Comments on the Current-Algebra Derivation of the $\Delta I=\frac{1}{2}$ Rule in Nonleptonic K Decay*

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It is shown that the conventional current-algebra derivation of the $\Delta I=\frac{1}{2}$ rule for nonleptonic K decay is invalid within the model of $SU(3)\times SU(3)$ summetry-breaking given by Gell-Mann, Oakes, and Renner. A method for retaining a $\Delta I=\frac{1}{2}$ rule and this model without invoking either neutral currents or octet dominance is discussed.

IN a recent paper,[1] we constructed models for $K\to 2\pi$ and $K\to 3\pi$ decay which were consistent with all constraints of current algebra[2] and partially conserved axial-vector current (PCAC).[3] We found that using two phenomenological parameters[4]—the $\Delta I=\frac{1}{2}$ amplitude $\mathrm{Re}G_{1/2}$ and the $\Delta I=\frac{3}{2}$ amplitude $\mathrm{Re}G_{3/2}$—which can be determined from the experimental results for $K\to 2\pi$ decay, the predicted values for amplitudes and slopes in $K\to 3\pi$ decay were in approximate agreement with experimental numbers. A simple theoretical estimate for $\mathrm{Re}G_{3/2}$, given in Sec. V of A, was also found to agree fairly well with its empirically determined value. We have in such a model a somewhat "natural" explanation for the $\Delta I=\frac{1}{2}$ rule, in that $\mathrm{Re}G_{3/2}$ depends upon the coefficient—$B_{3/2}$—of a term which is first order in the pion momentum q, whereas $\mathrm{Re}G_{1/2}$ depends primarily upon the coefficient, $A_{1/2}$, of a term which is zeroth order in q and which we expect to be dominant. This type of derivation of the $\Delta I=\frac{1}{2}$ rule in nonleptonic kaon decay has been given previously by several authors.[5] We emphasize that this calculation employed the conventional Cabibbo current-current model of the nonleptonic weak Hamiltonian[6] and that a $\Delta I=\frac{1}{2}$ rule was obtained without requiring neutral currents,[7] octet dominance,[8] or accidental cancellations.

Such a result did depend, however, upon specific assumptions made concerning matrix elements of the σ terms.[9] In Appendix B of A we generalized these assumptions by introducing a new parameter λ in terms of which the σ matrix elements are[10]

$$\langle \pi_{q_b}{}^b|\sigma(0)|\pi_{q_a}{}^a\rangle = -im_\pi{}^2\delta^{ab}\,,$$
$$\langle K_{k'}{}^m|\sigma(0)|K_k{}^n\rangle = -\tfrac{1}{2}i\lambda m_\pi{}^2\delta^{mn}. \tag{1}$$

Then $\lambda=0$ characterizes the assumptions employed in the main text of A, while $\lambda=1$ is the value suggested by the model of $SU(3)\times SU(3)$ symmetry breaking given by Gell-Mann, Oakes, and Renner (GOR),[11] which has received some confirmation from studies of meson-baryon scattering,[12] although this is far from conclusive. With $\lambda=1$, we showed in Appendix B that the zeroth-order coefficient $A_{1/2}$ cannot contribute to the "physical"[13] $K\to 2\pi$ and $K\to 3\pi$ matrix elements. Our simple theoretical model now predicts that $|\mathrm{Re}G_{1/2}|\simeq|\mathrm{Re}G_{3/2}|$, so that we now lose a "natural" explanation for the $\Delta I=\frac{1}{2}$ rule. Of course, since this theoretical model includes only the vacuum state in an intermediate-state sum, it is certainly possible that other intermediate states produce an accidental cancellation in the $\Delta I=\frac{3}{2}$ channel, but this is purely speculation. Wallace[14] has also noted the fact that with the GOR model expression for the K-π scattering amplitude[15]

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[1] B. R. Holstein, Phys. Rev. **183**, 1228 (1969), hereafter referred to as A.

[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[3] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960). Note that, although in A these results were only explicitly shown to be consistent with single soft-pion limits, it is possible to show consistency also with two and three soft pions. See B. R. Holstein, PhD thesis, Carnegie-Mellon University, 1969 (unpublished).

[4] These parameters are defined by

$$\langle \pi_{q+}{}^+\pi_{q-}{}^-|S|K_{+k}{}^0\rangle = -i(2\pi)^4\delta^4(k-q_+-q_-)$$
$$\times[-\sqrt{2}\,\mathrm{Re}G_{1/2}M_Ke^{i\delta_0}+(\sqrt{\tfrac{1}{2}})\mathrm{Re}G_{3/2}M_Ke^{i\delta_2}],$$
$$\langle \pi_{q_{01}}{}^0\pi_{q_{02}}{}^0|S|K_{+k}{}^0\rangle = -i(2\pi)^4\delta^4(k-q_{01}-q_{02})$$
$$\times[\sqrt{2}\,\mathrm{Re}G_{1/2}M_Ke^{i\delta_0}+\sqrt{2}\,\mathrm{Re}G_{3/2}M_Ke^{i\delta_2}],$$

where $|K_+{}^0\rangle=(\sqrt{\tfrac{1}{2}})(|K^0\rangle+|\bar{K}^0\rangle)$ is the CP-even linear combination of $|K^0\rangle$ and $|\bar{K}^0\rangle$, and δ_0, δ_2 are the $I=0$, 2 s-wave π-π phase shifts evaluated at c.m. energy M_K.

[5] See, e.g., M. Suzuki, Phys. Rev. **144**, 1154 (1966); W. Alles and R. Jengo, Nuovo Cimento **42A**, 419 (1966); C. Bouchiat and Ph. Meyer, Phys. Letters **22**, 198 (1966); B. R. Holstein, Phys. Rev. **171**, 1668 (1968).

[6] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[7] Such currents have been used to construct explicitly $\Delta I=\frac{1}{2}$ Hamiltonians by D. F. Greenberg, Phys. Rev. **178**, 2190 (1969); Y. Nambu and Y. Hara, Phys. Rev. Letters **16**, 875 (1966), and others.

[8] See, e.g., R. Dashen, S. Frautschi, M. Gell-Mann, and Y. Hara, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1965).

[9] Here the σ operator is defined via the commutation relation

$$\delta(x^0-y^0)[A_a{}^0(x),\partial^\mu A_\mu{}^b(y)]=\delta^4(x-y)\delta^{ab}\sigma(x).$$

[10] In the following matrix elements, the upper indices (a,b,m,n) refer to isospin indices, while the lower indices (q_a,q_b,k,k') are the respective four-momenta.

[11] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968); see also R. W. Griffith, *ibid.* **176**, 1705 (1968) for the application of this model to π-π and K-π scattering.

[12] C. C. Chan and F. T. Meiere, Phys. Rev. Letters **22**, 737 (1969); F. von Hippel and J. K. Kim, *ibid.* **22**, 740 (1969).

[13] By physical matrix elements we mean those with all particles on-mass-shell and over-all energy-momentum conservation imposed.

[14] D. J. Wallace, Nucl. Phys. **B12**, 245 (1969).

[15] The GOR model K-π scattering amplitude has been derived by R. W. Griffith (Ref. 11) and has also been given by J. Cronin, Phys. Rev. **161**, 1483 (1967). Such an amplitude is needed in order to compute the Feynman diagrams shown in Figs. 1 and 2 of A.

$A_{1/2}$ cannot contribute to the physical nonleptonic K decay amplitudes. He then attempts to negate this result by arguing that when we take $\langle 0|\mathcal{H}_w(0)|K_k{}^n\rangle$ as independent of k^2, where $\mathcal{H}_w(0)$ is the weak Hamiltonian density, we are in effect saying that $\mathcal{H}_w$ can be used as a suitable interpolating field for the kaon, which is necessarily inconsistent with the GOR-model assumption of PCAC for kaons. We are not in agreement with Wallace's technique in two respects, however. First, we do not believe that there is obvious mathematical inconsistency in assuming, as a first approximation, the k^2 independence of $\langle 0|\mathcal{H}_w(0)|K_k{}^n\rangle$. Whether such an assumption proves to be in disagreement with experiment is a separate question. Secondly, if disagreement does develop and the GOR model is to be retained, then one should attempt to rectify the situation by including such k^2 dependence. Wallace's method, which effectively employs a K-π scattering amplitude which violates the Adler consistency condition for pions, is not convincing to us.[16]

Thus, if the GOR model is shown to be a reasonable description of nature, then the derivation of a $\Delta I=\frac{1}{2}$ rule in K decay appears to be lost. However, we wish to point out that at least one mechanism exists wherein the GOR assumptions about σ terms and a more or less natural explanation for the $\Delta I=\frac{1}{2}$ rule are retained. This method involves giving k^2 dependence to the K-vacuum matrix element. Since our technique always remains on the initial kaon mass shell, this might not seem to alter our results, but off-mass-shell effects can appear in the K-pole contribution to $K\to 2\pi$ and can produce important effects.

We follow the same procedure employed in A[17]—taking one soft pion at a time with the remaining particles on their respective mass shells. We begin by defining

$$\langle 0|\mathcal{H}_w{}^i(0)|K_k{}^n\rangle=\text{prev. result}+\delta_{i\frac{1}{2}}E_{1/2}M_K\bar{s}_{1/2}K^nk^2, \quad (2)$$

where $\mathcal{H}_w{}^i(0)\,(i=\frac{1}{2},\frac{3}{2})$ is the $\Delta I=i$ component of the weak Hamiltonian density, "prev. result" refers to the contribution to this matrix element given in A, and $E_{1/2}$ is a dimensionless constant. $\bar{s}_{1/2}$ is the isospurion (0,1) and has been described in A. Continuing, we find

$$\langle \pi_{q_a}{}^a|\mathcal{H}_w{}^i(0)|K_k{}^n\rangle=\text{prev. result}+\delta_{i\frac{1}{2}}\frac{E_{1/2}M_K}{2F_\pi}\bar{s}_{1/2}\tau^aK^nk^2,$$

$$\begin{aligned}\langle \pi_{q_a}{}^a\pi_{q_b}{}^b|\mathcal{H}_w{}^i(0)|K_k{}^n\rangle=\text{prev. result}+\delta_{i\frac{1}{2}}\Big(&{}^{1/2}A_n{}^{ab}k^2+{}^{1/2}B_n{}^{ab}k\cdot q_a+{}^{1/2}B_n{}^{ba}k\cdot q_b+{}^{1/2}C_n{}^{ab}q_a{}^2+{}^{1/2}C_n{}^{ba}q_b{}^2\\ &-\frac{E_{1/2}M_K}{4F_\pi{}^2}\frac{(k-q_a-q_b)^2}{(k-q_a-q_b)^2-M_K{}^2}\{\lambda\delta^{ab}\bar{s}_{1/2}K^n[M_K{}^2-(k-q_a-q_b)^2-(q_a+q_b)^2\\ &+(m_\pi{}^2-q_a{}^2)+(m_\pi{}^2-q_b{}^2)]+i\epsilon^{abc}\bar{s}_{1/2}\tau^cK^n[2k\cdot(q_b-q_a)-q_b{}^2+q_a{}^2]\}\Big),\end{aligned}$$

with

$${}^{1/2}A_n{}^{ab}=\frac{E_{1/2}M_K}{4F_\pi{}^2}\delta^{ab}\bar{s}_{1/2}K^n(1-\lambda),$$

$${}^{1/2}B_n{}^{ab}=-2\times{}^{1/2}C_n{}^{ab}=\frac{E_{1/2}M_K}{2F_\pi{}^2}[\lambda\delta^{ab}\bar{s}_{1/2}K^n-i\epsilon^{abc}\bar{s}_{1/2}\tau^cK^n];$$

$$\begin{aligned}&\langle \pi_{q_a}{}^a\pi_{q_b}{}^b\pi_{q_c}{}^c|\mathcal{H}_w{}^i(0)|K_k{}^n\rangle\\ &=\text{prev. result}+\delta_{i\frac{1}{2}}\Big({}^{1/2}A_n{}^{abc}k^2+{}^{1/2}B_n{}^{abc}k\cdot q_a+{}^{1/2}B_n{}^{bca}k\cdot q_b+{}^{1/2}B_n{}^{cab}k\cdot q_c+{}^{1/2}C_n{}^{abc}q_a{}^2+{}^{1/2}C_n{}^{bca}q_b{}^2+{}^{1/2}C_n{}^{cab}q_c{}^2\\ &+\frac{E_{1/2}M_K}{2F_\pi{}^3}\frac{k^2}{(q_a+q_b+q_c)^2-m_\pi{}^2}\{\delta^{ab}\bar{s}_{1/2}\tau^cK^n[(q_a+q_b)^2-m_\pi{}^2]+\delta^{ac}\bar{s}_{1/2}\tau^bK^n[(q_a+q_c)^2-m_\pi{}^2]\\ &+\delta^{bc}\bar{s}_{1/2}\tau^aK^n[(q_b+q_c)^2-m_\pi{}^2]\}-\frac{E_{1/2}M_K}{8F_\pi{}^3}\frac{(k-q_b-q_c)^2}{(k-q_b-q_c)^2-M_K{}^2}\{\lambda\delta^{bc}\bar{s}_{1/2}\tau^aK^n[M_K{}^2-(k-q_b-q_c)^2-(q_b+q_c)^2\\ &+(m_\pi{}^2-q_b{}^2)+(m_\pi{}^2-q_c{}^2)]+i\epsilon^{bcd}\bar{s}_{1/2}\tau^a\tau^dK^n[2k\cdot(q_c-q_b)-q_c{}^2+q_b{}^2]\}+\text{perm.}\Big), \quad (3)\end{aligned}$$

[16] We thank Professor S. B. Treiman for a useful discussion concerning these points.

[17] Identical results can be obtained by means of a hard-pion approach to nonleptonic K decay. See B. R. Holstein (Ref. 3).

with

$$^{1/2}A_n{}^{abc}=-\frac{E_{1/2}M_K}{8F_\pi{}^3}(1+\lambda)(\delta^{ab}\mathfrak{s}_{1/2}\tau^cK^n+\delta^{ac}\mathfrak{s}_{1/2}\tau^bK^n+\delta^{bc}\mathfrak{s}_{1/2}\tau^aK^n),$$

$$^{1/2}B_n{}^{abc}=-2\times{}^{1/2}C_n{}^{abc}=\frac{E_{1/2}M_K}{4F_\pi{}^3}[(\lambda-1)(\delta^{cb}\mathfrak{s}_{1/2}\tau^aK^n+\delta^{ac}\mathfrak{s}_{1/2}\tau^bK^n)+2\delta^{bc}\mathfrak{s}_{1/2}\tau^aK^n],$$

and "perm." indicates terms obtained via cyclic permutation of the indices a, b, and c in the previous term.

For the physical decay amplitudes—all particles are on mass shell and energy-momentum conservation is imposed—we find

$$\mathrm{Re}G_{1/2}=\frac{M_K{}^2}{4F_\pi{}^2}[A_{1/2}(1-\lambda)+B_{1/2}+E_{1/2}(1+\lambda-2\lambda\eta)],\quad\text{with}\quad \eta=m_\pi{}^2/M_K{}^2,$$

$$\mathrm{Re}G_{3/2}=\frac{M_K{}^2}{4F_\pi{}^2}B_{3/2},$$

$$\lambda_{+-0}{}^{1/2}=-3\eta\left[\frac{A_{1/2}(1-\lambda)+B_{1/2}(1-\lambda\eta)+E_{1/2}(1+\lambda-2\lambda\eta)}{[A_{1/2}(1-\lambda)+B_{1/2}(1-\lambda\eta)](1+3\eta)+E_{1/2}[1+\lambda+\eta(3-5\lambda)]}\right],\tag{4}$$

$$\bar{A}_{+-0}{}^{1/2}=\frac{\mathrm{Re}G_{1/2}M_K}{3\sqrt{2}F_\pi}\frac{1}{1-\eta}\left[\frac{[A_{1/2}(1-\lambda)+B_{1/2}(1-\lambda\eta)](1+3\eta)+E_{1/2}[1+\lambda+\eta(3-5\lambda)]}{A_{1/2}(1-\lambda)+B_{1/2}+E_{1/2}(1+\lambda-2\lambda\eta)}\right],$$

where $\bar{A}_{abc}{}^{1/2}$ and $\lambda_{abc}{}^{1/2}$ are the $\Delta I=\frac{1}{2}$ contributions to the amplitude and slope in $K\rightarrow 3\pi$ decay and are defined by

$$|\langle\pi^a\pi^b\pi^c|\mathcal{H}_w{}^{1/2}(0)|K^n\rangle_{\mathrm{physical}}|^2=|\bar{A}_{abc}{}^{1/2}|^2\left(1-2\lambda_{abc}{}^{1/2}\frac{s_3-s_0}{m_\pi{}^2}\right),\tag{5}$$

where $s_3=(k-q_3)^2$, q_3 being the four-momentum of the "odd" pion, and $s_0=\frac{1}{3}M_K{}^2+m_\pi{}^2$. For $\lambda=0$, $E_{1/2}$ and $A_{1/2}$ can be grouped together and $(A_{1/2}+E_{1/2})$ is expected to be much larger than $B_{1/2}$ and $B_{3/2}$ as before, so that a "natural" $\Delta I=\frac{1}{2}$ rule is obtained. On the other hand, if $\lambda=1$, we can still have $|B_{1/2}|\simeq|B_{3/2}|$ as predicted by our theoretical model and a $\Delta I=\frac{1}{2}$ rule if we assume $|E_{1/2}|\gg|B_{3/2}|$. This is not as desirable a solution for two reasons, however. Since $E_{1/2}$ and $B_{3/2}$ are both coefficients of momentum-dependent terms, we might expect that they are of the same order of magnitude. Also, appreciable k^2 dependence is generally considered to violate the spirit of the PCAC hypothesis for kaons.

We have emphasized that in the model of $SU(3)\times SU(3)$ symmetry proposed by Gell-Mann, Oakes, and Renner supplemented by the work on K-π scattering due to Griffith,[11] the conventional derivation of the $\Delta I=\frac{1}{2}$ rule in nonleptonic K decay is no longer valid. Since there is no way at present to derive this rule for nonleptonic hyperon decay[18]—yet experimentally such a rule is observed[19]—it may be that neutral currents or some type of octet enhancement are in fact needed. However, we have seen that for K decay we may retain both the standard current-current model for the weak Hamiltonian *and* a "natural" $\Delta I=\frac{1}{2}$ rule if we are willing to abandon the k^2 independence of the K-vacuum matrix element.

[18] The current algebra seems to imply a pseudo-$\Delta I=\frac{1}{2}$ rule for the s-wave decay; see H. Sugawara, Phys. Rev. Letters **15**, 870 (1965). However, this can be distinguished from a $\Delta I=\frac{1}{2}$ rule when final-state interactions are taken into account; see O. E. Overseth, *ibid.* **19**, 395 (1967).

[19] See, e.g., H. Filthuth, Invited Talk at the Topical Conference on Weak Interactions, CERN, Geneva, 1969 (unpublished).