In (A4) the dependence of the wave function ψ on k can be ignored, since—as discussed in Sec. II—the nucleus is well localized compared to its charge radius, and ψ is therefore essentially constant over the range of k for which the form factor f_V is appreciable. Also, we can replace E_{p-k} by E_p. Once these replacements are made, we can write

$$f_V = 1 + (f_V - 1) \tag{A5}$$

and note that it is the first term in (A5) which gives the Coulomb correction to zero order in the isovector charge radius. Retaining only this term in (A4), and employing the identities

$$\gamma_4 \boldsymbol{k} \gamma_4 = \boldsymbol{k}\,, \tag{A6a}$$

$$\gamma_4(m+\boldsymbol{e})\gamma_4 v(e) = E_e \gamma_0 v(e)\,, \tag{A6b}$$

the matrix element in (A4) becomes identical to the expression in (5.17).

PHYSICAL REVIEW D VOLUME 1, NUMBER 5 1 MARCH 1970

Magnetic Radiation in $K^\pm \to \pi^\pm \pi^0 \gamma$ Decays*

G. V. Dass†
Rutherford Laboratory, Chilton, Berkshire, United Kingdom

AND

A. N. Kamal
Physics Department, University of Alberta, Edmonton, Canada

(Received 30 October 1969)

An estimate of the strength of the magnetic radiation term in $K^\pm \to \pi^\pm\pi^0\gamma$ decays is made by relating it to the τ^+ decay rate and using the Veneziano model to give the off-mass-shell dependence of the amplitudes. We find this strength to be small ($|x_m| \leq 0.14$) and practically constant in the kinetic-energy range 55 MeV $\leq T_{\pi^+} \leq 80$ MeV. The results are discussed in the light of $K^\pm \to \pi^\pm\pi^0\gamma$ data, particularly for possible *CP*-violating effects.

The possibility[1] of *CP*-noninvariant effects in $K^\pm \to \pi^\pm\pi^0\gamma$ decays has led to considerable experimental[2–6] and theoretical[7,8] activity. The results of extensive experiments at Brookhaven National Laboratory, CERN, and Berkeley are expected to be available soon. Since the possible charge asymmetries in $K^\pm \to \pi^\pm\pi^0\gamma$ decays arise from interference between inner bremsstrahlung amplitude and the direct amplitude,[9] it would be desirable to have a reliable theoretical estimate of the strength of the direct amplitude in order to get an estimate of the expected asymmetry. In the present work we have estimated the strength of the magnetic part of the direct amplitude by relating the amplitude for $K^+ \to \pi^+\pi^0\gamma$ to the τ^+ decay amplitude which we take from experiment. If one sums over the photon polarizations, the charge asymmetry is directly proportional[1] to Imx_e (x_e is the strength[1] of the direct electric amplitude). Though we cannot estimate the strength x_e of the direct electric term, a knowledge of the strength x_m of the direct magnetic term is important for at least two reasons: First, attempts to determine x_e from the present experimental data would no longer require arbitrary assumptions about x_m, and second, if the polarizations are observed, one may observe effects[10] due to x_m.

Using the Veneziano model[11] to give the off-mass-shell extrapolations of various amplitudes, Lovelace[12] has been able to reproduce many of the results known from experiments, PCAC (partial conservation of axial-vector current), and/or current algebra. In particular, considering the pole-model type of diagram shown in Fig. 1 (symbol A in this diagram stands for a strangeness-zero 0^- system, not just a pion), Lovelace was able to reproduce the experimental decay spectra in $\eta \to 3\pi$ and $K \to 3\pi$ decays. However, the process shown in Fig. 2 would also contribute[13,14] to these spectra. It has

* Work supported in part by the National Research Council of Canada.

† On attachment from the Atomic Energy Research Establishment, Harwell, Berkshire, United Kingdom.

1 G. Costa and P. K. Kabir, Phys. Rev. Letters **18**, 429 (1969); **18**, 526 (1967).

2 D. Cline and W. F. Fry, Phys. Rev. Letters **13**, 101 (1964).

3 D. Cline, Phys. Rev. Letters **16**, 367 (1966).

4 J. McL. Emmerson and T. W. Quirk, Phys. Rev. Letters **23**, 393 (1969).

5 P. K. Kijewski, UCRL Report No. 18433 (unpublished).

6 B. Wolff and B. Aubert, Phys. Letters **25B**, 624 (1967).

7 Y. Singh, Phys. Rev. **175**, 2214 (1968).

8 J. D. Dorren, D. Phil. thesis, Oxford University, 1969 (unpublished).

9 J. D. Good, Phys. Rev. **113**, 352 (1959).

10 In fact, if the final-state interaction factor is unfavorable for appreciable asymmetry after polarization summation, it could be very favorable for detecting polarization asymmetry effects due to x_m. See Ref. 1, Eqs. (7) and (8).

11 G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

12 C. Lovelace, Phys. Letters **28B**, 264 (1968).

13 M. Jacob, C. H. Llewellyn Smith, and S. Pokorski, Nuovo Cimento **63A**, 574 (1969).

14 D. G. Sutherland, Nucl. Phys. **B13**, 45 (1969).

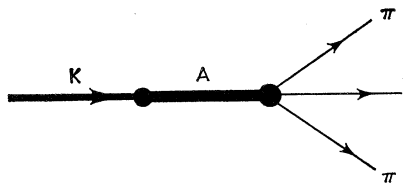

FIG. 1. A pole-model type of diagram. A is a zero-strangeness 0^- system.

been shown[13] that if the off-mass-shell extrapolations are ignored, then the inclusion of Fig. 2 leads to a slightly positive slope for τ^+ amplitude, in contrast to a substantial negative slope observed experimentally. The data would therefore suggest a suppression of the contribution of Fig. 2 relative to that of Fig. 1, possibly due to large off-mass-shell corrections. In what follows we shall assume that Fig. 1 represents a reasonable model for the structure of τ^+-decay amplitude.

Our model for the magnetic part of the direct amplitude in $K^\pm \to \pi^\pm\pi^0\gamma$ is that shown in Fig. 3. The charged system A is the same 0^- system which appears in Fig. 1. We expect the contribution of the analog of

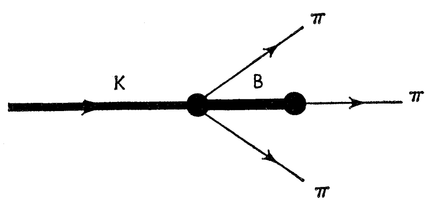

FIG. 2. Another diagram contributing to $K \to 3\pi$ decay. B is a strangeness-unity 0^- system.

Fig. 2 to be suppressed in $K^\pm \to \pi^\pm\pi^0\gamma$ decay for the same reasons as it is in the τ^+ decay. The amplitude for $K^+ \to \pi^+\pi^0\gamma$ decay is then written as

$$A(K^+ \to \pi^+\pi^0\gamma) = f(m_K{}^2)\beta_\gamma[B/(m_K{}^2 - m_A{}^2)]\epsilon_{\mu\nu\sigma\lambda}p^\mu q^\nu k^\sigma \epsilon^\lambda, \quad (1)$$

where m_A is an "effective mass" of the system labeled A. To be more general, we could write the propagator for the system A in a spectral representation form, and the subsequent algebra would go through just as well. f is the weak transition amplitude $K \to A$. β_γ is the strength of the $\pi A \to \pi\gamma$ Veneziano amplitude; p, q, and

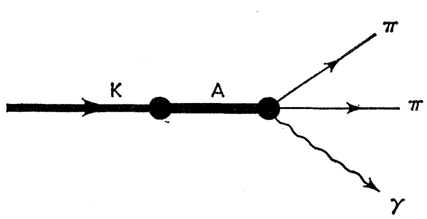

FIG. 3. Model for $K^\pm \to \pi^\pm\pi^0\gamma$. A is a zero-strangeness 0^- system.

k are the four-momenta of π^+, π^0, and γ, respectively; and ϵ^λ is the photon polarization four-vector. B generates the dynamical structure of the amplitude,

$$B = B_{st} + B_{tu} + B_{su}, \quad (2)$$

with

$$B_{xy} = \Gamma(1-\alpha_x)\Gamma(1-\alpha_y)/\Gamma(2-\alpha_x-\alpha_y). \quad (3)$$

α_x is the ρ trajectory,[13] and s, t, u are the Mandelstam variables for $\pi A \to \pi\gamma$. Normalizing the amplitude at the ρ pole, one gets

$$\beta_\gamma = \alpha_\rho' g_{\rho\gamma\pi} g_{\rho\pi A}, \quad (4)$$

where $g_{\rho\gamma\pi}$ is defined with all particles on their mass shells, while $g_{\rho\pi A}$ has system A off-mass-shell at $m_K{}^2$. $g_{\rho\gamma\pi}$ is defined via the vertex function

$$A(\rho^\alpha(P) \to \pi^\beta + \gamma(k)) = g_{\rho\gamma\pi}\delta^{\alpha\beta}\epsilon_{\mu\nu\sigma\lambda}\epsilon_{(\rho)}{}^\mu \epsilon_{(\gamma)}{}^\nu P^\sigma k^\lambda \quad (5)$$

and

$$\alpha_\rho' = 0.89 \text{ GeV}^{-2}. \quad (6)$$

Defining a symbol

$$\Psi = f(m_K{}^2) g_{\rho\pi A}/(m_K{}^2 - m_A{}^2), \quad (7)$$

we could write the strength of the direct magnetic term[1] as

$$|x_m| = \frac{m_\pi{}^4 \alpha_\rho' B \Psi}{|A(K^+ \to \pi^+\pi^0)|(4\pi\alpha)^{1/2}} g_{\rho\gamma\pi}. \quad (8)$$

Here α (without any subscript) is the fine-structure constant. Finally, we relate Ψ to the τ^+-decay amplitude represented in Fig. 1. This amplitude is given by

$$A(K^+ \to \pi^+\pi^+\pi^-) = f(m_K{}^2)[\beta/(m_K{}^2 - m_A{}^2)]V(\rho,\rho). \quad (9)$$

β is the strength of the Veneziano amplitude for $\pi A \to \pi\pi$ and $V(\rho,\rho)$ is defined in Eq. (5) of Ref. 13. The assumption is that $V(\rho,\rho)$ gives the entire structure of the amplitude. By going to the ρ pole, one finds

$$\beta = 2 g_{\rho\pi\pi} g_{\rho\pi A}. \quad (10)$$

Here $g_{\rho\pi\pi}$ involves all particles on their mass shells and hence is related to the experimental ρ-width. With[15] $\Gamma_\rho = 120$ MeV,

$$g_{\rho\pi\pi} = 5.5. \quad (11)$$

In terms of Ψ one has

$$A(K^+ \to \pi^+\pi^+\pi^-) = 2\Psi g_{\rho\pi\pi} V(\rho,\rho). \quad (12)$$

Matching this amplitude to the experimental one, we determine Ψ. This value of Ψ is fed into Eq. (8) for $|x_m|$. $|A(K^+ \to \pi^+\pi^0)|$ is taken from the partial decay rate[15] and we use[15] $\Gamma(\rho \to \pi\gamma) \leqslant 0.5$ MeV. We then find

$$|x_m| \leqslant 0.14. \quad (13)$$

If one uses the recent DESY result[16] $\Gamma(\rho \to \pi\gamma) \leqslant 0.24$

[15] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[16] R. Erbe *et al.*, DESY Report No. 69/19, 1969 (unpublished).

MeV, we get instead

$$|x_m|\leqslant 0.1. \tag{14}$$

We also did the calculation within the context of the vector-dominance model. The photon in this case is coupled to the ω meson. The direct magnetic amplitude in this case would be given by Eq. (1) with only one modification, namely that $g_{\rho\gamma\pi}$ in Eq. (4) is replaced[17–19] by

$$(4\pi\alpha)^{1/2}g_{\omega\rho\pi}(2f_y)^{-1}\sin\theta_y\approx 0.3\ \text{GeV}^{-1}.$$

On making this replacement in Eq. (8), one gets

$$|x_m|\approx 0.07. \tag{15}$$

In the derivation of the result in Eq. (15), we have assumed that the strength function for the $\pi A\to\pi\omega$ Veneziano amplitude does not change under the extrapolation $m_\omega^2\to 0$. We emphasize that by relating $K^+\to\pi^+\pi^0\gamma$ to the experimental $K^+\to\pi^+\pi^+\pi^-$, we have tried to minimize the uncertainties of mass extrapolation of the transition vertex $K\to A$. It is worth noting that if we do not do so, but instead use the pole model where the system A is taken to be the pion, and then relate the $K^+\to\pi^+$ amplitude to $K^+\to\pi^+\pi^0$ via PCAC and current algebra,[20] we get a value of $|x_m|$ smaller than the estimate of Eq. (15) by a factor of 6. We do not expect[13,14] our results to be much affected by the use of a complex trajectory.

Since experimental information on charge asymmetries in $K^{\pm}\to\pi^{\pm}\pi^0\gamma$ is still lacking, there have been many attempts to place limits on $|x_e|$ and $|x_m|$ by studying only K^+ decays, so that one could predict how much asymmetry to expect. There are, however, three parameters involved: $|x_e|$, $|x_m|$, and a CP-violating phase ϕ (the phase of x_e relative to the inner bremsstrahlung amplitude), in addition to the strong interaction phase, which is not well known either. The practice followed is to assign values to one or more of these parameters and thereby to limit the allowed range of the remaining parameter(s). The experimental values are[3] $|x_m|=0.0\pm0.27$, with some assumptions about the phases, $|x_m|<0.42$ for[4] $|x_e|=0$, and $|x_m|=0.21\pm0.15$ for[5] $|x_e|=0$. Our limit on $|x_m|$ is consistent with all these numbers and implies a very small contribution from the direct magnetic term relative to the inner bremsstrahlung term. For example, with $|x_m|=0.1$ it is only 2% of the inner bremsstrahlung branching ratio for all allowed π^0 energies and π^+ kinetic energy between 55 and 80 MeV (a typical range studied). With our estimate of $|x_m|$, the results of previous attempts to determine $|x_e|$ and ϕ, assuming $|x_m|=0$, will remain almost unchanged. It should be noted that a large value for $|x_m|$ would tend to suppress the charge asymmetry after polarization summation [Eq. (11) of Ref. 1, for example]. It is gratifying that our estimate of $|x_m|$ turns out to be small.

We also looked at the variation of $|x_m|$ in the kinetic energy range of π^+ between 55 and 80 MeV and found $|x_m|$ constant to within a few percent, thereby implying a dipole dominance of the direct magnetic term.

There have been some theoretical calculations[21–24] on $K^{\pm}\to\pi^{\pm}\pi^0\gamma$. Our result predicting a small branching ratio from $|x_m|$ agrees qualitatively with some of them[22–24] but disagrees with that of Pepper and Ueda,[21] who predict a rate already much too large compared with the known data.[4,5]

Note added in manuscript. The statement regarding the use of current algebra [see the paragraph following Eq. (15)] is subject to large ambiguities and is possibly incorrect. Current algebra was used in the most naive way by relating the $K^{\pm}\to\pi^{\pm}$ amplitude to $K^{\pm}\to\pi^{\pm}\pi^0$ with one pion soft. This relation was then used in the physical situation of both pions on their mass shells. On the other hand, if we relate[20] $K^+\to\pi^+$ to $K_1^0\to\pi^+\pi^-$ with one pion soft and use this relationship when both pions are on their mass shells, we get a value of $|x_m|$ about three to four times *larger* than the estimate of Eq. (15). This difference arises from the fact that although with one pion made soft, both the $K^+\to\pi^+\pi^0$ and $K_1^0\to\pi^+\pi^-$ amplitudes may have the same magnitudes, they extrapolate to the physical region quite differently, $K^+\to\pi^+\pi^0$ being a $\Delta I\geqslant\frac{3}{2}$ decay. We wish to thank D. G. Sutherland for bringing this point to our attention.

One of us (G.V.D.) would like to thank J. Baacke, R. H. Dalitz, M. Jacob, P. K. Kabir, and T. W. Quirk for discussions. He would also like to thank the Theory Group at CERN, where the work was started, and the University of Alberta for their hospitality. The other (A.N.K.) would like to thank his colleagues in the Theory Group at Rutherford Laboratory for the hospitality extended to him.

[17] J. J. Sakurai, *Currents and Fields* (The University of Chicago Press, Chicago, 1969), p. 155.

[18] R. Dashen and D. Sharp, Phys. Rev. **133**, B1585 (1964).

[19] H. Goldberg and Y. Srivastava, Phys. Rev. Letters **22**, 749 (1969).

[20] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).

[21] S. V. Pepper and Y. Ueda, Nuovo Cimento **33**, 1614 (1964).

[22] C. Itzykson, M. Jacob, and G. Mahoux, Nuovo Cimento Suppl. **5**, 978 (1967).

[23] S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. **136**, B1064 (1964).

[24] K. Tanaka, Phys. Rev. **136**, B1813 (1964).