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Study of $\omega \to \pi^+\pi^-$ in $K^-p \to \Lambda\omega$ from 1.2 to 2.7 GeV/c†

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A general phenomenological method for studying a two-pion mass spectrum is developed; it is shown that, without assumptions about the production mechanism of the two-pion system, no significant upper limit on the $\omega \to \pi^+\pi^-$ branching ratio can be set with present-day experiments. In general, a lower limit may be set if a significant effect is seen. This method is applied to data containing more than 8000 $\omega \to \pi^+\pi^-\pi^0$ events. The lower-momentum half of the sample, which shows a significant $\omega \to \pi^+\pi^-$ signal, was published previously but is here reanalyzed to set a lower limit on the $\omega \to \pi^+\pi^-$ branching ratio. The new data at higher momenta show no significant $\omega \to \pi^+\pi^-$ signal. The results from the various momenta are shown to be consistent.

I. INTRODUCTION

THE decay of ω into $\pi^+\pi^-$ has been of continuing theoretical and experimental interest[1] because of its possible revelations concerning electromagnetic mixing between the ρ and the ω. For the most part, in theoretical calculations the $\omega \to \pi^+\pi^-$ amplitude is related to the ω-ρ transition matrix element, which in turn is related by $SU(3)$-breaking theory to the other electromagnetic effects in the vector-meson octet, namely, the K^{*0}-K^{*+} mass difference, the ρ^0-ρ^+ mass difference, or both. These calculations yield very rough predictions, somewhere between 0.1 and 5% for $(\omega \to \pi^+\pi^-)/(\omega \to \pi^+\pi^-\pi^0)$.

Experimentally, although it is generally agreed that $\omega \to \pi^+\pi^-$ has been seen,[2] no quantitatively precise results have been obtained because of the complication of interference between the production of the two-pion state via ω and via other channels.

The experimental results have been not only imprecise, but even somewhat mysterious; though significant results have been reported by several individual experiments,[2] when compilations[3,4] are made, no significant $\omega \to \pi^+\pi^-$ signal is seen, even at a much smaller level.

The experiment reported here contains what is probably the largest individual analyzed sample of $\omega \to \pi^+\pi^-\pi^0$ events in existence, namely, a total of about 8000 events. Data corresponding to 5900 events are used in the two-pion-decay analysis; this can be compared with the compilation by Lütjens and Steinberger,[3] which had about 3500 events from six different reactions.

The events discussed in this paper are from the reaction $K^-p \to \Lambda\omega$ as seen in the 72-in. hydrogen bubble chamber. About half the events, in the momentum region 1.2–1.7 GeV/c, have been previously published.[5] They show a significant $\omega \to \pi^+\pi^-$ signal, which was reported to imply a branching ratio $R = \Gamma(\omega \to \pi^+\pi^-)/\Gamma(\omega \to \pi^+\pi^-\pi^0)$ between 1 and 10% (90% confidence level). The other half of the events, in the momentum region 1.7–2.7 GeV/c, are analyzed here. The present analysis shows the following:

(1) In general, without assumptions about the production mechanism of the two-pion system, no significant upper limit on R can be set by any present-day experiment. This, of course, applies to compilations as well.

(2) From the preceding statement, one must conclude that the previous analysis for the lower-momentum half of the data was incorrect. It was also limited

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[1] See references listed in a previous publication (Ref. 5). Also, more recent theoretical analyses are treated in A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters **30B**, 249 (1969). V. S. Mathur and S. Okubo, Phys. Rev. **188**, 2435 (1969). A recent experimental result is in G. Goldhaber *et al.*, Phys. Rev. Letters **23**, 1351 (1969).

[2] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[3] G. Lütjens and J. Steinberger, Phys. Rev. Letters **12**, 517 (1964).

[4] J. Pisut and M. Roos, Nucl. Phys. **B6**, 325 (1968).

[5] S. M. Flatté, D. O. Huwe, J. J. Murray, J. B. Shafer, F. T. Solmitz, M. L. Stevenson, and C. G. Wohl, Phys. Rev. **145**, 1050 (1966).

in generality. It is not possible to set a significant upper limit on R at all, and the present analysis shows that the lower limit should be lowered to 0.2%.

(3) The new data show no significant $\omega \to \pi^+\pi^-$ signal.

(4) When the data are separated into four momentum regions (1.5, 1.7, 2.1, and 2.6 GeV/c), the only significant $\omega \to \pi^+\pi^-$ signal is seen in the 1.5-GeV/c sample.

(5) Since no sample can give an upper limit, the absence of a signal can never be contradictory to the observation of a signal in another sample; therefore, all momentum regions are consistent. This follows from the understanding that the effect arises from an interference between amplitudes, which can depend strongly on the production mechanism, and hence on production variables such as incident momentum. If no assumption about the production mechanism is made, no limits can be set on the violence of this dependence.

(6) The authors of compilations[3,4] have assumed that the ω amplitudes in their samples were completely incoherent with all other amplitudes. This assumption about the production mechanisms allowed them to set an upper limit on R. Although the upper limits from these compilations are consistent with the lower limit derived here, it should be emphasized that on theoretical grounds[1,6] the assumption they made is questionable.

II. DATA

Between 1961 and 1965, more than 1.5 million pictures of K^- incident on hydrogen in the 72-in. bubble chamber were gathered. The K^- momenta were spread from 1.2 to 2.7 GeV/c. Many results have come from this film, and it is still proving fruitful today. The analysis of the vee–two-prong topology has been described in detail elsewhere[5]; here only the measurements pertinent to a study of $\omega \to \pi^+\pi^-$ are discussed.

The two reactions of interest are $K^-p \to \Lambda\pi^+\pi^-\pi^0$, where the dominant ω decay mode into $\pi^+\pi^-\pi^0$ is seen, and $K^-p \to \Lambda\pi^+\pi^-$, where the two-pion mass spectrum is studied. In the latter reaction there is strong production of $\Sigma(1385)\pi$; in order to raise the signal-to-noise ratio in the two-pion spectrum, the $\Sigma(1385)$ events are eliminated by requiring both $\Lambda\pi$ masses to be greater than 1430 MeV. If the incident beam momentum and the two-pion mass are fixed, then the $\Lambda\pi$ mass cutoffs correspond to restrictions on the angle between one of the pions and the Λ in the two-pion rest frame (see Fig. 1). In order to find out how many $\omega \to \pi^+\pi^-\pi^0$ events correspond to a given two-pion mass spectrum, it is necessary to place the same restrictions on the angle between the normal to the ω decay plane and the Λ in the ω rest frame. This has reduced the effective $\omega \to \pi^+\pi^-\pi^0$ events by about 30%, but has reduced background considerably. The cutoff is much less damaging to the high-momentum samples

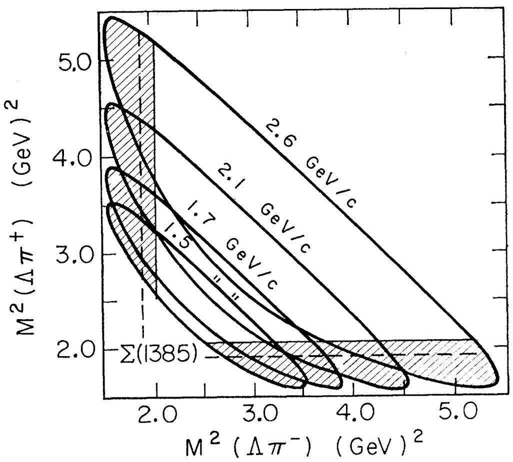

FIG. 1. Dalitz plots for $\Lambda\pi^+\pi^-$ events in $K^-p \to \Lambda\pi^+\pi^-$ for the four-momentum regions of the data. The unshaded region contains the events used in the search for the $\omega \to \pi^+\pi^-$ decay.

than to those at low momentum because the $\Sigma(1385)$ covers a significantly smaller portion of the Dalitz plot at high momentum.

Table I lists the total number of $\omega \to \pi^+\pi^-\pi^0$ events in the samples, the number of $\omega \to \pi^+\pi^-\pi^0$ after restrictions on the decay angle have been applied, and the number of $\Lambda\pi^+\pi^-$ events after elimination of $\Sigma(1385)$. As mentioned in the Introduction, some of the data (the "old" sample) have been previously published and are here reanalyzed. Those data covered 1.2–1.7-GeV/c beam momenta, while the "new" data cover 1.7–2.7 GeV/c. In Sec. III an explanation is given for the division of the data by incident beam momentum into four samples—1.5, 1.7, 2.1, and 2.6 GeV/c—where the 1.5-GeV/c sample contains data

TABLE I. Number of events in the experiment. The column labeled "$\omega \to 3\pi$" lists the total number of $\omega \to \pi^+\pi^-\pi^0$ events in that subsample after background subtraction. The column labeled "$\omega \to 3\pi$ with restriction" lists the number of $\omega \to \pi^+\pi^-\pi^0$ events remaining after a restriction is made on the ω decay angle (the normal to the ω decay plane with respect to the ω line of flight) that corresponds to the elimination of $\Sigma(1385)$ events in the $\Lambda\pi^+\pi^-$ samples. The third data column lists the number of events of $K^-p \to \Lambda\pi^+\pi^-$ with $m^2(\pi^+\pi^-)<1.2$ GeV^2 and $m^2(\Lambda\pi)>2.05$ GeV^2 in the subsample. The samples at individual energies, taken together, do not represent the total sample because events at 1.6 GeV/c were eliminated. The "new data" events have been weighted for Λ escape from the chamber, which accounts for the larger number than listed in Ref. (a). The number of unweighted new $\omega \to 3\pi$ events is 4020.

Sample	$\omega \to 3\pi$	$\omega \to 3\pi$ with restriction	$\Lambda\pi^+\pi^-$ without Σ (1385)
Total	9132	5920	10 479
Old data	3706	2050	2997
New data	5426	3870	7482
1.5 GeV/c	2980	1650	2218
1.7 GeV/c	1919	1160	1857
2.1 GeV/c	1581	1080	2426
2.6 GeV/c	2283	1840	3697

[a] S. M. Flatté, Phys. Rev. 155, 1517 (1967).

[6] J. Harte and R. G. Sachs (private communication).

labeled 1.4 and 1.5 GeV/c in previous publications, and events at 1.6 GeV/c have been omitted.

Figure 2 shows the histograms of the two-pion mass squared for the various samples of the data. In all the figures, clear evidence for the ρ meson is seen. In Figs. 2(a), 2(b), and 2(d), a definite spike at the ω mass is seen [remember that Fig. 2(d) is a subsample of Fig. 2(b), which is a subsample of Fig. 2(a)].

Before the analysis is discussed, the following should be stated: No dependence on the polarization of the Λ or on the production angle of the Λ in the center-of-mass system that would distinguish them from the ρ-meson events has been discovered for the events in the spike.

III. ANALYSIS

In the past, many methods have been used to analyze two-pion mass spectra. Originally fits were made with ρ, ω, and background terms adding incoherently. However, it has been pointed out by Harte and Sachs[6] that the ρ and ω have a "natural" coherence due to their electromagnetic mixing, and that this coherence would be washed out only by a fortuitous cancellation. Some more recent analyses considered the possibility that the ρ and ω were *completely* coherent, with background added incoherently. And several different expressions for the amplitudes themselves have been used.

An attempt is made here to be as general as possible. The only qualification that should be stated immediately is that no concerted effort is made to understand the ρ^0 meson, beyond finding a formula which fits the shape reasonably well. The sole purpose of the analysis is to discover and parametrize any anomaly in the ω mass region. Because of the narrowness of the ω, the task is made much simpler by this point of view.

A general amplitude for two-pion production may be written

$$A_{2\pi}=B+\psi_1\frac{|m_\rho^2-\rho_0|}{k^2-\rho}\left(\frac{\Gamma_\rho}{\Gamma_{\rho0}}\right)^{1/2}+\psi_2\frac{|m_\omega^2-\omega_0|}{k^2-\omega}\left(\frac{\Gamma_\omega}{\Gamma_{\omega0}}\right)^{1/2},$$

where k^2 is the two-pion mass squared, m_ρ and m_ω are the masses of the ρ and ω, B is the background amplitude, and ψ_1 and ψ_2 are complex numbers (in general, functions of k^2). Also,

$$\Gamma_\omega=\Gamma_{\omega0}\left(\frac{k^2-4m_\pi^2}{m_\omega^2-4m_\pi^2}\right)^{3/2},\quad \Gamma_\rho=\Gamma_{\rho0}\left(\frac{k^2-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2},$$

$$\rho=(m_\rho-i\Gamma_\rho/2)^2,\qquad \omega=(m_\omega-i\Gamma_\omega/2)^2;$$

ρ_0 is ρ evaluated when $k^2=m_\rho^2$ and ω_0 is ω evaluated when $k^2=m_\omega^2$.

The square of the amplitude is now

$$|A_{2\pi}|^2=\alpha_1+\alpha_2\frac{|m_\rho^2-\rho_0|^2}{|k^2-\rho|^2}\frac{\Gamma_\rho}{\Gamma_{\rho0}}+\alpha_3\frac{|m_\omega^2-\omega_0|^2}{|k^2-\omega|^2}\frac{\Gamma_\omega}{\Gamma_{\omega0}}$$
$$+\mathrm{Re}\left[C_1\frac{|m_\rho^2-\rho_0|}{k^2-\rho}\left(\frac{\Gamma_\rho}{\Gamma_{\rho0}}\right)^{1/2}+C_2\frac{|m_\omega^2-\omega_0|}{k^2-\omega}\left(\frac{\Gamma_\omega}{\Gamma_{\omega0}}\right)^{1/2}\right.$$
$$\left.+C_3\frac{|m_\rho^2-\rho_0|\,|m_\omega^2-\omega_0|}{(k^2-\rho)(k^2-\omega)}\left(\frac{\Gamma_\rho\Gamma_\omega}{\Gamma_{\rho0}\Gamma_{\omega0}}\right)^{1/2}\right].$$

Unfortunately, two very important simplifications can now be made. The word "unfortunately" is used because the simplifications are the result only of the lack of precision in presently possible experiments. First, present experiments lack statistics; second, they lack perfect mass resolution. Both these effects mean that the *exact* shape of the experimental mass spectrum is not known well, which allows the following simplifications:

(1) The terms multiplied by C_2 and C_3 are indistinguishable. They are the interference terms between the ω and either the ρ or background; and because of the small width of the ω, the shape of these terms is overwhelmed by the ω Breit-Wigner amplitude (BW). ("Shape" refers to the distribution in k^2.) Therefore the C_3 term can be dropped, and its effects are incorporated in the C_2 term.

(2) The parameters C_1 and C_2 are complex, and they multiply BW amplitudes. Thus

$$\mathrm{Re}[C(\mathrm{BW})]=(\mathrm{Re}C)(\mathrm{ReBW})-(\mathrm{Im}C)(\mathrm{ImBW}).$$

Because of the experimental limitations already mentioned, it is a fact that the imaginary part of a BW is *indistinguishable* from the BW-squared plus a small background. But the BW-squared is already included in the amplitude squared (the α_2 and α_3 terms).

The square of the amplitude can now be succinctly presented:

$$|A_{2\pi}|^2=\alpha_1+\alpha_2|\mathrm{BW}|_\rho^2+\alpha_3|\mathrm{BW}|_\omega^2$$
$$+\alpha_4\,\mathrm{Re}(\mathrm{BW})_\rho+\alpha_5\,\mathrm{Re}(\mathrm{BW})_\omega,$$

where

$$|\mathrm{BW}|_\rho^2=\frac{|m_\rho^2-\rho_0|^2}{|k^2-\rho|^2}\frac{\Gamma_\rho}{\Gamma_{\rho0}},$$

$$|\mathrm{BW}|_\omega^2=\frac{|m_\omega^2-\omega_0|^2}{|k^2-\omega|^2}\frac{\Gamma_\omega}{\Gamma_{\omega0}},$$

$$\mathrm{Re}(\mathrm{BW})_\rho=\frac{|m_\rho^2-\rho_0|}{|k^2-\rho|^2}\left(\frac{\Gamma_\rho}{\Gamma_{\rho0}}\right)^{1/2}(k^2-m_\rho^2+\tfrac{1}{4}\Gamma_\rho^2),$$

$$\mathrm{Re}(\mathrm{BW})_\omega=\frac{|m_\omega^2-\omega_0|}{|k^2-\omega|^2}\left(\frac{\Gamma_\omega}{\Gamma_{\omega0}}\right)^{1/2}(k^2-m_\omega^2+\tfrac{1}{4}\Gamma_\omega^2).$$

Figure 3 shows these four universal functions of k^2,

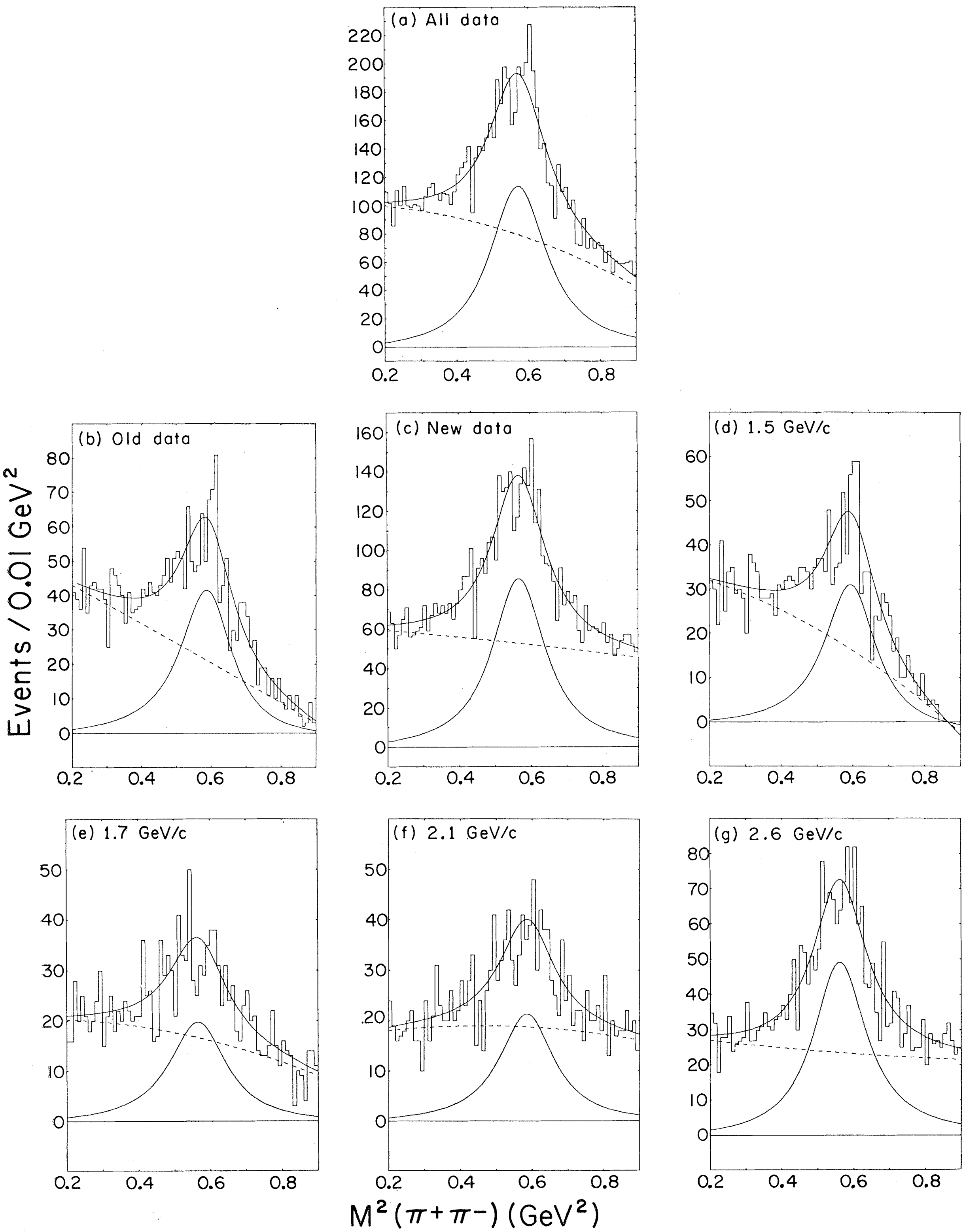

FIG. 2. Two-pion mass-squared spectra exactly as in Fig. 2. Only the curves have changed. The curves represent the fits including the ω meson as described in the text. The top solid curves are the fits; the dashed curves are, first, the background and, second, the ρ (including the ρ interference term) contribution; the bottom solid curves are the ω (including the ω interference term) contribution.

with the masses and widths of the ρ and ω set equal to 765, 783.4, 120, and 12.2 MeV.

Thus far a pure state has been assumed; that is, all variables other than k^2 have been fixed (for example, momentum transfers, polarizations, etc.). The mixed-state case is treated by taking the expectation value of $|A_{2\pi}|^2$ over all variable s other than k^2.

Since variables other than k^2 appear only in the α parameters, the form of the expression for $|A_{2\pi}|^2$ remains the same when expectation values are taken; the only change is in the relative size of the α parameters. In general, the α parameters can also be functions of k^2, but the assumption is made that they are slowly varying near $k^2=m_\omega^2$. Hence the α parameters are assumed to be constants, and the form of $|A_{2\pi}|^2$ as a function of k^2 remains as valid for a mixed state as it was for a pure state. However, for a pure state the α parameters have a definite algebraic relationship; for a mixed state only inequalities can be given.

The actual two-pion spectrum is obtained by multiplying by phase space. Because of the cuts on the $\Lambda\pi$ mass, phase space is a linear function of k^2; however, for simplicity in parametrizing the background, the final two-pion spectrum is obtained by the equation

$$dN/dk^2=|A_{2\pi}|^2[1+\alpha_6(k^2-m_\omega^2)+\alpha_7(k^2-m_\omega^2)^2].$$

Now the final assumption that all the α's are constant makes the above a simple expression which reproduces the salient features of any two-pion spectrum and has the unique feature that it is capable of representing *any degree of coherence of the* ω with the other amplitudes.

Finally, the experimental mass resolution ($\simeq$10 MeV full width at half-maximum) is folded in.

Several important characteristics of the final result should be emphasized:

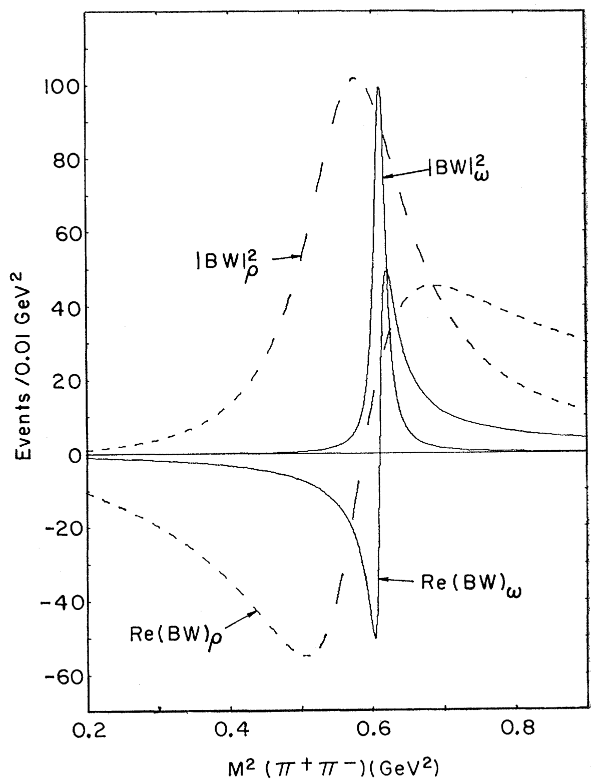

FIG. 3. The four universal functions which can be used to represent a two-pion mass spectrum near the ω mass. Each curve is the corresponding function described in the text, multiplied by 100 events/0.01 GeV². $|\mathrm{BW}|_\rho^2$ and $|\mathrm{BW}|_\omega^2$ are the BW-squared contributions of the ρ and ω, respectively, while $\mathrm{Re(BW)}_c$ and $\mathrm{Re(BW)}_\omega$ are the real parts of the BW formulas for the ρ and ω, respectively. The $\mathrm{Re(BW)}_\rho$ term represents the interference of the ρ with background, and the $\mathrm{Re(BW)}_\omega$ represents the interference of the ω with either the background or the ρ amplitude.

(1) The parameters α_2 and α_3 may be negative. One might think that a BW-squared must make a positive contribution, but one must remember that these terms contain contributions from the imaginary parts of the BW amplitudes which can give *negative contributions*. Therefore, dips could be seen in the two-pion spectra instead of peaks. If no peak or dip is seen, it could be that a negative contribution has canceled a positive contribution, as Lütjens and Steinberger point out; therefore, without assumptions, *no conclusion can be drawn from the absence of a peak*. This is a consequence of the experimental inability to distinguish the imaginary part of the ω BW amplitude from its square. Of course, if one assumes complete incoherence or coherence of the ω with other amplitudes, then an upper limit can be set on the ω production.

(2) Observation of either an α_3 or α_5 term allows one to set a lower limit on the $\omega\to\pi^+\pi^-$ branching ratio.

(3) The α_1 through α_5 have dimensions M^{-2}, so that, for example, α_3 represents the actual height (in events/0.01 GeV²) of the α_3 term's contribution to the spectrum at the ω mass-squared.

(4) The hypothesis of no ω production can be easily treated by setting $\alpha_3=\alpha_5=0$.

(5) If it were possible to determine a pure state for the production of ρ and ω, without background, so that complete coherence could be assured, then one could solve for the amplitude of pure ω production. Let $\alpha_2'=\alpha_2|\mathrm{BW}|_\rho^2$ evaluated at $k^2=m_\omega^2$. Then,

$$|\psi_\omega|^2=(\alpha_3+2\alpha_2')\pm[(\alpha_3+2\alpha_2')^2-(\alpha_3^2+\alpha_5^2)]^{1/2}.$$

Then the branching ratio $\omega\to\pi^+\pi^-/\omega\to\pi^+\pi^-\pi^0$ is $|\psi_\omega|^2(\pi m_\omega\Gamma_{\omega 0})/N_\omega$, where N_ω is the number of $\omega\to\pi^+\pi^-\pi^0$ events corresponding to the fitted sample.

Unfortunately, even if present-day experiments had enough data to restrict s, t, the decay angle of the two-pion system, and all decay angles of other final-state particles (in this case, the Λ), a pure state would still not necessarily be achieved because of background in the two-pion system, with one exception.

There is one experiment, which may be feasible in the near future, where the production mechanism is indeed well known and a pure state is formed. That is the experiment with colliding electron-positron beams to give $e^+e^- \to \pi^+\pi^-$. Thus, from this experiment, an unambiguous $\omega \to \pi^+\pi^-$ amplitude can be extracted, with enough statistics.

On the other hand, one might attempt to create a completely incoherent case, where all interference effects have washed out. There are two objections to this: First, it is quite difficult to be assured of having an incoherent sample (as mentioned previously, Harte and Sachs[6] maintain that even if the production processes of ρ and ω were incoherent, which would not be easy to prove, the final two-pion spectrum would in general exhibit interference from the very nature of ρ-ω mixing); second, the effect may be so small as to be undetectable, whereas in an interference term, small amplitudes can have large effects.

Therefore it seems worthwhile to try to restrict as many kinematical variables as possible, in order to see what effect they have. For this reason the data have been split into four parts, each part having a particular value of s. The variable s was chosen because the data are very close to the threshold of the reaction, and are, therefore, perhaps more susceptible to s-channel rapidly varying effects than anything else.

(6) The branching ratio calculated in point (5) can be used, without assumption of a pure state, to find a lower limit on the $(\omega \to 2\pi)/(\omega \to 3\pi)$ ratio.

IV. RESULTS

In order to determine whether a significant anomaly exists in the data at the ω mass, the following is done: The terms representing the ω are set equal to zero, and the other five α's, along with the ρ mass and width, are allowed to vary in a fit to the data, which yields a minimized χ_ρ^2. Then another similar fit is made, but with the ω parameters free to vary, which yields a minimized χ_ω^2. Since the ω mass and width are fixed at their accepted values (783.4 and 12.2 MeV), this second fit has only two more parameters than the first. The significance of an ω signal is measured directly by the difference between the two χ^2; that is, $\chi^2 = \chi_\rho^2 - \chi_\omega^2$, which is a χ^2 for two degrees of freedom.

A confidence level can be calculated from this $\Delta\chi^2$ for two degrees of freedom. The confidence level thus determined is *the confidence level for the theory that no ω signal exists in the data.* This should not be confused with the confidence level for the "ρ alone" fit, which has 63 degrees of freedom. Even if the "ρ alone" fit failed by a tremendous χ^2, it would not prove the existence of the $\omega \to 2\pi$ decay, since the reason for failure may be unassociated with the ω. On the other hand, just looking at the goodness of the "ρ alone" fit is not a sensitive test of an ω signal, because the fit could be relatively quite good in χ^2 but fail miserably in the bins near the ω. Therefore, the most sensitive

TABLE II. Fitted parameters and χ^2 in the $\omega \to \pi^+\pi^-$ analysis. The parameters are defined in Sec. III; their physical identification is indicated above each one. "ρ int" means the term representing ρ interference with background. "ω int" represents ω interference with background or the ρ. The units of α_1–α_5 are events per 0.01 GeV^2. The units of α_6 and α_7 are GeV^{-2} and GeV^{-4}, respectively. The units of m_ρ and Γ_{ρ_0} are MeV. Each sample has two rows: The first represents the fit without the ω, the second with the ω. The numbers of degrees of freedom are 63 and 61, respectively. The column labeled N_σ lists the number of standard deviations from zero for the ω signal, as derived from the differences in χ^2 between the two rows of each sample. The column labeled CL lists the confidence level for the theory that no ω signal exists in the data.

Sample	Bkgd α_1	ρ α_2	ω α_3	ρ int α_4	ω int α_5	Bkgd α_6	Bkgd α_7	m_ρ	Γ_{ρ_0}	χ^2	$\Delta\chi^2$	CL (%)	N_σ
Total	77	108	···	−8	···	−1.2	−1.1	771	134	77.5	21.0	0.005	4.2
	76	98	62	−10	−36	−1.1	−0.9	770	142	56.5			
Old	20	40	···	−2	···	−2.8	−0.1	782	120	88.5	10.0	0.7	2.7
	22	33	26	1	−7	−2.7	−0.6	775	124	78.5			
New	52	82	···	−14	···	−0.4	−0.1	767	124	62.1	7.3	2.5	2.2
	55	69	38	−7	−27	−0.4	−0.7	763	141	54.8			
1.5 GeV/c	16	31	···	0	···	−3.2	−2.0	785	116	75.0	14.7	0.07	3.4
	19	22	20	1	−18	−2.9	−3.4	787	122	60.3			
1.7 GeV/c	16	18	···	−3	···	−1.1	−1.2	771	134	69.7	2.4	30	1.0
	16	17	11	−2	3	−1.1	−1.0	760	135	67.3			
2.1 GeV/c	19	20	···	−4	···	−0.2	−0.8	780	125	57.9	0.7	70	0.4
	19	18	8	−3	2	−0.3	−1.0	772	129	57.2			
2.6 GeV/c	23	47	···	−6	···	−0.3	−0.3	763	124	59.9	5.2	7.5	1.7
	24	43	19	−6	−13	−0.2	0.4	759	126	54.7			

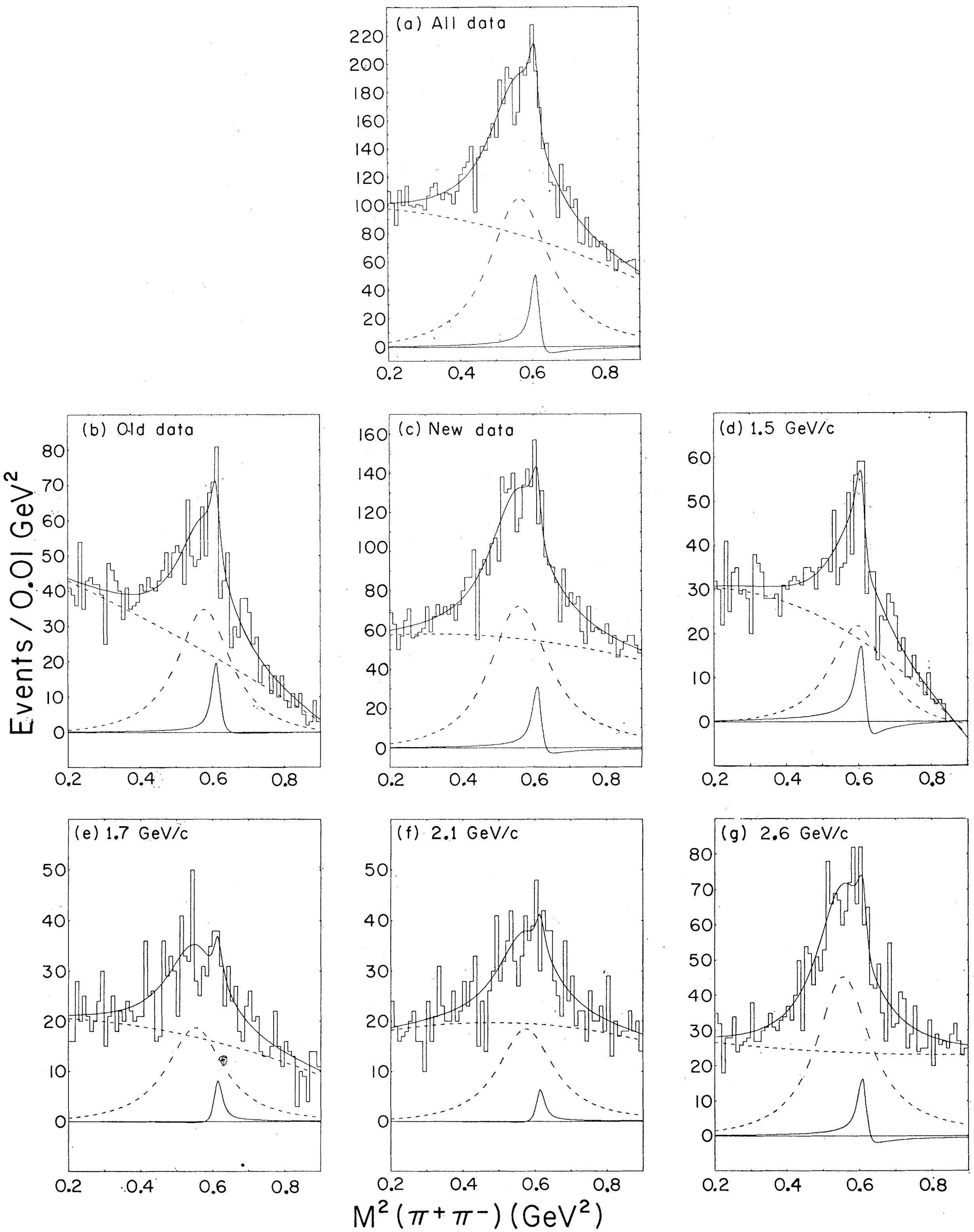

FIG. 4. Two-pion mass-squared spectra for various samples of events. In each case the $\Sigma(1385)$ has been removed. The curves represent fits including a ρ meson and background, as described in the text. The top solid curves are the complete fits; the bottom solid curves are the contributions from the ρ plus the interference term between the ρ and background; the dashed curves are the background contributions. (a) Total sample, 1.4–2.7 GeV/c, corresponding to 5920 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (b) Previously published data, 1.4–1.7 GeV/c, 2050 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (c) New data, 1.7–2.7 GeV/c, 3870 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (d) 1.5-GeV/c data: subsample of (b), 1650 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (e) 1.7-GeV/c data: subsample of both (b) and (c), 1160 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (f) 2.1-GeV/c data: subsample of (c), 1080 $\omega \rightarrow \pi^+\pi^-\pi^0$ events. (g) 2.6-GeV/c data: subsample of (c), 1840 $\omega \rightarrow \pi^+\pi^-\pi^0$ events.

test of the ω is the ΔX^2 test. The number of standard deviations from zero, N_σ, for an ω signal can also be computed from ΔX^2 (remember, two degrees of freedom).

Table II lists the various subsamples of analyzed data, with the fitted values of α_1–α_7, the fitted ρ mass and width, the X^2, the confidence level, and the number of standard deviations from zero for the ω signal. The fits without ω, along with the separate contributions from the ρ and from background, are shown in Fig. 2. The fits with the ω parameters included are shown in Fig. 4, with the ω contribution also shown. It is clear that the only undeniably significant ω effect appears in the 1.5-GeV/c data (and of course exhibits itself in the "old" and "total" samples). The $X_\omega{}^2$ contours for the 1.5-GeV/c data, plotted in (α_3,α_5) space, are shown in Fig. 5.

From Fig. 5 a lower limit for the $\omega\rightarrow\pi^+\pi^-$ branching ratio can be found with the help of the equation derived in Sec. V. First, since the ω can interfere with background as well as the ρ, the quantity α_2' must be replaced by $\alpha_2'+\alpha_1$. Secondly, if $|\psi_\omega|^2$ is imaginary from the equation (which is physically impossible), then a pure state is not allowed by the data, and a minimum $|\psi_\omega|^2$ is found by a search of all mixed-state possibilities. Thus,

$$\Gamma(\omega\rightarrow\pi^+\pi^-)/\Gamma(\omega\rightarrow\pi^+\pi^-\pi^0)>0.2\%$$

at a 90% confidence level, which is different from, and lower than, the value given in Ref. 5. The difference arises solely from the analysis. In Ref. 5, $m_{\rho 0}$ and $\Gamma_{\rho 0}$ were fixed; in this analysis, they were allowed to vary;

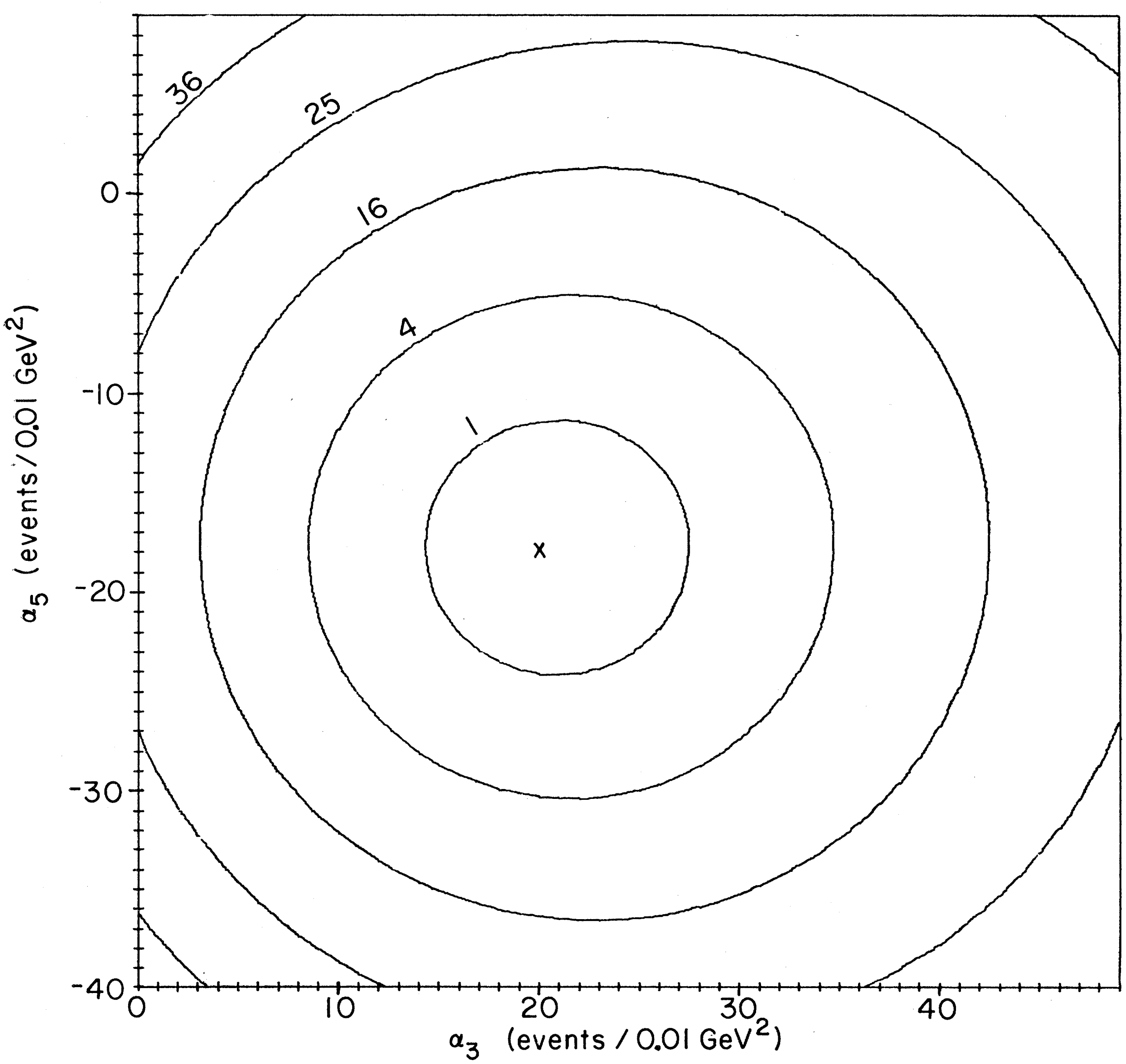

FIG. 5. Contours of X^2 for the 1.5-GeV/c data. The variables α_3 and α_5 represent the $|BW|_\omega{}^2$ and $Re(BW)_\omega$ terms; they are not strongly correlated with the other variables in the fit. The contours are labeled by the differences of X^2 from the best-fit value, which is 60.3 for 61 degrees of freedom.

thus the limit is weakened here. Also, interference with background was completely neglected in Ref. 5. Both effects were important.

The other three samples of data show no significant ω signal. An investigation into the dependence on momentum transfer, and also on Λ polarization, by splitting the data into smaller subsamples also yielded no significant ω signals. Since we have shown that any given sample can set only a lower limit on $\omega \to \pi^+\pi^-$, never an upper limit, naturally there is no contradiction between samples. In fact, it is not surprising to see the ω signal appear at only one energy, since the effect is probably due not to a simple ω signal, but to interference between a very small ω amplitude and the ρ+background amplitude.

One might ask why the 2.2–standard-deviation effect in the new data is ignored, while the 2.7–standard-deviation effect in the old data is considered significant. First, the 1.5-GeV/c subsample of the old data has a 3.4–standard-deviation effect that is difficult to ignore. Second, the effect in the new data is in fact associated with the two bins in the middle of the ρ that are so low; and though the ω fit lowers the curve somewhat in this area, it really seems that these two bins have little to do with an ω anomaly.

These results should be compared with those of a large compilation of pion-induced reactions: In 1967 Roos[7] published a compilation which claimed a 3–standard-deviation effect in $\pi^-p \to \pi^+\pi^-n$, but no branching ratio could be set because of the unknown $\omega \to \pi^+\pi^-\pi^0$ rate. However, in a later paper[4] the claim was withdrawn because of changes in some of the experimental data.

Lütjens and Steinberger,[3] in an earlier compilation, set an upper limit of 0.8% on $\omega \to \pi^+\pi^-$. Even though their limit is consistent with the result of this paper, it should be mentioned that they assumed no interference. (They had to assume something about interference; otherwise, as the present analysis shows, and as they specifically pointed out, they could not set any significant upper limit.)

[7] M. Roos, Nucl. Phys. **B2**, 615 (1967).

V. CONCLUSIONS

Harte and Sachs[6] have shown, within a simple and believable interpretation of the behavior of quantum-mechanical states under mixing, that, in a reaction where ρ and ω are produced, the $\pi^+\pi^-$ system will almost always produce non-negligible interference effects, no matter how many reactions are added together. Thus, assumptions of no interference may not be valid. Consideration of this problem has led to the development of a general method for analyzing the ω contribution to a two-pion spectrum, without any assumptions about coherence.

It has been shown that if no assumptions about coherence are made, it is impossible for present experiments to set a significant upper limit on $\omega \to \pi^+\pi^-/\omega \to \pi^+\pi^-\pi^0$, with one exception: A colliding-beam experiment, $e^+e^- \to \pi^+\pi^-$, could, with enough statistics, unambiguously determine the $\omega \to \pi^+\pi^-$ amplitude.

The method has been used to analyze a sample of the reaction $K^-p \to \Lambda\pi^+\pi^-$, where the sample corresponds to 5900 $\omega \to \pi^+\pi^-\pi^0$ events. An $\omega \to \pi^+\pi^-$ signal is seen ($>3\sigma$), and the final result at a 90% confidence level is

$$\Gamma(\omega \to \pi^+\pi^-)/\Gamma(\omega \to \pi^+\pi^-\pi^0) > 0.2\%.$$

This result is based in part on a reanalysis of previously published data,[5] and supersedes all previous upper and lower limits stated in previous publications; differences are solely a result of the more general analysis employed here.

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