ρ^0-ω Interference*

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(Received 27 October 1969)

The structure of ρ^0-ω interference is discussed. Applications are made to photoproduction of 3π states, $\pi^+\pi^-$ production in electron-positron colliding beams, as well as in other processes, and lepton-pair photoproduction. The various parameters that appear in all these experiments are discussed, and the correlations of the various results are pointed out.

I. GENERAL FORMALISM

In this paper we investigate the structure of ρ^0-ω interference in various reactions. We base our discussion on the formalism developed in Ref. 1 which is suitable for treating this problem of two overlapping resonances. A general T-matrix element between some initial and final state can be written in the form

$$\langle f|T|i\rangle=\frac{\pi(\rho_i\rho_f)^{1/2}}{(1-|\chi|^2)^{1/2}}\left\langle f\left|H'\left\{\frac{|\mathrm{I}\rangle\langle\mathrm{I}'|}{E-M_{\rho^0}+\frac{1}{2}i\Gamma_{\rho^0}}+\frac{|\mathrm{II}\rangle\langle\mathrm{II}'|}{E-M_\omega+\frac{1}{2}i\Gamma_\omega}\right\}H'\right|i\right\rangle. \quad (1)$$

H' is an interaction Hamiltonian that is responsible for both strong and weak decays and productions of the resonances and E is the c.m. energy of the resonating configuration. The states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ are right-eigenvectors of an effective Hamiltonian $\mathbf{M}-\frac{1}{2}i\mathbf{\Gamma}$ with eigenvalues $M_{\rho^0}-\frac{1}{2}i\Gamma_{\rho^0}$ and $M_\omega-\frac{1}{2}i\Gamma_\omega$, respectively. The states $\langle\mathrm{I}'|$ and $\langle\mathrm{II}'|$ are left-eigenvectors with the same eigenvalues, respectively. Let us write them explicitly in a two-dimensional space spanned by the two states $|\rho^0\rangle$ and $|\omega\rangle$ which are chosen to be eigenstates of the strong-interaction Hamiltonian in the limit $H'=0$. To first order in e^2, we find then

$$|\mathrm{I}\rangle=|\rho^0\rangle-e^2\beta|\omega\rangle,\qquad |\mathrm{II}\rangle=e^2\beta|\rho^0\rangle+|\omega\rangle, \quad (2)$$

$$|\mathrm{I}'\rangle=T|\mathrm{I}\rangle=|\rho^0\rangle-e^2\beta^*|\omega\rangle,\quad |\mathrm{II}'\rangle=T|\mathrm{II}\rangle=e^2\beta^*|\rho^0\rangle+|\omega\rangle, \quad (3)$$

$$\chi=\langle\mathrm{II}|\mathrm{I}\rangle=-2ie^2\,\mathrm{Im}\beta, \quad (4)$$

$$\mathbf{M}-\tfrac{1}{2}i\mathbf{\Gamma}=\begin{pmatrix} M_{\rho^0}-\frac{1}{2}i\Gamma_{\rho^0} & e^2\beta[M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_{\rho^0})] \\ e^2\beta[M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_{\rho^0})] & M_\omega-\frac{1}{2}i\Gamma_\omega \end{pmatrix}. \quad (5)$$

Note that M_{ρ^0}, Γ_{ρ^0}, M_ω, and Γ_ω designate the observed masses and widths of the resonances. The states $|\mathrm{I}'\rangle$ and $|\mathrm{II}'\rangle$ can be reached from $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ by the time-reversal operation T, and thus we took time-reversal invariance into account.[1] Therefore χ, the overlap of the two states $\langle\mathrm{II}|\mathrm{I}\rangle$, is purely imaginary. We have one complex parameter β, related to the off-diagonal matrix element of the effective Hamiltonian in terms of which all the relevant states are defined.

It is now straightforward to apply this formalism to various reactions that may be of interest to us. Let us concentrate on the following four possible reactions:

(a) $e^+e^-\to\pi^+\pi^-$,
(b) some production $\to\pi^+\pi^-$,
(c) $\pi^+\pi^-\to\pi^+\pi^-$,
(d) some production $\to e^+e^-$.

In each case we will observe the square of the absolute value of a matrix element of the form of (1). This involves two interfering Breit-Wigner distributions. The quantity that is of interest to us can be defined as

$$R=(\text{residue of }\omega\text{ pole})/(\text{residue of }\rho^0\text{ pole}) \quad (6)$$

and it does vary from one reaction to another.

The experimental curve of the cross section should be proportional to

$$f=\left|\frac{1}{E-M_{\rho^0}+\frac{1}{2}i\Gamma_{\rho^0}}+\frac{R}{E-M_\omega+\frac{1}{2}i\Gamma_\omega}\right|^2. \quad (7)$$

The question that confronts us is to what extent can R be calculated and what are the parameters needed to fit experiment. Let us designate the couplings of the states

* Work supported in part by Contract No. AT(11-1)-68 of the San Francisco Operations Office, U. S. Atomic Energy Commission.
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[1] Y. Dothan and D. Horn, Phys. Rev. D 1, 916 (1970). This paper discusses the phenomenon of overlapping resonances in the case that they appear as poles in the s channel. We use here a generalization of this result by assuming that the structure of the propagator [the term in the curly brackets in our Eq. (1)] remains the same in general production processes.

$|\rho^0\rangle$ and $|\omega\rangle$ to the photon by $g_{\gamma\rho^0}$ and $g_{\gamma\omega}$, respectively. (Often one finds the alternative notation $g_{\gamma\rho^0} = -eM_{\rho^0}{}^2/2\gamma_{\rho^0}$, which we do not use here.) Similarly, let us denote the general production amplitudes of $\langle\rho^0|$ and $\langle\omega|$ [cases (b) and (d)] by f_{ρ^0} and f_ω, respectively. Then we find, to leading order in e^2,

$$R_{\mathrm{a}}=\left(e^2\beta+\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right)\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}, \tag{8}$$

$$R_{\mathrm{b}}=\left(e^2\beta+\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right)\frac{f_\omega}{f_{\rho^0}}, \tag{9}$$

$$R_{\mathrm{c}}=e^4\beta^2+\left|\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right|^2 +2e^2\beta\,\mathrm{Re}\left(\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right), \tag{10}$$

$$R_{\mathrm{d}}=\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\frac{f_\omega}{f_{\rho^0}}. \tag{11}$$

The corrections to these expressions are

$$\Delta R_{\mathrm{a}}=\left(e^2\beta+\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right)e^2\beta\left[1+\left(\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\right)^2\right], \tag{12}$$

$$\Delta R_{\mathrm{b}}=\left(e^2\beta+\frac{\langle\pi^+\pi^-|H'|\omega\rangle}{\langle\pi^+\pi^-|H'|\rho^0\rangle}\right)e^2\beta\left[1+\left(\frac{f_\omega}{f_{\rho^0}}\right)^2\right], \tag{13}$$

$$\Delta R_{\mathrm{c}}=O(e^8), \tag{14}$$

$$\Delta R_{\mathrm{d}}=e^2\beta\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}+e^2\beta\frac{f_\omega}{f_{\rho^0}}+\frac{g_{\gamma\omega}f_\omega}{g_{\gamma\rho^0}f_{\rho^0}}\left(e^2\beta\frac{f_\omega}{f_{\rho^0}}+e^2\beta\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\right). \tag{15}$$

Equations (8)–(15) are derived by direct substitutions of Eqs. (2) and (3) into the form (1) for the four different processes.

Let us now discuss the term $e^2\beta$ that appears in all these equations. We use Eq. (5) to rewrite it in terms of the off-diagonal matrix element of the effective Hamiltonian:

$$e^2\beta=\frac{M_{\rho^0\omega}-\frac{1}{2}i\Gamma_{\rho^0\omega}}{M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_{\rho^0})}. \tag{16}$$

We can estimate $M_{\rho^0\omega}$ from SU_3 relations. Thus, assuming this electromagnetic mass term to be pure $U=0$, we find that

$$M(\rho^0)-M(\rho^+)+M(K^{*+})-M(K^{*0}) =\sqrt{3}\langle\rho^0|M|\omega_8\rangle. \tag{17}$$

If we further choose the SU_6 mixing angles for ω and ϕ and assume that $\langle\rho^0|M|\phi\rangle=0$, as suggested from the quark model, then

$$\langle\rho^0|M|\omega\rangle=M(\rho^0)-M(\rho^+)+M(K^{*+})-M(K^{*0}). \tag{18}$$

The mass difference between the K^{*+} and K^{*0} is quoted in the Rosenfeld tables[2] as -6.3 ± 4.1 MeV. The mass difference $\rho^0-\rho^+$ is hard to estimate but looks negative. The tadpole contribution à la Coleman and Glashow[3] is about -2.5 MeV and, following Harari's reasoning,[4] this might very well be almost the full value of this term. We can therefore estimate $M_{\rho^0\omega}$ to be around -3 MeV. See also the discussions in Refs. 5 and 6.

The next term to discuss is $\Gamma_{\rho^0\omega}$. That is defined as

$$\Gamma_{\rho^0\omega}=\sum_i 2\pi\rho_i\langle\rho^0|H'|i\rangle\langle i|H'|\omega\rangle=\sum_i(\gamma_{\rho^0}{}^{i*}\gamma_\omega{}^i)^{1/2} \tag{19}$$

in the notation of Ref. 1. $(\gamma_V{}^i)^{1/2}$ denotes the direct-decay matrix element of resonance V into channel i. Since the main decay mode of ρ^0 is into 2π and that of ω is into 3π, we assume that these two channels will give the main contributions to Eq. (19):

$$\Gamma_{\rho^0\omega}\approx(\gamma_{\rho^0}{}^{2\pi*}\gamma_\omega{}^{2\pi})^{1/2}+(\gamma_{\rho^0}{}^{3\pi*}\gamma_\omega{}^{3\pi})^{1/2}. \tag{20}$$

The phases of the various terms in Eq. (20) are not completely arbitrary since $\Gamma_{\rho^0\omega}$ is a real number. Thus we can write

$$\begin{aligned}(\gamma_{\rho^0}{}^{2\pi*}\gamma_\omega{}^{2\pi})^{1/2}&=a_2e^{i\phi}(\Gamma_\rho\Gamma_\omega)^{1/2},\\ (\gamma_{\rho^0}{}^{3\pi*}\gamma_\omega{}^{3\pi})^{1/2}&=a_3e^{i\theta}(\Gamma_\rho\Gamma_\omega)^{1/2}.\end{aligned} \tag{21}$$

Imposing the condition that $\Gamma_{\rho^0\omega}$ be real, we find

$$\Gamma_{\rho^0\omega}=[a_2\cos\phi\pm(a_3{}^2-a_2{}^2\sin^2\phi)^{1/2}](\Gamma_\rho\Gamma_\omega)^{1/2}, \tag{22}$$

which leaves us with three different continuous parameters (a_2,a_3,ϕ) and the choice of the sign of the square root. Note that $a_2{}^2$ and $a_3{}^2$ correspond to the branching ratios of the direct $\omega\to2\pi$ and $\rho^0\to3\pi$ decays, respectively. There might be a confusion here with regard to the terms of partial widths and branching ratios. Usually one refers to the decay probabilities of the decaying resonances ($|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$), whereas we referred here to the pure states $|\rho^0\rangle$ and $|\omega\rangle$. It sometimes makes a big difference. Thus even if $a_2=0$, the state $|\mathrm{II}\rangle$, which can be justly referred to as the decaying state of the ω resonance, might very well have a decay probability into the 2π states that is caused by the $e^2\beta|\rho^0\rangle$ component. In order to avoid confusion, we refer to the decay that might be caused by $|\omega\rangle$ as the direct-decay term.

This discussion of $\Gamma_{\rho^0\omega}$ assumed that the 2π and 3π modes are dominant in Eq. (19). This is true if the branching ratio of the direct decay of $\omega\to2\pi$, i.e., $a_2{}^2$, or that of $\rho^0\to3\pi$ $(a_3{}^2)$ is of order of 1% or so. If it is much lower, then one has to take also all other channels into account in (19). However, in that case, the value of

[2] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[3] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).
[4] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).
[5] A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters **30B**, 249 (1969).
[6] V. S. Mathur and S. Okubo, Phys. Rev. **188**, 2435 (1969).

$\Gamma_{\rho^0\omega}$ will be much smaller than $M_{\rho^0\omega}$, so that it can be neglected altogether.

We saw that β depends on $M_{\rho^0\omega}$ and on $\langle\pi\pi|H'|\omega\rangle$ and $\langle\pi\pi\pi|H'|\rho^0\rangle$. The first decay amplitude appears again in R_a, R_b, and R_c as an independent parameter. For the sake of completeness, we have to add a reaction where also $\langle\pi\pi\pi|H'|\rho^0\rangle$ appears as an independent parameter. That is readily achieved by looking at a process of the form

(e) some production $\rightarrow 3\pi$.

Here the ω pole would be the dominant one and therefore we should look at R^{-1}, which, in terms of the various parameters introduced before, is

$$R_e^{-1}=\left(\frac{\langle 3\pi|H'|\rho^0\rangle}{\langle 3\pi|H'|\omega\rangle}-e^2\beta\right)\frac{f_{\rho^0}}{f_\omega}, \tag{23}$$

and includes the $\langle 3\pi|H'|\rho^0\rangle$ term, as expected.

Before continuing with the examination of the various parameters, let us note that the incoming and outgoing states ($\pi^+\pi^-$, 3π, "some production," and e^+e^-) are already the combinations that have the right relevant quantum numbers, i.e., $J=1$, $P=-$. Note that there is only one such 3π combination for low relative angular momenta; therefore, we are justified in considering it as a single channel.

II. PHOTOPRODUCTION OF 3π

A recent SLAC experiment[7] on photoproduction at 5.25 GeV observed a clean ρ^0 peak in the 2π channel and a clean ω peak in the 3π channel. The latter fact has direct relevance to Eq. (23). If we think about the production process as being pure diffractive, we might expect along the usual reasoning that $f_{\rho^0}:f_\omega\simeq 3$. As a matter of fact, by comparing the total number of observed events of ρ^0 versus ω, one is led to $|f_{\rho^0}/f_\omega|^2\approx 12$, which is even slightly larger than the expected value.

This means that

$$|R_e^{-1}|\simeq\left|\frac{\langle 3\pi|H'|\rho^0\rangle}{\langle 3\pi|H'|\omega\rangle}-\frac{M_{\rho^0\omega}-\frac{1}{2}i\Gamma_{\rho^0\omega}}{M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_\rho)}\right|\sqrt{12}. \tag{24}$$

Let us estimate the order of magnitude of the two terms in the bracket. We know that

$$-\frac{M_{\rho^0\omega}}{M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_\rho)}\simeq\frac{3}{19+54i},$$

and hence has absolute value of ~ 0.05. We do not expect $\Gamma_{\rho^0\omega}$ to change this result significantly. The other term is

$$\frac{\langle 3\pi|H'|\rho^0\rangle}{\langle 3\pi|H'|\omega\rangle}=\frac{\gamma_{\rho^0}{}^{3\pi}}{\gamma_\omega{}^{3\pi}}=a_3e^{-i\theta}(\Gamma_\rho/\Gamma_\omega)^{1/2},$$

which has an absolute value of about $3a_3$. Experimentally, one observes no event in the ρ region. Since about 33 events are recorded in the ω region, that means that

$$|R_e^{-1}|^2(\Gamma_\omega/\Gamma_\rho)\times 33\lesssim 1,\quad |R_e^{-1}|\lesssim 0.55. \tag{25}$$

The $M_{\rho^0\omega}$ terms leads to a contribution of $0.05\sqrt{12}\simeq 0.17$ which is much smaller than the upper limit. Thus Eq. (25) can be used to restrict severely a_3 since now we have to conclude that

$$3a_3(\sqrt{12})\lesssim 0.55,\quad a_3\lesssim 0.055,\quad a_3^2\lesssim 0.003. \tag{26}$$

The last result simply means that the direct (as well as indirect) branching ratio of $\rho^0\rightarrow 3\pi$ is experimentally of order 0.3% or less, which can also be deduced directly from the above quoted numbers. Since it is quite difficult to estimate the exact error of this statement, let us be more conservative and, in the following, use the upper limit

$$a_3<0.1,\quad \Gamma(\rho^0\rightarrow 3\pi)/\Gamma(\rho^0\rightarrow 2\pi)<0.01. \tag{27}$$

III. $\pi^+\pi^-$ PRODUCTION

In Sec. II we saw that it is possible to put quite a strong bound on a_3 from the photoproduction of 3π systems. We will see that the situation is different for a_2.

We study first the $\pi^+\pi^-$ production in e^+e^- colliding beams. Let us go back to Eq. (8) for R_a and substitute in it Eqs. (16) and (22). Thus we find

$$R_a=\left[\frac{M_{\rho^0\omega}-\frac{1}{2}i[a_2\cos\phi\pm(a_3^2-a_2^2\sin^2\phi)^{1/2}](\Gamma_\rho\Gamma_\omega)^{1/2}}{M_\omega-M_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_\rho)}+a_2e^{i\phi}\frac{\Gamma_\omega}{\Gamma_\rho}\right]\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}. \tag{28}$$

We choose $a_2\gg a_3$ and assume for a moment that $M_\omega-M_{\rho^0}\ll\frac{1}{2}\Gamma_\rho$ and $\Gamma_\omega\ll\Gamma_\rho$ (numerical values for these last two approximations are $19\ll 60$ and $13\ll 120$, respectively). In these limits, it is easy to see that $\sin\phi=0$, the two terms that contain a_2 cancel each other, and we are left with the $M_{\rho^0\omega}$ term only. That is the result obtained by Goldhaber, Fox, and Quigg[5] which leads to the structure of a "shoulder" in the ω region. In Fig. 1 we plot f of Eq. (7) with the present R_1 under the assumption $a_3=a_2=0$ and in Fig. 2 we show some other characteristic results for the case $a_3=0$. It is clear that the results do not change considerably with the variation of a_2 provided a_3 is much smaller. How-

[7] J. Ballam *et al.*, Phys. Letters **30B**, 421 (1969).

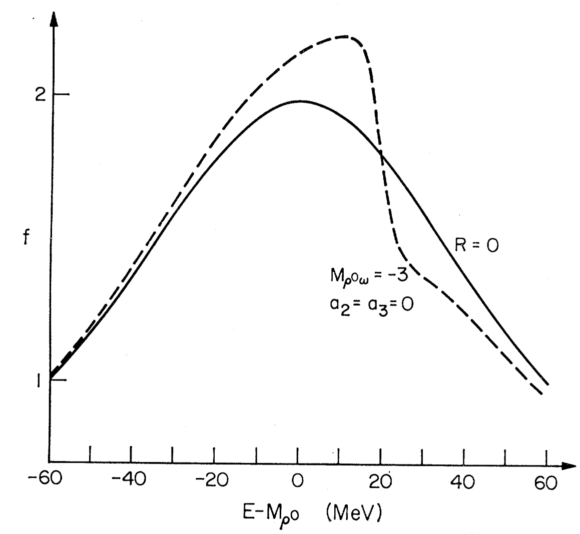

FIG. 1. Plot of f for R_a using $g_{\gamma\omega}/g_{\gamma\rho^0}=\frac{1}{3}$. Arbitrary scale for f. The parameters used are $M_\omega - M_{\rho^0}=19$ MeV, $\Gamma_{\rho^0}=120$ MeV, and $\Gamma_\omega=13$ MeV.

ever, if we include an a_3 term that is of order 0.1, then almost arbitrary results can be achieved (Fig. 3).

The various figures show f as a function of E for $g_{\gamma\omega}/g_{\gamma\rho^0}=\frac{1}{3}$, $M_{\rho^0\omega}=-3$ MeV. Note that the choice of a positive sign for $g_{\gamma\omega}/g_{\gamma\rho^0}$ is essentially arbitrary. This corresponds to the arbitrariness in the sign of the state $|\omega\rangle$. The same is also true for the value of $M_{\rho^0\omega}$. Indeed, the above quoted values are true for a specific choice of phases. However, this arbitrariness cancels out in the product $(g_{\gamma\omega}/g_{\gamma\rho^0})M_{\rho^0\omega}$. Therefore, the freedom in the sign of $g_{\gamma\omega}/g_{\gamma\rho^0}$ in Eq. (28) can be shifted to the arbitrariness in ϕ and the sign of the square root.

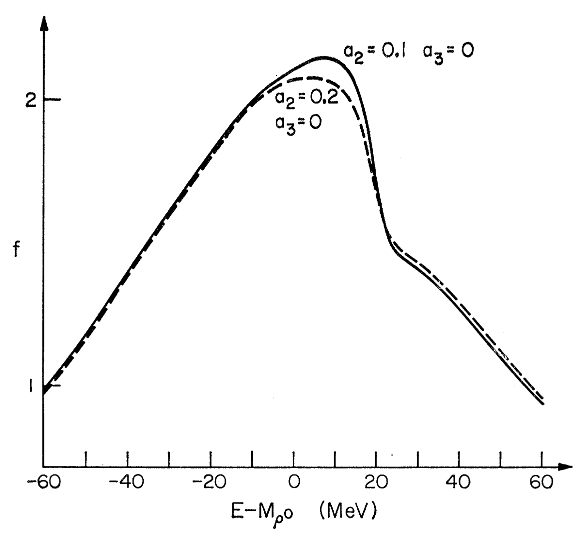

FIG. 2. Plot of f for R_a using $g_{\gamma\omega}/g_{\gamma\rho^0}=\frac{1}{3}$, $M_{\rho^0\omega}=-3$ MeV, and $a_3=0$.

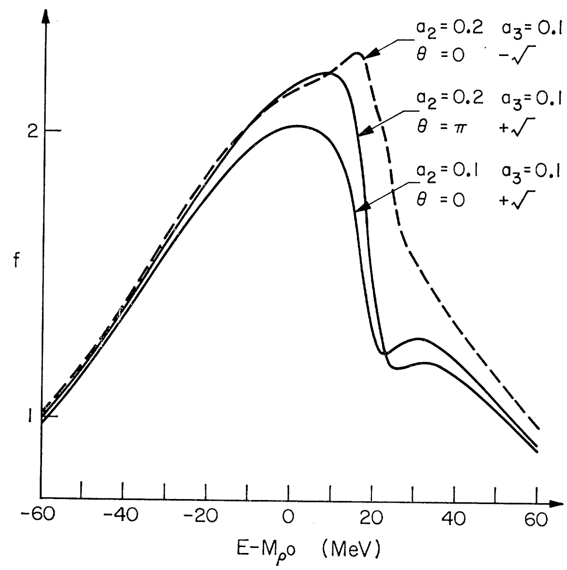

FIG. 3. Plot of f for R_a using $g_{\gamma\omega}/g_{\gamma\rho^0}=\frac{1}{3}$, $M_{\rho^0\omega}=-3$ MeV, and various values of the different parameters.

Experimental results of the Orsay group[8] are shown in Fig. 4 for comparison. The situation is still somewhat discouraging. Note that in order to compare the data with our formulas, one has to apply to them the suitable radiative corrections that depend on the ex-

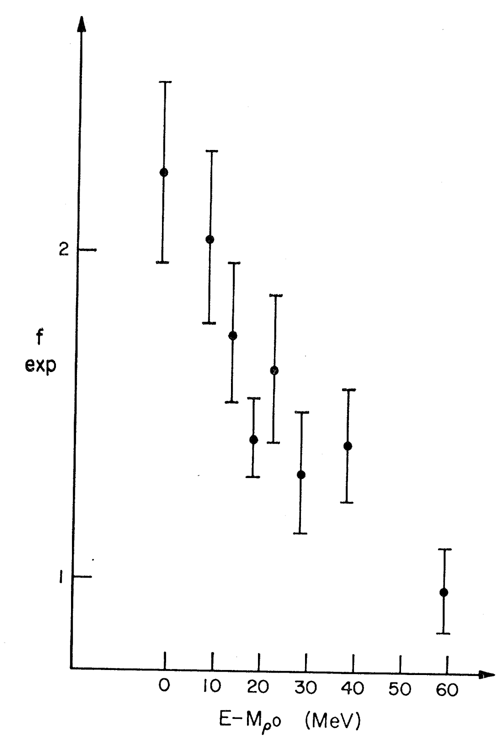

FIG. 4. The experimental data of Ref. 8 on the same scale as the previous figures.

[8] J. E. Augustin *et al.*, Nuovo Cimento Letters **2**, 214 (1969).

perimental setup as well as on the predicted curves. Since such corrections can be of the order of 10%, we can use only the gross features of the data to find what types of curves might be the right candidates. Figures 2 and 3 show the behavior of f for various choices of a_2, a_3, and ϕ. It seems that even if the data will permit a very precise determination of f, we will still be unable to disentangle the various parameters since different combinations lead to similar curves. We saw that it is reasonable to assume that a_3 is very small and therefore the expected curve would be the one exhibiting a shoulder. A dip in the ω region would call for the existence of sizable a_2 as well as a_3 parameters and, therefore, would contradict the information from the 3π photoproduction data.

The discussion of other production modes of $\pi^+\pi^-$ pairs proceeds along similar lines. A general production mode [process (b)] is described by R_b and differs from the previous discussion just by the appearance of f_ω/f_{ρ^0} in the place of $g_{\gamma\omega}/g_{\gamma\rho^0}$. Thus if one looks at processes that have relative real production amplitudes for the ω and ρ^0, one expects the same structure as in the colliding-beam experiment. Such might be the photoproduction situation. The question of the relative phase of the two photoproduction amplitudes will be discussed in Sec. IV. We would like to point out that the SLAC experiment[7] does not see any particular local effect at the ω. Nevertheless, they do see a somewhat distorted Breit-Wigner that they fit with the Ross-Stodolsky resonance form. It is possible that the shoulder of the ω is responsible for this distortion.

A. S. Goldhaber *et al.*[5] discussed the production of $\rho^0\omega$ in various πN and KN reactions. This is in particular related to the recent paper of G. Goldhaber *et al.*[9] that reports destructive interference of the ω and ρ^0 in $\pi^+p \to \pi^+\pi^-\Delta^{++}$. We saw above that the size of a_2 will not change considerably the dominant features of R as long as a_3 is small. Therefore, we agree with the conclusion of Ref. 5 that such a dip calls for a relative imaginary phase in the production of ρ^0 and ω. We refer to Ref. 5 for the discussion of interference in other πN and KN reactions.

We note that the corrections of the next order in e^2 to R [Eqs. (12) and (13)] are of a relative magnitude determined essentially by $e^2\beta(f_{\rho^0}/f_\omega)$, which, according to the discussion following Eq. (24), should be somewhere around 10–20%. Therefore, although it may affect the fine details, it will not change the general consequences of this and the previous sections.

The last $\pi^+\pi^-$ production process that we discuss is the elastic scattering, reaction (c). Although this experiment cannot be carried out, one can nevertheless assume that ρ production in πN processes will show the corresponding behavior since the one-pion-exchange models usually work quite well in this case. A look at R_c [Eq. (10)] tells us that we should not expect any ω effect at all. The shoulder plot in Fig. 1 is determined by R_a that has the absolute value of $0.05\times\frac{1}{3}$. Here, the corresponding R_c will be 0.05×0.05, which is therefore an order of magnitude smaller. Hence we cannot hope for the experimental verification of the details of Eq. (10) in the near future.

[9] G. Goldhaber *et al.*, Phys. Rev. Letters **23**, 1351 (1969).

IV. PHOTOPRODUCTION OF LEPTON PAIRS

We turn now to the last application of our formalism —photoproduction of lepton pairs. We may expect this R_d to be an order of magnitude bigger than R_a and therefore have a sizable ρ^0-ω interference effect. In particular, we can hope for a precise determination of the phases involved. We will therefore take into account in the following discussions both R_d as well as ΔR_d since the correction might amount here to a sizable effect in the phase. If the photoproduction of ω and ρ^0 would be purely diffractive, then we could expect that in the asymptotic limit

$$g_{\gamma\omega}/g_{\gamma\rho^0}=f_\omega/f_{\rho^0}, \tag{29}$$

for which $SU(6)$ predicts the value $\frac{1}{3}$. If we expect the deviations from the $SU(6)$ relations to be small, then the leading contributions to $R_d+\Delta R_d$ can be written as

$$R_d+\Delta R_d \approx \frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\frac{f_\omega}{f_{\rho^0}}+e^2\beta\frac{f_\omega}{f_{\rho^0}}+e^2\beta\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}. \tag{30}$$

Thus, even if f_ω/f_{ρ^0} is real, we have a sizable imaginary part given by

$$i\,\mathrm{Im}(e^2\beta)\left(\frac{f_\omega}{f_{\rho^0}}+\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\right). \tag{31}$$

Experimentally, we know that $g_{\gamma\omega}/g_{\gamma\rho^0}$ seems to be somewhat bigger than $\frac{1}{3}$ and f_ω/f_{ρ^0} varies quite rapidly with energy in photoproduction off nucleons. We can select out the diffractive production by scattering on heavy nuclei. We note that the value $2\,\mathrm{Im}(e^2\beta)\approx 0.1$ is quite independent of the value of $\Gamma_{\rho^0\omega}$ since even if $\Gamma_{\rho^0\omega}$ were of the order of $M_{\rho^0\omega}$, its contribution to $2\,\mathrm{Im}(e^2\beta)$ would not exceed 0.02. Therefore, one does expect in the diffractive production off nuclei an imaginary part of $R_d+\Delta R_d$ of the order of

$$i\,\mathrm{Im}(e^2\beta)\left(\frac{f_\omega}{f_{\rho^0}}+\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\right)\approx i\frac{g_{\gamma\omega}}{g_{\gamma\rho^0}}\times 0.05\times\left(\frac{\sigma_{\omega A}}{\sigma_{\rho^0 A}}+1\right). \tag{32}$$

If $\Gamma_{\rho^0\omega}$ is small, then $e^2\beta$ is almost purely imaginary. In that case, the imaginary part, Eq. (32), is about a third of the real part $(g_{\gamma\omega}/g_{\gamma\rho^0})(f_\omega/f_{\rho^0})$.

V. SUMMARY

We discussed the phenomenon of ρ^0-ω mixing and the way it is manifested in several reactions. Our discussion was essentially independent of any specific dynamical model and rested just on general quantum-mechanical principles. We did use some theoretical ideas to estimate $M_{\rho^0\omega}$ but left a_2, a_3, and ϕ as free parameters.

We saw that experiment seems to imply that a_3 is small, which simplifies many results. Should these experimental results change in the future, then one would require all these parameters to fit the data. We correlated the various experiments and pointed out the parameters which they really measure. The main aim was to put order in assumptions as well as results in this field.

The smallness of a_3 implies the shoulder shape in the $\pi^+\pi^-$ production by e^+e^- colliding beams. Therefore, the shapes of the interference effects in other production experiments give direct information about the relative phases of the production amplitudes. Since this is discussed in more detail in Ref. 5, we did not list all possible results and predictions. We have seen that the exact determination of the phases is affected also by the correction term ΔR. In particular, we found that in the photoproduction of leptonic pairs we might expect a relative phase even if the production amplitudes are relatively real, as discussed in Sec. IV.

ACKNOWLEDGMENTS

This work was carried out while the author was a summer visitor at the Center for Particle Theory at Austin, Texas and the Theory Group of SLAC at Stanford University. The author would like to thank Professor T. Griffy and Professor S. Drell for the kind hospitality extended to him at Austin and Stanford, respectively. The author would like to thank J. Ballam, Y. Dothan, R. G. Parsons, A. H. Rosenfeld, Y. Tsai, and R. Weinstein for helpful discussions and remarks.

PHYSICAL REVIEW D VOLUME 1, NUMBER 5 1 MARCH 1970

Electromagnetic Meson Decays from an Effective Lagrangian with Chiral Symmetry*

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(Received 7 November 1969)

A nonet-symmetrical and PCAC-violating term is introduced into a Lagrangian with $SU_3 \otimes SU_3$ chiral symmetry and PCAC (hypothesis of partially conserved axial-vector current). Breaking of the nonet symmetry of this term is achieved through the renormalization of the fields. This renormalization is necessary because of the requirement that the kinetic-energy terms of the Lagrangian be correctly normalized after the removal of the tadpole terms. Using as input the decay rate for $\pi^0 \to 2\gamma$, we predict $\Gamma(\eta \to 2\gamma) = 0.41 \pm 0.08$ keV and $\Gamma(X^0 \to 2\gamma) = 6 \pm 1$ keV. The result for $\Gamma(\eta \to 2\gamma)$ agrees with experiment within 2 standard deviations. It is argued that errors of about 20% should be expected in our calculations, mainly because of the neglect of the effects due to the finite width of the various particles.

I. INTRODUCTION

OVER the past three years, many models[1–7] have been suggested which have attempted to explain, or at least incorporate in a scheme, the experimental fact[8] that the ratio

$$\Gamma(\eta \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma) \tag{1.1}$$

is about six times larger than the SU_3 prediction. The variety of the assumptions made in these models is rather large, but of course one can discriminate among them by measuring the electromagnetic decay rates of some other mesons; in particular, $X^0 \to 2\gamma$. Meanwhile Gasiorowicz and Geffen[9] (hereafter referred to as GG) have studied in detail a Lagrangian which is invariant under chiral $U_3 \otimes U_3$ apart from a term which is breaking it, in such a way that we have exact PCAC (partial conservation of axial-vector current) for all nonconserved current except, of course, for the zeroth axial-vector current.[10] They have found that when the

* Work performed under the auspices of U. S. Atomic Energy Commission.

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