

Inelastic Scattering of Polarized Leptons from Polarized Nucleons*

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A previously derived sum rule, based on $U(6) \otimes U(6)$ equal-time commutation relations for the space components of the electromagnetic current, implies mean polarization asymmetries of greater than 20% throughout most of the inelastic continuum.

I. INTRODUCTION

SOME time ago, a high-energy sum rule involving electromagnetic scattering of longitudinally polarized leptons from polarized protons and neutrons was derived¹ and then dismissed as "worthless." However, it turns out to be interesting to reconsider that negative conclusion in light of the present experimental and theoretical situation.^{2,3} We find that given "naive" quark-model equal-time commutation relations for the space components of the electromagnetic current⁴ and reasonable estimates for the convergence of the related sum rule, there must be parallel-antiparallel asymmetry effects of greater than 20% over a large region of the "deep inelastic" continuum. It appears that the relevant experiments with electrons—or even muons—may be feasible.

In Sec. II we review the kinematics of polarized lepton scattering from a polarized target⁵ and make contact with the sum rule previously derived. Our main result is Eq. (2.10) and its consequences. In Sec. III, we estimate the magnitude of the asymmetry effects, given the present data. The Appendix provides more details of the kinematics and a simplified derivation of the sum rule.

II. KINEMATICS

The differential cross section for electroproduction of a hadron system Γ from a left-handed incident lepton⁵ can be written at high Q^2 and ν (specifically, $\nu \gg M$, $\nu^2 \gg Q^2$) as follows:

$$\frac{d\sigma_L}{dQ^2 d\nu d\Gamma} = \frac{\pi}{EE'} \frac{d\sigma_L}{d\Omega dE' d\Gamma} = \frac{4\pi\alpha^2 E'}{Q^4 E} \times \sum_{n \text{ in } d\Gamma} |\langle n | j^{\text{lept.}} \cdot J | Ps \rangle|^2 (2\pi)^3 \delta^4(P_n - P - q), \quad (2.1)$$

where E = energy of incident lepton, E' = energy of

scattered lepton, θ = angle of scattered lepton relative to incident lepton, q = four-momentum of incident virtual photon, $Q^2 = -q^2 = 4EE' \sin^2(\frac{1}{2}\theta)$, $\nu = E - E'$, and P, S = proton four-momentum and spin.

The polarization of the virtual photon is determined in terms of the lepton current, which can be computed explicitly. For $\nu \gg M$ and $\nu^2 \gg Q^2$ it is especially simple:

$$j^{\text{lept.}} \cong \frac{\sqrt{Q^2}}{\nu} \left[\epsilon_S^\mu + \left(\frac{E}{2E'} \right)^{1/2} \epsilon_{L^\mu} + \left(\frac{E'}{2E} \right)^{1/2} \epsilon_{R^\mu} \right]. \quad (2.2)$$

The ϵ_i ($\epsilon_S^2 = +1$, $\epsilon_R^2 = \epsilon_L^2 = -1$) are normalized polarization vectors for longitudinal (S), right-handed (R), and left-handed (L) virtual-photon helicity states.

If the final hadron system which is detected is rotated rigidly about the direction of \mathbf{q} by angle ϕ (in laboratory frame) and if the initial hadron is polarized along the direction of \mathbf{q} , the cross section is modified only by the following replacement in (2.2):

$$\begin{aligned} \epsilon_{R,L}^\mu &\rightarrow \epsilon_{R,L}^\mu e^{\pm i\phi}, \\ \epsilon_S^\mu &\rightarrow \epsilon_S^\mu. \end{aligned} \quad (2.3)$$

Upon averaging over ϕ , the interference terms between amplitudes of differing helicity vanish⁶ and the cross section becomes

$$\int \frac{d\phi}{2\pi} \frac{d\sigma_L}{dQ^2 d\nu d\Gamma} \cong \frac{\alpha}{\pi Q^2 \nu} \frac{E'}{E} \left(1 - \frac{Q^2}{2M\nu} \right) \times \left[\frac{d\sigma_S}{d\Gamma} + \frac{E}{2E'} \frac{d\sigma_L}{d\Gamma} + \frac{E'}{2E} \frac{d\sigma_R}{d\Gamma} \right], \quad (2.4)$$

where we use the Hand-Berkelman⁷ notation

$$\frac{d\sigma_i}{d\Gamma} = \frac{4\pi^2\alpha}{\nu - Q^2/2M} \times \sum_{n \text{ in } d\Gamma} |\langle n | \epsilon_i \cdot J | Ps \rangle|^2 (2\pi)^3 \delta^4(P_n - P - q). \quad (2.5)$$

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¹ J. Bjorken, Phys. Rev. **148**, 1467 (1966) [Eq. (6.16)].

² E. Bloom, D. Coward, H. DeStaeble, J. Drees, G. Miller, L. Mo, R. Taylor, M. Breidenbach, J. Friedman, G. Hartmann, and H. Kendall, Phys. Rev. Letters **23**, 930 (1969), and references quoted therein.

³ J. Bjorken, Stanford Linear Accelerator Center Report No. SLAC-PUB-571 (unpublished).

⁴ R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964).

⁵ We follow the approach of N. Christ and T. D. Lee, Phys. Rev. **143**, 1310 (1966); see also N. Dombey, Rev. Mod. Phys. **41**, 236 (1969).

⁶ If \mathbf{s} is not parallel to \mathbf{q} , this azimuthal average cannot be made. There then exists an additional interference term between the longitudinal amplitude and transverse amplitudes. This term is most likely small because of the observed smallness of σ_S ; in any case it can be independently determined by varying E' and θ at fixed Q^2 and ν . A superconvergence sum rule for this interference term has been discussed by H. Burkhardt and W. Cottingham [University of Birmingham Report (unpublished)]; see also Ref. 5.

⁷ L. Hand, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies* (Stanford Linear Accelerator Center, Stanford, Calif., 1967); Phys. Rev. **185**, 1975 (1969).

For the present application, we sum over all Γ but do not average over the initial polarization s . For fixed ν and Q^2 , and for $E \rightarrow \infty$, polarization effects vanish and

$$\lim_{E \rightarrow \infty} \frac{d\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} W_2(Q^2, \nu). \quad (2.6)$$

Comparison with Eq. (2.4) yields (for $\nu^2 \gg Q^2$)

$$W_2 = \frac{Q^2}{4\pi^2\alpha\nu} \left(1 - \frac{Q^2}{2M\nu}\right) (\sigma_T + \sigma_S), \quad (2.7)$$

with

$$2\sigma_T = \sigma_L + \sigma_R, \quad (2.8)$$

and we can rewrite the integrated version of (2.4) as

$$\frac{d\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2 E'}{Q^4 E} W_2(Q^2, \nu) \left[1 + \frac{\nu}{E'} \frac{\sigma_L}{\sigma_L + \sigma_R + 2\sigma_S} - \frac{\nu}{E} \frac{\sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right], \quad \nu \gg M, \nu^2 \gg Q^2. \quad (2.9)$$

With this form for the cross section, we can make contact with the sum rule derived in Ref. 1. Using subscripts P and A (instead of R and L) to denote parallel and antiparallel configurations of virtual-photon spin with respect to nucleon spin, we see that the sum-rule equation (6.16) of Ref. 1 may be written

$$\begin{aligned} \lim_{Q^2 \rightarrow \infty} \int_0^\infty d\nu W_2(\nu, Q^2) \left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S} \right) \\ = Z = \lim_{P_z \rightarrow \infty} \frac{1}{2} i \int d^3x \langle P | [J_x(\mathbf{x}, 0), J_y(0)] | P \rangle \\ = \bar{Z} + \frac{1}{6} \left| \frac{g_A}{g_V} \right| \text{ for quark algebra, proton target} \\ = \bar{Z} - \frac{1}{6} \left| \frac{g_A}{g_V} \right| \text{ for quark algebra, neutron target.} \end{aligned} \quad (2.10)$$

Here $|g_A/g_V| \approx 1.2$ is the ratio of β -decay coupling constants and \bar{Z} is an isotopic-scalar contribution which depends upon the model of the nucleon. Equation (2.10) is our principal result. In the Appendix, we produce a simplified derivation of the result, along with more general kinematical considerations.

III. LOWER BOUND FOR POLARIZATION EFFECTS

Most theoretical models⁸⁻¹² anticipate a near equality between W_{2n} and W_{2p} . If they are significantly different,

⁸ R. P. Feynman (unpublished); J. Bjorken and E. Paschos, Phys. Rev. **185**, 1975 (1969).

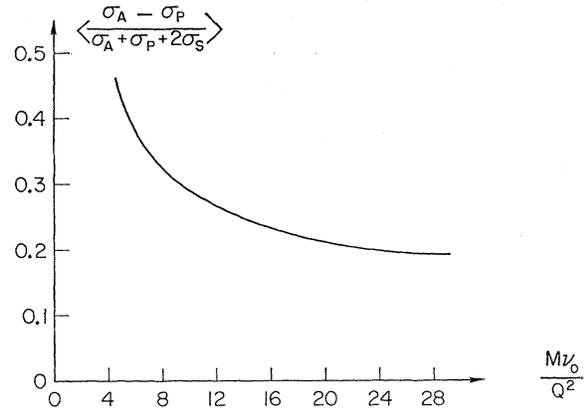


FIG. 1. Mean asymmetry $\bar{\epsilon}$ for polarized electron-nucleon scattering as function of $M\nu_0/Q^2$, where ν_0 is the energy at which the sum-rule equation (2.10) converges. $\bar{\epsilon}$ is given by (3.2) with $Z' = 0$.

this in itself would most likely imply a "parton" interpretation^{8,9} of the deep-inelastic experiments and, consequently, nonvanishing polarization effects of the type exhibited in (2.10). If it turns out experimentally that $W_{2n} \approx W_{2p}$, it will be harder to decide between the two classes of theories—those based on a parton interpretation and those based on a diffraction mechanism (Pomeranchuk exchange,^{10,11} vector dominance,¹² etc.)—on the basis of unpolarized data alone. Depending upon the sign of \bar{Z} , the magnitude of the right-hand side of (2.10) must be greater than 0.2 for either the proton or the neutron target. Because the integral over W_{2p} experimentally is rather small, the polarization effects must be large. Let us suppose that the sum rule equation (2.10) converges at some value $2M\nu_0/Q^2 \equiv \omega_0$. Then, using the premise² $\sigma_T \gg \sigma_S$ and $\lim_{\nu \rightarrow \infty} \nu W_2(\nu, Q^2) = 0.33$, we find from the data² that

$$\int_0^{\omega_0} d\nu W_{2p}(\nu, Q^2) \approx 0.33 [\ln \omega_0 - 0.9] \quad (\omega_0 > 2). \quad (3.1)$$

We define the mean asymmetry $\bar{\epsilon}_p$ as

$$\begin{aligned} \bar{\epsilon}_p &= \int_0^{\omega_0} d\nu W_{2p}(\nu, Q^2) \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S} / \int_0^{\omega_0} d\nu W_{2p}(\nu, Q^2) \\ &= \frac{\bar{Z} + \frac{1}{6} |g_A/g_V|}{0.33 [\ln \omega_0 - 0.9]} \cong Z' + \frac{0.6}{\ln \omega_0 - 0.9}. \end{aligned} \quad (3.2)$$

Thus either $|\bar{\epsilon}_p|$ or $|\bar{\epsilon}_n|$ must be greater than $0.6 [\ln \omega_0 - 0.9]^{-1}$. The predicted value of $\bar{\epsilon}$, assuming $Z' = 0$, is plotted in Fig. 1.

⁹ S. Drell, D. Levy, and T. Yan, Phys. Rev. Letters **22**, 744 (1969).

¹⁰ H. Harari, Phys. Rev. Letters **22**, 1078 (1969).

¹¹ H. Abarbanel, M. Goldberger, and S. Treiman, Phys. Rev. Letters **22**, 500 (1969).

¹² J. Sakurai, Phys. Rev. Letters **22**, 978 (1969).

Notice that for given E , E' and θ (and $\nu^2 \gg Q^2$) the experimental asymmetry Δ is

$$\Delta = \frac{(d\sigma_A/d\nu dQ^2) - (d\sigma_P/d\nu dQ^2)}{(d\sigma_A/d\nu dQ^2) + (d\sigma_P/d\nu dQ^2)} \\ \cong \frac{\nu(E+E')}{2EE'} \left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S} \right) \\ \times \left[1 + \frac{\nu^2}{2EE'} \left(\frac{\sigma_T}{\sigma_T + \sigma_S} \right) \right]^{-1}. \quad (3.3)$$

The most advantageous case for observing an experimental asymmetry occurs for large scattering angles, for which $E' \ll E$. Then, from (3.3), Δ becomes

$$\Delta \approx \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P}, \quad (3.4)$$

and, from (3.2) and Fig. 1, for either proton or neutron,

$$\Delta \geq \epsilon \geq 0.2, \quad (3.5)$$

provided $W_0 \leq 60$. That large scattering angles are most favorable follows simply from the fact that this situation corresponds to backward scattering of electron and nucleon in the center-of-mass frame. Under this circumstance the virtual-photon helicity must be the same as that of the incident lepton.

It appears to be possible to produce electron¹³ or muon polarized beams which have nearly 100% longitudinal polarization. Polarized targets of ~4% polarization per nucleon are at present in use.¹⁴ Therefore, nearly 1% raw asymmetries are predicted; this may well be within range of muon scattering as well as electron scattering experiments in the future.

The use of "naive" commutation relations of space components of currents has been criticized.¹⁵⁻¹⁸ It has been shown that, given the validity of the perturbation expansion of a renormalizable field theory, such equal-time commutators are modified from their naive (canonical) values by the effects of the interactions. It can also be argued that, since the perturbation expansion gives unreliable results for asymptotic behavior of matrix elements of currents (e.g., elastic form factors), this may also be the case for the commutators. But in

¹³ V. Hughes, M. Lubell, M. Posner, and W. Raith, in *Proceedings of the Sixth International Conference on High-Energy Accelerators* (unpublished).

¹⁴ O. Chamberlain (private communication); M. Borghini, S. Mango, O. Runoffsson, and J. Vermeulen, in *Proceedings of the International Conference on Polarized Targets and Ion Sources, Saclay, France, 1966* (Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, France, 1967).

¹⁵ A. Vainstein and B. Ioffe, *Zh. Eksperim. i Teor. Fiz. Pisma v Redaktsiyu* **6**, 917 (1967) [English transl.: *Soviet Phys.—JETP Letters* **6**, 341 (1967)].

¹⁶ S. Adler and W. Tung, *Phys. Rev. Letters* **22**, 978 (1969).

¹⁷ R. Jackiw and G. Preparata, *Phys. Rev.* **185**, 1748 (1969).

¹⁸ R. Jackiw, CERN Report No. TH-1065 (unpublished).

any case, the commutator $[J_x, J_y]$ in question is an observable and this polarization experiment *measures* its matrix element between nucleons. Any reasonable nonvanishing value should be detectable experimentally.

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APPENDIX: KINEMATICAL DETAILS AND SUM RULE

The approximation $\nu^2 \gg Q^2$ used in the text is valid provided $\nu \gg M$, because from kinematics

$$\nu \geq Q^2/2M, \quad (A1)$$

which implies

$$\nu^2/Q^2 \geq \nu/2M. \quad (A2)$$

Likewise, because the sum rule equation (2.10) is valid only as $Q^2 \rightarrow \infty$, the inequality $\nu \gg M$ will be satisfied in that limit as a consequence of (A1). Therefore, in principle, the formulas quoted in the text should suffice. However, in practice, the neglected terms may contribute and we here present the correct formulas, which are somewhat more opaque. Instead of (2.2), the correct expression (neglecting only lepton mass) is

$$j_\mu^{\text{lept}} = \left(\frac{Q^2}{\nu^2 + Q^2} \right)^{1/2} \left[\left(1 - \frac{Q^2}{4EE'} \right)^{1/2} \right. \\ \left. + \frac{E+E'}{(8EE')^{1/2}} (\epsilon_\mu^L + \epsilon_\mu^R) + \left(\frac{\nu^2 + Q^2}{8EE'} \right)^{1/2} (\epsilon_\mu^L - \epsilon_\mu^R) \right]. \quad (A3)$$

Equations (2.4)–(2.6) remain correct, but (2.7) is replaced by

$$W_2 = \frac{Q^2}{4\pi^2\alpha\nu} \left(1 + \frac{Q^2}{\nu^2} \right)^{-1} \left(1 - \frac{Q^2}{2M\nu} \right) (\sigma_T + \sigma_S). \quad (A4)$$

Also, the important equation (2.9) becomes

$$\frac{d\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2 E'}{Q^4 E} W_2(Q^2, \nu) \\ \times \left[1 - \frac{Q^2}{4EE'} + \frac{\nu^2 + Q^2}{2EE'} \left(\frac{\sigma_L + \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right) \right. \\ \left. + \frac{(\nu^2 + Q^2)^{1/2} (E+E')}{2EE'} \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right) \right]. \quad (A5)$$

The experimental asymmetry Δ defined in (3.3) becomes

$$\Delta = \frac{(\nu^2 + Q^2)^{1/2}(E + E')}{2EE'} \left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S} \right) \times \left[1 - \frac{Q^2}{4EE'} + \frac{\nu^2 + Q^2}{2EE'} \left(\frac{\sigma_T}{\sigma_T + \sigma_S} \right) \right]^{-1}. \quad (\text{A6})$$

To derive the sum rule, we observe that

$$\begin{aligned} J_{xy} &\equiv \sum_n |\langle n | \epsilon_L \cdot J | Ps \rangle|^2 (2\pi)^3 \delta^4(P_n - P - q) \\ &\quad - \sum_n |\langle n | \epsilon_R \cdot J | Ps \rangle|^2 (2\pi)^3 \delta^4(P_n - P - q) \\ &= \frac{i}{2\pi} \int d^4x e^{iq \cdot x} \langle Ps | [J_x(x), J_y(0)] \\ &\quad - [J_y(x), J_x(0)] | Ps \rangle \\ &= \frac{i}{\pi} \int d^4x e^{iq \cdot x} \langle Ps | [J_x(x), J_y(0)] | Ps \rangle, \end{aligned} \quad (\text{A7})$$

where in the last line, a 90° rotation about the z axis was used to relate the two commutators. The z axis is taken along \mathbf{q} , which is also the direction of $\pm \mathbf{s}$.

From (2.5) and (2.7), and in the laboratory frame,

$$\begin{aligned} J_{xy} &= \left(\nu - \frac{Q^2}{2M} \right) \frac{\sigma_L - \sigma_R}{4\pi^2 \alpha} \\ &\approx \frac{2\nu^2}{Q^2} W_2(\nu, Q^2) \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right). \end{aligned} \quad (\text{A8})$$

Under a Lorentz transformation in the z direction,

$(P_0/M)J_{xy}$ remains invariant. Let $\mathbf{q} = 0$, $P_z \rightarrow \infty$, $q_0 \rightarrow \infty$, and $q_0/P_0 = -\omega = -Q^2/M\nu$ fixed. In that limit,¹⁹ letting $\tau = P_0 t$,

$$\begin{aligned} \lim_{P_z \rightarrow \infty} \frac{i}{\pi} \int d^3x dt e^{-i\omega\tau} \langle Ps | [J_x(\mathbf{x}, \tau/P_0), J_y(0)] | Ps \rangle \\ = \lim_{P_z \rightarrow \infty, q_0 \rightarrow \infty, \omega \text{ fixed}} \frac{2M\nu^2}{Q^2} W_2(\nu, Q^2) \\ \times \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right). \end{aligned} \quad (\text{A9})$$

As in Ref. 19, νW_2 (times the cross-section ratio in parentheses) becomes a function of ω alone. Integrating over all ω , we get

$$\begin{aligned} \lim_{Q^2 \rightarrow \infty} 2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \nu W_2 \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right) \\ = \lim_{Q^2 \rightarrow \infty} 4 \int_0^{\infty} d\nu W_2(\nu, Q^2) \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \right) \\ = \lim_{P_z \rightarrow \infty} 2i \int d^3x \langle Ps | [J_x(\mathbf{x}, 0), J_y(0)] | Ps \rangle \equiv 4Z. \end{aligned}$$

This reproduces the sum rule equation (6.16) of Ref. 1. The "justification" of the equality of the limit for time-like and spacelike $Q^2 \rightarrow \infty$ can be done along the lines used in deriving general asymptotic sum rules.¹⁹ But it must be said that the derivation used in Ref. 1 remains the most reliable.

¹⁹ J. Bjorken, Phys. Rev. **179**, 1547 (1969).