TABLE II. K_{l4} decay rates (10^3 sec^{-1}).

Decay mode	Current-algebra predictions: Present calculations	Soft pion[a]	Hard pion[b]	Dispersion relations[c]	Analysis of Berends *et al.*[d]	Expt.[e]
$K^+ \to \pi^+\pi^-e^+\nu$	2.43	1.87	2.81	2.06	2.9 ±0.6	2.9±0.6
$K^+ \to \pi^0\pi^0e^+\nu$	1.15	0.78	1.33	0.80	1.13±0.25	···
$K_2^0 \to \pi^-\pi^0e^+\nu$	0.14	0.30	0.16	0.46	0.64±0.25	···
$K^+ \to \pi^+\pi^-\mu^+\nu$	0.31	···	0.45	0.33	···	1.1±0.7
$K^+ \to \pi^0\pi^0\mu^+\nu$	0.15	···	0.22	0.15	···	···
$K_2^0 \to \pi^-\pi^0\mu^+\nu$	0.01	···	0.015	0.037	···	···

[a] Reference 13. [b] Reference 5. [c] Reference 6. [d] Reference 12. [e] Reference 11.

$\Delta I=\frac{1}{2}$ rule.[10] For comparison, the results of other calculations[5,6] and experimental determinations[11] are also given in this table. We neglect the small variations of the form factors with respect to their arguments in the physical region and compute the decay rate[12] from the constant form factors at $s=t=u=0$ quoted in Table I. The predicted decay rates are given in Table II along with the results obtained by different current-algebra methods[5,6,13]

[10] The form factors and the decay rates in the three charge states of K_{l4} decay are related by the following $\Delta I=\frac{1}{2}$ rule: $F_i{}^A=F_i{}^B+F_i{}^C$, $\Gamma^A=2\Gamma^B+\Gamma^C$, $i=1, 2, 3$.

[11] R. Birge *et al.*, Phys. Rev. **139**, 1600 (1965).

[12] The analysis of Berends *et al.* has been used. See F. A. Berends, A. Donnachie, and G. C. Oades, Phys. Letters **26B**, 109 (1967); Nucl. Phys. **B3**, 569 (1967).

[13] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966); **18**, 1178(E) (1967).

It is interesting to note that our results are in better agreement with the experiment than those of the soft-pion calculations[13] and are nearer to the hard-pion results.[5,6] Thus we have obtained some improvements over the chiral Lagrangian calculations of Cronin[3] which essentially give the soft-pion results of Weinberg.[13] This modification is certainly due to the inclusion of possible vector and axial-vector meson dominance of the currents associated with the decay. We may conclude that the main common point underlying all these calculations is the basic validity of the approximate chiral $SU_3 \otimes SU_3$ symmetry and dominance by the vector and axial-vector mesons. These features are most readily introduced through the formalism described in the present analysis.

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Compton Terms in Inelastic Photoproduction of Muon Pairs*

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Given the parton model of the nucleon, the cross section for photoproduction of high-energy muon pairs of small invariant mass and high transverse momentum is estimated.

In a recent paper[1] we proposed an experimental check of the parton model[2] by comparing inelastic electron-proton scattering to inelastic Compton scattering. It was found that the ratio of the two processes is independent of the parton spin and distribution of longitudinal momentum, but depends sensitively on the parton charge. A related experiment with perhaps better signature is to consider the "Compton" terms in inelastic photoproduction of μ pairs. The contributing diagrams in this case are shown in Fig. 1. In this case, one would look for μ pairs produced with small invariant mass and at large angles to the incident γ rays. Hopefully, the angular correlation of the two muons provides a signature sufficiently unique to distinguish this "Compton" process from the two major sources of background: muons from $\pi^\pm$ or $K^\pm$ decay and μ pairs

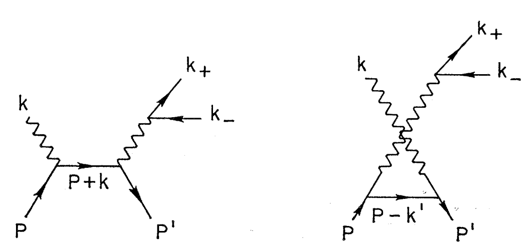

FIG. 1. Feynman diagrams for the Compton terms in the photoproduction of muon pairs.

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[1] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).
[2] R. P. Feynman (unpublished); also, see Ref. 1.

from Bethe-Heitler process. We have not calculated these backgrounds in detail; they depend upon the specific experimental configuration to be used. However, as the energy and transverse momentum of the Compton μ pairs increase, the relative importance of both backgrounds will decrease.

We have calculated this Compton process in the parton model. To this end, we calculated first the point cross section for the diagrams in Fig. 1. All variables are in the laboratory frame and are defined as follows:

$k'=k_+ + k_- =$ energy of virtual photon,
$\nu=k-k'$,
$\Delta^\mu=k_+{}^\mu-k_-{}^\mu$: four-momentum difference between final muons,
M, m_μ, σ: masses of proton, muon, and virtual photon,
θ: angle of virtual photon relative to incident photon,
$q^2=(k-k')^2=\sigma^2-2k[k'-(k'^2-\sigma^2)^{1/2}]$,
ϵ_μ: polarization of the initial photon.

The point cross section from a spin-$\frac{1}{2}$ particle of unit charge is given by

$$\frac{d\sigma}{d\Omega_+d\Omega_-dE_+dE_-}=\frac{\alpha^3}{\pi^2}\frac{1}{\sigma^4}\frac{E_+E_-}{Mk}\Bigg\{(m_\mu{}^2+p_+\cdot p_-)\left[\frac{k'}{k}\left(1-\frac{\sigma^2}{2Mk'}\right)+\frac{k}{k'}\frac{1}{(1-\sigma^2/2Mk')}-2\left(\frac{\epsilon\cdot k'}{k'}\right)^2\frac{1+\sigma^2/2M^2}{(1-\sigma^2/2Mk')}\right]$$
$$+\tfrac{1}{4}(\Delta\cdot\Delta)\frac{k'}{k}\left(1-\frac{\sigma^2}{2Mk'}\right)\left(1-\frac{k}{k'}\frac{1}{1-\sigma^2/2Mk'}\right)^2+\frac{1}{4}\frac{(k\cdot\Delta)^2}{M^2kk'}\frac{\sigma^2}{(1-\sigma^2/2Mk')}$$
$$-\left((\epsilon\cdot\Delta)-\frac{\Delta_0}{k'}\frac{\epsilon\cdot k'}{1-\sigma^2/2Mk'}\right)^2-\frac{\sigma^2\Delta\cdot\Delta}{4M^2k'^2}\frac{(\epsilon\cdot k')^2}{(1-\sigma^2/2Mk')}\Bigg\}\delta(2M\nu+q^2). \quad (1)$$

The final result for the scattering of unpolarized photons is obtained by averaging over the initial photon polarizations.

In checking the trace calculation of the hadronic part of the diagrams, we explicitly verified that $T^{\mu\nu}$

(i) satisfies gauge invariance, $T^{\mu\nu}k_\mu'k_\nu'=0$;
(ii) gives the ordinary Compton cross section when we interpret Δ_μ as a polarization vector ϵ_μ' and take the limit of $\sigma^2\to 0$.

To obtain the cross section from a proton in the parton model we replace M by Mx, then multiply by the distribution function $f_N(x)$ and by $P(N)$, and then integrate over x as has been discussed in Ref. 1. The final result is given below for the case of spin-$\frac{1}{2}$ integer-charge partons. With $x=-q^2/2M\nu$ and $\xi=1-\sigma^2/2Mxk'$, one has

$$\frac{d\sigma}{d\Omega_+d\Omega_-dE_+dE_-}=\frac{\alpha^3}{2\pi^2}\frac{E_+E_-}{M^2x^2k\nu\sigma^4}\Bigg\{(m_\mu{}^2+p_+\cdot p_-)\left[\frac{k'}{k}\xi+\frac{k}{k'}\frac{1}{\xi}-2\left(\frac{\epsilon\cdot k'}{k'}\right)^2\frac{1+\sigma^2/2M^2x^2}{\xi}\right]+\tfrac{1}{4}(\Delta\cdot\Delta)\frac{k'}{k}\left(1-\frac{k}{k'}\frac{1}{\xi}\right)^2\xi$$
$$+\frac{1}{4}\frac{(k\cdot\Delta)^2}{M^2x^2}\frac{\sigma^2}{kk'}\frac{1}{\xi}-\left(\epsilon\cdot\Delta-\frac{\Delta_0}{k'}\frac{\epsilon\cdot k'}{\xi}\right)^2-\frac{\Delta\cdot\Delta}{4M^2x^2}\frac{\sigma^2}{k'^2}\frac{(\epsilon\cdot k')^2}{\xi}\Bigg\}F(x), \quad (2)$$

where $F(x)$ is the structure function $\nu W_2(q^2,\nu)$ measured in inelastic electron-proton scattering.[3] We can also integrate over the relative phase space between the muons to obtain a triple differential cross section with respect to the solid angle, energy, and mass squared of the outgoing virtual photon.

$$\frac{d\bar{\sigma}}{d\Omega dk'd\sigma^2}=\frac{\alpha^3}{6\pi}\frac{1}{M^2x^2k\nu\sigma^2}\left(1-\frac{4m_\mu{}^2}{\sigma^2}\right)^{1/2}\left(1+\frac{2m_\mu{}^2}{\sigma^2}\right)(k'^2-\sigma^2)^{1/2}\left[\frac{k'}{k}\xi+\frac{k}{k'}\frac{1}{\xi}-2\left(1+\frac{\sigma^2}{2M^2x^2}\right)\left(\frac{\epsilon\cdot k'}{k'}\right)^2\frac{1}{\xi}\right]F(x). \quad (3)$$

We have calculated the counting rates at large momentum transfers using a bremsstrahlung spectrum and Eq. (3) for several experimental situations at SLAC and Cornell and we found that they are small but measurable. A more formidable problem, perhaps, is the elimination of the backgrounds. The accidental muons from $\pi^\pm$ and $K^\pm$ depend critically on the experimental apparatus and the duty cycle of the accelerator. On the other hand, the μ pairs from the Bethe-Heitler process are completely predictable[4] once the inelastic form factors $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ become available.

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