Analysis of 17 898 τ^+ Decays at Rest*

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The energy spectrum of the decay pions in τ^+ decay was obtained from a sample of 17 898 events. It is found that the spectrum is adequately fitted by a squared matrix element of linear form, $|M|^2 \propto 1+\alpha_\tau \times (M_K Q/m_\pi^2)Y$, with $\alpha_\tau = 0.131 \pm 0.008$. No evidence for CP noninvariance is found for $\tau^\pm$ decays. A significant violation of the $\Delta I = \frac{1}{2}$ rule is observed for $K^+ \to 3\pi$ decays. Comparison with several theoretical models for $K \to 3\pi$ decays shows fair agreement between experiment and theory.

I. INTRODUCTION

THE three-pion decay modes of the K mesons have drawn considerable theoretical and experimental interest in recent years. The data have shown systematic deviations in the secondary-pion spectra from the predictions of phase-space alone.[1,2] The precise experimental determination of the spectral shapes, using a large number of events with improved statistics, provides tests of a variety of theoretical predictions.

In this paper a detailed analysis of 17 898 τ^+ decays at rest is presented. Preliminary results have been previously reported.[3,4] The experimental procedures are discussed in Sec. II. Section III contains a description of the data. The details of the analysis are presented in Sec. IV. The results are discussed and compared to predictions of certain theoretical models and to other experiments involving $K \to 3\pi$ decays in Sec. V. In particular, CP conservation is tested by comparing τ^+ to τ^- spectra and deviations from the $\Delta I = \frac{1}{2}$ rule are examined by comparing τ^+ to τ'^+ and K_2^0 spectra.

II. EXPERIMENTAL PROCEDURE

A. Exposure

A beam of K^+ mesons from the alternating gradient synchrotron (AGS) of Brookhaven National Laboratory was stopped in the Columbia-BNL 30-in. propane bubble chamber. The chamber was photographed by three 70-mm cameras. Approximately 150 000 pictures containing about 10^6 K^+'s were taken, of which some 37 000 pictures were scanned for the experiment reported here.

B. Scanning and Preliminary Selection Procedure

Each roll of film was subjected to a systematic frame-by-frame scan and rescan. On the first scanning of the film the scanners were instructed to locate all possible candidates for τ decays. This included all events in which the incoming particle decayed into or produced more than one charged secondary particle. The non-τ background included $K_{\pi 2}$ decays with electron-positron pairs, τ' decays with pairs, strong interactions such as $\pi^+ + n \to \pi^+ + \pi^- + p$, etc. The scanners were encouraged to consult a physicist when they had any doubts that an event was a τ, and in this manner much of the obvious non-τ background was eliminated. All remaining events, including doubtful events, were measured as described below with two minor exceptions. These were the "obvious" τ decays in flight and the "unmeasurable" τ decays. An "obvious" τ decay in flight was defined as one where the secondary particles were emitted in the forward direction with respect to the incoming K meson, and where at least one secondary had a projected range longer than the maximum allowed range for a pion from a τ decay at rest ($\sim$20 cm in the present experiment). An "unmeasurable" τ was one in which the K decay point was visible in a single view only or for which the fiducial marks were missing and hence which could not be geometrically reconstructed. All "unmeasurables"

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[1] A bibliography of early experimental and theoretical results on $K \to 3\pi$ decays is included in Ref. 2.

[2] T. Huetter, S. Taylor, E. L. Koller, P. Stamer, and J. Grauman, Phys. Rev. **140**, B655 (1965). The data have been corrected by one event.

[3] See J. Grauman, E. L. Koller, S. Taylor, D. Pandoulas, S. Hoffmaster, P. Stamer, A. Kanofsky, and V. Mainkar, Bull. Am. Phys. Soc. **13**, 586 (1968). In addition, all τ^+ decays in Ref. 2 are included in the results of the present work.

[4] J. Grauman, S. Taylor, E. L. Koller, D. Pandoulas, S. Hoffmaster, O. Raths, L. Romano. P, Stamer, A. Kanofsky, and V. Mainkar, Phys. Rev. Letters **23**, 737 (1969).

were checked by two physicists. These two types of events were classified and recorded but not measured.

After a roll was scanned and measured for the first time, the measurements were processed by a modified version of the Columbia University NP54[5] spatial reconstruction program using the computing facilities at Columbia and Stevens. The output was returned to the scanners, who were instructed to rescan the roll frame by frame and to check whether the events they found were recorded or measured on the first scan. The rescan was usually carried out by scanners other than those who performed the initial scan. Upon finding new events the scanners were to treat them as described above for the first scan. In the process of rescanning the film the scanners also measured events which failed to meet certain kinematical and geometrical constraints discussed below. A physicist was consulted on events which were originally measured but suspected to be τ decays in flight or non-τ's. The physicist then classified these events after examining them both qualitatively from their geometrical configuration on the scanning table and quantitatively from their kinematics as calculated by the computer program.

C. Scanning Efficiency

Three rolls were subjected to a completely independent second scan in order to determine scanning efficiencies. It was found that the single-scan scanning efficiency ranged from 95 to 99%, giving combined double-scan efficiencies of over 99%.

Owing to these high scanning efficiencies, it was decided that no more than one rescan of the film was necessary to keep the data reasonably free of biases due to particular geometrical configurations of some events.

D. Measurement Procedure and Determination of Secondary-Pion Energies

The secondary-pion ranges were measured in all three views with two or three points per track for a "straight" or a "curved" track, respectively. Where the K decay point or the whole event was obscured in one view, the measurements were performed in the remaining two views. The kinetic energies of the secondaries were obtained from the calculated ranges of the tracks utilizing the range-energy relations incorporated in the NP54[5] spatial reconstruction program, since, at low energies, range measurements are much more accurate than curvature measurements. In cases where one of the secondary tracks was "incomplete" (i.e., the pion interacted, decayed in flight, or left the chamber), the kinetic energy of that track was obtained by imposing momentum conservation along its measured direction. In the case of two "incomplete" tracks, over-all momentum conservation was imposed in order to obtain the energies.

[5] R. J. Plano and D. H. Tycko, Nucl. Instr. Methods **20**, 458 (1963).

E. Selection Criteria for τ Decays Included in Analysis

1. Fiducial Region

A fiducial "volume" was chosen such that for a decay point and decay plane corresponding to that of each kaon under consideration, no kaon could have more than one secondary track leave the chamber. Also rejected were those events in which the decay point lay within 0.5 cm from any wall of the chamber, and events in which the normal to the decay plane made an angle of less than 5° with the horizontal.

2. Kinematical Constraints

The following quantities were calculated for each measured event.

(a) The three kinetic energies t_1, t_2, and t_3 corresponding to the two positive pions and the negative pion, respectively, were calculated, and the Q value was determined from their sum, i.e.,

$$Q_c=\sum_{i=1}^{3} t_i.$$

The value of Q_c was required to be in the range 75.08 $\pm$6.00 MeV.[6] The kinetic energy of each track was required not to exceed 52 MeV.

(b) The residual momentum

$$p=|\sum_{i=1}^{3} \mathbf{p}_i|$$

was calculated, where the $\mathbf{p}_i$ are the momenta of the secondary pions. The quantity p was required to be less than 30 MeV/c.

(c) A coplanarity measure

$$\phi=\frac{(\mathbf{p}_i\times\mathbf{p}_j)\cdot\mathbf{p}_k}{|\mathbf{p}_i|\,|\mathbf{p}_j|\,|\mathbf{p}_k|}$$

was calculated and required to differ from the zero by no more than $\pm$0.160. In the case where one of the secondary pions was too short to be measured in a meaningful way, and hence classified as being "insignificant," a collinearity measure

$$\phi'=\frac{|\mathbf{p}_i\times\mathbf{p}_j|}{|\mathbf{p}_i|\,|\mathbf{p}_j|}$$

was calculated and required to be less than 0.200.

The limits on the ranges of the above quantities were based on the measurements of an initial sample of approximately 800 events. Since the quantities Q_c, p, and ϕ or ϕ' are not independent, it was required that the percentage of events which passed each test sepa-

[6] All particle masses, Q values, etc., were taken from N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).

rately be approximately the same. This requirement was met at the roughly 90% passing level.

Any event in which one or more of the above-mentioned quantities failed to lie within the specified ranges was returned to the scanners for remeasurement, possibly with a new interpretation. If successive measurements of an event failed one or more of the tests, but were, within statistics, internally consistent and compatible with the event being a τ decay at rest, the criteria were relaxed first by 25%, then, depending on the number of remeasurements, by 50 or 100%, if necessary. The procedure was, however, to exhaust all of the possibilities of reinterpreting an event before large relaxations of the criteria were permitted.

For some events, even 100% expansions of the test criteria were not sufficient. Such events were usually those which had a poor kinematical configuration (e.g., events with two "incomplete" secondary pions and a low-energy "complete" secondary), or those which had a distorted geometrical configuration (such as events in which one of the secondary pions scattered so close to the K decay point that its measured direction was meaningless). Events of this kind were subjected to a special set of "hand calculations" in which the fact that the kinematics for τ decay are overdetermined is utilized. The "hand calculations" are of two types.

(a) Calculations using conservation of momentum. In this case the input, consisting of the kinetic energies of the "good" secondary pions and the direction cosines, was used to calculate the kinetic energies of the "bad" secondary pions by requiring that the vector sum of the secondary momenta vanish, as it must for a τ decay at rest. However, a different method of averaging over the experimental space angles was used than that of the regular calculation. The Q value was then calculated from the sum of the secondary kinetic energies as a check.

(b) Calculations using conservation of energy. In this case the input, consisting of the direction cosines of the three secondary pions, was used, together with the requirement that the Q value be the accepted value of 75.08 MeV, to generate the kinetic energies of the secondary pions.

The results of the "hand calculations" were then examined for internal consistency and compared to the results of the measurements obtained from NP54. A decision was then made, by at least two physicists, as to whether to accept one of the versions of the "hand calculations" or one of the NP54 versions, or to further remeasure the event.

III. DATA

A. Tabulation

Approximately 37 000 frames were scanned and a total number of 23 275 events was located. Of this number 6034 were found to lie outside the prescribed fiducial "volume," 2888 were classified as τ decays in flight, 37 were "unmeasurable" events, and 6 remained "unidentified." The "unidentified" events were τ-like decays which failed to meet the maximum expandable ranges of one or more of the tests described above, and failed to give results consistent with a τ decay in the various "hand calculation" schemes. Since all of these events had abnormally low Q values, they are most likely radiative τ decays. Using the value 8.5×10^{-4} for the branching ratio[7–9] $\Gamma(3\pi+\gamma)/\Gamma(3\pi)$ with a photon of energy >12 MeV, 12 radiative τ decays are expected. Comparing this with the number of "unidentified" events, it is estimated that our final sample contains approximately six undetected radiative τ's. "Unidentified" events constitute a possible source of bias, but the number here is obviously negligible.

TABLE I. Classification of the data.

	Accepted	Rejected
Outside fiducial volume		6034
Decays in flight		2888
Unmeasurable		37
Unidentified		6
Passed all tests	13 718	
Passed relaxed tests	559	
"Hand calculations"	33	
Total	14 310	8965

The remaining events, 14 310 in number, were included in the final analysis. Of these, 13 718 passed all the tests described above, in 559 cases the limits on the deviations of Q_c, p, and ϕ or ϕ', or a combination thereof, had to be relaxed, and 33 were settled by the use of "hand calculations." These figures are summarized in Table I.

B. Renormalization of Secondary-Pion Kinetic Energies

An average Q value Q_0 was calculated for all events which had all three secondary pions "complete" and passed all the unrelaxed criteria described above. The value obtained was $Q_0=76.1$ MeV. The standard deviation of a single event of this type was 1.6 MeV. Likewise, an average Q value Q_{0I} was calculated for all events with an "insignificant" or "zero-energy" track which passed all unrelaxed tests. The value $Q_{0I}=74.7$ MeV was obtained. The difference of the two means, $\Delta Q=Q_0-Q_{0I}=1.34$ MeV, was assigned as the unrenormalized kinetic energy of all "insignificant" secondary pions.

In order to eliminate small systematic effects (such as an error in the assumed density of the liquid in the bubble chamber), and in order to give more weight to the directly measured energies of "complete" tracks as compared with the indirectly measured energies of "in-

[7] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).
[8] I. R. Lapidus and M. J. Tausner, Phys. Rev. **140**, B1620 (1965).
[9] P. Stamer, T. Huetter, E. L. Koller, S. Taylor, and J. Grauman, Phys. Rev. **138**, B440 (1965).

complete" tracks, all events were renormalized as follows:

(a) Events in which all secondary pions were "complete," including those with an "insignificant" pion, had their kinetic energies renormalized as

$$t_{i0}=(Q_0/Q_c)t_i \quad (i=1,2,3).$$

(b) Events with one "incomplete" secondary pion had their kinetic energies renormalized as

$$t_{i0}=t_i,$$
$$t_{j0}=t_j,$$
$$t_{k0}=Q_0-t_i-t_j,$$

where t_i and t_j are the measured kinetic energies of the two "complete" secondary pions.

(c) Events with two "incomplete" secondary pions had their kinetic energies renormalized as

$$t_{i0}=t_i,$$
$$t_{j0}=\frac{Q_0-t_i}{t_j+t_k}t_j,$$
$$t_{k0}=\frac{Q_0-t_i}{t_j+t_k}t_k,$$

where t_i is the calculated kinetic energy of the "complete" secondary pion.

Finally, all kinetic energies were renormalized to the accepted Q value, $Q=75.08$ MeV,[6] by

$$T_i=(Q/Q_0)t_{i0} \quad (i=1,2,3).$$

The 14 310 bubble-chamber events were then combined with the 3588 emulsion events previously reported[2] to give a total sample of 17 898 events for the analysis.

IV. ANALYSIS OF DATA

The final-state kinematics of $K\rightarrow 3\pi$ decays may be described by two independent variables such as the Lorentz-invariant quantities S_3-S_0 and S_1-S_2, where the S_i are the Mandelstam variables defined by

$$S_i=(P-q_i)^2 \quad (i=1,2,3),$$

where P is the four-momentum of the kaon and q_i is the four-momentum of the ith pion. The index $i=3$ refers to the π^-. In the rest frame of the decaying particle,

$$S_i=(M_K-m_i)^2-2M_KT_i$$

and

$$S_0=\tfrac{1}{3}\sum_{i=1}^{3}S_i=\tfrac{1}{3}\sum_{i=1}^{3}(M_K-m_i)^2-\tfrac{2}{3}M_KQ.$$

M_K is the mass of the kaon and m_i is the mass of the ith pion. For convenience the variables X and Y are introduced:

$$X=(\sqrt{3})(S_1-S_2)/2M_KQ,$$
$$Y=-3(S_3-S_0)/2M_KQ,$$

which, in the case of $\tau^\pm$ decay, are identical with the Dalitz variables[10]

$$x=(\sqrt{3})(T_2-T_1)/Q,$$
$$y=(3T_3-Q)/Q.$$

The differential decay probability may be written as

$$w(X,Y)dXdY=|M(X,Y)|^2\phi(X,Y)C(X,Y)dXdY,$$

where $\phi(X,Y)$ is the invariant phase space, $M(X,Y)$ is the matrix element neglecting final-state Coulomb interactions, and $C(X,Y)$ is a factor to account for the final-state Coulomb interactions[11] between pairs of pions.

The physical ranges in X and Y were divided into ten equal intervals, giving 100 "bins," not all of which are physically allowed. The events were sorted into these bins by a sorting program. It was found that 333 events lay outside the physical region, but all lay within ~ 5 MeV of the boundary. These events were moved to the nearest physical bin. Bins which had only a small part within the physical region were consolidated with neighboring bins, thus reducing the number of physical bins from 89 to 80 (see Fig. 1).

The "theoretical" number of events in the i-j bin is given by

$$N_{ij}^{\text{theor}}=N_0\int_{X_{i\,\min}}^{X_{i\,\max}}\int_{Y_{j\,\min}}^{Y_{j\,\max}}w(X,Y)dXdY\Big/\int_R\int w(X,Y)dXdY,$$

where N_0 is the total number of events, R is the entire phase-space region, and $X_{i\,\min}$, $X_{i\,\max}$, $Y_{j\,\min}$, and $Y_{j\,\max}$ give the physical ranges of the variables in the i-j bin.

Following Weinberg,[12] the square of the matrix element, $|M(X,Y)|^2$, may be expanded in a power series in the variables X and Y defined above:

$$|M(X,Y)|^2\propto 1+\alpha_{\tau^+}\left(\frac{M_KQ}{m_\pi^2}\right)Y+\beta_{\tau^+}\left(\frac{M_KQ}{m_\pi^2}\right)^2Y^2+\gamma_{\tau^+}\left(\frac{M_KQ}{m_\pi^2}\right)^2X^2+\cdots.$$

Because of the symmetry under interchange of the like pions, the expansion contains only even powers of X.

[10] R. H. Dalitz, Phil. Mag. **44**, 1068 (1953).
[11] R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956); see footnote on p. 537.
[12] S. Weinberg, Phys. Rev. Letters **4**, 87 (1960); **4**, 585(E) (1960).

The spectrum was examined using the two forms:

$$\text{(a)}\quad |M|^2 \propto 1+\alpha_{\tau^+}\left(\frac{M_K Q}{m_\pi^2}\right)Y$$

and

$$\text{(b)}\quad |M|^2 \propto 1+\alpha_{\tau^+}\left(\frac{M_K Q}{m_\pi^2}\right)Y+\beta_{\tau^+}\left(\frac{M_K Q}{m_\pi^2}\right)^2 Y^2 +\gamma_{\tau^+}\left(\frac{M_K Q}{m_\pi^2}\right)^2 X^2.$$

The dependence of the observed spectrum on the variables X and Y is found by fitting the resultant N_{ij}^{theor} to the observed frequencies N_{ij}^{obs} by means of a χ^2 test. Specifically,

$$\chi^2=\sum_{i,j}\frac{(N_{ij}^{\text{obs}}-N_{ij}^{\text{theor}})^2}{N_{ij}^{\text{theor}}}$$

is minimized with respect to parameters appearing in the various forms of $w(X,Y)$ used to generate the quantities N_{ij}^{theor}.

The χ^2 test, using form (a) above for $|M|^2$, gives $\chi^2=156$ for 78 degrees of freedom, with a χ^2 probability $<0.01\%$. This large value of χ^2 dictated a reexamination of the data. It was found that in the region of the Dalitz plot corresponding to low π^+ energy, nine bins made contributions to χ^2 of 4 or greater. Of these bins, those lying on the border of the Dalitz plot seemed to contain too few events, while those lying one bin away from the border seemed to contain too many events. A possible explanation is that in the case of very short positive pions, the measurers sometimes inadvertently measured the decay muons. (This is consistent with the observation that the muons are not always clearly distinguishable.) To allow for this possibility, four pairs of adjacent bins were consolidated, reducing the number of bins to 76 (see Fig. 1).

A χ^2 test with the consolidated bins, using form (a) for $|M|^2$, gives $\chi^2=114$ for 74 degrees of freedom, with a χ^2 probability of $\sim 0.2\%$. The fit yields[13] $\alpha_{\tau^+}=0.131 \pm 0.008$. While the value of χ^2 is still somewhat large, two widely separated bins contribute 27 to χ^2, the remaining bins corresponding to a $\sim 10\%$ fit. Hence it is felt that the linear form (a) for $|M|^2$ is an adequate fit to the data.

The corresponding χ^2 test using form (b) above gives $\chi^2=111$ for 72 degrees of freedom, with a χ^2 probability of $\sim 0.2\%$. Hence the inclusion of quadratic terms in $|M|^2$ gives no significant improvement in fit. The results are summarized in Table II.

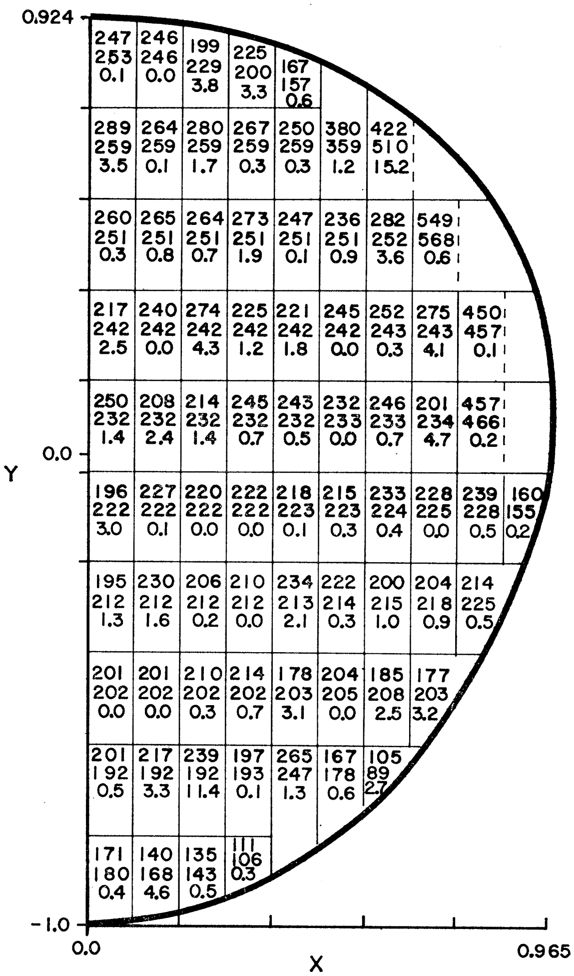

FIG. 1. Dalitz plot, showing division into 76 bins. The dashed lines show the bins prior to the correction for low-energy positive pions. The numbers in each bin are the experimental (top) and theoretical (center) numbers of events, and the contribution to χ^2, for the best-fit linear squared matrix element.

For purposes of display, graphical approximations to $|M|^2(Y)$ and $|M|^2(X)$, both experimental (points with errors) and "theoretical" (solid curves), are presented in Figs. 2 and 3. These were obtained by dividing the experimental and calculated numbers of events, summed over X or Y, by the appropriate integral over phase

TABLE II. Fits to $|M|^2 \propto 1+\alpha_{\tau^+}(M_KQ/m_\pi^2)Y+\beta_{\tau^+}(M_KQ/m_\pi^2)^2Y^2+\gamma_{\tau^+}(M_KQ/m_\pi^2)^2X^2$.

	α_{τ^+}	β_{τ^+}	γ_{τ^+}	Degrees of freedom	χ^2	Probability (%)
Linear fit	0.131 ± 0.008	$\cdots$	$\cdots$	74	114	~ 0.2
Quadratic fit	0.131 ± 0.008	-0.002 ± 0.009	-0.018 ± 0.009	72	111	~ 0.2

[13] Before the consolidation of the four pairs of bins, the value of α_{τ^+} was the same within three significant figures.

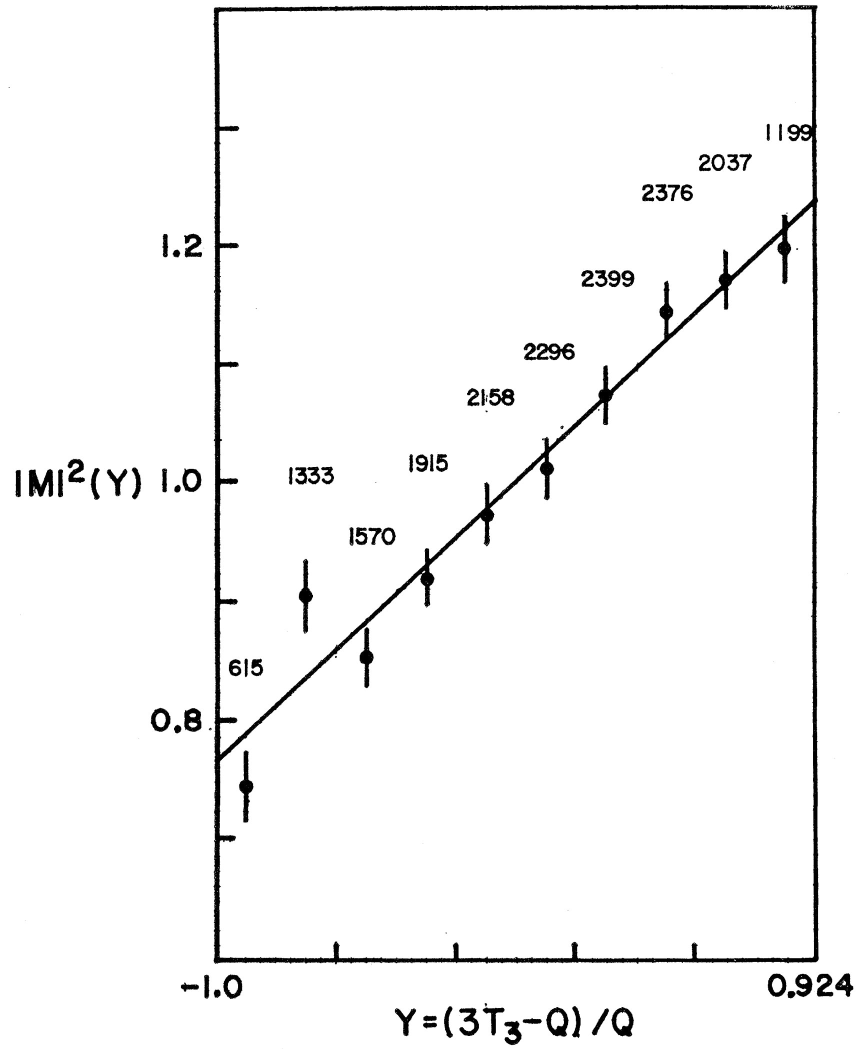

FIG. 2. Y dependence of $|M|^2$. See text. The number of events in each bin is given above the experimental points.

space, including the Coulomb correction, and renormalizing to unity. The linear form, (a) above, was used for $|M|^2$. In the X distribution the phase-space integral included the Y-dependent term of $|M|^2$, since the aver-

TABLE III. Compilation of experimental $K\rightarrow 3\pi$ slope parameters.

Decay mode	Number of events	Authors	Ref.	α
τ^+	17 898	This experiment		0.131±0.008
	9 994	W. R. Butler *et al.*	14	0.146±0.011
	5 428	A. Zinchenko	15	0.147±0.016
τ^-	50 919	T. S. Mast *et al.*	16	0.130±0.005
	5 778	M. L. Moscoso	17	0.127±0.015
	1 347	M. Ferro-Luzzi *et al.*	18	0.147±0.024
τ'^+	4 048	D. Davison *et al.*	19	−0.344±0.012
	1 874	V Bisi *et al.*	20	−0.400±0.067
	1 792	G E. Kalmus *et al.*	21	−0.320±0.027
K_0^2	2 446	P. Basile *et al.*	22	−0.245±0.026
	1 350	H. W. K. Hopkins *et al.*	23	−0.377±0.023
	1 198	B. M. K. Nefkens *et al.*	24	−0.265±0.032

TABLE IV. Predictions of various models.

Author(s)	Ref.	α_τ	$\alpha_{\tau'}$	$\alpha_{K_2^0}$	$\Delta I=\frac{1}{2}$ rule
H. D. J. Abarbanel	27	0.17	−0.35	−0.35	yes
Y. T. Chiu *et al.*	28	0.16	−0.32	−0.32	yes
L. J. Clavelli	29	0.17	−0.37	−0.37	no[a]
D. F. Greenberg	30	0.16	−0.32	−0.32	yes
K. C. Gupta *et al.*	31	0.16	−0.34	−0.30	no[a]
Y. Hara *et al.*	32	0.16	−0.34	−0.30	yes[a]
C. Itzykson *et al.*	33	0.16	−0.32	−0.32	yes
M. C. Li	34	0.16	−0.32	−0.32	yes
P. McNamee	35	0.13	−0.26	−0.26	yes
D. A. Nutbrown	36	0.16	−0.32	−0.32	yes
C. Bouchiat *et al.*	37	0.131[b]	−0.34	−0.33	no
B. Holstein	38	0.131[b]	−0.38	−0.36	no
H. T. Nieh	39	0.131[b]	−0.35	−0.34	no
R. N. Chaudhuri	40	0.15	−0.31	−0.31	yes
R. H. Graham *et al.*	41	0.14	−0.27	−0.25	yes[a]
		0.14	−0.35	−0.18	no[a]

[a] Mass differences within isotopic-spin multiplets are taken into account.
[b] The value obtained by present experiment used as input.

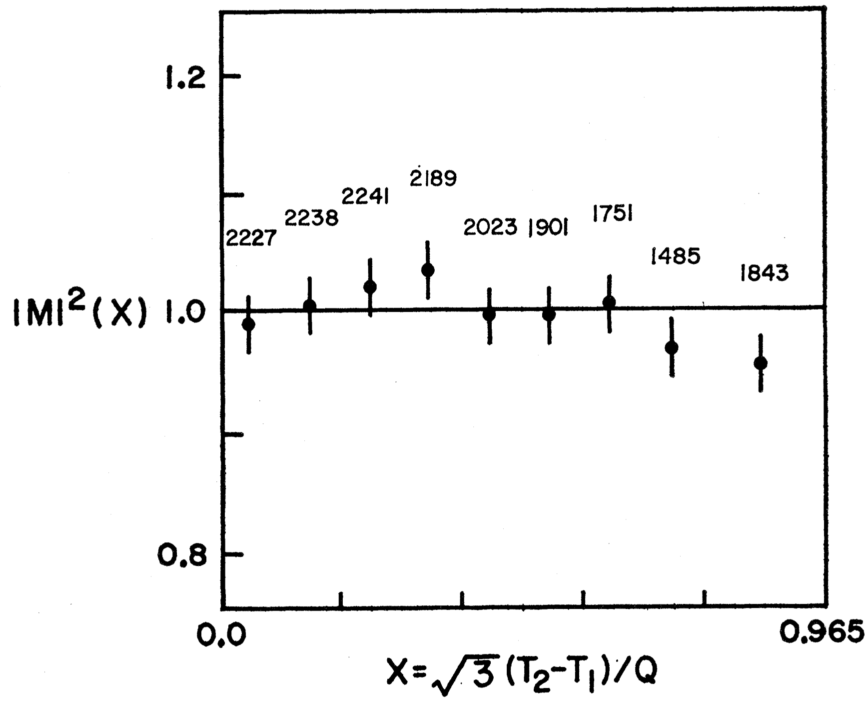

FIG. 3. X dependence of $|M|^2$. See text. The number of events in each bin is given above the experimental points.

age value of the term is not zero. Also in that distribution, the last two points have been consolidated in order to approximate the correction for low-energy π^+'s described above.

As a check, the results obtained from the bubble-chamber data alone have been calculated and are consistent with those obtained from the emulsions data alone.

V. DISCUSSION OF RESULTS

No convincing evidence of quadratic or higher-order terms was found in the squared matrix element for the decay.

A compilation of $K \to 3\pi$ data is presented in Table III.[14–24] The results of the present experiment are consistent with the results of other experiments on τ^+ decay.

Following Wolfenstein,[25] the deviations from CP conservation may be calculated by comparing τ^+ and τ^- slopes through the quantity

$$\Delta = \frac{\alpha_{\tau^+} - \alpha_{\tau^-}}{\alpha_{\tau^+} + \alpha_{\tau^-}}.$$

Using the results of Mast *et al.*,[16] together with the present data, the value $\Delta = 0.004 \pm 0.036$ is obtained, which is consistent with CP conservation. A number of models[25] for CP violation predict $\Delta < 10^{-3}$.

The linear approximation[12,26] with the $\Delta I = \frac{1}{2}$ rule predicts

$$\alpha_{K_2^0}/\alpha_\tau = -2 \quad \text{and} \quad \alpha_{\tau'}/\alpha_\tau = -2.$$

These ratios are to be compared with the experimental values

$$\alpha_{K_2^0}/\alpha_{\tau^+} = -1.92 \pm 0.19 \quad \text{and} \quad \alpha_{\tau'^+}/\alpha_{\tau^+} = -2.63 \pm 0.18,$$

where the average value of $\alpha_{K_2^0}$ from Refs. 22 and 24 and the value of $\alpha_{\tau'^+}$ of Ref. 19 were used. As reported previously,[4] the first of these, $\alpha_{K_2^0}/\alpha_{\tau^+}$, is consistent with the predictions of the $\Delta I = \frac{1}{2}$ rule, while the other, $\alpha_{\tau'^+}/$

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α_{τ^+}, indicates a $3\frac{1}{2}$-standard-deviation violation of the predicted value. This result is consistent with the results obtained from the τ^- experiment of Mast *et al.*[16]

Theoretical treatments of $K \to 3\pi$ decays utilizing current algebra with partial conservation of axial-vector current (PCAC) predict actual values for the slope parameter α[27–38] (see Table IV). These values are in fair agreement with the data. Nieh,[39] using a phenomenological model with $\Delta I = \frac{3}{2}$ admixtures, predicts relationships between the α's in fair agreement with the data (see Table IV).

Chaudhuri,[40] and Graham and Yun,[41] using pole models, calculate values of the α's similar to those obtained from current algebra (see Table IV).

A number of models based on the strong interactions of the pions in the final state have been proposed. Barbour and Schult[42] and Dunn and Ramachandran[43] used the Faddeev equations to calculate $K \to 3\pi$ decay spectra. Eliezer and Singer,[44] Lapidus and Dutta-Roy,[45] and Mitra and Ray[46] have proposed resonance models. Veneziano[47] proposed a model for the pion-pion scattering amplitude based on the Regge pole = resonance duality. Lovelace[48] applied this model to K and $\eta \to 3\pi$ decays. All these models have at least one free parameter. Any of these models should fit the data reasonably well for some value or range of values of the parameters.[49] In fact, since the data require only a linear form for the square of the matrix element, any theoretical model (a) which is capable of being expanded in a linear approximation, (b) where the coefficient of the linear term is not fixed, and (c) where the higher-order terms are not too large will be in fair agreement with the data. In all of the above models, where quadratic and higher-order terms are predicted, these terms are small and are consistent with the data.

In conclusion, (a) there is no convincing evidence for quadratic terms in the τ^+ decay spectrum, (b) there is no evidence for a violation of CP invariance in $\tau^\pm$ decay, (c) a significant violation of the $\Delta I = \frac{1}{2}$ rule has been established for $K^+ \to 3\pi$ decay,[4] and (d) the data are in fair agreement with the models mentioned above.

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