Very-High-Energy Inelastic Hadron-Hadron Collisions*

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A framework is suggested for interpreting processes of the type $a+b \to a+b+(n \text{ pions})$ at energies from tens to hundreds of GeV. The model predicts, without any adjustable parameters, the observed average transverse momenta of the pions in the entire range, as well as the existence and observed masses of "fire-balls." The advantages of this model over the multi-Regge models is discussed.

IN this paper, we wish to present what we believe may be a suitable framework for analyzing very-high-energy inelastic hadronic collisions—in particular, reactions of the type $a+b \to a+b+$pions. These reactions are gathering increasing importance for several reasons. With the advent of accelerators of more and more energy in the coming years, theorists will be presented with a vast amount of such multiple-production data to be analyzed and understood. In addition, cosmic-ray experimenters long accustomed to such ultrahigh-energy production processes, are making available data of increasing detail and accuracy.

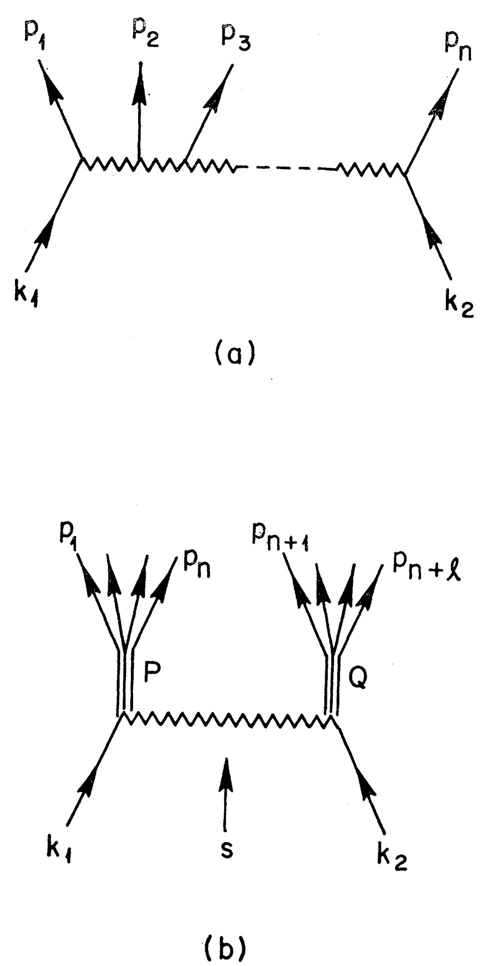

FIG. 1. (a) Multi-Regge model and (b) our model.

Several worthy models have been suggested to explain these processes, including statistical and multi-Regge models. In particular, the multi-Regge model[1] has been very much in vogue lately. We believe, however, that the multi-Regge model is not applicable to the majority of such multiple-production events. Let us first justify this statement in order to motivate the need for our alternative model.

By the "multi-Regge model" we mean the model depicted in Fig. 1(a), where a Regge pole is exchanged between neighboring stable particles. Although some applications of this model fit the data well,[2] reasonable proponents of multi-Regge theories will agree that the success is limited to that minority of events where the number of particles is small. This is because in order for the multi-Regge formula to be applicable, the subenergies $s_i=(p_i+p_{i+1})^2$ of every pair of neighboring particles must be large. But, for the majority of the events the average number of particles for any given total energy is large enough so that the s_i are typically small. This is found to be true in analyses of accelerator data.[3] Besides, it has been shown[4] that even at cosmic-ray energies, the typical s_i is kept at about 1 GeV^2, owing to the increasing multiplicity. Thus, for the majority of the multiple-production events at energies from tens to thousands of GeV, the s_i are not in the multi-Regge domain. Of course, one *could* pick those events which contain few final-state particles, in which case the s_i are likely to be large. For these events, we would have no objections to trying a multi-Regge fit. But such events will be in the minority.

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[1] G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968); Phys. Rev. Letters **20**, 1078 (1968); N. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. **163**, 1572 (1967); G. F. Chew, M. Goldberger, and F. Low, Phys. Rev. Letters **22**, 208 (1969).
[2] See, for instance, Chan Hong-Mo, J. Loskiewicz, and W. W. M. Allison, Nuovo Cimento **57A**, 93 (1968). See also O. Czyzewski, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 367.
[3] R. Lipes, G. Zweig, and W. Robertson, Phys. Rev. Letters **22**, 433 (1969).
[4] Shau-Jin Chang and R. Rajaraman, Phys. Rev. **183**, 1442 (1969).

It has been suggested that one might continue to use the multi-Regge amplitude for all the events even though the s_i are small, by involving duality. This we believe to be severely optimistic. We are aware of the successes of the duality concept, Veneziano formulas, etc. These indicate that, in some sense, cross-channel Reggeons are related to direct-channel resonances, and that even in low-s_i regions the Regge formula may be used, but only in an average sense. Thus, finite-energy sum rules show that integrals of the amplitude over resonance regions may be replaced by integrals over Regge forms. However, "local" duality in the strict sense, i.e., replacing the resonant amplitude by an $s^{\alpha(t)}$ form, is clearly not true. Similarly, extending the multi-Regge form for the $2\to n$ process, where the s_i are typically in the resonant region, would be optimistic except for some happily averaged-out predictions.

In contrast to the multi-Regge model, a purely statistical model would yield isotropic emission of particles in the center-of-mass frame, in blatant contradiction to observed distributions. We therefore suggest a hybrid model, which incorporates some aspects of statistical as well as Regge or peripheral models, but is in much better agreement with the experimental facts.

We may note that at the present time experimental data on multiple-particle production are nowhere nearly as good as elastic scattering data. One might expect that cosmic-ray experiments would provide the best source of information about asymptotic trends in the multiparticle processes. Of the information available from this source,[5] the following features seem to be relatively well established:

(1) The remarkably small value of $\langle p_t\rangle$, the average transverse momentum of the particles. As the incoming energy varies from tens to thousands of GeV, the $\langle p_t\rangle$ varies only from about 300 to about 450 MeV/c.

(2) The observation of "fireballs," i.e., clusters of particles, with a cluster mass peaked around 2 or 3 GeV, for a wide range of incoming energies.

The smallness of $\langle p_t\rangle$ is observed in accelerator experiments as well. The fireballs are not, but as we will see below, at accelerator energies one would expect the fireballs, if any, to overlap with one another.

Our hybrid model predicts, without any adjustable parameters, all the above features, including the approximate quantitative values of $\langle p_t\rangle$ and fireball masses.

Very simply, we suggest the following model:

Separate the outgoing particles into two clusters, the separation being based on the sign of p_l, the longitudinal c.m. momentum of the particle. It will be seen that such a separation is very natural in our model for lab energies in the 50–200-GeV range. The model is primarily designed for this range, since we believe that the multiple-production processes become asymptotic here, although the model seems consistent with data even at lower energies. We have, for the cross section for producing n particles in one cluster and l in the other,

$$\sigma_{n,l}(s)=\int d^4p_1\theta(p_1{}^0)\delta(p_1{}^2-\mu_1{}^2)\cdots \int d^4p_{n+l}\theta(p_{n+l}{}^0)\delta(p_{n+l}{}^2-\mu_{n+l}{}^2) \times|M(p_1,\ldots,p_{n+l})|^2\delta^4\Big[\sum_{i=1}^{n+l}p_i-K\Big].$$

Here p_i and μ_i are the momenta and masses of the final particles, M is the matrix element, $K=k_1+k_2$ is the total initial momentum, and $s=K^2$. Let us write this as

$$\sigma_{n,l}(s)=\int d^4P\int d^4Q\delta^4(P+Q-K)|A(P,Q,k_1,k_2)|^2 \times\Big[\int d^4p_1\theta(p_1{}^0)\delta(p_1{}^2-\mu_1{}^2)\cdots\int d^4p_n\theta(p_n{}^0)\delta(p_n{}^2-\mu_n{}^2)\delta^4(p_1+\cdots+p_n-P)f_1(p_1,\ldots,p_n)\Big] \times\Big[\int \delta^4p_{n+1}\theta(p_{n+1}{}^0)\delta(p_{n+1}{}^2-\mu_{n+1}{}^2)\cdots\int d^4p_{n+l}\theta(p_{n+l}{}^0)\delta(p_{n+l}{}^2-\mu_{n+l}{}^2) \times\delta^4(p_{n+1}+\cdots+p_{n+l}-Q)f_2(p_{n+1},\ldots,p_{n+l})\Big].\quad(1)$$

In other words, we have assumed that the two clusters are produced with an amplitude $A(P,Q,k_1,k_2)$ which depends only on their total four-momenta. These clusters then decay into the final particles with probability

[5] Our statements about data in the 100-GeV range are obtained from cosmic-ray experiments. A collection of such data can be found in Can. J. Phys. **46**, No. 10 (1968). See, in particular, X. Y. Akashi *et al.*, *ibid.* **46**, No. 10, s660 (1968); N. M. Gerasimova, *ibid.* **46**, No. 10, s715 (1968); J. F. de Boer *et al.*, *ibid.* **46**, No. 10, s737 (1968).

distributions $f_1(p_1\ldots,p_n)$ and $f_2(p_{n+1},\ldots,p_{n+l})$. The model is pictorially represented in Fig. 1(b). Let

$$\int d^4p_1\theta(p_{10})\delta(p_1{}^2-\mu_1{}^2)\cdots \delta(p_1+p_2+\cdots+p_n-P)f_1(p_1,\ldots,p_n)=g_1(M_1{}^2,n). \quad (2)$$

$g_1(M_1{}^2,n)$ depends on P only via the mass $M_1{}^2=P^2$ from Lorentz invariance and can be evaluated in the rest frame of the cluster. Similarly define $g_2(M_2{}^2,l)$, where $M_2{}^2=Q^2$. Equation (1) can then be written

$$\sigma_{n,l}(s)=\int dM_1{}^2\int dM_2{}^2g_1(M_1{}^2,n)g_2(M_2{}^2,l)\times\int\frac{d^3P}{2P_0}\int\frac{d^3Q}{2Q_0}|A(P,Q,k_1,k_2)|^2\delta^4(P+Q-K). \quad (3)$$

As a candidate for $A(P,Q,k_1,k_2)$, let us try a simple Regge formula $g^2(s/s_0)^{\alpha(t)}$, where g^2 and s_0 are constants. This form has been successfully used, not only for elastic scattering, but also when the final "particles" are resonances, such as $N+N\to N^*+N$, etc. It will be seen that our results do not need this specific Regge form. Any reasonable amplitude with such an exponential t dependence will do. This property is more general than Regge theory. It will also be shown that the masses of the clusters are peaked at about 2–3 GeV, so that we are likely to be in the Regge domain for their production.

Now, $\alpha(t)\simeq\alpha_0+\alpha_1 t$, where α_0 depends on the quantum numbers of the exchanged trajectory, which in turn depend on the quantum numbers of the two clusters. However, our conclusions will depend only on the trajectory slope, α_1, which seems to be common to all trajectories. Our results are hence valid for all combinations of charge, etc., among the two clusters.

Using the above Regge form for $A(P,Q,k_1,k_2)$ in Eq. (3), and performing the straightforward integrations over d^3P and d^3Q, we get

$$\sigma_{n,l}(s)=\frac{\pi g^4(s/s_0)^{2\alpha_0}}{2\alpha_1\ln(s/s_0)[s(s-4m^2)]^{1/2}}\times\int dM_1{}^2\int dM_2{}^2g_1(M_1{}^2,n)g_2(M_2{}^2,l)\left(\frac{s}{s_0}\right)^{2\alpha_1 t_0}, \quad (4)$$

where the initial particles are taken to have mass m. Here t_0 is the momentum transfer in the forward direction, and arises from the $\theta=0$ limit of the angular integration. The backward term is negligible for large s, as usual. For elastic scattering t_0 is zero, but here it is given by

$$t_0=-(1/s)[(M_1{}^2-m^2)(M_2{}^2-m^2)] \quad \text{to order } 1/s. \quad (5)$$

The functions $g_1(M_1{}^2,n)$ and $g_2(M_2{}^2,l)$ cannot be calculated exactly, since they involve the relativistic phase space of many massive particles. This once again is an unavoidable problem in the multiparticle processes. However, the dominant features of our model can be estimated as follows:

Suppose the decay of each cluster in its rest frame were isotropic and governed just by phase space. There is evidence from cosmic-ray studies of fireballs[5] that their decay is highly isotropic. Further, let the particles be massless. This is not bad for the pions. (If a nucleon is present in the cluster as in p-p collisions, we will take it to be nearly at rest in the fireball frame. This is also reasonable in view of the typically low cluster mass obobtained below.) Then g_1 and g_2 simply become the phase space for the decay of masses M_1 and M_2 into n and l massless particles, respectively. These have precise expressions:

$$g(M^2,n)=\left(\frac{M^2}{16\pi^2}\right)^{n-2}\frac{8\pi}{(n-1)!(n-2)!}. \quad (6)$$

On inserting (6) and (5) into (4), we see that at a given s, the mass distribution of clusters is given by the function

$$F_{n,l}(M_1{}^2,M_2{}^2)=(M_1{}^2)^{n-2}(M_2{}^2)^{l-2}\times\exp\left[-\frac{2\alpha_1\ln(s/s_0)}{s}(M_1{}^2-m^2)(M_2{}^2-m^2)\right]. \quad (7)$$

Note that the phase-space factors prefer high masses, while the exponential t dependence of the Regge amplitude prefers low masses. As a result of these opposing factors, $F(M_1{}^2,M_2{}^2)$ peaks at an optimal value of $(M_1{}^2,M_2{}^2)$. By differentiating Eq. (7), one obtains the following equations for the most probable masses:

$$\frac{n-2}{M_1{}^2}=\frac{2\alpha_1\ln(s/s_0)}{s}(M_2{}^2-m^2)$$

and

$$\frac{l-2}{M_2{}^2}=\frac{2\alpha_1\ln(s/s_0)}{s}(M_1{}^2-m^2). \quad (8)$$

Equations (7) and (8) are the main results of our model. Several conclusions can be drawn from them, depending on how literally one wishes to take the above simple model.

Let us start with a given total energy and multiplicity. As an example, let $E_{\text{lab}}=100$ GeV ($s=200$), and let $n=l=5$, which is a typical multiplicity. The trajectory slope α_1 is experimentally about 1 GeV^{-2}, regardless of the charge and strangeness structure of the exchanged object. Thus, for our example, we get from Eq. (8)

$$M_1=M_2\simeq 2.8\text{ GeV}. \quad (9)$$

The width of the distribution of fireball masses [Eq. (7)] comes out to be about 1 GeV. These numbers are well in the range of observed fireball characteristics.[5]

Next, we can calculate the typical transverse momentum. In our example, since the 2.8-GeV fireball decays via pure phase space into the five pions, each pion gets an energy of about 560 MeV. Since the decay is isotropic, this gives a $\langle p_t \rangle$ of about $530/\sqrt{2}$ MeV/c. Note that the transverse momentum is the same in laboratory, center-of-mass, or fireball rest frames. Thus our model naturally gives rise to a remarkably small value of transverse momentum despite the large incoming energy.

The numerical estimates above were for an incoming laboratory energy of 100 GeV. Using Eq. (8), one can also calculate the fireball masses and $\langle p_t \rangle$ as a function of E_{lab} (or s), for a wide range of energy from tens to nearly a thousand GeV, where the number of particles $n+l$ is small and a two fireball picture might satisfactorily describe most events. Thus, for $n=l$,

$$M^2(M^2-m^2)=\frac{s(n-2)}{2\alpha_1 \ln(s/s_0)}$$

and

$$\langle p_t \rangle \simeq \frac{1}{n} M \simeq \frac{1}{n}\left(\frac{s(n-2)}{2\alpha_1 \ln(s/s_0)}\right)^{1/4}. \tag{10}$$

Note that as E_{lab} varies from 10 to 1000 GeV, $\langle p_t \rangle$ increases only by a factor of 1.5, once again predicting the near constancy of $\langle p_t \rangle$ with energy. In the above equations, the number of particles in each fireball, and hence the total multiplicity $n+l$, has been used as input. This is not a serious problem, since at energies up to hundreds of GeV, the typical range of multiplicities is fairly well known.[5] But, if one wished to take this model literally, one could insert the precise functions for $g(M^2,n)$ in Eq. (6) into Eq. (4), integrate over M_1^2 and M_2^2, and obtain the cross section for generating a given set (n,l) of particles. Thus, one could predict the relative abundances of different multiplicity combinations. However, in view of the several simplifications introduced in this model (chiefly the pure zero-mass phase-space approximation for the fireball wave functions g_1 and g_2), we feel that multiplicity predictions are not reliable. If one were to improve our model by introducing resonances, nonisotropy, and other refinements into g_1 and g_2, their number dependence would alter significantly.

By contrast, our approximate prediction of fireball mass, the typical transverse momentum $\langle p_t \rangle$, and the smallness and slow variation of $\langle p_t \rangle$ will survive any refinements in the model. These conclusions depend only on the basic physics behind our model, viz., that the masses of the fireballs and the energy per particle in each fireball are determined by two competing factors—the decay of the fireballs into constituents, which favors larger masses, and the production mechanism for the fireballs which depends steeply on t, and prefers low masses. Thus, it is not necessary to have the "Regge form" $(s/s_0)^{\alpha(t)}$ for the production amplitude $A(P,Q,k_1,k_2)$ in Eq. (3). Any exponential dependence e^{at}, where a is of the order of 1 GeV^{-2}, would produce similar results.

Our point in stating all this is to emphasize that refinements and alterations of this very simple two-fireball picture are clearly needed and are welcome. However, the virtues of this model, few but distinct, namely, the smallness and near constancy of $\langle p_t \rangle$ and the prediction of small fireball masses, are likely to survive.

This model is clearly designed for 100 GeV and beyond, where there is enough energy and enough multiplicity for nearly isotropic fireballs to appear. Perhaps in the 70-GeV data from Serpukhov such clustering may already show up. In the 30-GeV-or-below region, where most of the accelerator data presently exist, such a division into two fireballs is not seen. Thus the longitudinal momentum distribution in the c.m. frame does not reveal two bumps.[6] However, one can see that our model, when applied to the 10–20-GeV region, does not predict two peaks in p_l either. From Eq. (8), one can calculate the fireball mass (and velocity in the c.m. frame) at, say, 12 GeV, where Krisch has his data, and see that the fireball velocities are not large enough to separate the two peaks in the p_l distribution of the pions. Further, $\langle p_t \rangle$ in this energy range is well in the region of our prediction. Thus our model, while primarily intended for the ("ultra-accelerator" or cosmic-ray energies of hundreds of GeV, does not contradict data at present accelerator energies.

We conclude by noting some advantages of such a hybrid model over a purely multi-Regge model. Our model, which has a single Regge-like exchange, does not involve dependences on Toller angles or on internal Reggeon-Reggeon-particle vertices. Further, since only the trajectory slope α_1 is used and not the intercept α_0, the predictions are valid for all quantum numbers exchanged. Finally, we are using a Regge amplitude in a situation where s is much larger than the masses, so that no additional justifications of local duality, etc., are needed.

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[6] An earlier work of Ratner *et al.* [Phys. Rev. **166**, 1353 (1968)] indicated the presence of two peaks with a dip at $p_l=0$ in the p_l distribution. However, they have now retracted this claim in the light of more recent work [A. Krisch (private communication)]. As mentioned in the text, our model would not lead to two separate peaks at their laboratory energy, in agreement with their corrected statement.