sensitive to the choice of the energy intervals. In each slice the "age" parameter s is computed as a function of E, ρ, and t by finding the roots of Eq. (15). However the contribution from $\Gamma_{N\pi}\hat{\Pi}_1^{(n)}(E,\rho,t)$ is negligible at energies lower than about 10^5 GeV.[3] In that case, $\hat{N}_1^{(n)}$ is independent of ρ, and by (3),

$$N_1(E,r,t)=N_1(E,t)\delta(r)/2\pi r.$$

Results were obtained for primary nucleons of 10^4, 10^5, and 10^6 GeV for showers developing in the atmosphere as well as in an ionization calorimeter, they are represented in Figs. 2–8. All figures refer to the number of particles with energy $E>2$ GeV.

V. DISCUSSION AND CONCLUSIONS

The pion-links method has already proved to be a useful guide in the physical interpretation of the development of the nuclear active component of one-dimensional extensive air showers.[4] It gives a most important role to pion-nucleon collisions.

Comparing the lateral structure function for the same primary energy in the atmosphere and in the ionization calorimeter, it can be seen that in the last case the curves are more flat. This is a consequence of the fact that the decay affects more those pions which are farther away from the shower axis and are less energetic. The effect is smaller at greater depth as could be expected.

Figure 8 shows that the number of particles per unit area grows faster with the primary energy for small values of r, where the $S_{\pi\pi}$ contribution is more important. In Fig. 2 comparison is made with the Monte Carlo calculation of Thielheim and Beiersdorf.[2] The Monte Carlo curve drops out a little too fast compared to the measurements of Abrosimov *et al.*,[9] Nikolsky *et al.*,[10] and Chatterjee *et al.*[11] The curves calculated here are in good agreement with those experimental data. Finally, the method used here has proven to be fast: For $E_0=10^6$ GeV, the computing time is less than 30 min.

ACKNOWLEDGMENTS

The author is indebted to G. B. Yodh, J. E. Moyal, I. K. Abu-Shumays, and J. R. Wayland for many helpful discussions and encouragement during the course of this work. He also thanks B. Garbow for useful suggestions on the computer program.

[9] A. I. Abrosimov *et al.*, Zh. Eksperim. i Teor. Fiz. **29**, 693 (1955).
[10] S. I. Nikolsky *et al.*, Dokl. Akad. Nauk SSSR **111**, 71 (1956).
[11] B. K. Chatterjee *et al.*, Can J. Phys. **46**, S136 (1968).

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Energy Loss of High-Energy Cosmic Rays in Pair-Producing Collisions with Ambient Photons

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General and essentially exact formulas for the energy-loss process $A+\gamma\to A+e^++e^-$ are given for high-energy cosmic-ray nuclei traversing an isotropic radiation field of arbitrary energy spectrum. These formulas are obtained by integrating the appropriate differential cross section in the rest frame of the cosmic-ray particle. Results are then given for the case of a Planckian photon distribution at arbitrary temperature. The effect of this energy loss on a universal flux of cosmic rays is then investigated along with the possibility that the observed "knee" at high energies is due to pair production. Finally, the energy-loss formula is applied to quasistellar objects, and it is shown that if these objects are at cosmological distances, they are opaque to particles above a certain energy which depends upon the size of the object.

I. INTRODUCTION

THE discovery of a large flux of microwave radiation by Penzias and Wilson[1] has led to the investigation of numerous interactions between high-energy cosmic rays and a background of low-energy photons. These reactions occur essentially because a low-energy photon can Lorentz transform into a γ ray in the rest frame of a very-high-energy particle. Greisen[2] showed that a cosmic-ray proton with energy above 10^{18} eV loses its energy in slightly less than one Hubble time due to pair production in collisions with a 3°K blackbody background. At higher energies, just under 10^{20} eV for a proton, photomeson production becomes the dominant energy-loss mechanism. At this energy cosmic rays are able to travel a distance of about 10 Mpc without being severely attenuated.[2–4]

[1] A. A. Penzias and R. W. Wilson, Astrophys. J. **142**, 419 (1965).

[2] K. Greisen, Phys. Rev. Letters **16**, 748 (1966).
[3] G. T. Zatsepin and V. A. Kuzmin, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **4**, 114 (1966) [English transl.: Soviet Phys.—JETP Letters **4**, 78 (1966)].
[4] F. W. Stecker, Phys. Rev. Letters **21**, 1016 (1968).

Finally, for heavy nuclei at high energies, photodisintegration in collisions with dilute optical radiation becomes important.[2,5]

All of these processes may have a significant effect upon the spectrum of high-energy cosmic rays. Greisen[2] and Kuzmin and Zatsepin[6] showed that if high-energy primaries are universal, their spectrum should cut off at 10^{20} eV because of photomeson production, and below this cutoff the attenuation due to pair production may cause a steepening of the spectrum. Encrenaz and Partridge[7] then interpreted the observed flattening of the primary spectrum above 10^{18} eV, in spite of pair and photomeson production, as indicating that these cosmic rays are produced locally; they also showed that the results of Shivanandan *et al.*,[8] that the microwave background near 1 mm is two orders of magnitude higher than that predicted for a 3°K blackbody background, would probably produce a severe steepening of the cosmic-ray spectrum above 10^{17}–10^{18} eV. Finally, Hillas[9] proposed a model for the evolution of cosmic rays produced at cosmological distances and attempted to explain the form of the cosmic-ray spectrum at high energies as due to pair production at an earlier epoch in which the temperature of the universal blackbody radiation is higher and in which the energy of a relativistic cosmic ray is greater because of red shift; thus the threshold for pair production *then* corresponds to a much lower energy at the present epoch.

This paper is concerned with the process of pair production. In Sec. II general formulas are derived for the energy loss of a high-energy nucleus due to pair production with ambient photons. A similar calculation was performed by Feenberg and Primakoff[10] using the extreme relativistic approximation for the relevant pair-production cross section. In the following calculation the exact cross section from quantum electrodynamics is used and, therefore, more accurate results are obtained. These results are then applied in Sec. III to the case of a blackbody distribution at arbitrary temperature. Section IV then treats various applications of these general formulas. The effect of pair production upon the various models for the origin of high-energy cosmic rays is investigated. Also, the attenuation of high-energy nuclei traversing the dense radiation fields found in quasistellar objects is calculated, and it is shown that a cutoff occurs above an energy which depends upon the size and distance of these objects.

[5] F. W. Stecker, Phys. Rev. **180**, 1264 (1969).

[6] V. A. Kuzmin and G. T. Zatsepin, Can. J. Phys. **46**, S617 (1968).

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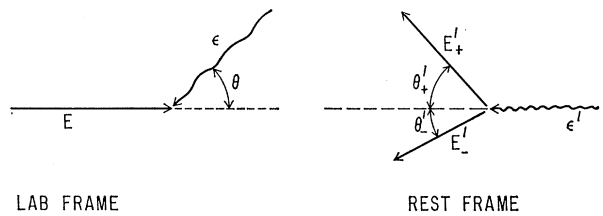

FIG. 1. Collision of a photon and cosmic-ray nucleus viewed in both the laboratory and rest frames.

II. DERIVATION OF GENERAL FORMULAS

Consider the situation shown in Fig. 1: a high-energy nucleus with energy E and Lorentz factor $\gamma=(1-v^2/c^2)^{-1/2}$ colliding with a photon of energy ϵ at angle θ. Then, the energy of the photon in the rest frame of the cosmic-ray particle is

$$\epsilon'=\gamma\epsilon(1+\beta\cos\theta)\,, \qquad (1)$$

and clearly the threshold condition for pair production is that $\epsilon'>2mc^2$. Furthermore, since it is assumed throughout this calculation that $\gamma\gg1$, it follows that an isotropic photon distribution in the laboratory frame will, in the rest frame, mainly be confined to an angle $\sim1/\gamma$ with the direction of the transformation.[11] Therefore, the head-on approximation is adopted, and it is assumed that in the rest frame all photons approach the nucleus from its direction of motion. This approximation causes little error at the energies considered here so long as the photons are distributed isotropically in the lab. Then, in the rest frame, an electron-position pair is produced with energies E_-' and E_+' making angles θ_-' and θ_+', respectively, with the direction of the incident photon.

To obtain the rate of energy loss, use is made of the fact that in the laboratory, the rate at which the nucleus loses energy equals the rate in which the electrons and positrons gain energy minus the rate at which energy is lost by the ambient photons. But as long as $\epsilon\ll mc^2$ this last contribution is completely negligible, and thus

$$-dE/dt=dE_+/dt+dE_-/dt\,. \qquad (2)$$

Upon transforming to the rest frame and noting that $dt=\gamma dt'$, Eq. (2) becomes

$$-dE/dt=(d/dt')[E_+'+E_-'-\beta c(p_+'\cos\theta_+'+p_-'\cos\theta_-')]\,. \qquad (3)$$

However, for $\gamma\gg1$, it follows that if $\epsilon^2\ll m^2c^4$, then β can be set equal to 1 in the above formula.

The differential cross section for pair production given in the rest frame of the target nucleus was obtained originally by Bethe and Heitler[12] and Racah[13];

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[12] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).

[13] G. Racah, Nuovo Cimento **11**, 461 (1934).

it also follows readily from covariant quantum electrodynamics.[14] All treatments, however, neglect the recoil energy of the nucleus which is down by a factor of $m/M<10^{-3}$. This cross section, $d\sigma/dE_-'d\cos\theta_+'d\cos\theta_-'$, is completely invariant with respect to interchange of the plus and minus subscripts, and therefore, in the Born approximation, the electron and positron behave identically. Equation (3) then becomes

$$-dE/dt=2d\Delta_-'/dt', \tag{4}$$

where $\Delta_-'=E_-'-cp_-'\cos\theta_-'$. The total energy loss is then given by an integral over the cross section as follows:

$$-\frac{dE}{dx}=2\int_{2mc^2}^{\infty}d\epsilon'\,n'(\epsilon')\int_{mc^2}^{\epsilon'-mc^2}dE_-'\int_{-1}^{+1}d\cos\theta_-' \times\Delta_-'\frac{d\sigma}{dE_-'d\cos\theta_-'}, \tag{5}$$

where $n'(\epsilon')$ is the differential number density of photons in the rest frame. Note that Eq. (5) represents energy lost per unit distance traveled ($dx=cdt$). Using the fact that for an isotropic photon distribution in the lab,

$$\frac{n'(\epsilon')d\epsilon'}{\epsilon'}=\frac{n(\epsilon)d\epsilon d\cos\theta}{2\epsilon}, \tag{6}$$

the ϵ' integration can be transformed to lab quantities. This yields

$$-\int\frac{dE}{dx}=\gamma\int_{mc^2/\gamma}^{\infty}d\epsilon\,n(\epsilon)\int_{2mc^2/\gamma\epsilon-1}^{+1}d\cos\theta\,(1+\cos\theta) \times\int_{mc^2}^{\epsilon'-mc^2}dE_-'\int_{-1}^{+1}d\cos\theta_-'\,\Delta_-'\frac{d\sigma}{dE_-'d\cos\theta_-'}. \tag{7}$$

Finally, by dropping the primes and expressing all energies in terms of mc^2, one can define the dimensionless function

$$A(k,E_-)=\frac{2k}{\alpha Z^2r_0^2}\int_{-1}^{+1}d\cos\theta_-\,\Delta_-\frac{d\sigma}{dE_-d\cos\theta_-}, \tag{8}$$

where r_0 is the classical electron radius, $k=\epsilon'/mc^2$, and Z is the charge on the nucleus under consideration. Then Eq. (7) is simplified and becomes

$$-\frac{dE}{dx}=\alpha r_0^2Z^2(mc^2)^2\int_2^{\infty}\frac{d\xi}{\xi^2}n\left(\frac{\xi mc^2}{2\gamma}\right) \times\int_2^{\xi}dk\int_1^{k-1}dE_-\,A(k,E_-). \tag{9}$$

To evaluate the energy loss, one must first integrate over the cross section to obtain $A(k,E_-)$. From Eq. (8) it is clear that the differential pair-production cross section integrated over θ_+ is required. This integration was performed rather elegantly by Gluckstern and Hull,[15] who obtained[16]

$$\frac{d\sigma}{dE_-d\cos\theta_-}=\left(\frac{\alpha Z^2r_0^2p_-p_+}{2k^3}\right)\Bigg[-4\sin^2\theta_-\frac{2E_-^2+1}{p_-^2\Delta_-^4}+\frac{5E_-^2-2E_+E_-+3}{p_-^2\Delta_-^2}+\frac{p_-^2-k^2}{T^2\Delta_-^2}+\frac{2E_+}{p_-^2\Delta_-}+\frac{Y}{p_-p_+} \times\left(2E_-\sin^2\theta_-\frac{3k+p_-^2E_+}{\Delta_-^4}+\frac{2E_-^2(E_-^2+E_+^2)-7E_-^2-3E_+E_--E_+^2+1}{\Delta_-^2}+\frac{k(E_-^2-E_-E_+-1)}{\Delta_-}\right) -\frac{\delta_+^T}{p_+T}\left(\frac{2}{\Delta_-^2}-\frac{3k}{\Delta_-}-\frac{k(p_-^2-k^2)}{T^2\Delta_-}\right)-\frac{2y_+}{\Delta_-}\Bigg], \tag{10}$$

where

$$T=|\mathbf{k}-\mathbf{p}_-|,\quad Y=(2/p_-^2)\ln[(E_+E_-+p_+p_-+1)/k], \quad y_+=p_+^{-1}\ln[(E_++p_+)/(E_+-p_+)],\quad \delta_+^T=\ln[(T+p_+)/(T-p_+)]. \tag{11}$$

Note that the primes are dropped in Eq. (10); E_- is still the rest-frame electron energy, but expressed in units of mc^2. Using this cross section in Eq. (8), the integral over θ_- becomes elementary but somewhat tedious, and yields

$$k^2A(k,E_-)=\left(\frac{4p_+}{p_-}\right)\left(E_+-4E_--\frac{6E_-}{p_-^2}\right)+\left(\frac{2p_+y_-}{p_-}\right)\left(4E_-^2-E_-E_++4+\frac{6}{p_-^2}\right)-2p_-p_+y_++Y\Bigg[4E_+E_-^2+y_- \times\left(2E_-^2(E_-^2+E_+^2)-7E_-^2-5E_-E_+-E_+^2+1-\frac{6kE_-}{p_-^2}\right)+2k\left(1-E_-^2-E_-E_++\frac{6E_-^2}{p_-^2}\right)-p_-^2y_+\Bigg]. \tag{12}$$

Here, y_- is defined in analogy with Eq. (11).

[14] J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1955).
[15] R. L. Gluckstern and M. H. Hull, Phys. Rev. **90**, 1030 (1953); R. L. Gluckstern, M. H. Hull, and G. Breit, *ibid.* **90**, 1026 (1953).
[16] The pair-production cross section given in Ref. 15 is too small by an over-all factor of 2. When this factor is removed, the integral of their cross section over θ_- checks with the formula given by W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, England, 1954). The expression for $d\sigma/dE_-$ given in Ref. 14 contains a misprint.

The total cosmic-ray energy loss thus follows when Eq. (12) is substituted into Eq. (9). However, now the integration cannot be performed analytically but must be done numerically. This gives for the energy loss

$$-\frac{dE}{dx}=\alpha r_0{}^2Z^2(mc^2)^2\int_2^\infty d\xi\, n\left(\frac{\xi mc^2}{2\gamma}\right)\frac{\phi(\xi)}{\xi^2}, \qquad (13)$$

where

$$\phi(\xi)=\int_2^\xi dk\int_1^{k-1} dE_-\, A(k,E_-). \qquad (14)$$

Figure 2 contains a plot of the function $\phi(\xi)$.

Until now it has been assumed that the Born approximation to the pair-production cross section is exact. In reality, just above threshold, the produced electron and positron do not behave identically, because of the Coulomb force with the nucleus. For this reason, Coulomb rather than plane-wave states should be used to calculate the cross section. Such calculations have been carried out, and to a good approximation the corrected cross section differs from that obtained from the Born approximation by an over-all factor[14]

$$F(E_+,E_-)=\frac{\eta_+\eta_-}{(e^{\eta_+}-1)(1-e^{-\eta_-})}, \qquad (15)$$

with $\eta_\pm=2\pi\alpha Zc/v_\pm$. For relativistic particles, η and therefore $F(E_+,E_-)$ are nearly constant. Thus, since the electron and positron do not act identically, a factor of $\frac{1}{2}[F(E_+,E_-)+F(E_-,E_+)]$ should be included in the integral in Eq. (14). However, for $Z\sim1$, this factor causes little change in $\phi(\xi)$; indeed, for $Z=1$, virtually no change occurs in the graph of $\phi(\xi)$ shown in Fig. 2. At larger Z, $\phi(\xi)$ will be lowered by an almost constant factor.

Various treatments of energy loss due to pair production[2,10] have used an extreme relativistic, $\gamma\epsilon\gg mc^2$, approximation for the cross section. To obtain an asymptotic formula for the energy loss given by Eq. (13), it is not sufficient to use an extreme relativistic approximation to the cross section given in Eq. (10); such an approximation, which is identical to that obtained by Schiff,[17] when substituted into Eq. (8) for $A(k,E_-)$, leads to a logarithmically divergent integral. It is reasonable, however, to approximate the function $A(k,E_-)$, Eq. (12), in the extreme relativistic limit, since for $k=\epsilon'/mc^2\gg1$, the function goes to zero at the end points $E_-=1$, $k-1$. Then there is very little contribution from the nonrelativistic parts of the integrals in Eq. (14). Doing this, one obtains an extreme relativistic asymptotic formula for $\phi(\xi)$ with $\xi\gg1$:

$$\phi(\xi)\rightarrow\xi[-86.07+50.95\ln\xi-14.45(\ln\xi)^2+2.667(\ln\xi)^3]. \qquad (16)$$

When $n(\epsilon)$ is such that for all contributing photons,

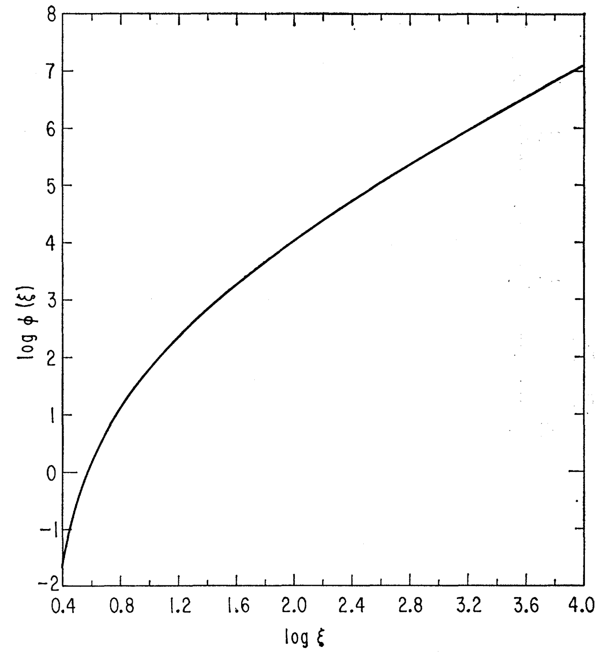

FIG. 2. Graph of the function $\phi(\xi)$ given in Eq. (14). Logarithms are to the base ten.

$\gamma\epsilon\gg mc^2$, then this value of $\phi(\xi)$ when substituted into Eq. (13) yields a good approximation for the total energy loss.

III. INTERACTION WITH BLACKBODY PHOTON GAS

Equation (13) gives an expression for the energy loss of a high-energy cosmic ray due to pair production

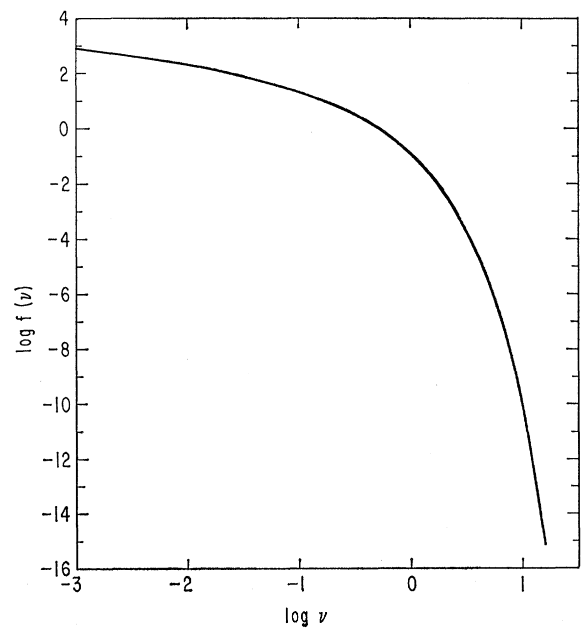

FIG. 3. Graph of $\log_{10}f(\nu)$ versus $\log_{10}\nu$, where $f(\nu)$ is defined by Eq. (20).

[17] L. I. Schiff, Phys. Rev. **83**, 252 (1951).

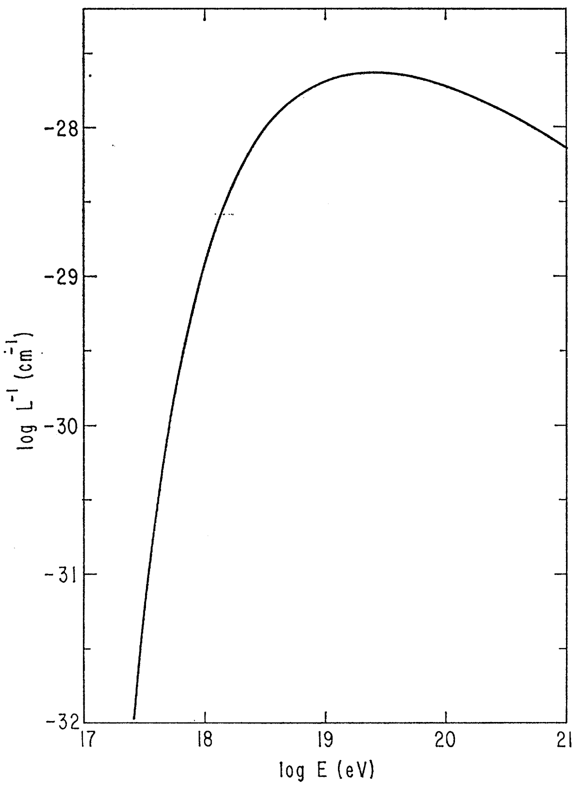

FIG. 4. Inverse radiation length, $E^{-1}dE/dx$, of cosmic-ray protons against pair production. The logarithms in this figure are to be base ten. Above 10^{20} eV, the form of their curve is unimportant since photomeson production dominates.

as an integral over the ambient photon spectrum. When this spectrum is that of a blackbody at temperature T,

$$n(\epsilon)=(\hbar c)^{-3}(\epsilon/\pi)^2(e^{\epsilon/kT}-1)^{-1}, \quad (17)$$

the energy loss can be cast into a more useful form. Using the dimensionless parameter

$$\nu=mc^2/2\gamma kT, \quad (18)$$

the energy loss becomes

$$-\frac{dE}{dx}=\frac{\alpha r_0{}^2Z^2(mc^2kT)^2}{\pi^2\hbar^3c^3}f(\nu), \quad (19)$$

where

$$f(\nu)=\nu^2\int_2^\infty d\xi\,\phi(\xi)(e^{\nu\xi}-1)^{-1}. \quad (20)$$

$f(\nu)$ is calculated for the $(Z\sim1)$ $\phi(\xi)$ found earlier and is shown in Fig. 3. It is clear from this figure that the threshold for pair production is at $\nu\sim1$ and that the energy loss then increases at a decreasing rate as ν decreases (or as γ increases).

For particles traversing a diluted blackbody photon gas of temperature T and energy density w, Eq. (19) is still applicable. However, this equation must then be multiplied by the dilution factor w/aT^4, where $a=7.57\times10^{-15}$ erg deg^{-4}.

IV. APPLICATIONS

A. Effect on Spectrum of High-Energy Cosmic Rays

In order to gauge the importance of pair-producing collisions with the microwave background upon the cosmic-ray spectrum, it is instructive to calculate an effective opacity for these particles. Using Eq. (19) and assuming a value of $T=2.7$°K for the microwave background,[18] the inverse radiation length

$$L^{-1}=E^{-1}dE/dx \quad (21)$$

is calculated and shown in Fig. 4. L is the distance over which a particle will lose essentially all of its energy. From Fig. 4 it is clear that the radiation length becomes less than the Hubble radius well above 10^{18} eV and that the opacity reaches a maximum near 10^{19} eV. The form of this curve above 10^{20} eV is irrelevant, since losses due to photomeson production dominate above this energy.[2,4] However, the form of this curve may have important consequences upon the various theories for the origin and evolution of cosmic rays.

Observational data, collected by Greisen,[19] on the integral spectrum of high-energy cosmic rays are shown in Fig. 5(a). This spectrum follows essentially a 1.6 power law until it steepens at about 10^{15} eV. The flattening of the spectrum above 10^{18} eV is probably still in doubt, but there do exist observations of extensive air showers well above this energy.[20,21] Some theories for the spectrum at high energies contain a transition from galactic to extragalactic cosmic rays. For example, Brecher and Morrison[22] assume that the break at 10^{15} eV is associated with the sources of these particles, since this will explain the break in the spectrum of the x-ray background. They then conclude that the flattening of the spectrum above 10^{18} eV represents a transition from galactic to extragalactic cosmic rays. From Fig. 4, these particles would not be attenuated within one Hubble time until nearly 10^{19} eV. Another explanation due to Hillas[9] assumes that cosmic-ray production has occurred since an early epoch in the universe when the universal blackbody temperature was higher; then, since very relativistic protons are red-shifted like photons, the threshold for pair production in an earlier epoch corresponds to a lower proton energy now.

This hypothesis can be studied using Eq. (19) for the energy loss due to a blackbody photon gas. The integral spectrum of cosmic rays, $n(E,z)$ = number of particles per unit proper volume with energy above E

[18] R. A. Stokes, R. B. Partridge, and D. T. Wilkinson, Phys. Rev. Letters 19, 1199 (1967).

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[21] D. Andrews, A. C. Evans, R. J. Reid, R. M. Tennent, A. A. Watson, and J. G. Wilson, Nature 219, 343 (1968).

[22] K. Brecher and P. Morrison, Phys. Rev. Letters 23, 802 (1969).

at red shift z, satisfies a continuity equation with z as an independent variable:

$$\frac{\partial n}{\partial z}+\left[\frac{E}{1+z}+\lambda(E,z)\right]\frac{\partial n}{\partial E}-\frac{3n}{1+z}=g(E,z), \quad (22)$$

for a homogeneous isotropic universe. Here, $\lambda(E,z)$ represents energy loss per unit red shift due to pair production and follow from Eq. (19) using the fact that in a Friedmann universe,

$$\frac{dx}{dz}=\frac{-cH_0^{-1}}{(1+z)^2(1+2q_0z)^{1/2}}, \quad (23)$$

and the fact that $T=T_0(1+z)$ for the microwave background. The $g(E,z)$ represents sources of cosmic rays at red shift z. Equation (22) is obtained by equating gains to losses inside a fixed coordinate volume and then transforming to proper volume. When $\lambda=0$, this equation follows directly from Liouville's equation. The equation essentially describes the energy distribution of a highly relativistic proton gas during the adiabatic expansion of the universe; the $\lambda(E,z)$ term includes the effects of continuous but nonadiabatic energy losses such as pair production. If magnetic fields are present in intergalactic space, then these cosmic rays are deflected and do not travel in straight lines. However, this fact does not alter the validity of Eq. (22) so long as the relativistic proton gas occupies a region of space which partakes in the general expansion of the universe. The assumption is therefore made here that these cosmic rays are not confined to a region of constant *proper* volume (such as the galaxy).

A solution of Eq. (22) for the present spectrum is obtained by solving the equation

$$dE/dz=\lambda(E,z)+E/(1+z) \quad (24)$$

for the function $E(E_0,z)$ such that $E(E_0,0)=E_0$. The spectrum then becomes

$$n(E_0)=-\int_0^y dz\,(1+z)^{-3}g(E(E_0,z),z), \quad (25)$$

where y is the lesser of $z_{\max}$ and that z at which photomeson production becomes important [e.g., where $(1+z)^2E_0\sim10^{20}$ eV]. It is assumed here that $g(E,z)$ is always a 1.6 power law of the form

$$g(E,z)=kE^{-1.6}(1+z)^{m+1}(1+2q_0z)^{-1/2}, \quad (26)$$

where m corresponds to a density evolution of cosmic-ray sources and is identical to the evolution introduced by Longair[23] in analyzing the radiosource count data.

From the above equations it is clear that there are, unfortunately, four free parameters in this calculation: the deceleration parameter q_0, the cosmic-ray source evolution m, the red shift at which sources "turn on," and the over-all normalization k. All but the last determine the shape of the present spectrum. In order that pair production steepen the present spectrum above 10^{15} eV, the evolution m must be near 4 and $z_{\max}\gtrsim15$. Two (unnormalized) spectra obtained from Eq. (25) are shown in Fig. 5. Figure 5(b) contains the solution for $q_0=\frac{1}{2}$, $m=4$, and $z_{\max}=15$, while Fig. 5(c) contains the spectrum when $q_0=0$, $m=3$, and $z_{\max}=50$. These spectra flatten between 10^{18} and 5×10^{19} eV because pair-production losses during the present epoch essentially cut off protons in this energy range from reaching us from large z, where photomeson production may be important. Thus, in this range the energy loss is roughly proportional to E, and the injection spectrum index is preserved. As long as $0\leq q_0\lesssim1$, it is possible to obtain a reasonably good fit to the experimental air-shower data with $m\approx4$ and $z_{\max}\gtrsim15$. The advantage of this cosmic-ray model is that one injection spectrum may account for the entire observed spectrum up to an energy of about 5×10^{19} eV. However, it is difficult for this model to account for the spectrum above 5×10^{19} eV, and if the existence of these very-high-energy showers is confirmed, then it may mean that these cosmic rays are produced locally.[2,4,9]

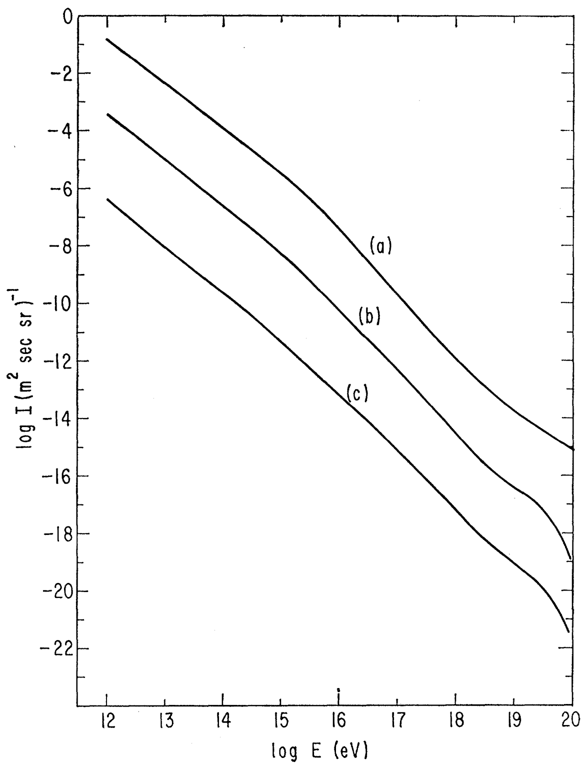

FIG. 5. (a) Logarithmic (base ten) plot of the integral spectrum of primary cosmic rays at high energies; (b) unnormalized spectrum for $q_0=\frac{1}{2}$, $m=4$, $z_{\max}=15$; (c) unnormalized spectrum for $q_0=0$, $m=3$, $z_{\max}=50$.

[23] M. S. Longair, Monthly Notices Roy. Astron. Soc. **133**, 421 (1966).

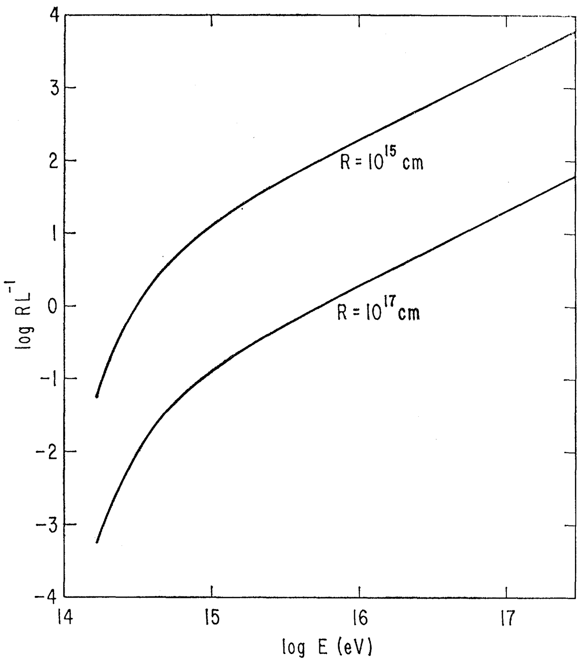

FIG. 6. Graph of the ratio of the radius of a quasistellar object to radiation length versus energy of high-energy protons. The logarithms are to the base ten. Above about 10^{17} eV, photomeson production will dominate over pair production.

B. Opacity of Quasistellar Objects to High-Energy Particles

Besides interactions with the microwave background, the process of pair production can be important in any region of space containing a large density of ambient photons. Quasistellar objects are likely candidates because of both their high luminosities and small radii.

This opacity can be calculated if the photon spectrum inside these objects is known. If quasistellar objects are at cosmological distances, then, typically, they have an optical luminosity of around 10^{46} erg/sec; furthermore, some objects such as *3C 273B* emit nearly 10^{47} erg/sec in the infrared between 10^{11}–10^{13} Hz.[24] Therefore, a total luminsoity of the form $\mathcal{L}(\epsilon)=5\times10^{57}\epsilon^{-1}$ (eV sec^{-1} eV^{-1}) between 5×10^{-4} and 5 eV fits the observations of the luminosity of *3C 273B* fairly well and is also consistent with *UBV* filter observations of the flux from a number of these objects. Then, noting that approximately $\epsilon n(\epsilon)\approx\mathcal{L}(\epsilon)/\pi R^2c$, the spectral photon density becomes

$$n(\epsilon)=5\times10^{46}\epsilon^{-2}R^{-2}\quad(\text{eV}^{-1}\text{ cm}^{-3}), \qquad (27)$$

where R is the radius of the object. However, because of the fast time variations of the flux from these objects, the radius of the emitting region must be in the range 10^{15}–10^{17} cm.[24] Using the above formula for $n(\epsilon)$, the ratio of the radius to the radiation length, Eq. (21), is calculated for protons and shown in Fig. 6. When $RL^{-1}\sim1$, then a particle cannot traverse the entire object without losing essentially all its energy. From this figure it is clear that pair production becomes important just above 10^{14} eV for $R=10^{15}$ cm and at about 4×10^{15} eV for $R=10^{17}$ cm. Again, above 10^{17} eV, photomeson production becomes the dominant energy-loss mechanism.

Thus, if quasistellar objects are at cosmological distances, one would expect any spectrum of high-energy protons within them to exhibit a cutoff at 10^{14}–10^{16} eV owing to pair production. This could be a source of very-high-energy electrons within these objects. The assumption that the ambient photon spectrum is isotropic has been made in the above calculation, but a more careful consideration would require a detailed knowledge of the nature of the emitting region and would not be likely to change the results by more than a factor of 2. Finally, Rees[25] has constructed a model for these objects in which the radius obtained from time variations is smaller than the actual radius because of relativistic expansion. This would dilute the photon density within the quasistellar objects and thus decrease RL^{-1} (Fig. 6).

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