Equation (39c) was obtained by Harari[17] from (39a) and (39b) through dispersion relations, which has not been necessary here.

For the sake of interest, we write down here a few more relations which follow from the above conjecture:

$$\begin{aligned}\mathrm{Im}(K^-p \to \bar{K}^{*0}n) &= (5/2\sqrt{2})\,\mathrm{Im}(K^-p \to \bar{K}^{*0}N^{*0}) \\ &= (5\sqrt{2}/3\sqrt{3})\,\mathrm{Im}(K^-p \to \phi\Lambda)\,, \end{aligned} \tag{40}$$

$$\mathrm{Im}(\pi^-p \to \pi^0 n) = \tfrac{1}{2}\sqrt{2}\,\mathrm{Im}(K^-p \to \bar{K}^0 n) \quad \text{(Ref. 19)}, \tag{41}$$

$$\mathrm{Im}(\bar{K}^0 p \to \phi\Sigma^+) = (1/2\sqrt{2})\,\mathrm{Im}(\bar{K}^0 p \to \phi Y_1^{*+})\,, \tag{42}$$

$$\mathrm{Im}(K^-p \to \bar{K}^0\Lambda) = -3\,\mathrm{Im}(K^-p \to \bar{K}^0\Sigma^0)\,, \tag{43}$$

$$\mathrm{Im}(\pi^-p \to K^{*0}\Lambda) = 3\,\mathrm{Im}(K^-p \to \bar{K}^{*0}\Sigma^0)\,. \tag{44}$$

We note that since spin is included, the present conjecture has more dynamical content and seems to give all the results of Harari and Rosner.

Again, for high-energy processes, when we assume that quark-quark and quark-antiquark forces are equal in magnitude, we get

$$\mathrm{Re}(K^-p \to \bar{K}^0 n)/\mathrm{Im}(K^0 p \to K^+ n) = 1\,, \tag{45}$$

$$\mathrm{Re}(\pi^-p \to \pi^0 n)/\mathrm{Im}(\pi^-p \to \pi^0 n) = 1\,, \tag{46}$$

and

$$\mathrm{Re}(K^-p \to \bar{K}^0 n)/\mathrm{Im}(K^-p \to \bar{K}^0 n) = 0\,. \tag{47}$$

We may compare these results with the results of the Regge-pole model[19,20]:

$$\begin{aligned}&\mathrm{Re}(K^-p \to \bar{K}^0 n)/\mathrm{Im}(K^0 p \to K^+ n) = 1\,, \\ &\mathrm{Re}(\pi^-p \to \pi^0 n)/\mathrm{Im}(\pi^-p \to \pi^0 n) = \tan\tfrac{1}{2}\pi\alpha_\rho\,, \\ &\mathrm{Re}(K^-p \to \bar{K}^0 n)/\mathrm{Im}(K^-p \to \bar{K}^0 n) = \cot\pi\alpha_\rho\,, \end{aligned} \tag{48}$$

where $\alpha_\rho = 0.5$. This result, of course, could be accidental, although amusing.

ACKNOWLEDGMENT

One of the authors (C. V. S.) would like to thank the Ministry of Education of the Government of India for financial help.

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PHYSICAL REVIEW D VOLUME 1, NUMBER 1 1 JANUARY 1970

Test of Duality in π^+p Backward Angular Distributions*

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A model in which the backward π^+p angular distribution is assumed to arise from a sum of direct-channel resonances is compared with the data at intermediate energies. Many as yet undiscovered resonances are assumed to exist as Regge recurrences of experimentally known resonances. The agreement between the results of the model and experiment is qualitatively quite good at angles near the backward direction for incident pion momenta between 2.2 and 5.0 GeV/c. In particular, the position and the shape of the minimum around $u \simeq -0.2$ (GeV/c)^2 in the differential cross section is correctly obtained. Since a similar minimum has previously been obtained in a model with Regge-pole exchange in the crossed channel, the result of the calculation gives support to the concept of duality at intermediate energies.

1. INTRODUCTION

One of the most striking features of π^+p elastic scattering at intermediate[1–4] and high[5] energies near the backward direction[6] is a minimum in the differential cross section near $u = -0.2$ (GeV/c)^2, where u is the square of the four-momentum transfer from the

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[6] For a summary of the experimental information, see M. Derrick, in *Proceedings of the Topical Conference on High Energy Collisions on Hadrons, CERN, 1968* (Scientific Information Service, Geneva, 1968), p. 111.

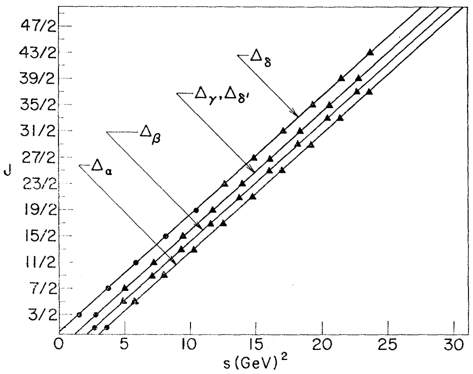

FIG. 1. Regge trajectories showing the resonances used in our model. The solid circles denote observed resonances, while the triangles denote postulated resonances.

pion to the nucleon.[7] Using a Regge-pole model with baryon exchange, Chiu and Stack[8] and Barger and Cline[9] have obtained qualitative agreement with experiment at incident pion momenta above 4 GeV/c.

It is the purpose of this paper to point out that at intermediate pion momenta (2.2–5.0 GeV/c), this dip can be obtained in a model in which the differential cross section away from the forward diffraction peak is assumed to arise solely from direct-channel resonances. This result is consistent with the idea of duality.[10] By this we mean that at intermediate energies, the amplitude can be approximated *either* by Regge-pole exchange in the crossed channel *or* by a sum of direct-channel resonances. In this sense, our definition of duality differs from the one of Ref. 10 in that we represent the entire amplitude, and not just its imaginary part, by resonance contributions. The near-backward direction is the best angular region in which to test duality, since in this region no appreciable contribution is expected to come from the Pomeranchuk Regge trajectory, which is usually associated with the nonresonant forward diffractive peak.

In our model, the dip in the differential cross section arises from destructive interference between resonances on different trajectories. In a previous calculation using only direct-channel resonances, Barger and Cline[11] used only one low-energy resonance not on the $\Delta(1236)$ trajectory. Such a model cannot give a minimum in the angular distribution near the backward direction at energies appreciably higher than the energy of this one additional resonance. Although Barger and Cline obtained rather good agreement with the observed momentum dependence of the cross section at 180°, their predicted angular distribution shows no minimum near the backward direction.[12]

At present, there is experimental evidence for six π^+p resonances not on the $\Delta(1236)$ trajectory,[13] and all of these are at relatively low energy. Therefore, at intermediate energies, in order to obtain the dip in the angular distribution in a resonance model, we must assume the existence of resonances which have not yet been discovered.

In Sec. 2 we discuss how we choose the postulated resonances. In Sec. 3 we show a comparison of the results of the model with experimental data at intermediate momenta. Finally, in Sec. 4 we discuss our results.

2. DETAILS OF MODEL

Since a π^+p amplitude constructed from the known resonances is inadequate to explain the data at intermediate energies, we postulate that there exist additional as yet undiscovered resonances which are the Regge recurrences of the established ones.

The compilation of the Particle Data Group[13] lists five π^+p resonances on the well-established Δ_δ trajectory, although the spins of the two resonances of highest mass have not been measured, and the parity of the highest resonance is not known. In addition, the compilation lists six other possible states. Of these, two are considered good, or well-established, two are considered fair, and two are considered poor. Of these six states, four can be considered to be the lowest-mass states of four different Regge trajectories. These states are

$$\Delta(1630,\tfrac{1}{2}^-)\,,\quad \Delta(1670,\tfrac{3}{2}^-)\,,\quad \Delta(1690,\tfrac{3}{2}^+)\,,\quad \Delta(1930,\tfrac{1}{2}^+)\,.$$

Our notation is that Δ stands for a π^+p resonance whose mass (in MeV), spin, and parity are given in parentheses. We shall assume the existence of the resonances lying on these four Regge trajectories, which we take parallel to the Δ_δ trajectory, as shown in Fig. 1. We do this in order to avoid introducing adjustable parameters to characterize the intercepts of the new trajectories. This procedure seems reasonable since there is evidence that all Regge trajectories have the universal slope of $\sim$1 GeV^{-2}. In Fig. 1, the resonances for which experimental evidence exists are shown by small solid circles and the postulated resonances by triangles. We use subscripts α, β, γ, and δ on the symbol Δ to denote the signature and parity of a trajectory, where

$$\alpha=++\,,\quad \beta=+-\,,\quad \gamma=--\,,\quad \delta=-+\,,$$

with the first plus or minus sign standing for the signature and the second for the parity. Since we are assuming the existence of two Δ_δ trajectories, we distinguish the second from the first by a prime.

In addition to the four resonances above, the Particle Data Group lists two possible Δ resonances of spin $\frac{5}{2}$, one

[7] We let s, t, and u be the usual Mandelstam variables; S. Mandelstam, Phys. Rev. **115**, 1741 (1959).

[8] C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).

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[12] N. Dikmen [Phys. Rev. Letters **18**, 798 (1967)] has fitted the backward π^-p (180°) data using only direct-channel resonances, and has argued the need for fitting the angular distribution as well as the 180° data. He has also commented [Phys. Rev. Letters **22**, 622 (1969)] that the forward π^-p angular distribution is sensitive to the spins and parities of resonances in the direct channel.

[13] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969), see especially pp. 123, 164, and 165.

of which is fair and the other poor. If these states exist, they are expected to be the Regge recurrences of two states of spin $\frac{1}{2}$. Since no such states have been seen, and since low-mass states are more likely to be observed than their recurrences, we shall not include these possible trajectories in our model. From Fig. 1 we can read the masses, spins, and parities of the postulated new resonances. However, there are two additional properties of a resonance which we need to know: its total width Γ and its elasticity x, which is the ratio of the partial width for elastic scattering to the total width. In assigning these parameters to the postulated resonances, we are again guided by the behavior of the corresponding parameters for the resonances on the known Δ_δ trajectory. If we plot the widths Γ_i of the known resonances which lie on the Δ_δ trajectory against their masses M_i, we find that they fall approximately on a straight line of the form

$$\Gamma_i=\Gamma_1+a(M_i-M_1)\,, \tag{1}$$

where M_1 is the mass, Γ_1 is the width of the lowest-energy resonance on the trajectory, and a is a constant which is characteristic of the trajectory. For the Δ_δ trajectory, we have

$$M_1=1236\text{ MeV}\,,\quad \Gamma_1=120\text{ MeV}\,,\quad a=0.161\,.$$

The extent of the agreement with a straight line is shown in Fig. 2.

By analogy, we assume that the widths of the resonances on the other trajectories also lie on straight lines of the same form as that of Eq. (1), but we let the constants a be parameters which we can vary so as to improve the agreement with experiment. In Fig. 2 we also show the widths of the resonances on the other trajectories, using adjusted parameters. In Sec. 3 we describe the data used in obtaining the values of the parameters a for the different trajectories.

The elasticities x_i of the known resonances on the Δ_δ trajectory exhibit an approximate exponential falloff in the variable s, which is the square of the center-of-mass energy. We therefore assume that the elasticities can be

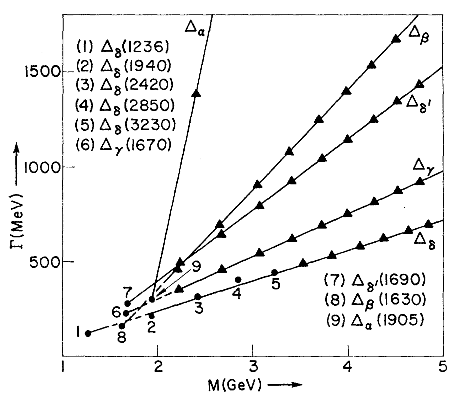

FIG. 2. Widths of the resonances versus their masses. In the case of the Δ_δ trajectory, the slope of the line is taken from experiment, while the other slopes are adjusted to fit the data of Ref. 1.

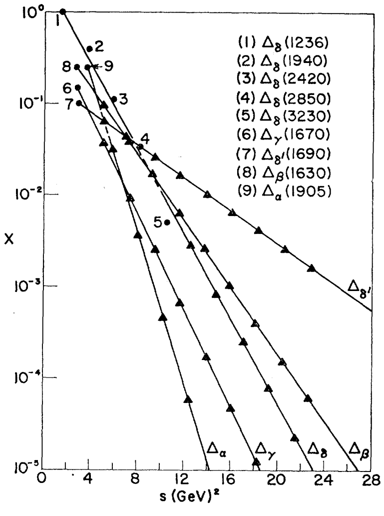

FIG. 3. Elasticities of the resonances versus the squares of their masses. The slope of the exponential falloff is taken from experiment in the case of the Δ_δ trajectory, while for the others the slopes are adjusted to fit the data of Ref. 1.

approximated by the equation

$$x_i=x_1\exp[-b(s_i-s_1)]\,, \tag{2}$$

where x_1 is the elasticity and s_1 the square of the mass of the lowest-energy resonance on the trajectory. The constant b is assumed to be characteristic of a particular trajectory. For the Δ_δ trajectory we have

$$x_1=1\,,\quad s_1=(1236\text{ MeV})^2\,,\quad b=0.54\text{ GeV}^{-2}.$$

The agreement between the observed elasticities of the Δ_δ trajectory and the values from Eq. (2) is shown in Fig. 3. We assume that the elasticities of the resonances on the other trajectories are also given by Eq. (2), but the parameters b are varied so as to obtain a best fit to the data. The adjusted elasticities of the other trajectories are also shown in Fig. 3.

Altogether, therefore, we allow eight parameters to vary, four determining the widths of the postulated resonances and four determining the elasticities. As can be seen from Figs. 2 and 3, none of the postulated resonances has a large enough elasticity or a small enough width to make it very likely that it would have been seen.

One difficulty in using a model with broad overlapping resonances is that we do not have any guide for the behavior of the scattering amplitude at energies appreciably different from the resonance energies. In particular, there is no evidence that the resonance "tails" have the characteristic Breit-Wigner shape. Following Feshbach, Peaslee, and Weisskopf,[14] we have parametrized a resonance such that its contribution to the amplitude vanishes for energies E more than a full width away from the resonance energy. We assume a resonance amplitude A, given by

$$A=x(\cot\delta-i)^{-1}\,, \tag{3}$$

[14] H. Feshbach, D. C. Peaslee, and V. F. Weisskopf, Phys. Rev. **71**, 145 (1947).

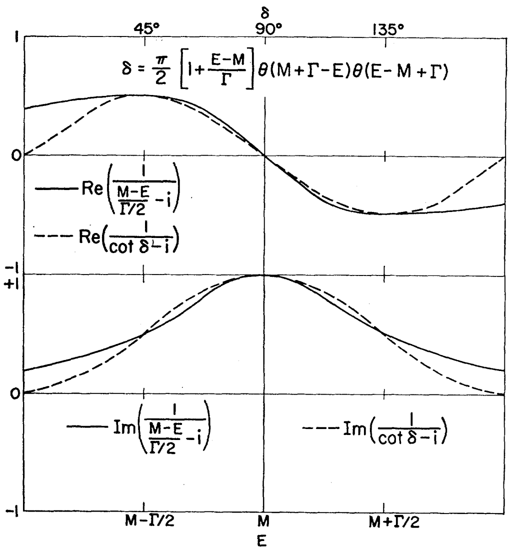

FIG. 4. Comparison of the shapes of the real and imaginary parts of the resonant amplitude given by Eqs. (3) and (4) with the usual Breit-Wigner amplitude of Eq. (5). We have taken $x=1$ for both amplitudes.

where

$$\delta=\tfrac{1}{2}\pi[1+(E-M)/\Gamma]\ \theta(M+\Gamma-E)\ \theta(E-M+\Gamma). \quad (4)$$

Here θ is the step function.

In Fig. 4 we show a comparison of the real and the imaginary part of a resonant amplitude parametrized by Eqs. (3) and (4) as compared to the usual Breit-Wigner form in which A is given by

$$A=x[2(M-E)/\Gamma-i]^{-1}. \quad (5)$$

We see that there is little difference between the two curves in the resonance region, but the long-tail contribution present in (5) is eliminated using (3) and (4).

3. COMPARISON WITH DATA

Since we have specified the masses, spins, parities, widths, elasticities, and shapes of the resonances, the parametrization of our amplitude is fixed with eight adjustable parameters. We vary these parameters so as to obtain a best fit to the data of Chandler *et al.*[1] We use the data of this experiment because it covers the intermediate momentum range (2.2–5 GeV/c) in the backward region in rather small intervals of momentum and angle, and because it should have a consistent over-all normalization. The extent of the agreement of the model with the experiment is shown in Fig. 5. The fit to 210 data points has a χ^2 of 720. Thus, the model does not give quantitative agreement with experiment, but has the correct qualitative features, especially the dip in the angular distribution at $u=-0.2$ (GeV/c)^2. The position of the dip comes out correctly, and is remarkably insensitive to our choice of parameters for the postulated resonances. The shape of the dip, however, is rather sensitive to this choice of parameters.

The final values of the eight adjustable parameters are given in Table I. These values have been used in plotting the widths and elasticities of the resonances on the Δ_α, Δ_β, Δ_γ, and Δ_δ' trajectories, as shown in Figs. 2 and 3. For comparison we also list the fixed values of these parameters for the Δ_δ trajectory. Table I also contains fixed values of M_1, Γ_1, and x_1 for all the trajectories.

We can now compare the results of the model with the other available data in the same momentum and angular range.[2–4] This is done in Fig. 6. The model is in rather good agreement with the data, but becomes progressively worse at lower momenta.

In Fig. 7 we show a comparison of the predicted momentum dependence of the 180° cross section with the data.[2–5,15–17] We have extended the momentum range outside the region where we expect the model to work best. Again, the model gives a remarkably good qualitative agreement with experiment, with the minima occurring at the right momenta. This is especially interesting in view of the fact that, in adjusting our parameters, we did not use any of the considerable amount of data that exists for $\cos\theta<-0.98$, or any of the data outside the momentum range 2.2–5 GeV/c.

4. DISCUSSION

Using a model in which the π^+p amplitude is composed of a sum of resonant amplitudes, we have obtained qualitative agreement with the observed elastic scattering cross section near the backward direction in the momentum interval 2.2–5.0 GeV/c. In particular, we have obtained the minimum in the angular distribution near $u=-0.2$ (GeV/c)^2, a result that has been previously obtained only in a model with Regge-pole exchange. This result gives support to the concept of duality as we have used it in the Introduction. In this connection, Chiu and Der Sarkissian,[18] in a study of

TABLE I. Values of the parameters identifying the Regge trajectories. [For the definition of these parameters, see Eqs. (1) and (2) and the accompanying text.] In the first three columns the masses, widths, and elasticities of the first resonance on each trajectory are taken from Ref. 11. The values of the parameters a and b are fixed for the Δ_δ trajectory, but are varied for the other trajectories in order to obtain a best fit to the data. The adjusted parameters are italicized.

Trajectory	M_1 (MeV)	Γ_1 (MeV)	x_1	a	b
Δ_δ	1236	120	1	0.161	0.540
Δ_α	1905	300	0.25	*2.139*	*0.970*
Δ_β	1630	160	0.25	*0.524*	*0.417*
Δ_γ	1670	225	0.15	*0.226*	*0.632*
Δ_δ'	1690	280	0.10	*0.374*	*0.207*

[15] J. A. Helland, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and C. D. Wood, Phys. Rev. **134**, B1062 (1964).
[16] P. J. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, and J. J. Thresher, Phys. Rev. **149**, 1077 (1966).
[17] T. Drobrowolski, B. N. Gus'kov, M. F. Likhachev, A. L Lubimov, Yu. A. Matulenko, V. S. Stavinsky, and A. S. Vovenko, Phys. Letters **24B**, 203 (1967).
[18] C. B. Chiu and M. Der Sarkissian, Nuovo Cimento **55**, 396 (1968).

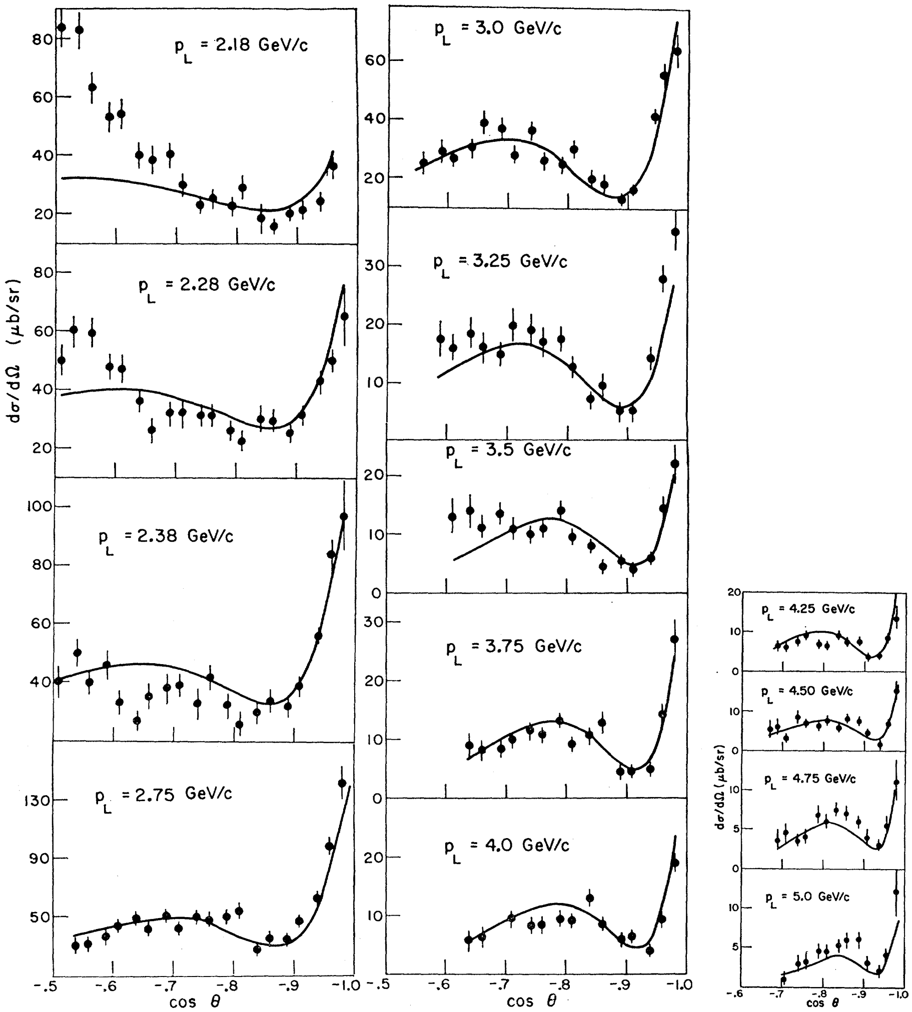

FIG. 5. Eight-parameter fit of the model to the π^+p elastic scattering of Ref. 1 in the angular region $-0.98 \lesssim \cos\theta \lesssim -0.6$.

finite-energy sum rules,[10] have found that the low-energy backward data require a dip-producing factor α_N in the nucleon Regge residue. This result also supports duality in the sense of Dolen, Horn, and Schmid.[10]

In our model, we have included only contributions from the resonances given in Fig. 1. Since the elasticities of these resonances are assumed to decrease exponentially as their masses squared, our results are not significantly altered by omitting a few of the highest-energy resonances of Fig. 1 or by including additional resonances on these trajectories.

We expect the model to be useful only at intermediate energies. At low energies, complete phase-shift analyses have been done and show that a nonresonant background exists. At high energies, on the other hand, since the model contains resonances with exponentially decreasing elasticities on only a finite number of trajectories, it cannot lead to the observed Regge asymptotic behavior.

Also, we do not expect this model to be applicable near the forward direction. Processes in which no prominent resonances have been observed, such as K^+p scattering, exhibit a forward diffraction peak. Therefore, we expect that the π^+p forward diffraction peak arises mostly by the unitarity effects of a nonresonant absorptive amplitude. At low energy, diffractive effects may not be confined to the forward direction, and this can account in part for the fact that at 2.18 GeV/c our fit is rather poor for $\cos\theta > -0.7$. The predictions of the model at still lower incident momenta are in even greater disagreement with the data.

Although the model is based on the assumption that only direct-channel resonances contribute to the amplitude, the model does in effect include a contribution from a slowly varying amplitude. We see this because the widths of most of the resonances on the Δ_α trajectory are larger than their masses. On the other hand, these resonances are very inelastic (the parameter b is large), so that the contribution from this smooth amplitude is small.

It is worthwhile to comment on the reason we used resonance amplitudes without long tails rather than the

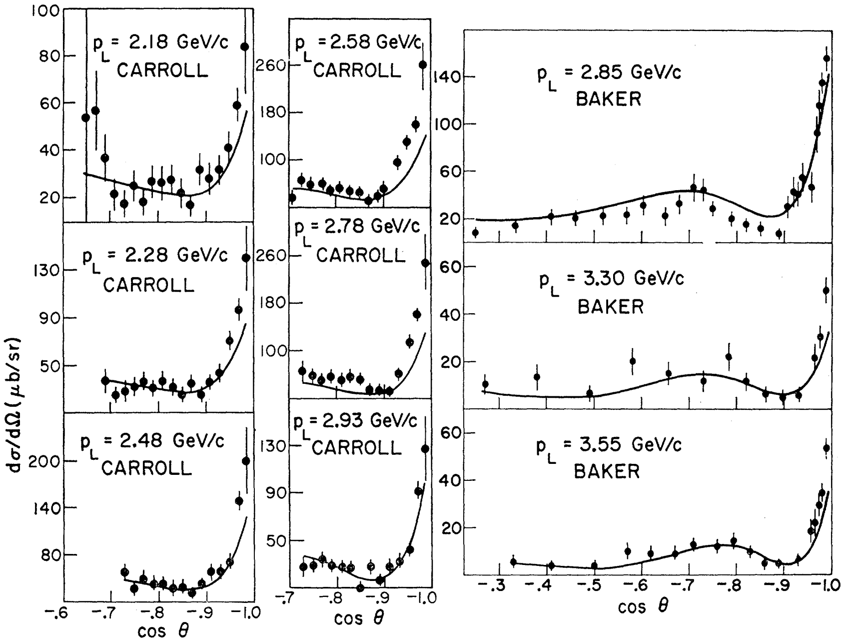

FIG. 6. Comparison of the results of the model to the π^+p elastic scattering data of Ref. 2 and Ref. 3.

usual Breit-Wigner form. If a Breit-Wigner form is used, the high-energy contribution from the tail of a low-energy resonance goes as $s^{-1/2}$. (Some authors use a modified Breit-Wigner form with a tail that goes like $1/s$.) On the other hand, the contribution from high-energy resonances decreases like e^{-bs} because of the elasticity factor. Thus, if we use a Breit-Wigner form, we find that at high energy the amplitude is dominated by the tails of the low-energy resonances. We do not believe that such a result is in the spirit of a resonance model. On the other hand, in the present model the contribution to the amplitude at any energy is dominated by the nearby resonances, with a small background from the broad-width resonances of the Δ_α trajectory.

We should remark that not all the resonances we have taken as "known" are well established. Furthermore, the masses, widths, and elasticities of the observed resonances are not known with precision. The Particle Data Group does not list errors in these parameters, but an examination of Table II of the compilation of this group[13] shows that the masses may be in error by as much as 30 MeV, the widths by 50 MeV, and the elasticities by 20%. In our fit to the data, we have taken the quoted masses, widths, and elasticities as fixed numbers. This means that at low energies we have in effect a zero-parameter fit to the data except for a small background arising from the tails of the broad high-energy resonances. This fact accounts in part for the poorness of the fit at low energies. If we had allowed the parameters of the low-energy resonances to vary within their experimental errors, we would have improved the agreement with experiment. Furthermore, the parameters of the higher-energy resonances would also have changed.

For these reasons, if the resonances which we have postulated really exist, their widths and elasticities may be very different from our values. Nevertheless, we can comment that if our parameters are approximately correct, the resonances on the Δ_α trajectory will not be

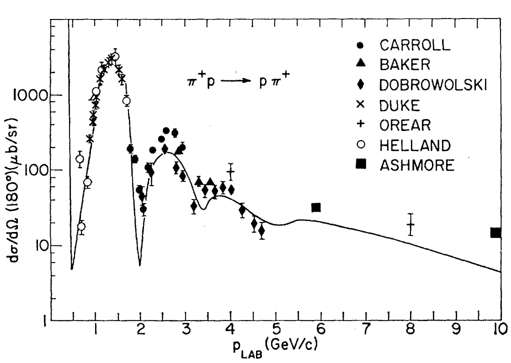

FIG. 7. Comparison of the observed momentum dependence of the 180° π^+p cross section with the results of the model. The data are taken from Refs. 2–5 and 15–17.

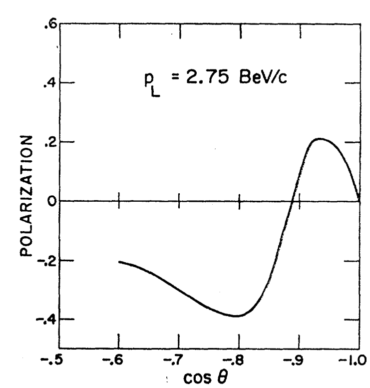

FIG. 8. Polarization at 2.75 GeV/c as predicted by the model.

seen because of their very broad widths and low elasticities. Several of the resonances on the other trajectories might be seen, however. The resonances of the Δ_δ' trajectory have relatively high elasticities in our model, a property that favors observation, but they have large widths, a property which makes observation difficult. The situation is the opposite for the resonances of the Δ_γ trajectory, which have relatively small widths (favoring observation) and small elasticities (making observation difficult).

It is interesting, however, that the use of the set of resonances proposed in this paper gives[19] a fair agreement to the observed π^+p total cross section once the diffraction contribution has been subtracted out.

An important test of the model would be a comparison with polarization data. Unfortunately, only preliminary π^+p backward polarization data exist.[20] Therefore, without attempting to make a fit to these preliminary data, we simply predict the polarization for an incident momentum of 2.75 GeV/c. This is shown in Fig. 8. The remarkable feature of this prediction is that it qualitatively agrees with the preliminary data of Ref. 18 in that it is positive in the backward direction and changes sign near the position of the dip in the angular distribution. Since the polarization depends very sensitively on the exact values of the parameters, we expect that a slight change in the parameters could lead to a more quantitative agreement with the final data.

Lacking more complete experimental data on higher recurrences of the Regge trajectories, one might consider comparing our elasticities (Fig. 3) with those predicted by the Veneziano model.[21] At the moment the conclusion seems to be that the Veneziano model "...fails to provide an adequate extrapolation from the scattering data to the widths of the Δ_δ (1238) and its recurrences."[22]

We do not wish to claim that all the trajectories of our model really exist. Our main point is that by including resonances lying on trajectories other than the Δ_δ, we can obtain the qualitative characteristic behavior of the observed π^+p cross section in an angular range near the backward direction. We therefore take it as quite plausible that at least one additional Δ trajectory exists. Such a trajectory is necessary if duality is meaningful at these energies.

Because of the reasonable success of the model, we are at present extending the calculations to include the angular distributions and polarization at all angles, adding the forward diffraction peak as a separate contribution.

ACKNOWLEDGMENTS

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Electroproduction of Nucleon Resonances and Implications for Coincidence Experiments*

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The general form of the coincidence cross section for electroproduction of nucleon resonances is reviewed and discussed using the helicity formalism. The predictions of a coupled-channel relativistic N/D model are discussed for nucleon levels in the first four resonance regions. These predictions serve to indicate important questions concerning the structure of the nucleon, and consideration is given as to how these questions may be answered by means of coincidence experiments.

I. INTRODUCTION

ELECTRON excitation has now become a practical means for studying the detailed structure of the excited states of the nucleon. In most experiments performed so far, only the final electron has been detected. Some experiments detecting the final charged pion or nucleon in coincidence with the electron have been performed in the $N^*(1236)$ region,[1–5] and extensive coincidence experiments are planned for the higher

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