T_{\mu\nu}\}_+)\\&\quad+i(\bar{B}\gamma_5\{B,P\}_+)]-\sqrt{3}if[3\bar{B}_0\gamma_5B_0P_0-\mathrm{Tr}(\bar{B}\gamma_5B)P_0-\mathrm{Tr}(\bar{B}\gamma_5P)B_0-\bar{B}_0\gamma_5\,\mathrm{Tr}(BP)]+\cdots\\&\qquad-\tfrac{1}{2}f^2[\mathrm{Tr}(\bar{B}\{B,P^2\}_+)+3\bar{B}_0B_0\,\mathrm{Tr}(P^2)-\mathrm{Tr}(\bar{B}B)\,\mathrm{Tr}(PP)-\sqrt{3}\,\mathrm{Tr}(\bar{B}P^2)B_0-\sqrt{3}\bar{B}_0\,\mathrm{Tr}(BP^2)]+\cdots,\end{aligned} \tag{5.5}$$

$$\begin{aligned}\mathfrak{L}_{MB}{}^3&=-\tfrac{1}{2}[(B_R{}^\dagger)_\alpha{}^\beta\gamma_4M_\beta{}^\zeta M_\eta{}^\alpha(B_L)_\zeta{}^\eta+(B_L{}^\dagger)_\alpha{}^\beta(M^\dagger)_\beta{}^\zeta(M^\dagger)_\eta{}^\alpha\gamma_4(B_R)_\zeta{}^\eta\\&\quad+(B_R{}^\dagger)_\beta{}^\alpha\gamma_4M_\zeta{}^\beta M_\alpha{}^\eta(B_L)_\eta{}^\zeta+(B_L{}^\dagger)_\beta{}^\alpha(M^\dagger)_\zeta{}^\beta(M^\dagger)_\alpha{}^\eta\gamma_4(B_R)_\eta{}^\zeta]\\&=-\sum_{i=0}^{8}\bar{B}_iB_i-f\,\mathrm{Tr}[(\bar{B}\{S,B\}_+)+\tfrac{1}{2}(\bar{B}\sigma_{\mu\nu}\{T_{\mu\nu},B\}_+)\\&\qquad+i(\bar{B}\gamma_5\{P,B\}_+)]+f^2\,\mathrm{Tr}(\bar{B}PBP)+\tfrac{1}{2}f^2\,\mathrm{Tr}(\bar{B}\{P^2,B\}_+)+\cdots,\end{aligned} \tag{5.6}$$

$$\begin{aligned}\mathfrak{L}_{MB}{}^4&=\tfrac{1}{2}[(B_R{}^\dagger)_\alpha{}^\beta\gamma_4(\partial_\mu M)_\beta{}^\zeta\gamma_\mu\gamma_4(M^\dagger)_\zeta{}^\eta\gamma_4(B_R)_\eta{}^\alpha-(B_R{}^\dagger)_\alpha{}^\beta\gamma_4M_\beta{}^\zeta\gamma_\mu\gamma_4(\partial_\mu M^\dagger)_\zeta{}^\eta\gamma_4(B_R)_\eta{}^\alpha\\&\quad+(B_L{}^\dagger)_\alpha{}^\beta(\partial_\mu M^\dagger)_\beta{}^\zeta\gamma_4\gamma_\mu M_\zeta{}^\eta(B_L)_\eta{}^\alpha-(B_L{}^\dagger)_\alpha{}^\beta(M^\dagger)_\beta{}^\zeta\gamma_4\gamma_\mu(\partial_\mu M)_\zeta{}^\eta(B_L)_\eta{}^\alpha]\\&=if\,\mathrm{Tr}[\bar{B}\gamma_\mu(mV_\mu)B+\bar{B}\gamma_5\gamma_\mu(\partial_\mu P)B]-\tfrac{1}{2}f^2\,\mathrm{Tr}[\bar{B}\gamma_\mu(P\overleftrightarrow{\partial}_\mu P)B]+\cdots,\end{aligned} \tag{5.7}$$

$$\begin{aligned}\mathfrak{L}_{MB}{}^5&=\tfrac{1}{2}[(B_R{}^\dagger)_\beta{}^\alpha\gamma_4(\partial_\mu M)_\zeta{}^\beta\gamma_\mu\gamma_4(M^\dagger)_\eta{}^\zeta\gamma_4(B_R)_\alpha{}^\eta-(B_R{}^\dagger)_\beta{}^\alpha\gamma_4M_\zeta{}^\beta\gamma_\mu\gamma_4(\partial_\mu M)_\eta{}^\zeta\gamma_4(B_R)_\alpha{}^\eta\\&\quad+(B_L{}^\dagger)_\beta{}^\alpha(\partial_\mu M^\dagger)_\zeta{}^\beta\gamma_4\gamma_\mu M_\eta{}^\zeta(B_L)_\alpha{}^\eta-(B_L{}^\dagger)_\beta{}^\alpha(M^\dagger)_\zeta{}^\beta\gamma_4\gamma_\mu(\partial_\mu M)_\eta{}^\zeta(B_L)_\alpha{}^\eta]\\&=if\,\mathrm{Tr}[\bar{B}\gamma_\mu B(mV_\mu)+\bar{B}\gamma_5\gamma_\mu B\partial_\mu P]+\tfrac{1}{2}f^2\,\mathrm{Tr}[\bar{B}\gamma_\mu B(P\overleftrightarrow{\partial}_\mu P)]+\cdots.\end{aligned} \tag{5.8}$$

The Lagrangian $\mathfrak{L}_{MB}{}^1$ is the kinetic-energy baryon Lagrangian. The $\mathfrak{L}_{MB}{}^2$ is the only meson-baryon Lagrangian of first order in M. It implies that the mass of the B_0 is negative and its absolute value is twice as large as the masses of the other baryons. A negative-mass baryon can be interpreted as a baryon with positive mass and opposite parity.[28] The ninth baryon will be identified with the $Y_0{}^*(1405)$. Since its mass is far from twice the average baryon mass, the Lagrangian $\mathfrak{L}_{MB}{}^3$ is considered. However, the Lagrangians $\mathfrak{L}_{MB}{}^2$ and $\mathfrak{L}_{MB}{}^3$ give, when they are expanded, only the tensor coupling of the vector mesons to baryons and not the vector coupling. We get this coupling from the Lagrangians $\mathfrak{L}_{MB}{}^4$ and $\mathfrak{L}_{MB}{}^5$.

The above Lagrangians give a degenerate baryon octet. To introduce mass splitting within this octet and at the same time allow a mixing of the $SU(3)$ singlet with the $I=0$ member of the octet, we consider the following

[26] N. Cabibbo [Phys. Rev. Letters 10, 531 (1963)] found an admixture of the f type in the axial-vector pattern of the order of 30%.

[27] R. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965).

[28] P. G. O. Freund and Y. Nambu, Phys. Rev. Letters 12, 714 (1964).

Lagrangians:

$$\begin{aligned}\mathcal{L}_{MB}{}^{6}=&\tfrac{1}{2}[(B_R{}^{\dagger})_{\alpha}{}^{\beta}\gamma_4 M_{\beta}{}^{\zeta}M_{\eta}{}^{\alpha}(B_L\lambda_8)_{\zeta}{}^{\eta}+(B_L{}^{\dagger})_{\alpha}{}^{\beta}(M^{\dagger})_{\beta}{}^{\zeta}(M^{\dagger})_{\eta}{}^{\alpha}\gamma_4(B_R\lambda_8)_{\zeta}{}^{\eta}\\&+(\lambda_8 B_R{}^{\dagger})_{\beta}{}^{\alpha}\gamma_4 M_{\zeta}{}^{\beta}M_{\alpha}{}^{\eta}(B_L)_{\eta}{}^{\zeta}+(\lambda_8 B_L{}^{\dagger})_{\beta}{}^{\alpha}(M^{\dagger})_{\zeta}{}^{\beta}(M^{\dagger})_{\alpha}{}^{\eta}\gamma_4(B_R)_{\eta}{}^{\zeta}]\\=&\mathrm{Tr}(\bar{B}B\lambda_8)+\tfrac{1}{2}f\,\mathrm{Tr}[(\bar{B}\{S,B\lambda_8\}_+)+(\lambda_8\bar{B}\{S,B\}_+)+\tfrac{1}{2}(\bar{B}\sigma_{\mu\nu}\{T_{\mu\nu},B\lambda_8\}_+)\\&+\tfrac{1}{2}(\lambda_8\bar{B}\sigma_{\mu\nu}\{T_{\mu\nu},B\}_+)+i(\bar{B}\gamma_5\{P,B\lambda_8\}_+)+i(\lambda_8\bar{B}\gamma_5\{P,B\}_+)]\\&-\tfrac{1}{2}f^2\,\mathrm{Tr}[(\bar{B}PB\{\lambda_8,P\}_+)+(\bar{B}P^2B\lambda_8)+\tfrac{1}{2}(\bar{B}B\{\lambda_8,P^2\}_+)]+\cdots,\end{aligned}\tag{5.9}$$

$$\begin{aligned}\mathcal{L}_{MB}{}^{7}=&\tfrac{1}{2}[(B_R{}^{\dagger})_{\alpha}{}^{\beta}\gamma_4 M_{\beta}{}^{\zeta}M_{\eta}{}^{\alpha}(\lambda_8 B_L)_{\zeta}{}^{\eta}+(B_L{}^{\dagger})_{\alpha}{}^{\beta}(M^{\dagger})_{\beta}{}^{\zeta}(M^{\dagger})_{\eta}{}^{\alpha}\gamma_4(\lambda_8 B_R)_{\zeta}{}^{\eta}\\&+(B_R{}^{\dagger}\lambda_8)_{\beta}{}^{\alpha}\gamma_4 M_{\zeta}{}^{\beta}M_{\alpha}{}^{\eta}(B_L)_{\eta}{}^{\zeta}+(B_L{}^{\dagger}\lambda_8)_{\beta}{}^{\alpha}(M^{\dagger})_{\zeta}{}^{\beta}(M^{\dagger})_{\alpha}{}^{\eta}\gamma_4(B_R)_{\eta}{}^{\zeta}]\\=&\mathrm{Tr}(\bar{B}\lambda_8 B)+\tfrac{1}{2}f\,\mathrm{Tr}[(\bar{B}\{S,\lambda_8 B\}_+)+(\bar{B}\lambda_8\{S,B\}_+)+\tfrac{1}{2}(\bar{B}\sigma_{\mu\nu}\{T_{\mu\nu},\lambda_8 B\}_+)\\&+\tfrac{1}{2}(\bar{B}\lambda_8\sigma_{\mu\nu}\{T_{\mu\nu},B\}_+)+i(\bar{B}\gamma_5\{P,\lambda_8 B\}_+)+i(\bar{B}\lambda_8\gamma_5\{P,B\}_+)]\\&+\tfrac{1}{2}f^2\,\mathrm{Tr}[(\bar{B}\{P,\lambda_8\}_+BP)+\tfrac{1}{2}(\bar{B}\{\lambda_8,P^2\}_+B)+\bar{B}\lambda_8 BP^2]+\cdots.\end{aligned}\tag{5.10}$$

The Lagrangians $\mathcal{L}_{MB}{}^6$ and $\mathcal{L}_{MB}{}^7$ are invariant only under the group $U(2)_L\otimes U(2)_R$. So the meson-baryon Lagrangian destroys kaon PCAC, while it does not affect pion PCAC.

The complete meson-baryon Lagrangian $\mathcal{L}_{MB}$ will be formed as a linear combination of the above Lagrangians:

$$\mathcal{L}=\sum_{i=1}^{7}\alpha_i\mathcal{L}_{MB}{}^i.\tag{5.11}$$

We determine the coefficients α_i from the following assumptions:

(α) To zero order in f, the Lagrangian $\mathcal{L}_{MB}$ represents the free Lagrangian of a nonet of baryons, all of which have their physical masses, apart from electromagnetic mass splitting (which we ignore). An obvious consequence of this assumption is that

$$\alpha_1=1.\tag{5.12}$$

(β) The coupling of the vector mesons to baryons is pure F-type and the coupling of the ρ mesons to nucleons is $f_{\rho NN}$. This is a consequence of the assumption that the ρ mesons and the K^* mesons are coupled to the baryonic part of the isospin current and the strangeness-changing current, respectively. In the spirit of our phenomenological approach, we shall not assume that $f_{\rho\pi\pi}=-f_{\rho NN}$ but we shall consider the experimental value for $f_{\rho NN}$.

(γ) The coupling constant $G_{\pi NN}$ of the π mesons to baryons has the well-known value

$$G_{\pi NN}{}^2/4\pi=15,\tag{5.13}$$

which is given by the dispersion-relation approach to pion-nucleon scattering.[29] In our $\mathcal{L}_{MB}$ we have both the nonderivative and the derivative coupling of the pions to nucleons. The $G_{\pi NN}$ of Eq. (5.13) will be the total pion-nucleon coupling constant obtained after transforming the derivative coupling to the usual pseudoscalar coupling using the Dirac equation.[30]

[29] J. Hamilton and W. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[30] This transformation will be done simply to fix the $G_{\pi NN}$ and not to eliminate the derivative coupling in our interaction Lagrangian.

From Eqs. (5.5), (5.6), and (5.9)–(5.11), we find the following baryon mass Lagrangian:

$$-\alpha_2\Big(\sum_{i=1}^{8}\bar{B}_iB_i-2\bar{B}_0B_0\Big)-\alpha_3\Big(\sum_{i=1}^{8}\bar{B}_iB_i+\bar{B}_0B_0\Big)+\alpha_6\,\mathrm{Tr}(\bar{B}B\lambda_8)+\alpha_7\,\mathrm{Tr}(\bar{B}\lambda_8 B).\tag{5.14}$$

The above expression has off-diagonal terms of the type $\bar{B}_0B_8+\bar{B}_8B_0$. To diagonalize it, we shall assume B_0-B_8 mixing with a mixing angle λ_B according to

$$\begin{aligned}\Lambda&=\cos\lambda_B\,B_8+\sin\lambda_B\,B_0,\\Y'&=-\sin\lambda_B\,B_8+\cos\lambda_B\,B_0.\end{aligned}\tag{5.15}$$

As we argued before, the physical $\frac{1}{2}^-$ isosinglet $Y_0{}^*(1405)$ will be represented by the field γ_5Y'. Then from Eqs. (5.14), (5.15), and assumption (α), we find the following expressions for the parameters α_2, α_3, α_6, and α_7:

$$\begin{aligned}\alpha_2&=(m_N+m_\Xi+m_Y-m_\Lambda)/3,\\\alpha_3&=(m_\Sigma-m_Y+m_\Lambda)/3,\\\alpha_6&=(m_N-m_\Sigma)/\sqrt{3},\\\alpha_7&=(m_\Xi-m_\Sigma)/\sqrt{3}.\end{aligned}\tag{5.16}$$

As expected, the parameters α_6 and α_7 are at the order of Δm. The mixing angle λ_B is given by

$$\tan\lambda_B=-\frac{4(\alpha_6+\alpha_7)}{(\sqrt{27})\alpha_2+\alpha_6+\alpha_7}=0.067.\tag{5.17}$$

So we get $\lambda_B=1°\,55'$, a quite small mixing angle.

From Eqs. (5.7) and (5.8), we easily see that assumption (β) requires

$$\alpha_5=-\alpha_4.\tag{5.18}$$

Then from Eqs. (5.5)–(5.11) and (5.18) we find the following interaction Lagrangian if we keep only some terms which are first-order in f and which will be of interest in the present case:

$$\begin{aligned}&-(\alpha_2+\alpha_3)f\,\mathrm{Tr}[\tfrac{1}{2}(\bar{B}\sigma_{\mu\nu}\{T_{\mu\nu},B\}_+)+i(\bar{B}\gamma_5\{P,B\}_+)]\\&+i\alpha_4 f\,\mathrm{Tr}\{(\bar{B}\gamma_\mu[(mV_\mu),B])+(\bar{B}\gamma_5\gamma_\mu[\partial_\mu P,B])\}\\&+\tfrac{1}{2}i\alpha_6 f\,\mathrm{Tr}[(\bar{B}\gamma_5\{P,B\lambda_8\}_+)+(\lambda_8\bar{B}\gamma_5\{P,B\}_+)]\\&+\tfrac{1}{2}i\alpha_7 f\,\mathrm{Tr}[(\bar{B}\gamma_5\{P,\lambda_8 B\}_+)+(\bar{B}\lambda_8\gamma_5\{P,B\}_+)].\end{aligned}\tag{5.19}$$

From expression (5.19), we find that assumption (β) implies

$$\alpha_4 = f_{\rho NN}/\sqrt{2}m_\rho f. \qquad (5.20)$$

Then, using the Dirac equation, we get from expression (5.19) and Eq. (5.20)

$$-i\{[\alpha_2+\alpha_3+(2/\sqrt{3})(\alpha_6-\tfrac{1}{2}\alpha_7)]f/\sqrt{2}-f_{\rho NN}m_N/m_\rho\} \times \boldsymbol{\pi}\cdot(\bar{N}\gamma_5\boldsymbol{\tau}N), \qquad (5.21)$$

where only the πNN coupling is shown. It is interesting to observe that, using Eqs. (5.16), we get

$$\alpha_2+\alpha_3+(2/\sqrt{3})(\alpha_6-\tfrac{1}{2}\alpha_7)=m_N, \qquad (5.22)$$

so only one baryon mass enters into expression (5.21). According to assumption (γ), we get from (5.21)

$$f=\frac{\sqrt{2}(f_{\rho NN}m_N-G_{\pi NN}m_\rho)}{m_\rho m_N}. \qquad (5.23)$$

From Eqs. (5.20) and (5.23), we get

$$\alpha_4=\frac{f_{\rho NN}m_N}{2(f_{\rho NN}m_N-G_{\pi NN}m_\rho)}. \qquad (5.24)$$

Thus all parameters α_i are determined.

From the analysis of s-wave pion-nucleon scattering, the value of $f_{\rho\pi\pi}f_{\rho NN}$ can be calculated. Hamilton[31] finds

$$f_{\rho\pi\pi}f_{\rho NN}/4\pi=2.85\pm0.3. \qquad (5.25)$$

Thus we get, for

$$f_{\rho\pi\pi}^2/4\pi=2.1, \qquad (5.26)$$

if we omit the ±0.3 in Eq. (5.25),

$$f_{\rho NN}^2/4\pi=3.9. \qquad (5.27)$$

From Eqs. (5.13), (5.23), and (5.27), we find

$$|f|=1.06/m_\pi \qquad (5.28)$$

$$=4.6/m_\pi. \qquad (5.29)$$

The upper value corresponds to $f_{\rho NN}G_{\pi NN}>0$ and the lower value corresponds to $f_{\rho NN}G_{\pi NN}<0$.

The value of $|f|$ given by Eq. (5.28) is identical with the value of $|f|$ which is obtained from the leptonic decays $\pi^+\to\mu^++\nu_\mu$ and $K^+\to\mu^++\nu_\mu$ if the axial-vector current is given by Eq. (4.10). We find[6]

$$|f|=(1.03\pm0.05)m_\pi^{-1}. \qquad (5.30)$$

Another independent way to determine $|f|$ is the following. Consider the tensor coupling of the ρ mesons to nucleons and write the interaction Hamiltonian

$$\mathcal{H}_{\rho NN}{}^T=(g_{\rho NN}/4m_N)\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N\cdot(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu). \qquad (5.31)$$

Using dispersion relations, Hamilton[31] has found

$$g_{\rho NN}=-1.85f_{\rho NN}. \qquad (5.32)$$

[31] J. Hamilton, in *High Energy Physics*, edited by E. H. S. Burhop (Academic Press Inc., New York, 1967), Vol. 1.

In our case the tensor coupling of the ρ mesons to nucleons, which we obtain from the Lagrangian $\mathcal{L}_{MB}$ of Eq. (5.11) if we use Eqs. (5.5), (5.6), (5.9), (5.10), and (5.16), is given by

$$\mathcal{L}_{\rho NN}{}^T=-[m_Nf/(\sqrt{8})m_\rho](\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N) \times(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu). \qquad (5.33)$$

As we shall see in Sec. VI, the interaction Hamiltonian coming from the above Lagrangian is $-\mathcal{L}_{\rho NN}{}^T$ to lowest order. So we obtain from Eqs. (5.31) and (5.33)

$$|f|=|g_{\rho NN}|m_\rho/\sqrt{2}m_N^2.$$

From Eqs. (5.27), (5.32), and the above equation, we obtain

$$|f|=1.08/m_\pi, \qquad (5.34)$$

in remarkable agreement with the value of $|f|$ given by Eq. (5.28).

As we see from expression (5.19), a feature of the model is that the nonderivative Yukawa coupling of the pseudoscalar mesons to baryons goes together with the tensor coupling of the vector mesons to baryons. Both are pure D-type in the Lagrangians $\mathcal{L}_{MB}{}^2$ and $\mathcal{L}_{MB}{}^3$, while in the Lagrangians $\mathcal{L}_{MB}{}^6$ and $\mathcal{L}_{MB}{}^7$ the pure D-type $U(3)$ symmetry is broken by means of λ_8. (Remember that α_2, $\alpha_3\sim m_{\text{baryon}}$; α_6, $\alpha_7\sim\Delta m_{\text{baryon}}$.) Experimentally the tensor coupling of the vector mesons to baryons is predominantly D-type[32] with a D/F ratio

$$D/F\sim1.5\text{–}2.3. \qquad (5.35)$$

VI. INTERACTION HAMILTONIAN

Some of the usual applications of phenomenological chiral Lagrangians refer to processes of the type $PP\to PP$ and $PB\to PB$. The amplitudes for such processes are calculated to first and second order in the coupling constants. To proceed in such calculations by perturbation theory, we need the interaction Hamiltonian in the interaction representation. We have shown previously[16] how to obtain the interaction Hamiltonian when our interaction Lagrangian involves the fields $T_{\mu\nu}$ and $\partial_\mu T_{\mu\nu}$. This procedure was applied to obtain the interaction Hamiltonian for a specific interaction Lagrangian. It was also shown that the S matrix obtained from this $\mathcal{H}_{\text{Int}}$ is covariant to any order in perturbation theory. In the present case, we shall start from a general interaction Lagrangian and we shall calculate the $\mathcal{H}_{\text{Int}}$ to second order in the coupling constants which appear in the Lagrangian. Then we shall show that the second-order S matrix coming from this $\mathcal{H}_{\text{Int}}$ is covariant.

It is shown in Appendix A that from the interaction Lagrangian $\mathcal{L}_{\text{Int}}(P,\partial_\mu P,T_{\mu\nu},\partial_\rho T_{\rho\sigma},B)$, we obtain to second order in the coupling constants the following inter-

[32] P. Carruthers, *Introduction to Unitary Symmetry* (Wiley-Interscience, Inc., New York, 1966), p. 122.

action Hamiltonian in the interaction representation:

$$\mathcal{H}_{\rm Int}[x,n] = -\mathcal{L}_{\rm Int}(x)+\sum_i \{(1/4m_i^2)[J_{\mu\lambda}{}^{T^i}(x)J_{\mu\lambda}{}^{\bar{T}^i}(x) +2J_{\mu\lambda}{}^{T^i}(x)J_{\mu\sigma}{}^{\bar{T}^i}(x)n_\lambda n_\sigma] +\tfrac{1}{2}[J_\mu{}^{T^i}(x)J_\mu{}^{\bar{T}^i}(x)+J_\lambda{}^{T^i}(x)J_\sigma{}^{\bar{T}^i}(x)n_\lambda n_\sigma] +\tfrac{1}{2}J_\lambda{}^{p^i}(x)J_\sigma{}^{\bar{p}^i}(x)n_\lambda n_\sigma\}, \quad (6.1)$$

where the currents $J_{\mu\nu}{}^{T^i}$, $J_\mu{}^{T^i}$, and $J_\mu{}^{p^i}$ are defined by

$$J_{\mu\lambda}{}^{T^i}=\frac{\partial\mathcal{L}_{\rm Int}}{\partial\bar{T}_{\mu\lambda}{}^i}, \quad (6.2)$$

$$J_\mu{}^{T^i}=\frac{\partial\mathcal{L}_{\rm Int}}{\partial(\partial_\nu\bar{T}_{\nu\mu}{}^i)}, \quad (6.3)$$

$$J_\mu{}^{p^i}=-\frac{\partial\mathcal{L}_{\rm Int}}{\partial(\partial_\mu\bar{P}^i)}, \quad (6.4)$$

the summation refers to all meson fields, and the fields $\bar{P}^i$ and $\bar{T}_{\mu\nu}{}^i$ are the Hermitian conjugates of the fields P^i and $T_{\mu\nu}{}^i$, respectively.

Obviously the first order in the coupling-constant S matrix, coming from the term $-\mathcal{L}_{\rm Int}$, is covariant. We want to show that the S matrix is again covariant to second order in the coupling constants. Such second-order processes come from

$$\tfrac{1}{2}(-i)^2\int\int d^4x_1d^4x_2\, T[\mathcal{L}_{\rm Int}(x_1)\mathcal{L}_{\rm Int}(x_2)] \quad (6.5)$$

and from

$$(-i)\int d^4x_1\, N\Big(\sum_i \Big\{\frac{1}{4m_i^2}[J_{\mu\lambda}{}^{T^i}(x_1)J_{\mu\lambda}{}^{\bar{T}^i}(x_1)+2J_{\mu\lambda}{}^{T^i}(x_1)J_{\mu\sigma}{}^{\bar{T}^i}(x_1)n_\lambda n_\sigma] +\tfrac{1}{2}[J_\mu{}^{T^i}(x_1)J_\mu{}^{\bar{T}^i}(x_1)+J_\lambda{}^{T^i}(x_1)J_\sigma{}^{\bar{T}^i}(x_1)n_\lambda n_\sigma]+\tfrac{1}{2}J_\lambda{}^{p^i}(x_1)J_\sigma{}^{\bar{p}^i}(x_1)n_\lambda n_\sigma\Big\}\Big), \quad (6.6)$$

where T stands for Dyson time-ordered product and N for normal product. We find

$$\Big\langle T\Big[P^i(x),\frac{\partial}{\partial y_\nu}\bar{P}^i(y)\Big]\Big\rangle_0 N(J^{\bar{p}^i}(x)J_\nu{}^{p^i}(y))=\frac{1}{2}\Big[\frac{\partial}{\partial y_\nu}\Delta_F(x-y)\Big]N(J^{\bar{p}^i}(x)J_\nu{}^{p^i}(y)), \quad (6.7)$$

$$\Big\langle T\Big[\frac{\partial}{\partial x_\mu}P^i(x),\frac{\partial}{\partial y_\nu}\bar{P}^i(y)\Big]\Big\rangle_0 N(J_\mu{}^{\bar{p}^i}(x)J_\nu{}^{p^i}(y))=\Big[\frac{1}{2}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\Delta_F(x-y)-i\delta^4(x-y)n_\mu n_\nu\Big]N(J_\mu{}^{\bar{p}^i}(x)J_\nu{}^{p^i}(y)), \quad (6.8)$$

$$\tfrac{1}{4}\langle T[T_{\mu\nu}{}^i(x),\bar{T}_{\rho\sigma}{}^i(y)]\rangle_0 N(J_{\mu\nu}{}^{\bar{T}^i}(x)J_{\rho\sigma}{}^{T^i}(y))=\frac{i}{m_i^2}\Big[\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\rho}G_{F\nu\sigma}(x-y)-\delta^4(x-y)\delta_{\nu\sigma}n_\mu n_\rho\Big]N(J_{\mu\nu}{}^{\bar{T}^i}(x)J_{\rho\sigma}{}^{T^i}(y)), \quad (6.9)$$

$$\Big\langle T\Big[T_{\mu\nu}{}^i(x),\frac{\partial}{\partial y_\rho}\bar{T}_{\rho\sigma}{}^i(y)\Big]\Big\rangle_0 N(J_{\mu\nu}{}^{\bar{T}^i}(x)J_\sigma{}^{T^i}(y))=2i\Big[\frac{\partial}{\partial x_\mu}G_{F\nu\sigma}(x-y)\Big]N(J_{\mu\nu}{}^{\bar{T}^i}(x)J_\sigma{}^{T^i}(y)), \quad (6.10)$$

$$\Big\langle T\Big[\frac{\partial}{\partial x_\mu}T_{\mu\nu}{}^i(x),\frac{\partial}{\partial y_\rho}\bar{T}_{\rho\sigma}{}^i(y)\Big]\Big\rangle_0 N(J_\nu{}^{\bar{T}^i}(x)J_\sigma{}^{T^i}(y))=i[m_i^2G_{F\nu\sigma}(x-y)-n_\nu n_\sigma\delta^4(x-y)]N(J_\nu{}^{\bar{T}^i}(x)J_\sigma{}^{T^i}(y)), \quad (6.11)$$

where

$$G_{F\nu\rho}(x-y)=-\tfrac{1}{2}i\Big(\delta_{\nu\rho}-\frac{1}{m_i^2}\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial y_\rho}\Big)\Delta_F(x-y) \quad (6.12)$$

and $\Delta_F(x-y)$ is the usual Feynman propagator. The expression on the left-hand side of Eq. (6.8) appears in Eq. (6.5) since we have[33]

$$\mathcal{L}_{\rm Int}(x)=J_\mu{}^{\bar{p}^i}(x)\frac{\partial P^i(x)}{\partial x_\mu}+J_\nu{}^{p^i}(x)\frac{\partial\bar{P}^i(x)}{\partial x_\nu}+\cdots, \quad (6.13)$$

and so expression (6.5) becomes, if we use Eq. (6.8),

$$\begin{aligned}&\tfrac{1}{2}(-i)^2\int\int d^4x_1d^4x_2\, T\Big[\Big(J_\mu{}^{\bar{p}^i}(x_1)\frac{\partial P^i(x_1)}{\partial x_{1\mu}}+J_\nu{}^{p^i}(x_1)\frac{\partial\bar{P}^i(x_1)}{\partial x_{1\nu}}+\cdots\Big)\Big(J_\mu{}^{\bar{p}^i}(x_2)\frac{\partial P^i(x_2)}{\partial x_{2\mu}}+J_\nu{}^{p^i}(x_2)\frac{\partial\bar{P}^i(x_2)}{\partial x_{2\nu}}+\cdots\Big)\Big]\\ &=-\int\int d^4x_1d^4x_2\Big\langle T\Big[\frac{\partial}{\partial x_{1\mu}}P^i(x_1),\frac{\partial}{\partial x_{2\nu}}\bar{P}^i(x_2)\Big]\Big\rangle_0 N(J_\mu{}^{\bar{p}^i}(x_1)J_\nu{}^{p^i}(x_2))+\cdots\\ &=-\int\int d^4x_1d^4x_2\Big[\frac{1}{2}\frac{\partial}{\partial x_{1\mu}}\frac{\partial}{\partial x_{2\nu}}\Delta_{F\mu\nu}(x_1-x_2)\Big]N(J_\mu{}^{\bar{p}^i}(x_1)J_\nu{}^{p^i}(x_2))+i\int d^4x_1\, N(J_\mu{}^{\bar{p}^i}(x_1)J_\nu{}^{p^i}(x_1))n_\mu n_\nu+\cdots. \quad (6.14)\end{aligned}$$

[33] In Eq. (6.13) we have assumed for simplicity that there is only one $\partial_\nu P^i$ factor in the Lagrangian $\mathcal{L}_{\rm Int}$. If there are more than one such factor, say, n factors, the coefficient n should multiply both terms on the right-hand side of Eq. (6.13). But then in Eq. (6.14) we have to consider contractions of all these fields and our conclusion does not change.

The above expression holds if $P^i \neq \bar{P}^i$. If $P^i = \bar{P}^i$, there is only one term on the right-hand side of Eq. (6.13), and so both terms on the right-hand side of Eq. (6.14) are multiplied by the coefficient $\frac{1}{2}$. Remembering that in expression (6.6) we sum over all mesons and so for $P^i \neq \bar{P}^i$ we get the term $-i\int d^4x_1\, N[J_\mu^{\bar{P}^i}(x_1)J_\nu^{P^i}(x_1)]n_\mu n_\nu$, while for $P^i = \bar{P}^i$ we get the term $-\frac{1}{2}i\int d^4x_1\, N[J_\mu^{P^i}(x_1)J_\nu^{P^i}(x_1)]n_\mu n_\nu$, we find that the normal-dependent term of Eq. (6.14) exactly cancels out with a normal-dependent term of expression (6.6). In a similar fashion we find that the normal-dependent terms appearing in the S matrix according to Eqs. (6.9) and (6.11) cancel out, with corresponding terms appearing in expression (6.6). So we have found that the S matrix is covariant to second order in the coupling constants and can be calculated by using the effective interaction Hamiltonian

$$[\mathcal{H}_{\rm Int}(x)]_{\rm eff} = -\mathcal{L}_{\rm Int}(x) + \sum_i [(1/4m_i^2)J_{\mu\lambda}^{T^i}(x)J_{\mu\lambda}^{\bar{T}^i}(x) + \tfrac{1}{2}J_\mu^{T^i}(x)J_\mu^{\bar{T}^i}(x)], \tag{6.15}$$

assuming that for the propagators we shall use the expressions on the right-hand sides of Eqs. (6.8)–(6.11) but without the normal-dependent terms.

We shall apply Eq. (6.15) to calculate the interaction Hamiltonians we shall use.[34] The $\mathcal{H}_{\rm Int}$ coming from the Lagrangians of Eq. (3.19) is given by the Lagrangians themselves with an opposite sign. Their explicit expressions are given in Appendix B. From Eqs. (3.22) and (3.26) we get, if we drop the 4-divergence,

$$\mathcal{L}_M{}^3 = \frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\,{\rm Tr}[\partial_\mu T_{\mu\nu}(P\overleftrightarrow{\partial}_\nu P)] - \frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\epsilon_{\kappa\varphi\nu\sigma}\,{\rm Tr}[T_{\kappa\varphi}\{\partial_\nu P, \partial_\mu T_{\mu\sigma}\}_+] + \cdots. \tag{6.16}$$

The above Lagrangian gives the following interaction Hamiltonian:

$$-\frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\,{\rm Tr}[\partial_\mu T_{\mu\nu}(P\overleftrightarrow{\partial}_\nu P)] + \frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\epsilon_{\kappa\varphi\nu\sigma}\,{\rm Tr}(T_{\kappa\varphi}\{\partial_\nu P, \partial_\mu T_{\mu\sigma}\}_+) + \sum_i (1/4m_i^2)J_{\mu\nu}^{T^i}J_{\mu\nu}^{\bar{T}^i} + \tfrac{1}{2}\sum_i J_\mu^{T^i}J_\mu^{\bar{T}^i}. \tag{6.17}$$

Using Eq. (3.23), we obtain from the first term of the above expression

$$\mathcal{H}_{\rm Int}(VPP) = -\frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\,{\rm Tr}[(mV_\mu)(P\overleftrightarrow{\partial}_\mu P)] = f_{\rho\pi\pi}\boldsymbol{\rho}_\nu\cdot(\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi}) + \cdots. \tag{6.18}$$

So, apart from mass factors, we obtain the usual coupling of the vector mesons to pseudoscalar mesons. The coefficient which multiplies the Tr[$\cdots$] in Eq. (3.21) is taken in such a way that the $\rho\pi\pi$ coupling constant is $f_{\rho\pi\pi}$. The explicit expressions of $\mathcal{H}_{\rm Int}(VPP)$ and $\frac{1}{2}\sum_i J_\mu^{T^i}J_\mu^{\bar{T}^i}$ are given in Appendix B. The second term appearing in expression (6.17) becomes, if we use Eqs. (3.23), (3.25), and the antisymmetry of $\epsilon_{\kappa\varphi\nu\sigma}$,

$$\mathcal{H}_{\rm Int}(VVP) = \frac{\sqrt{2}if_{\rho\pi\pi}}{m_\rho}\epsilon_{\kappa\varphi\nu\sigma}\,{\rm Tr}[\partial_\kappa(V_\varphi/m)\{\partial_\nu P, mV_\sigma\}_+]. \tag{6.19}$$

The physical pseudoscalar mesons $\eta(549)$ and $\eta'(958)$ are mixtures of the fields η_8 and η_0. In Refs. 6 and 35 the mixing angle λ_η is estimated to be

$$\lambda_\eta = \pm 10.8°. \tag{6.20}$$

The quark model predicts[35] $\lambda_\eta = 10.8°$.

Since the Lagrangian of Eq. (3.21) is invariant under the group $U(3)_L \otimes U(3)_R$, we have obtained a VVP interaction in a chiral-invariant way. The VVP interaction, and particularly the $\pi\rho\omega$ vertex, has been discussed recently in connection with current algebra and chiral symmetries. It is argued[36–39] that if one insists upon the validity of the algebra of fields[5] and strict PCAC, no VVP vertex is possible. By relaxing these requirements, a VVP vertex can be found. In our case, strict PCAC does not hold.

According to Eq. (6.15), the $\bar{B}BP$ interaction Hamiltonian is simply given by $-\mathcal{L}_{\rm Int}(\bar{B}BP)$. Using Eqs. (5.5)–(5.11), (5.16), (5.18), the Dirac equation, and assuming $\eta_0-\eta_8$ and B_0-B_8 mixing according to Eqs. (3.18) and

[34] From now on, when we say interaction Hamiltonian we mean effective interaction Hamiltonian.
[35] R. H. Dalitz, in *High Energy Physics, Les Houches, 1965*, edited by C. DeWitt and M. Jacob (Gordon and Breach, Science Publishers, Inc., New York, 1965).
[36] R. Perrin, Phys. Rev. **170**, 1367 (1968).
[37] S. G. Brown and G. B. West, Phys. Rev. **174**, 1777 (1968).
[38] R. Arnowitt, M. H. Friedman, and R. Nath, Phys. Letters **27B**, 657 (1968).
[39] L. M. Brown and H. Munczek, in Proceedings of the Coral Gables Conference on Fundamental Interactions, 1969 (unpublished).

(5.15), respectively, we find the following expression for $\mathcal{H}_{\rm Int}(\bar{B}BP)$:

$$
\begin{aligned}
\mathcal{H}_{\rm Int}(\bar{B}BP)
&= -iG_{\pi NN}(\bar{N}\boldsymbol{\tau}N\cdot\boldsymbol{\pi}) - if\frac{m_\Xi(1+2\alpha_4)}{\sqrt{2}}(\bar{\Xi}\boldsymbol{\tau}\Xi\cdot\boldsymbol{\pi}) + f\frac{(m_N+m_\Sigma)(1+2\alpha_4)}{\sqrt{8}}(i\bar{N}\boldsymbol{\tau}K\cdot\boldsymbol{\Sigma}+\text{H.c.})\\
&\quad - f\frac{(m_\Sigma+m_\Xi)(1-2\alpha_4)}{\sqrt{8}}(i\bar{\Xi}\boldsymbol{\tau}K^c\boldsymbol{\Sigma}+\text{H.c.}) - if(\sqrt{8})\alpha_4 m_\Sigma(\bar{\boldsymbol{\Sigma}}\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}) - iG_{\Sigma\Sigma\eta}(\bar{\Sigma}\Sigma\eta) - iG_{\Sigma\Sigma\eta'}(\bar{\Sigma}\Sigma\eta')\\
&\quad - iG_{NN\eta}(\bar{N}N\eta) - iG_{NN\eta'}(\bar{N}N\eta') - iG_{\Xi\Xi\eta}(\bar{\Xi}\Xi\eta) - iG_{\Xi\Xi\eta'}(\bar{\Xi}\Xi\eta') - G_{\Sigma\pi\Lambda}(i\bar{\Sigma}\pi\Lambda+\text{H.c.}) - G_{\Sigma\pi Y}(i\bar{\Sigma}\pi Y+\text{H.c.})\\
&\quad - G_{NK\Lambda}(i\bar{N}K\Lambda+\text{H.c.}) - G_{NKY}(i\bar{N}KY+\text{H.c.}) - G_{\Xi K\Lambda}(i\bar{\Xi}K^c\Lambda+\text{H.c.}) - G_{\Xi KY}(i\bar{\Xi}K^cY+\text{H.c.})\\
&\quad - iG_{YY\eta}(\bar{Y}Y\eta) - iG_{YY\eta'}(\bar{Y}Y\eta') - iG_{\Lambda\Lambda\eta}(\bar{\Lambda}\Lambda\eta) - iG_{\Lambda\Lambda\eta'}(\bar{\Lambda}\Lambda\eta') - G_{\Lambda Y\eta}(i\bar{\Lambda}Y\eta+\text{H.c.}) - G_{\Lambda Y\eta'}(i\bar{\Lambda}Y\eta'+\text{H.c.}),
\end{aligned}
\tag{6.21}
$$

where

$$
Y=\gamma_5 Y',\quad K^c=\begin{pmatrix}\bar{K}^0\\ -K^-\end{pmatrix},
$$

the pion-nucleon coupling constant is given by Eq. (5.13), and the rest of the coupling constants are given by the following expressions:

$$G_{\Sigma\Sigma\eta}=f\left[\frac{m_N+m_\Xi+m_Y-m_\Lambda-2m_\Sigma}{\sqrt{3}}\sin\lambda_\eta-\frac{2m_\Sigma}{\sqrt{6}}\cos\lambda_\eta\right], \tag{6.22}$$

$$G_{\Sigma\Sigma\eta'}=f\left[\frac{m_N+m_\Xi+m_Y-m_\Lambda-2m_\Sigma}{\sqrt{3}}\cos\lambda_\eta+\frac{2m_\Sigma}{\sqrt{6}}\sin\lambda_\eta\right], \tag{6.23}$$

$$G_{NN\eta}=f\left[\frac{m_\Xi-m_N+m_Y-m_\Lambda}{\sqrt{3}}\sin\lambda_\eta+\frac{m_N(1+6\alpha_4)}{\sqrt{6}}\cos\lambda_\eta\right], \tag{6.24}$$

$$G_{NN\eta'}=f\left[\frac{m_\Xi-m_N+m_Y-m_\Lambda}{\sqrt{3}}\cos\lambda_\eta-\frac{m_N(1+6\alpha_4)}{\sqrt{6}}\sin\lambda_\eta\right], \tag{6.25}$$

$$G_{\Xi\Xi\eta}=f\left[\frac{m_N-m_\Xi+m_Y-m_\Lambda}{\sqrt{3}}\sin\lambda_\eta+\frac{m_\Xi(1-6\alpha_4)}{\sqrt{6}}\cos\lambda_\eta\right], \tag{6.26}$$

$$G_{\Xi\Xi\eta'}=f\left[\frac{m_N-m_\Xi+m_Y-m_\Lambda}{\sqrt{3}}\cos\lambda_\eta-\frac{m_\Xi(1-6\alpha_4)}{\sqrt{6}}\sin\lambda_\eta\right], \tag{6.27}$$

$$G_{\Sigma\pi\Lambda}=f\left[\frac{m_N+m_\Xi+m_Y-m_\Lambda-2m_\Sigma}{\sqrt{3}}\sin\lambda_B-\frac{2m_\Sigma}{\sqrt{6}}\cos\lambda_B\right], \tag{6.28}$$

$$G_{\Sigma\pi Y}=f\left[\frac{m_N+m_\Xi+m_Y-m_\Lambda-2m_\Sigma}{\sqrt{3}}\cos\lambda_B+\frac{2m_\Sigma}{\sqrt{6}}\sin\lambda_B\right], \tag{6.29}$$

$$G_{NK\Lambda}=f\left\{\frac{m_\Xi-m_N+2m_Y-2m_\Lambda}{\sqrt{12}}\sin\lambda_B+\left[\frac{3m_N+2m_\Xi-3m_\Sigma}{\sqrt{24}}-\frac{3\alpha_4(m_N+m_\Lambda)}{\sqrt{6}}\right]\cos\lambda_B\right\}, \tag{6.30}$$

$$G_{NKY}=f\left\{\frac{m_\Xi-m_N+2m_Y-2m_\Lambda}{\sqrt{12}}\cos\lambda_B-\left[\frac{3m_N+2m_\Xi-3m_\Sigma}{\sqrt{24}}-\frac{3\alpha_4(m_N-m_Y)}{\sqrt{6}}\right]\sin\lambda_B\right\}, \tag{6.31}$$

$$G_{\Xi K\Lambda}=f\left\{\frac{m_N-m_\Xi+2m_Y-2m_\Lambda}{\sqrt{12}}\sin\lambda_B+\left[\frac{2m_N+3m_\Xi-3m_\Sigma}{\sqrt{24}}+\frac{3\alpha_4(m_\Xi+m_\Lambda)}{\sqrt{6}}\right]\cos\lambda_B\right\}, \tag{6.32}$$

$$G_{\Xi KY}=f\left\{\frac{m_N-m_\Xi+2m_Y-2m_\Lambda}{\sqrt{12}}\cos\lambda_B-\left[\frac{2m_N+3m_\Xi-3m_\Sigma}{\sqrt{24}}+\frac{3\alpha_4(m_\Xi-m_Y)}{\sqrt{6}}\right]\sin\lambda_B\right\}, \tag{6.33}$$

$$G_{YY\eta}=f\left\{\frac{2(m_N+m_\Xi+m_\Sigma)}{\surd 27}\sin\lambda_\eta-(\surd 8)\frac{(m_N+m_\Xi-2m_\Sigma)}{\surd 27}\cos\lambda_\eta\right\}, \tag{6.34}$$

$$G_{YY\eta'}=f\left\{\frac{2(m_N+m_\Xi+m_\Sigma)}{\surd 27}\cos\lambda_\eta+(\surd 8)\frac{(m_N+m_\Xi-2m_\Sigma)}{\surd 27}\sin\lambda_\eta\right\}, \tag{6.35}$$

$$G_{\Lambda\Lambda\eta}=f\left\{\frac{-m_N-m_\Xi+3m_Y-3m_\Lambda+2m_\Sigma}{\surd 27}\sin\lambda_\eta+\frac{\sqrt{2}(4m_N+4m_\Xi-5m_\Sigma)}{\surd 27}\cos\lambda_\eta\right\}, \tag{6.36}$$

$$G_{\Lambda\Lambda\eta'}=f\left\{\frac{-m_N-m_\Xi+3m_Y-3m_\Lambda+2m_\Sigma}{\surd 27}\cos\lambda_\eta-\frac{\sqrt{2}(4m_N+4m_\Xi-5m_\Sigma)}{\surd 27}\sin\lambda_\eta\right\}, \tag{6.37}$$

$$G_{\Lambda Y\eta}=f\left\{\frac{-m_N-m_\Xi+3m_Y-3m_\Lambda+2m_\Sigma}{\surd 27}\cos\lambda_\eta+\frac{(\surd 8)(m_N+m_\Xi-2m_\Sigma)}{\surd 27}\sin\lambda_\eta\right\}, \tag{6.38}$$

$$G_{\Lambda Y\eta'}=f\left\{\frac{m_N+m_\Xi-3m_Y+3m_\Lambda-2m_\Sigma}{\surd 27}\sin\lambda_\eta+\frac{(\surd 8)(m_N+m_\Xi-2m_\Sigma)}{\surd 27}\cos\lambda_\eta\right\}. \tag{6.39}$$

For simplicity, Eqs. (6.34)–(6.39) give the coupling constants for $\lambda_B=0$.[40] The exact value of λ_B is 1°55′.

Several of the above coupling constants strongly depend on the sign of α_4. The choice of $|f|$ of Eq. (5.28) implies that $fG_{\pi NN}<0$, as we see from Eq. (5.23) and the experimental values of $|G_{\pi NN}|$ and $|f_{\rho NN}|$. So we have $ff_{\rho NN}<0$, which implies, according to Eq. (5.20), that

$$\alpha_4<0. \tag{6.40}$$

The experimental values of $G_{\Lambda NK}$ and $G_{\Sigma NK}$ have been estimated recently.[41–44] In Ref. 41, K^+p and K^-p forward dispersion relations were used and the following values were obtained[45]:

$$G_{pK\Sigma^0}{}^2/4\pi=0.3\pm0.5, \tag{6.41}$$

$$G_{pK\Lambda}{}^2/4\pi=16.0\pm2.5. \tag{6.42}$$

In our case, an estimate of the above coupling constants can be made by using Eqs. (5.20), (5.27), (5.28), (6.21), (6.30), and (6.40). We find

$$G_{NK\Sigma}{}^2/4\pi=1.2, \tag{6.43}$$

$$G_{NK\Lambda}{}^2/4\pi=29.2. \tag{6.44}$$

The value of the $NK\Sigma$ coupling constant that we obtained is reasonable, while the value of the $NK\Lambda$ coupling constant is larger than the experimental value. However, we should keep in mind that determination of the coupling constants requires an accurate value of $f_{\rho NN}$.[46]

Using the coupling constant $G_{\Sigma\pi Y}$ we have found, we calculate the decay width of the $Y_0^*(1405)$ resonance. We find

$$\Gamma(Y_0^*\to\Sigma\pi)=\frac{G^2{}_{Y\Sigma\pi}}{4\pi}\frac{3[(m_Y+m_\Sigma)^2-m_\pi{}^2]}{4m_Y{}^3}\times\{[m_Y{}^2-(m_\Sigma+m_\pi)^2][m_Y{}^2-(m_\Sigma-m_\pi)^2]\}^{1/2}=62\text{ MeV}, \tag{6.45}$$

where Eqs. (5.28) and (6.29) were used. The experimental value is[47]

$$\Gamma(Y_0^*\to\Sigma\pi)=40\pm10\text{ MeV}, \tag{6.46}$$

which is close to the calculated value.

Also, using Eqs. (5.28) and (6.31) we can calculate the coupling constant $|G_{NKY}|$ and compare with $|G_{\Sigma\pi Y}|$. We find

$$|G_{NKY}|/|G_{\Sigma\pi Y}|=2.0. \tag{6.47}$$

Since $SU(3)$ symmetry implies that the above ratio is equal to 1, this ratio is a measure of symmetry breaking. The value of $|G_{NKY}|/|G_{\Sigma\pi Y}|$ given by Gell-Mann, Oakes, and Renner[25] is also 2, while experimentally this ratio is estimated[48] to be about 3.

[40] The exact expressions for the coupling constants of Eqs. (6.34)–(6.39) can be easily found from the Hamiltonian

$$if[2(m_N+m_\Xi+m_\Sigma)/\surd 27]\bar{B}_0B_0\eta_0+f[(m_N+m_\Xi-3m_Y+3m_\Lambda-2m_\Sigma)/\surd 27][i\bar{B}_8B_8\eta_0+(i\bar{B}_8B_0\eta_8+\text{H.c.})]-f[(\surd 8)(m_N+m_\Xi-2m_\Sigma)/\surd 27]\times[i\bar{B}_0B_0\eta_8+i(\bar{B}_8B_0\eta_8+\text{H.c.})]-if[\sqrt{2}(4m_N+4m_\Xi-5m_\Sigma)/\surd 27]\bar{B}_8B_8\eta_8.$$

[41] J. K. Kim, Phys. Rev. Letters 19, 1079 (1967).

[42] M. Lusignoli, M. Restigonli, G. A. Snow, and G. Violini, Phys. Letters 21, 229 (1966); Nuovo Cimento 45A, 792 (1966). The values $G_{pK\Lambda}{}^2/4\pi=4.8\pm1.0$ and $G^2{}_{pK\Sigma^0}/4\pi\lesssim3.2$ were found.

[43] N. Zovko, Phys. Letters 23, 143 (1966). He found $G_{pK\Lambda}{}^2/4\pi=6.8\pm2.9$ and $G_{pK\Sigma^0}{}^2/4\pi=2.1\pm0.9$.

[44] A. D. Martin and F. Poole, Nucl. Phys. B4, 467 (1968). They obtained $G_{NK\Lambda}{}^2/4\pi=6.0\pm2.5$ and $G_{NK\Sigma^0}{}^2/4\pi\lesssim3.0$.

[45] The values for the coupling constants given by Eqs. (6.41) and (6.42) are also obtained from $SU(3)$ symmetry with $D/F\sim\frac{3}{2}$ and $G_{\pi NN}{}^2/4\pi\sim15$.

[46] The values of $f_{\rho\pi\pi}{}^2$ and $f_{\rho NN}{}^2$ of Eqs. (5.26) and (5.27) are inconsistent with the universality principle, which requires $f_{\rho\pi\pi}=-f_{\rho NN}$. So the real value of $f_{\rho NN}{}^2$ may be smaller than the one given by Eq. (5.27).

[47] Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

[48] C. Weil, Phys. Rev. 161, 1682 (1967).

We calculated the $\mathcal{H}_{\rm Int}$ starting from an interaction Lagrangian which does not contain the S^i fields. These fields vanish as free fields. In the presence of interactions they do not vanish and the Euler-Lagrange equations may be used to define them. If $\mathcal{L}_{\rm Int}{}^{\rm tot}$ is the total interaction Lagrangian, we get

$$m_{S^i}{}^2 S^i = \frac{\partial \mathcal{L}_{\rm Int}{}^{\rm tot}}{\partial(\bar{S}^i)} - \partial_\mu \frac{\partial \mathcal{L}_{\rm Int}{}^{\rm tot}}{\partial(\partial_\mu \bar{S}^i)} . \qquad (6.48)$$

By iteration of the above equations, we can express the S^i fields in terms of the other fields. Then we can use these expressions to eliminate the S^i fields from our $\mathcal{L}_{\rm Int}{}^{\rm tot}$. In that way, we find that the interaction Lagrangian from which the $\mathcal{H}_{\rm Int}$ should be calculated is not simply the one we obtain from $\mathcal{L}_{\rm Int}{}^{\rm tot}$ by dropping the terms which contain S^i fields, but it has some more terms which contribute to $\mathcal{H}_{\rm Int}$. The interesting additional terms of the $\mathcal{H}_{\rm Int}$ are given by[49]

$$-\sum_i (1/m_{S^i}{}^2) J^{s^i} J^{\bar{s}^i} , \qquad (6.49)$$

where the J^{s^i} is given by

$$J^{s^i} = -\frac{4 f_{\rho\pi\pi}}{\sqrt{2} m_\rho} \frac{\partial}{\partial \bar{S}^i} \mathrm{Tr}(\partial_\mu P \partial_\mu P S) . \qquad (6.50)$$

The $\mathcal{H}_{\rm Int}$ of expression (6.49) is explicitly given in Appendix B.

VII. MESON-MESON SCATTERING LENGTHS

To calculate the meson-meson scattering amplitude, we shall use the interaction Hamiltonians of Eqs. (B5)–(B7), (B10), and (B16) of Appendix B. It is interesting to note that the Hamiltonian (B7) coming from the Lagrangian $\mathcal{L}_M{}^m$ depends only on the masses of the pseudoscalar mesons, since the masses of the nonets 0^{--} and $1^{-\pm}$ which appear in the Lagrangian $\mathcal{L}_M{}^m$ are eliminated.

The invariant amplitude $\mathfrak{M}$ for meson-meson scattering in the center-of-mass system is defined by

$$S_{fi} = \delta_{fi} - \frac{i\delta^4(q_1+q_2-q_3-q_4)}{(2\pi)^2(16E_{q_1}E_{q_2}E_{q_3}E_{q_4})^{1/2}} \mathfrak{M}(s,t,u) , \qquad (7.1)$$

where for $m_1=m_3$ and $m_2=m_4$ we have

$$s=(q_1+q_2)^2=[(m_1{}^2+\mathbf{q}^2)^{1/2}+(m_2{}^2+\mathbf{q}^2)^{1/2}]^2 ,$$
$$t=-(q_1-q_3)^2=-2\mathbf{q}^2(1-\cos\theta) ,$$

and

$$u=-(q_1-q_4)^2=-2\mathbf{q}^2(1+\cos\theta) + [(m_1{}^2+\mathbf{q}^2)^{1/2}-(m_2{}^2+\mathbf{q}^2)^{1/2}]^2 .$$

For the above definition of $\mathfrak{M}$, the unitarity condition gives in the elastic region

$$\mathfrak{M}_I{}^l = -[2] 8\pi [(\sqrt{s})/|\mathbf{q}|] e^{i\delta_I{}^l} \sin\delta_I{}^l . \qquad (7.2)$$

The factor 2 within the brackets is present for $\pi\pi\to\pi\pi$ and $KK\to KK$ because identical particles are involved. So the S-wave scattering length in the isospin state I denoted by α_I is given by

$$\alpha_I = \lim_{|\mathbf{q}|\to 0} \frac{\delta_I{}^0}{|\mathbf{q}|} = -\frac{1}{[2]8\pi(m_1+m_2)} \lim_{|\mathbf{q}|\to 0} \mathfrak{M}_I{}^0 . \qquad (7.3)$$

A. $\pi\pi$ Scattering Lengths

The interaction Hamiltonian which contributes to the isospin amplitudes $\mathfrak{M}_I(\pi\pi)$ is the following:

$$\mathcal{H}_{\rm Int}(\pi\pi) = f_{\rho\pi\pi} \boldsymbol{\varrho}_\mu \cdot (\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) + \left(\frac{f_{\rho\pi\pi}{}^2}{2m_\rho{}^2} - \frac{f^2}{12}\right)(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) - \frac{f^2 m_\pi{}^2}{48}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2$$
$$- \frac{4 f_{\rho\pi\pi}{}^2}{3m_\rho{}^2}\left(\frac{1+\sqrt{2}\sin 2\lambda_S + \sin^2\lambda^2{}_S}{m_{\zeta'}{}^2} + \frac{1-\sqrt{2}\sin 2\lambda_S+\cos^2\lambda_S}{m_{\xi'}{}^2}\right)(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})^2 . \qquad (7.4)$$

From Eqs. (5.26), (5.28), (7.3), and (7.4), we get

$$\alpha_0(\pi\pi) = \frac{0.04}{m_\pi} + 0.47\left(\frac{1+\sqrt{2}\sin 2\lambda_S+\sin^2\lambda_S}{m_{\zeta}{}'^2} + \frac{1-\sqrt{2}\sin 2\lambda_S+\cos^2\lambda_S}{m_{\zeta}{}'^2}\right) m_\pi , \qquad (7.5)$$

$$\alpha_1(\pi\pi) = 0 , \qquad (7.6)$$

$$\alpha_2(\pi\pi) = -\frac{0.01}{m_\pi} + 0.19\left(\frac{1+\sqrt{2}\sin 2\lambda_S+\sin^2\lambda_S}{m_{\zeta'}{}^2} + \frac{1-\sqrt{2}\sin 2\lambda_S+\cos^2\lambda_S}{m_{\xi'}{}^2}\right) m_\pi . \qquad (7.7)$$

For $m_{\zeta'} = m_{\xi'} = 500$ MeV, the above equations give $\alpha_0(\pi\pi) = 0.15 m_\pi{}^{-1}$ and $\alpha_2(\pi\pi) = 0.10 m_\pi{}^{-1}$.

[49] I thank Dr. L. H. Chan for suggesting that such terms should be added, and for very interesting communications. In Eq. (6.49) only the terms which contain four pseudoscalar-meson fields have been considered. These terms, which come from the Lagrangian $\mathcal{L}_M{}^3$, contribute to the meson-meson scattering lengths.

The $\pi\pi$ scattering lengths have been calculated in several ways[6,50–53] and seem to be small. Assuming PCAC, Weinberg[50] found $\alpha_0(\pi\pi)=0.20m_\pi^{-1}$ and $\alpha_2(\pi\pi)=-0.06m_\pi^{-1}$. A pure dispersion-relation approach[53(a)] gives $\alpha_0(\pi\pi)=0.25m_\pi^{-1}$ and $\alpha_2(\pi\pi)\simeq 0$. Also, the successful calculation of the K_{l4} form factors by Weinberg,[54] who neglected final-state interactions, indicates that the $\pi\pi$ scattering lengths are small.

B. πK Scattering Lengths

The interaction Hamiltonian contributing to π-K scattering is

$$\begin{aligned}\mathcal{H}_{\mathrm{Int}}(\pi K)=&f_{\rho\pi\pi}\boldsymbol{\varrho}_\mu(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})+i\tfrac{1}{2}f_{\rho\pi\pi}\boldsymbol{\varrho}_\mu(K^\dagger\boldsymbol{\tau}\partial_\mu K)\\&+\frac{f_{\rho\pi\pi}m_{K^*}}{2m_\rho}(iK_\mu^{*\dagger}\boldsymbol{\tau}K\partial_\mu\boldsymbol{\pi}+\mathrm{H.c.})-\left(\frac{f_{\rho\pi\pi}}{4m_\rho^2}-\frac{f^2}{24}\right)\{[K^\dagger\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})][(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu K]\\&+\partial_\mu K^\dagger[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]K-K^\dagger[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]\partial_\mu K\}-\frac{f^2(m_\pi^2+m_K^2)}{24}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(K^\dagger K)\\&-\frac{8f_{\rho\pi\pi}^2}{m_\rho^2m_{K'}^2}\partial_\mu K^\dagger(\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi})(\boldsymbol{\tau}\cdot\partial_\nu\boldsymbol{\pi})\partial_\nu K+\frac{4f_{\rho\pi\pi}^2}{3m_\rho^2}\left(\frac{2-\sqrt{2}\sin2\lambda_S-10\sin^2\lambda_S}{m_{\zeta'}^2}\right.\\&\left.+\frac{2+\sqrt{2}\sin2\lambda_S-10\cos^2\lambda_S}{m_{\xi'}^2}\right)(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})(\partial_\nu K^\dagger\partial_\nu K).\end{aligned}\tag{7.8}$$

Using Eqs. (5.26), (5.28), (7.3), and (7.8), we get

$$\alpha_{1/2}(\pi K)=-\frac{0.09}{m_\pi}+\frac{5.4m_\pi}{m_{K'}^2}-0.26\left(\frac{2-\sqrt{2}\sin2\lambda_S-10\sin^2\lambda_S}{m_{\zeta'}^2}+\frac{2+\sqrt{2}\sin2\lambda_S-10\cos^2\lambda_S}{m_{\xi'}^2}\right)m_\pi,\tag{7.9}$$

$$\alpha_{3/2}(\pi K)=\frac{0.03}{m_\pi}+\frac{5.4m_\pi}{m_{K'}^2}-0.26\left(\frac{2-\sqrt{2}\sin2\lambda_S-10\sin^2\lambda_S}{m_{\zeta'}^2}+\frac{2+\sqrt{2}\sin2\lambda_S-10\cos^2\lambda_S}{m_{\xi'}^2}\right)m_\pi.\tag{7.10}$$

C. KK Scattering Lengths

The interaction Hamiltonian contributing to K-K scattering is

$$\begin{aligned}\mathcal{H}_{\mathrm{Int}}(KK)=&i\tfrac{1}{2}f_{\rho\pi\pi}\boldsymbol{\varrho}_\mu(K^\dagger\boldsymbol{\tau}\overleftrightarrow{\partial}_\mu K)+i\frac{f_{\rho\pi\pi}m_\omega}{2m_\rho}\omega_\mu(K^\dagger\overleftrightarrow{\partial}_\mu K)\\&-i\frac{f_{\rho\pi\pi}m_\varphi}{\sqrt{2}m_\rho}\varphi_\mu(K^\dagger\overleftrightarrow{\partial}_\mu K)-\tfrac{1}{12}f^2[(K^\dagger\overleftrightarrow{\partial}_\mu K)(\partial_\mu K^\dagger)K+K^\dagger(K\overleftrightarrow{\partial}_\mu K^\dagger)\partial_\mu K]\\&-\frac{f_{\rho\pi\pi}^2}{8m_\rho^2}[\partial_\mu K^\dagger(K\overleftrightarrow{\partial}_\mu K^\dagger)K-K^\dagger(K\overleftrightarrow{\partial}_\mu K^\dagger)\partial_\mu K+2K^\dagger(\partial_\mu K)(\partial_\mu K^\dagger)K-2(K^\dagger K)(\partial_\mu K^\dagger\partial_\mu K)]\\&-\tfrac{1}{12}f^2m_K^2(K^\dagger K)^2+\frac{4f_{\rho\pi\pi}^2}{m_\rho^2}\left(\frac{1}{m_{\pi'}^2}-\frac{1-2\sqrt{2}\sin2\lambda_S+7\sin^2\lambda_S}{3m_{\zeta'}^2}-\frac{1+2\sqrt{2}\sin2\lambda_S+7\cos^2\lambda_S}{3m_{\xi'}^2}\right)\\&\times(\partial_\mu K^\dagger\partial_\mu K)^2-\frac{8f_{\rho\pi\pi}^2}{m_\rho^2m_{\pi'}^2}(\partial_\mu K^\dagger\partial_\nu K)(\partial_\nu K^\dagger\partial_\mu K).\end{aligned}\tag{7.11}$$

---

[50] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).
[51] N. N. Khuri, Phys. Rev. **153**, 1477 (1967). In the calculation of the s-wave $\pi\pi$ scattering lengths, higher-order corrections are taken into account. The difference between the scattering lengths obtained in this paper and those of Weinberg, Ref. 50, is of the order of 10%.
[52] J. Sucher and C. H. Woo, Phys. Rev. Letters **17**, 723 (1967).
[53] (a) F. T. Meiere and M. Sugawara, Phys. Rev. **153**, 1702 (1967); (b) F. T. Meiere, *ibid.* **159**, 1462 (1967).
[54] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

From Eqs. (5.26), (5.28), (7.3), and (7.11), we get

$$\alpha_0(KK)=0\,, \tag{7.12}$$

$$\alpha_1(KK)=-\frac{0.04}{m_\pi}+6.1\left(\frac{1}{m_{\pi'}{}^2}+\frac{1-2\sqrt{2}\,\sin2\lambda_S+7\,\sin^2\lambda}{3m_{\zeta'}{}^2}+\frac{1+2\sqrt{2}\,\sin2\lambda_S+7\,\cos^2\lambda_S}{3m_{\xi'}{}^2}\right)m_\pi\,. \tag{7.13}$$

VIII. DECAYS $\eta'\to\eta+2\pi$, $K^*\to K\pi$, $\varphi\to K^+K^-$

In this section the decay rates of the strong decays $\eta'(958)\to\eta(549)+2\pi$, $K^*\to K\pi$, and $\varphi\to K^+K^-$ will be calculated.

A. Decay $\eta'\to\eta+2\pi$

The interaction Hamiltonian which contributes to this process comes from the $(PPPP)$ coupling terms of the Lagrangian $\mathfrak{L}_M{}^m$, which are given in (3.19) .We find

$$\mathcal{H}_{\rm Int}(\eta'\eta\pi\pi)=-\tfrac{1}{12}\sqrt{2}f^2(m_\eta{}^2+m_{\eta'}{}^2-m_\pi{}^2)\times[\cos2\lambda_\eta+(\sin2\lambda_\eta)/2\sqrt{2}](\boldsymbol{\pi}\cdot\boldsymbol{\pi})\eta\eta'\,, \tag{8.1}$$

which gives the decay rate

$$\Gamma(\eta'\to\eta+2\pi)=0.72(m_\pi f)^4[\cos2\lambda_\eta+(\sin2\lambda_\eta)/2\sqrt{2}]^2\ \text{MeV}. \tag{8.2}$$

If we take $\lambda_\eta=+10.8°$, which is predicted by the quark model,[35] and the value of f given by Eq. (5.28), we get

$$\Gamma(\eta'\to\eta+2\pi)=1.0\ \text{MeV}. \tag{8.3}$$

Experimentally we get[47]

$$\Gamma(\eta'\to\eta+2\pi)<2.8\ \text{MeV}. \tag{8.4}$$

B. Decay $K^*\to K\pi$

According to Eq. (B5), the interaction Hamiltonian which contributes to this process is

$$\mathcal{H}_{\rm Int}(K^*K\pi)=(f^2{}_{\rho\pi\pi}m_{K^*}/2m_\rho)[iK_\mu{}^{*\dagger}\boldsymbol{\tau}(K\overleftrightarrow{\partial}_\mu\boldsymbol{\pi})+\text{H.c.}]. \tag{8.5}$$

The decay rate is

$$\Gamma(K^*\to K\pi)=\frac{f_{\rho\pi\pi}{}^2}{8\pi m_\rho{}^2}\times\left\{\frac{[(m_{K^*}-m_\pi)^2-m_K{}^2][(m_{K^*}+m_\pi)^2-m_K{}^2]}{4m_{K^*}{}^2}\right\}^{3/2}. \tag{8.6}$$

Using Eq. (5.26), we get

$$\Gamma(K^*\to K\pi)=42\ \text{MeV}, \tag{8.7}$$

in reasonable agreement with the experimental value[47]

$$\Gamma(K^*\to K\pi)=49.7\pm1.1\ \text{MeV}. \tag{8.8}$$

C. Decay $\varphi\to K^+K^-$

From Eq. (B5), we get

$$\mathcal{H}_{\rm Int}(\varphi K^+K^-)=(if_{\rho\pi\pi}m_\varphi/\sqrt{2}m_\rho)\varphi_\mu(K^\dagger\overleftrightarrow{\partial}_\mu K^-)\,, \tag{8.9}$$

which gives the decay rate

$$\Gamma(\varphi\to K^+K^-)=(f_{\rho\pi\pi}{}^2/96\pi m_\rho{}^2)(m_\varphi{}^2-4m_K{}^2)^{3/2}. \tag{8.10}$$

Using Eq. (5.26), we obtain

$$\Gamma(\varphi\to K^+K^-)=2.32\ \text{MeV}, \tag{8.11}$$

while the experimental value is[47]

$$\Gamma(\varphi\to K^+K^-)=1.76\pm0.35\ \text{MeV}. \tag{8.12}$$

IX. DISCUSSION

We have tried to introduce the vector mesons in the phenomenological Lagrangians in the same way as the pseudoscalar mesons and not as Yang-Mills gauge fields. In this approach we had to introduce two more meson nonets. In all terms in the expansion of our Lagrangians, an even number of V' and A fields only can appear. So the V' and A mesons do not couple to baryons to lowest order. In fact, invariance under parity and charge conjugation does not allow a coupling of these mesons to any of the Dirac bilinear covariants.

We should emphasize that the choice of the Lagrangians is not unique. A strong requirement imposed upon the meson Lagrangian is that the free-meson Lagrangian we get to lowest order leads to a positive-definite Hamiltonian. Also the PCAC requirement crucially depends upon the choice of the meson mass Lagrangian. In the meson-baryon Lagrangian we have more freedom, particularly in its symmetry-breaking part[55,56]: We may choose a sufficient number of Lagrangians and adjust their coefficients in such a way that the baryons get their physical masses, and the D/F ratios their experimental values. From our Lagrangian $\mathfrak{L}_{MB}$ we see that the nonderivative coupling of the pseudoscalar mesons to baryons is pure D-type. Since, however, a large F-type derivative coupling comes from the Lagrangians $\mathfrak{L}_{MB}{}^4$ and $\mathfrak{L}_{MB}{}^5$ because of Eq. (5.18), no additional Lagrangians giving rise to F-type nonderivative coupling were introduced.

Weak interactions have not been considered at present. They can be introduced easily in the framework of the model. For example, the form factors of K_{l4} decays may be calculated using the axial-vector current of Eq.

[55] A. Kumar and R. Ramachandran, Tata Institute of Fundamental Research, Bombay, India, Report, 1968 (unpublished). The symmetry-breaking Lagrangians chosen in this paper are first-order in M, while our Lagrangians $\mathfrak{L}_{MB}{}^6$ and $\mathfrak{L}_{MB}{}^7$ are second-order in M. However, when the Lagrangians are expanded in powers of f, more terms of first order in f arise in their case.

[56] J. Schechter, Y. Ueda, and G. Venturi, Phys. Rev. **177**, 2311 (1969).

(4.10). The exchange particles are ρ, K^*, and K. Also, nonleptonic decays can be treated. The results of such calculations will be reported elsewhere.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Y. Nambu for many very interesting discussions and suggestions.

APPENDIX A: INTERACTION HAMILTONIAN

We want to calculate the interaction Hamiltonian corresponding to the interaction Lagrangian

$$\mathfrak{L}_{\rm Int}(\mathbf{P},\partial_\mu\mathbf{P},\mathbf{T}_{\rho\sigma},\partial_\lambda\mathbf{T}_{\lambda\tau},\mathbf{B}), \tag{A1}$$

which is a polynomial in the variables $\mathbf{P}$, $\partial_\mu\mathbf{P}$, $\mathbf{T}_{\rho\sigma}$, $\partial_\lambda\mathbf{T}_{\lambda\tau}$, and $\mathbf{B}$. In this appendix boldface letters denote fields in the Heisenberg representation. The fields in the interaction representation will be denoted by common letters. We shall calculate the interaction Hamiltonian to second order in the coupling constants which appear in $\mathfrak{L}_{\rm Int}$. The interaction Hamiltonian in the interaction representation $\mathcal{H}_{\rm Int}[x,n]$ is obtained as the solution of the equations[57,16]

$$\begin{aligned}[P^i(x),\mathcal{H}_{\rm Int}[x',n]]
&=S(\sigma)\mathbf{J}^{p^i}(x')S^{-1}(\sigma)[P^i(x),\bar{P}^i(x')]\\
&\quad+S(\sigma)\mathbf{J}_\mu{}^{p^i}(x')S^{-1}(\sigma)[P^i(x),\partial_\mu{}'\bar{P}^i(x')],\end{aligned} \tag{A2}$$

$$\begin{aligned}[\partial_\rho T_{\rho\kappa}{}^i(x),\mathcal{H}_{\rm Int}[x',n]]
&=-S(\sigma)\tfrac{1}{2}\mathbf{J}_{\mu\lambda}{}^{T^i}(x')S^{-1}(\sigma)[\partial_\rho T_{\rho\kappa}{}^i(x),\bar{T}_{\mu\lambda}{}^i(x')]\\
&\quad-S(\sigma)\mathbf{J}_\mu{}^{T^i}(x')S^{-1}(\sigma)[\partial_\rho T_{\rho\kappa}{}^i(x),\partial_\sigma{}'\bar{T}_{\sigma\mu}{}^i(x')],\end{aligned} \tag{A3}$$

$$[B^i(x),\mathcal{H}_{\rm Int}[x',n]]
=S(\sigma)\mathbf{J}^{B^i}(x')S^{-1}(\sigma)[B^i(x),\bar{B}^i(x')], \tag{A4}$$

where the index i labels the members of each nonet and is not summed on the right-hand side of the above equations. The currents $\mathbf{J}_{\mu\lambda}{}^{T^i}$, $\mathbf{J}_\mu{}^{T^i}$, and $\mathbf{J}_\mu{}^{p^i}$ are defined by Eqs. (6.2), (6.3) and (6.4), respectively,[58] while the currents $\mathbf{J}^{p^i}$ and $\mathbf{J}^{B^i}$ are given by

$$\mathbf{J}^{P^i}=-\frac{\partial\mathfrak{L}_{\rm Int}}{\partial\bar{\mathbf{P}}^i}, \tag{A5}$$

$$\mathbf{J}^{B^i}=-\frac{\partial\mathfrak{L}_{\rm Int}}{\partial\bar{\mathbf{B}}^i}. \tag{A6}$$

We find[16]

$$\begin{aligned}\partial_\rho\mathbf{T}_{\rho\lambda}{}^i(x)=(\partial_\rho T_{\rho\lambda}{}^i[x,\sigma])_{x1\sigma}-\mathbf{J}_\lambda{}^{T^i}(x)\\
-\mathbf{J}_\rho{}^{T^i}(x)n_\rho(x)n_\lambda(x),\end{aligned} \tag{A7}$$

$$\begin{aligned}\mathbf{T}_{\mu\lambda}{}^i(x)=(1/m_i{}^2)\{(\partial_\mu\partial_\rho T_{\rho\lambda}{}^i[x,\sigma])_{x1\sigma}
&-(\partial_\lambda\partial_\rho T_{\rho\mu}{}^i[x,\sigma])_{x1\sigma}-\mathbf{J}_{\mu\lambda}{}^{T^i}(x)\\
&+\mathbf{J}_{\rho\mu}{}^{T^i}(x)n_\lambda(x)n_\rho(x)\\
&-\mathbf{J}_{\rho\lambda}{}^{T^i}(x)n_\mu(x)n_\rho(x)\},\end{aligned} \tag{A8}$$

$$\mathbf{B}^i(x)=(B^i[x,\sigma])_{x1\sigma}=S^{-1}(\sigma)B^i(x)S(\sigma). \tag{A9}$$

Also, we find

$$\mathbf{P}^i(x)=(P^i[x,\sigma])_{x1\sigma}=S^{-1}(\sigma)P^i(x)S(\sigma), \tag{A10}$$

$$\partial_\mu\mathbf{P}^i(x)=(\partial_\mu P^i[x,\sigma])_{x1\sigma}+\mathbf{J}_\lambda{}^{p^i}(x)n_\lambda(x)n_\mu(x). \tag{A11}$$

So to first order in the coupling constants, denoted by the symbol $\approx$, we get, using the free-field equations (3.24),

$$\partial_\rho\mathbf{T}_{\rho\lambda}{}^i\approx S^{-1}(\sigma)\{\partial_\rho T_{\rho\lambda}{}^i-J_\lambda{}^{T^i}-J_\rho{}^{T^i}n_\rho n_\lambda\}S(\sigma), \tag{A12}$$

$$\begin{aligned}\mathbf{T}_{\mu\lambda}{}^i\approx S^{-1}(\sigma)\{T_{\mu\lambda}{}^i+(1/m_i{}^2)[-J_{\mu\lambda}{}^{T^i}+J_{\rho\mu}{}^{T^i}n_\lambda n_\rho\\
-J_{\rho\lambda}{}^{T^i}n_\mu n_\rho]\}S(\sigma),\end{aligned} \tag{A13}$$

$$\partial_\mu\mathbf{P}^i\approx S^{-1}(\sigma)(\partial_\mu P^i+J_\lambda{}^{p^i}n_\lambda n_\mu)S(\sigma). \tag{A14}$$

To obtain a current to first order in the coupling constants we must replace all the Heisenberg fields it contains with free fields, while to obtain a current to second order in the coupling constants we must replace one of the fields $\partial_\rho\mathbf{T}_{\rho\lambda}{}^i$, $\mathbf{T}_{\mu\rho}{}^i$, or $\partial_\mu\mathbf{P}^i$ it contains by its part which is first-order in the coupling constants, for example, $\partial_\mu\mathbf{P}^i\to S^{-1}(\sigma)J_\lambda{}^{p^i}n_\lambda n_\mu S(\sigma)$ according to Eq. (A14), etc., and all the other fields by free fields. In that way we get for the current $\mathbf{J}_\kappa{}^{p^i}$, by using Eqs. (A12)–(A14),

$$\begin{aligned}\mathbf{J}_\kappa{}^{p^j}=-\frac{\partial\mathfrak{L}_{\rm Int}}{\partial(\partial_\kappa\bar{\mathbf{P}}^j)}\approx S^{-1}(\sigma)\biggl\{&-\frac{\partial\mathfrak{L}_{\rm Int}}{\partial(\partial_\kappa\bar{P}^j)}-\sum_i\left[\frac{\partial}{\partial(\partial_\lambda\bar{P}^j)}\left(\frac{\partial\mathfrak{L}_{\rm Int}}{\partial(\partial_\kappa\bar{P}^j)}\right)\right]J_\tau{}^{p^i}n_\lambda n_\tau\\
&-\sum_i\left[\frac{\partial}{\partial T_{\mu\lambda}{}^i}\left(\frac{\partial\mathfrak{L}_{\rm Int}}{\partial(\partial_\kappa\bar{P}^j)}\right)\right]\frac{1}{2m_i{}^2}(-J_{\mu\lambda}{}^{T^i}+J_{\tau\mu}{}^{T^i}\,n_\lambda n_\tau-J_{\tau\lambda}{}^{T^i}n_\mu n_\tau)\\
&-\sum_i\left[\frac{\partial}{\partial(\partial_\nu T_{\nu\mu}{}^i)}\left(\frac{\partial\mathfrak{L}_{\rm Int}}{\partial(\partial_\kappa\bar{P}^j)}\right)\right](-J_\mu{}^{T^i}-J_\lambda{}^{T^i}n_\lambda n_\mu)\biggr\}S(\sigma).\end{aligned} \tag{A15}$$

[57] H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Co., Amsterdam, 1953).

[58] In the notation of this appendix all quantities appearing in Eqs. (6.2)–(6.4) should be written with boldface letters.

The above expression can be written as follows if we interchange the order of the two differentiations, and use Eqs. (6.2)–(6.4) and the antisymmetry of the tensor $T_{\mu\lambda}{}^{i}$:

$$\mathbf{J}_{\kappa}{}^{p^i}\approx S^{-1}(\sigma)\left\{-\frac{\partial\mathfrak{L}_{\text{Int}}}{\partial(\partial_\kappa\bar{P}^j)}+\sum_i\left[\frac{\partial J_\lambda{}^{\bar{p}^i}}{\partial(\partial_\kappa\bar{P}^j)}J_\tau{}^{p^i}n_\lambda n_\tau+\frac{1}{2m_i{}^2}\frac{\partial J_{\mu\lambda}{}^{\bar{T}^i}}{\partial(\partial_\kappa\bar{P}^j)}(J_{\mu\lambda}{}^{T^i}+2J_{\mu\tau}{}^{T^i}n_\lambda n_\tau)+\frac{\partial J^{\bar{T}^i}}{\partial(\partial_\kappa\bar{P}^j)}(J_\mu{}^{T^i}+J_\lambda{}^{T^i}n_\lambda n_\mu)\right]\right\}S(\sigma)$$

$$=S^{-1}(\sigma)\frac{\partial}{\partial(\partial_\kappa\bar{P}^j)}\left\{-\mathfrak{L}_{\text{Int}}+\sum_i\left[\tfrac{1}{2}J_\lambda{}^{\bar{p}^i}J_\tau{}^{p^i}n_\lambda n_\tau+\frac{1}{4m_i{}^2}(J_{\mu\lambda}{}^{\bar{T}^i}J_{\mu\lambda}{}^{T^i}+2J_{\mu\lambda}{}^{\bar{T}^i}J_{\mu\tau}{}^{T^i}n_\lambda n_\tau)+\tfrac{1}{2}(J_\mu{}^{\bar{T}^i}J_\mu{}^{T^i}+J_\mu{}^{\bar{T}^i}J_\lambda{}^{T^i}n_\mu n_\lambda)\right]\right\}S(\sigma).\quad\text{(A16)}$$

For the currents $\mathbf{J}_{\mu\lambda}{}^{T^i}$, $\mathbf{J}_\mu{}^{T^i}$, $\mathbf{J}^{p^i}$, and $\mathbf{J}^{B^i}$, we find an expression which is exactly like the above expression except that the operator $\partial/\partial(\partial_\kappa\bar{P}^j)$ is replaced by the operators $\partial/\partial\bar{T}_{\mu\lambda}{}^j$, $\partial/\partial(\partial_\nu\bar{T}_{\nu\mu}{}^j)$, $\partial/\partial\bar{P}^j$, and $\partial/\partial\bar{B}^j$, respectively. From these expressions and Eqs. (A2)–(A4) and (A16), we obtain Eq. (6.1).

APPENDIX B: EXPLICIT FORM OF INTERACTION HAMILTONIANS

In this appendix we shall give the explicit form of the interaction Hamiltonians we have used. We find

$$\mathfrak{H}(PPV)=-\frac{if_{\rho\pi\pi}}{\sqrt{2}m_\rho}\operatorname{Tr}[(mV_\mu)(P'\overleftrightarrow{\partial}_\mu P')],\quad\text{(B1)}$$

where

$$P=P'+(1/\sqrt{3})\eta_0,\quad\text{(B2)}$$

$$P'=\begin{bmatrix}\pi^0/\sqrt{2}+\eta_8/\sqrt{6} & \pi^+ & K^+\\ \pi^- & -\pi^0/\sqrt{2}+\eta_8/\sqrt{6} & K^0\\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6}\end{bmatrix},\quad\text{(B3)}$$

$$V=\begin{bmatrix}(\rho^0+\omega^0)/\sqrt{2} & \rho^+ & K^{*+}\\ \rho^- & (\omega^0-\rho^0)/\sqrt{2} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \varphi\end{bmatrix}.\quad\text{(B4)}$$

From Eq. (B1) we get for $K^\dagger=(K^-,\bar{K}^0)$

$$\mathfrak{H}(PPV)=f_{\rho\pi\pi}\boldsymbol{\rho}_\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})+i\tfrac{1}{2}f_{\rho\pi\pi}\boldsymbol{\rho}_\mu\cdot(K^\dagger\boldsymbol{\tau}\overleftrightarrow{\partial}_\mu K)+\frac{f_{\rho\pi\pi}m_{K^*}}{2m_\rho}[iK_\mu{}^{*\dagger}\boldsymbol{\tau}(K\overleftrightarrow{\partial}_\mu\boldsymbol{\pi})+\text{H.c.}]$$
$$+\frac{\sqrt{3}f_{\rho\pi\pi}m_{K^*}}{2m_\rho}[iK_\mu{}^{*\dagger}(K\overleftrightarrow{\partial}_\mu\eta_8)+\text{H.c.}]+\frac{if_{\rho\pi\pi}m_\omega}{2m_\rho}\omega_\mu(K^\dagger\overleftrightarrow{\partial}_\mu K)-\frac{if_{\rho\pi\pi}m_\varphi}{\sqrt{2}m_\rho}\varphi_\mu(K^\dagger\overleftrightarrow{\partial}_\mu K).\quad\text{(B5)}$$

The above Hamiltonian comes from the interaction Lagrangian which gives the three-meson coupling.

From the kinetic-energy part of the meson Lagrangian, we get the following four-pseudoscalar-meson interaction Hamiltonian:

$$(f^2/24)\operatorname{Tr}\{(P\overleftrightarrow{\partial}_\mu P)(P\overleftrightarrow{\partial}_\mu P)\}=(f^2/24)\operatorname{Tr}\{(P'\overleftrightarrow{\partial}_\mu P')(P'\overleftrightarrow{\partial}_\mu P')\}$$
$$=(f^2/24)\{-2(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})-2(K^\dagger\overleftrightarrow{\partial}_\mu K)(\partial_\mu K^\dagger)K-2K^\dagger(K\overleftrightarrow{\partial}_\mu K^\dagger)\partial_\mu K$$
$$+[K^\dagger\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})][(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu K]+(\partial_\mu K^\dagger)[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]K-K^\dagger[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]\partial_\mu K$$
$$+\sqrt{3}[K^\dagger\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})](\eta_8\overleftrightarrow{\partial}_\mu K)-\sqrt{3}(\eta_8\overleftrightarrow{\partial}_\mu K^\dagger)[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\partial_\mu K]+3(\eta_8\overleftrightarrow{\partial}_\mu K^\dagger)(K\overleftrightarrow{\partial}_\mu\eta_8)\}.\quad\text{(B6)}$$

The mass Lagrangian gives the following pseudoscalar-meson interaction Hamiltonian:

$$\tfrac{1}{12}f^2\operatorname{Tr}\{P^4[\tfrac{1}{6}(m_\pi{}^2+2m_K{}^2)+(1/\sqrt{3})(m_\pi{}^2-m_K{}^2)\lambda_8]\}.\quad\text{(B7)}$$

We find

$$\mathrm{Tr}(P^4)=2[\tfrac{1}{2}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+\tfrac{1}{3}(\eta_8/\sqrt{2}+\eta_0)^2]^2+2(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(K^\dagger K)+\tfrac{4}{3}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\eta_8/\sqrt{2}+\eta_0)^2+(12/\sqrt{6})\eta_0K^\dagger(\boldsymbol{\pi}\cdot\boldsymbol{\tau})K$$
$$+(K^\dagger K)(K^\dagger K)+[K^\dagger K+\tfrac{1}{3}(-\sqrt{2}\eta_8+\eta_0)^2]^2+\tfrac{2}{3}[(\eta_8/\sqrt{2}+\eta_0)^2+(-\eta_8/\sqrt{2}+2\eta_0)^2]K^\dagger K, \quad \text{(B8)}$$

$$\mathrm{Tr}(P^4\lambda_8)=(1/\sqrt{3})\{2[\tfrac{1}{2}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+\tfrac{1}{3}(\eta_8/\sqrt{2}+\eta_0)^2]^2+\tfrac{1}{2}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})(K^\dagger K)+\tfrac{4}{3}(\eta_8/\sqrt{2}+\eta_0)^2(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+\sqrt{3}\eta_8K^\dagger(\boldsymbol{\pi}\cdot\boldsymbol{\tau})K$$
$$+(K^\dagger K)(K^\dagger K)-2[\tfrac{1}{3}(-\sqrt{2}\eta_8+\eta_0)^2+K^\dagger K]^2+[\tfrac{2}{3}(\eta_8/\sqrt{2}+\eta_0)^2-\tfrac{1}{3}(-\eta_8/\sqrt{2}+2\eta_0)^2]K^\dagger K\}. \quad \text{(B9)}$$

The interaction Hamiltonian of Eq. (6.17) contains also the term

$$\tfrac{1}{2}\sum_i J_\mu{}^{Ti}J_\mu{}^{\bar{T}i}, \quad \text{(B10)}$$

which comes from the term

$$(if_{\rho\pi\pi}/\sqrt{2}m_\rho)\,\mathrm{Tr}[\partial_\mu T_{\mu\nu}(P\overleftrightarrow{\partial}_\nu P)] \quad \text{(B11)}$$

of the Lagrangian of Eq. (6.16). We find

$$\tfrac{1}{2}(J_\mu{}^{\rho^0}J_\mu{}^{\rho^0}+2J_\mu{}^{\rho^+}J_\mu{}^{\rho^-})$$
$$=-(f_{\rho\pi\pi}{}^2/2m_\rho{}^2)\{-(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})+\tfrac{1}{2}(\partial_\mu K^\dagger)[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]K-\tfrac{1}{2}K^\dagger[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})]\partial_\mu K$$
$$+\tfrac{1}{4}[(\partial_\mu K^\dagger)(K\overleftrightarrow{\partial}_\mu K^\dagger)K-K^\dagger(K\overleftrightarrow{\partial}_\mu K^\dagger)\partial_\mu K+2K^\dagger(\partial_\mu K)(\partial_\mu K^\dagger)K-2(K^\dagger K)(\partial_\mu K^\dagger\partial_\mu K)]\}, \quad \text{(B12)}$$

$$J_\mu{}^{K^{*+}}J_\mu{}^{K^{*-}}+J_\mu{}^{K^{*0}}J_\mu{}^{\bar{K}^{*0}}$$
$$=-(f_{\rho\pi\pi}{}^2/4m_\rho{}^2)\{[K^\dagger\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})][(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu K]+\sqrt{3}[K^\dagger\overleftrightarrow{\partial}_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})](\eta_8\partial_\mu K)$$
$$-\sqrt{3}(\eta_8\overleftrightarrow{\partial}_\mu K^\dagger)[(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\overleftrightarrow{\partial}_\mu K]+3(\eta_8\overleftrightarrow{\partial}_\mu K^\dagger)(K\partial_\mu\eta_8)\}, \quad \text{(B13)}$$

$$\tfrac{1}{2}J_\mu{}^\omega J_\mu{}^\omega=-(f_{\rho\pi\pi}{}^2/8m_\rho{}^2)(K^\dagger\overleftrightarrow{\partial}_\mu K)(K^\dagger\overleftrightarrow{\partial}_\mu K), \quad \text{(B14)}$$

$$\tfrac{1}{2}J_\mu{}^\varphi J_\mu{}^\varphi=-(f_{\rho\pi\pi}{}^2/4m_\rho{}^2)(K^\dagger\overleftrightarrow{\partial}_\mu K)(K^\dagger\partial_\mu K). \quad \text{(B15)}$$

The interaction Hamiltonian given by expression (B10) is obtained by summing the right-hand sides of Eqs. (B12)–(B15).

Finally, we have the interaction Hamiltonian of expression (6.49),

$$-\sum_i (1/m_{S^i}{}^2)J'^{s^i}J'^{\bar{s}^i}. \quad \text{(B16)}$$

We find

$$-(1/m_{\pi'}{}^2)(2J^{\pi'+}J^{\pi'-}+J^{\pi'0}J^{\pi'0})$$
$$=-(4f^2{}_{\rho\pi\pi}/m_{\pi'}{}^2m_\rho{}^2)\{(8/3)(\partial_\mu\eta_8/\sqrt{2}+\partial_\mu\eta_0)(\partial_\nu\eta_8/\sqrt{2}+\partial_\nu\eta_0)(\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi})$$
$$+(8/\sqrt{6})(\partial_\mu\eta_8/\sqrt{2}+\partial_\mu\eta_0)\partial_\nu K^\dagger(\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi})\partial_\nu K-(\partial_\mu K^\dagger\partial_\mu K)(\partial_\nu K^\dagger\partial_\nu K)+2(\partial_\mu K^\dagger\partial_\nu K)(\partial_\nu K^\dagger\partial_\mu K)\}, \quad \text{(B17)}$$

$$-(2/m_{K'}{}^2)(J^{K^+}J^{K^-}+J^{K^0}J^{\bar{K}^0})$$
$$=-(16f_{\rho\pi\pi}{}^2/m_{K'}{}^2m_\rho{}^2)\{\tfrac{1}{2}\partial_\mu K^\dagger(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau})(\partial_\nu\boldsymbol{\pi}\cdot\boldsymbol{\tau})\partial_\nu K+(1/\sqrt{6})(-\partial_\mu\eta_8/\sqrt{2}+2\partial_\mu\eta_0)[\partial_\nu K^\dagger(\boldsymbol{\tau}\cdot\partial_\nu\boldsymbol{\pi})\partial_\mu K$$
$$+\partial_\mu K^\dagger(\boldsymbol{\tau}\cdot\partial_\nu\boldsymbol{\pi})\partial_\nu K]+\tfrac{1}{3}(\partial_\mu\eta_8/\sqrt{2}-2\partial_\mu\eta_0)(\partial_\nu\eta_8/\sqrt{2}-2\partial_\nu\eta_0)(\partial_\mu K^\dagger\partial_\nu K)\}, \quad \text{(B18)}$$

$$-\frac{1}{m_{\zeta'}{}^2}J^{\zeta'}J^{\zeta'}-\frac{1}{m_{\xi'}{}^2}J^{\xi'}J^{\xi'}$$
$$=-\frac{4f_{\rho\pi\pi}{}^2}{3m_\rho{}^2}\left(\frac{\cos^2\lambda_S}{m_{\zeta'}{}^2}+\frac{\sin^2\lambda_S}{m_{\xi'}{}^2}\right)(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}-\partial_\mu K^\dagger\partial_\mu K-\partial_\mu\eta_8\partial_\mu\eta_8+2\sqrt{2}\partial_\mu\eta_8\partial_\mu\eta_0)^2-\frac{8f_{\rho\pi\pi}{}^2}{3\sqrt{2}m_\rho{}^2}\left(\frac{1}{m_{\zeta'}{}^2}-\frac{1}{m_{\xi'}{}^2}\right)$$
$$\times\sin 2\lambda_S(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}-\partial_\mu K^\dagger\partial_\mu K-\partial_\mu\eta_8\partial_\mu\eta_8+2\sqrt{2}\partial_\mu\eta_8\partial_\mu\eta_0)(\partial_\nu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi}+2\partial_\nu K^\dagger\partial_\nu K+\partial_\nu\eta_8\partial_\nu\eta_8+\partial_\nu\eta_0\partial_\nu\eta_0)$$
$$-\frac{8f_{\rho\pi\pi}{}^2}{3m_\rho{}^2}\left(\frac{\sin^2\lambda}{m_{\zeta'}{}^2}+\frac{\cos^2\lambda_S}{m_{\xi'}{}^2}\right)(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}+2\partial_\mu K^\dagger\partial_\mu K+\partial_\mu\eta_8\partial_\mu\eta_8+\partial_\mu\eta_0\partial_\mu\eta_0)^2. \quad \text{(B19)}$$

The interaction Hamiltonian given by expression (B16) is obtained by summing the right-hand sides of Eqs. (B17)–(B19).