

Effects of an induced three-body force in the incident channel of (d, p) reactions

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A widely accepted practice for treating deuteron breakup in $A(d, p)B$ reactions relies on solving a three-body $A + n + p$ Schrödinger equation with pairwise A - n , A - p and n - p interactions. However, it was shown in [Phys. Rev. C **89**, 024605 \(2014\)](#) that projection of the many-body $A + 2$ wave function into the three-body $A + n + p$ channel results in a complicated three-body operator that cannot be reduced to a sum of pairwise potentials. It contains explicit contributions from terms that include interactions between the neutron and proton via excitation of the target A . Such terms are normally neglected. We estimate the first-order contribution of these induced three-body terms and show that applying the adiabatic approximation to solving the $A + n + p$ model results in a simple modification of the two-body nucleon optical potentials. We illustrate the role of these terms for the case of $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ transfer reactions at incident deuteron energies of 11.8, 20, and 56 MeV, using several parametrizations of nonlocal optical potentials.

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I. INTRODUCTION

Transfer reactions provide a useful way of probing the structure of nuclei because differential cross sections show features that are sensitive to the shell structure of the target and residual nuclei. This is attractive for experimentalists looking to probe the structure of a nucleus, and (d, p) reactions are a popular choice at radioactive beam facilities for determining the spectroscopic strength of single-particle states for nuclei beyond stability. One theory available for analysis of (d, p) reactions is the adiabatic distorted-wave approximation (ADWA) [1]. The ADWA accounts for deuteron breakup effects through a three-body, $n + p + A$, description of the deuteron channel, represented by the wave function $\Psi_d^{(+)}$ in the (d, p) transition amplitude

$$T_{(d,p)} = \sqrt{S} \langle \chi_p^{(-)} \phi_n | V_{np} | \Psi_d^{(+)} \rangle, \quad (1)$$

where $\chi_p^{(-)}$ is the proton channel wave function, ϕ_n is the normalized bound-state wave function of the transferred neutron in the final state (more generally, the normalized neutron overlap function) and S is its spectroscopic factor.

It is standard to calculate $\Psi_d^{(+)}$ by using a three-body model consisting of the three V_{np} , V_{pA} , and V_{nA} pairwise interaction potentials between the n - p , p - A , and n - A pairs, respectively. A two-body p - A or n - A scattering model based on the Feshbach projection operator technique [2] involves an energy-dependent nonlocal optical potential, implicitly accounting for coupling to excited target states. Applying the Feshbach approach to the three-body $n + p + A$ channel has two implications [3]:

- (1) The n - A (p - A) optical potential in the $n + p + A$ system depends on the proton (neutron) dynamical variables and on the n - p interaction.
- (2) In addition to the pairwise interactions, new terms arise which correspond to an effective interaction

between the neutron and proton in the deuteron, via excitation of the target A , to all orders. These create a three-body interaction that has both diffractive and absorptive parts.

The dependence of the optical potential of each N - A subsystem within the $n + p + A$ three-body system on the position of the other nucleon can be averaged out if the wave function is expanded over some appropriate n - p basis states. It was shown in Ref. [3] that choosing the Weinberg expansion [1] and retaining leading-order terms only (which corresponds to the ADWA) results in an effective energy at which the N - A potential should be used in (d, p) calculations. However, applications of this idea in Refs. [3,4] treated only the pairwise interactions in the $n + p + A$ system, ignoring the additional three-body terms.

One way to deal with these induced three-body (I3B) terms is to explicitly include excited target states in the reaction model. This has, for example, been done within the CDCC [5,6] and Faddeev [7–9] approaches. However, these calculations include explicitly only a fraction of the model space needed to fully account for all the absorption known to be needed in the nucleon optical potentials. In this paper, we point out that the ADWA allows us to estimate the contribution of the effective I3B interaction, to first order, from all target excited states. We explain this procedure in Sec. II, showing that it results in a simple modification of the distorting potential in the deuteron channel and in Sec. III we describe their connection to the dynamical part of the nonlocal dispersive optical model (NLDOM). In Sec. VI we discuss nonlocal scattering inputs for the ADWA with NLDOM. In Sec. V we use the NLDOM potential with the required alterations and we present our numerical calculations using the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reaction as an example. For comparison, we also present calculations with two other nonlocal optical potentials. The discussion and conclusions are presented in Sec. VI.

II. EFFECTIVE INTERACTIONS IN THE ADIABATIC DISTORTED WAVE APPROXIMATION DEUTERON CHANNEL

For the case of two-body scattering of a nucleon N on a complex target A , Feshbach shows, by using the projection operators P_A and Q_A which project onto the ground and excited states of nucleus A , respectively, the total many-body scattering wave function is split into two parts, $\Psi_P = P_A |\Psi_N^{(+)}\rangle$ and $\Psi_Q = Q_A |\Psi_N^{(+)}\rangle$. Feshbach then shows that Ψ_P appears in a modified Schrödinger equation [10]

$$(E - H_{PP})\Psi_P = 0, \quad (2)$$

and this is governed by an effective Hamiltonian

$$H_{PP} = T_{NA} + V_{NA}^{\text{opt}}(E_N). \quad (3)$$

In the case of two-body scattering of a nucleon N Feshbach [2] demonstrates that for a nucleon with kinetic energy E_N the optical potential V_{NA}^{opt} takes the form

$$V_{NA}^{\text{opt}}(E_N) = \langle \phi_A | U_{NA}^{\text{opt}} | \phi_A \rangle, \quad (4)$$

where U_{NA}^{opt} is the optical model operator

$$U_{NA}^{\text{opt}} = v_{NA} + v_{NA} Q_A \frac{1}{e_N - Q_A v_{NA} Q_A} Q_A v_{NA}. \quad (5)$$

Here v_{NA} is the sum of interactions of the nucleon N with all nucleons in the target A and the energy denominator e_N is given by $e_N = E_N + i0 - T_{NA} - (H_A - E_A)$, where T_{NA} is the N - A relative kinetic energy, H_A and E_A are the internal Hamiltonian and the ground-state energy of the target A , respectively.

In previous work [3] this idea of the optical potential was extended for a three-body case. The three-body wave function in this case can be considered as the projection Ψ_P of the full many-body wave function, $\Psi_d^{(+)}$, of the $n + p + A$ system onto the ground state of the target A . This projection is governed by the effective Hamiltonian

$$H_{\text{eff}} = T_3 + V_{np} + \langle \phi_A | U | \phi_A \rangle, \quad (6)$$

where T_3 is the three-body kinetic-energy operator and V_{np} is a short-range n - p interaction. The operator U is an operator in all $A + 2$ coordinates of n , p , and A . The final term implies integration over the target nucleus coordinates to leave an operator in three-body coordinates only. We can express U , which accounts for the excitation of target nucleus degrees of freedom on the target ground-state projection of the scattering wave function, in terms of operators U_{pA} and U_{nA} that define excitations of A by n and p separately [3]. Up to second order in U_{NA} these terms are

$$\begin{aligned} U^{(0)} &= U_{pA} + U_{nA}, \\ U^{(1)} &= U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA}, \end{aligned} \quad (7)$$

where

$$U_{NA} = v_{NA} + v_{NA} \frac{Q_A}{e - Q_A v_{NA} Q_A} v_{NA}. \quad (8)$$

However, because of the definition of e , given by

$$e = E_3 + i0 - T_3 - V_{np} - (H_A - E_A), \quad (9)$$

the U_{nA} is an operator in both n and p coordinates despite including the n - A interaction only. The same is true of U_{pA} . In Eq. (9) E_3 is the three-body energy, related to the incident center-of-mass kinetic energy E_d and deuteron binding energy ϵ_d by $E_3 = E_d - \epsilon_d$. The U_{NA} is not equal to the Feshbach optical operator that describes the two-body N - A scattering.

To calculate $V_{np} |\Psi_d^{(+)}\rangle$ the ADWA expands the three-body wave function $\Psi_d^{(+)}(\mathbf{R}, \mathbf{r})$ in a discrete set of states using the Weinberg eigenstates [1], defined by

$$[-\epsilon_d - T_r - \alpha_i V_{np}] \phi_i(\mathbf{r}) = 0, \quad i = 1, 2, \dots, \quad (10)$$

where the ϕ_i satisfy the orthonormality relation

$$\langle \phi_i | V_{np} | \phi_j \rangle = -\delta_{i,j}. \quad (11)$$

The eigenvalue equation (10) features a fixed deuteron energy $-\epsilon_d$ and n - p kinetic-energy operator T_r . The α_i increase monotonically, with ϕ_i possessing i nodes within the range of V_{np} , such that ϕ_i becomes increasingly oscillatory. The $\Psi_d^{(+)}(\mathbf{R}, \mathbf{r})$ is thus expanded in this basis,

$$\Psi_d^{(+)}(\mathbf{R}, \mathbf{r}) = \sum_{i=1}^{\infty} \phi_i(\mathbf{r}) \chi_i^{(+)}(\mathbf{R}), \quad (12)$$

where

$$\chi_i^{(+)}(\mathbf{R}) = -\langle \phi_i | V_{np} | \Psi_d^{(+)} \rangle. \quad (13)$$

In the ADWA the first Weinberg component of $\Psi_d^{(+)}$, which provides the dominant contribution to the (d, p) stripping amplitude [11], is found by determining the distorted wave $\chi_d^{(+)}$, which is the solution to the Schrödinger equation

$$(E_d - T_R - \langle \phi_1 \phi_A | U | \phi_0 \phi_A \rangle) \chi_d^{(+)}(\mathbf{R}) = 0, \quad (14)$$

where T_R is the kinetic energy associated with the n - p center-of-mass coordinate $\mathbf{R} = (\mathbf{r}_n + \mathbf{r}_p)/2$, and ϕ_A and ϕ_0 are the target and deuteron ground-state wave functions, respectively, while $|\phi_1\rangle$ is given by

$$|\phi_1\rangle = \frac{V_{np} |\phi_0\rangle}{\langle \phi_0 | V_{np} | \phi_0 \rangle}. \quad (15)$$

The matrix element in $\langle \phi_1 \phi_A | U | \phi_0 \phi_A \rangle$ from Eq. (14) implies integration over all internal degrees of freedom of A and over n and p spin coordinates together with the relative n - p coordinate $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$. This leaves $\langle \phi_1 | U | \phi_0 \rangle$ to be an operator in the space of the coordinate \mathbf{R} and the triplet spin-space of the n and p [3].

In Ref. [3] it was assumed that $\langle \phi_1 | U | \phi_0 \rangle \approx \langle \phi_1 | U^{(0)} | \phi_0 \rangle$ and it was shown that, because of the short-range nature of ϕ_1 , the averaging procedure results in

$$\langle \phi_1 \phi_A | U^{(0)} | \phi_0 \phi_A \rangle \approx \sum_{N=n,p} \langle \phi_1 \phi_A | U_{NA}^{\text{opt}}(E_{\text{eff}}) | \phi_0 \phi_A \rangle, \quad (16)$$

where U_{NA}^{opt} is the optical model operator

$$U_{NA}^{\text{opt}}(E_{\text{eff}}) = v_{NA} + v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA}. \quad (17)$$

taken at energy

$$E_{\text{eff}} = \frac{1}{2} E_d + \frac{1}{2} \langle T_r \rangle, \quad (18)$$

which differs from the commonly used value of half the deuteron incident energy by half the n - p kinetic energy T_r averaged over the short range of the n - p interactions:

$$\langle T_r \rangle = \langle \phi_1 | T_r | \phi_0 \rangle. \quad (19)$$

This form (17) for the optical model operator differs from that in Eq. (5) only in the energy denominator, as now U_{NA}^{opt} describes the two-body N - A scattering the effective energy E_{eff} .

We want to show now that the same ideas allow us to recover some of the contributions from $U^{(1)}$. Approximating U_{NA} in Eq. (7) by its leading value of v_{NA} we obtain the contribution to the deuteron distorted potential given by

$$\begin{aligned} & \langle \phi_1 \phi_A, \mathbf{R} | U^{(1)} | \phi_A \phi_0, \mathbf{R}' \rangle \\ & \approx \langle \phi_1 \phi_A, \mathbf{R} | v_{nA} \frac{Q_A}{e} v_{pA} | \phi_A \phi_0, \mathbf{R}' \rangle \\ & + \langle \phi_1 \phi_A, \mathbf{R} | v_{pA} \frac{Q_A}{e} v_{nA} | \phi_A \phi_0, \mathbf{R}' \rangle \\ & \equiv U_{np}^{(1)}(\mathbf{R}, \mathbf{R}') + U_{pn}^{(1)}(\mathbf{R}, \mathbf{R}'). \end{aligned} \quad (20)$$

This potential is nonlocal in space \mathbf{R} . To estimate its magnitude we consider the case of local, spin-independent inter-

actions v_{NA} and ignore Coulomb contributions. We rewrite $U_{np}^{(1)}(\mathbf{R}, \mathbf{R}')$ as

$$\begin{aligned} U_{np}^{(1)}(\mathbf{R}, \mathbf{R}') &= \int d\xi_A d\mathbf{r} \phi_1^*(\mathbf{r}) \phi_A^*(\xi_A) v_{nA} \left(\mathbf{R} - \frac{\mathbf{r}}{2}, \xi_A \right) \\ & \times \tilde{\Psi}_p(\mathbf{r}_n, \mathbf{r}_p, \mathbf{R}', \xi_A), \end{aligned} \quad (21)$$

where $\mathbf{r}_n = \mathbf{R} - \frac{\mathbf{r}}{2}$, $\mathbf{r}_p = \mathbf{R} + \frac{\mathbf{r}}{2}$, and

$$\tilde{\Psi}_p(\mathbf{r}_n, \mathbf{r}_p, \mathbf{R}', \xi_A) = \langle \mathbf{r}_n, \mathbf{r}_p, \xi_A | \frac{Q_A}{e} v_{pA} | \phi_0 \phi_A, \mathbf{R}' \rangle. \quad (22)$$

Given the short range of ϕ_1 we can replace \mathbf{r}_p and \mathbf{r}_n in $\tilde{\Psi}_p$ by \mathbf{R} , and \mathbf{r} in v_{nA} by zero in Eq. (21). Then the expression for $U_{np}^{(1)}$ becomes very similar to the one for $U_{pp}^{(0)}$, arising from averaging the second term on the right in U_{pA} in Eq. (8) and treated by using the same approximation. The only difference is the presence of $v_{nA}(\mathbf{R}, \xi_A)$ instead of $v_{pA}(\mathbf{R}, \xi_A)$ in the integrand of the right-hand side of Eq. (21). If we further assume that the interaction of the proton p and neutron n with the nucleons of target A are the same we obtain

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \sum_{N=n,p} \langle \phi_1 \phi_A | v_{NA} + 2v_{NA} \frac{Q_A}{e - Q_A v_{NA} Q_A} v_{NA} | \phi_0 \phi_A \rangle. \quad (23)$$

Further reasoning along the lines in Ref. [3] leads to conclusion that e could be substituted by $E_{\text{eff}} + i\epsilon - T_N - H_A$, so that

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \sum_{N=n,p} \langle \phi_1 \phi_A | v_{NA} + 2v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} Q_A | \phi_0 \phi_A \rangle. \quad (24)$$

When compared with our expression for $\langle \phi_1 \phi_A | U^{(0)} | \phi_A \phi_0 \rangle$ in Eq. (16) we can see that $\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle$ includes a factor of two in the term corresponding to the second term in Eq. (8). As such, it is possible to rewrite $\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle$ in terms of $\langle \phi_1 \phi_A | U^{(0)} | \phi_A \phi_0 \rangle$ as

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx 2 \langle \phi_1 \phi_A | U^{(0)} | \phi_0 \phi_A \rangle - \sum_{N=n,p} \langle \phi_1 \phi_A | v_{NA} | \phi_0 \phi_A \rangle. \quad (25)$$

III. CONNECTION WITH DISPERSIVE OPTICAL MODEL

It has been shown in the previous section that I3B terms arising from $U^{(1)}$ can be accounted for in the ADWA to first order by doubling the adiabatic deuteron optical potential and subtracting from it the Johnson–Tandy potential calculated from nucleon-target (real) folding potentials. The nucleon optical potentials, $V_{NA}^{\text{opt}}(E) \equiv \langle \phi_A | U_{NA}^{\text{opt}} | \phi_A \rangle$, needed to construct the deuteron adiabatic distorting potential should be nonlocal, energy-dependent and complex since they arise due to projecting out the space of excited states of the target [12]. It is also known that, due to causality, optical potentials fulfill a dispersion relation [13]. This means that the optical potential consists of two terms,

$$V_{NA}^{\text{opt}}(E) = V_{NA}^{HF} + \Delta V_{NA}^{\text{dyn}}(E), \quad (26)$$

one of which, V_{NA}^{HF} , is a real energy-independent potential and the other, $\Delta V_{NA}^{\text{dyn}}(E)$, is generated dynamically though

coupling to inelastic channels and is energy dependent [14]. This complex term has an imaginary part $W_{NA}(E)$ and a real part that is related to $W_{NA}(E)$ by the dispersive relation, so that

$$\Delta V_{NA}^{\text{dyn}}(E) = iW_{NA}(E) + \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}. \quad (27)$$

We identify the real, energy-independent, folding term in Eq. (24), $\langle \phi_A | v_{NA} | \phi_A \rangle$, with V_{NA}^{HF} , ignoring the fact that the Feshbach formalism used here does not carry the exchange terms needed to generate V_{NA}^{HF} , and identify the second term in Eq. (24) with the dynamical term $\Delta V_{NA}^{\text{dyn}}(E)$ multiplied by two. Then,

$$\begin{aligned} & \langle \phi_A | U^{(0)} + U^{(1)} | \phi_A \rangle \\ & = V_{nA}^{HF} + 2\Delta V_{nA}^{\text{dyn}}(E) + V_{pA}^{HF} + 2\Delta V_{pA}^{\text{dyn}}(E). \end{aligned} \quad (28)$$

This equation provides an approximate practical approach to estimating the effect of the I3B terms if both the parts V_{NA}^{HF} and $\Delta V_{NA}^{\text{dyn}}(E)$ are known. These potentials should be used

at an energy $E = E_{\text{eff}}$ defined in Eq. (18). Recently, a phenomenological NLDOM parametrization has been proposed to forge the link between nuclear structure and reactions [14] for proton and neutron scattering from ^{40}Ca . Below, we use this potential for the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ calculations, employing the updated NLDOM parameters from Ref. [4].

IV. NONLOCAL SCATTERING

The ADWA calculations require knowledge of the distorted waves χ_d and χ_p in the entrance and exit channels. For nonlocal optical potentials they are found from the equations [15]

$$\begin{aligned} & (T_d^{(L)} + U_c(\mathbf{R}_d) - E_d)\chi_{LL}^J(\mathbf{R}_d) \\ &= - \sum_{L'} \int_0^\infty dR'_d R_d R'_d \mathcal{U}_{LL'}^J(\mathbf{R}_d, R'_d) \chi_{L'}^J(\mathbf{R}'), \end{aligned} \quad (29)$$

and

$$\begin{aligned} & (T_p^{(L)} + U_c(\mathbf{R}_p) - E_p)\chi_{LL}^J(\mathbf{R}_p) \\ &= - \int_0^\infty dR'_p R_p R'_p \mathcal{U}_{LL}^J(\mathbf{R}_p, R'_p) \chi_{L'}^J(\mathbf{R}'), \end{aligned} \quad (30)$$

where $T_\alpha^{(L)}$ is the kinetic-energy operator in a partial wave with orbital angular momentum L in the channel α , characterized by energy E_α and reduced mass μ_α :

$$T_\alpha^{(L)} = -\frac{\hbar^2}{2\mu_\alpha} \left[\frac{d^2}{dR_\alpha^2} - \frac{L(L+1)}{R_\alpha^2} \right], \quad \alpha = d, p. \quad (31)$$

Equations (29) and (30) contain the Coulomb interaction U_c and nonlocal deuteron-channel $\mathcal{U}_{LL'}^J(\mathbf{R}_d, R'_d)$ and proton-channel $\mathcal{U}_{LL}^J(\mathbf{R}_p, R'_p)$ potential kernels that depend on channel coordinates R_α and R'_α (note that $\mathbf{R}_d \equiv \mathbf{R}$). In these equations J is the total angular momentum in the scattering channels. We neglect spin-orbit interaction in the present work. The correct description of its effects within the ADWA requires spin-dependent tensor terms [16], for which no numerical implementations are yet available.

The nucleon NLDOM potential used in this work consists of seven terms,

$$\mathcal{U}^{\text{NLDOM}}(\mathbf{R}, \mathbf{R}') = \sum_{i=1}^7 U_i \left(\frac{|\mathbf{R} + \mathbf{R}'|}{2} \right) H_i(s), \quad (32)$$

where $s = \mathbf{R} - \mathbf{R}'$, each described by its own nonlocality range β_i in the nonlocal factor

$$H_i(s) = \frac{\exp(-s^2/\beta_i^2)}{\pi^{\frac{3}{2}} \beta_i^3}. \quad (33)$$

Details of nonlocal kernel calculations for one nonlocality range and the s -wave deuteron only are given in Ref. [17], while the generalization to a realistic deuteron wave function that includes the deuteron d state is available in Ref. [15]. Nonlocal kernels with several nonlocality ranges are just the sums of kernels calculated with one nonlocality range.

It has been shown in Ref. [18] that including the deuteron d -wave component in ADWA leads to enhanced sensitivity of (d, p) cross sections to high n - p momenta, which is an artifact of the adiabatic approach [19,20]. It has also been shown in

Ref. [19] that ADWA and beyond-ADWA calculations differ less when only a deuteron s state is included. For this reason we use the Hulthén model for ϕ_1 and ϕ_0 , which does not contain any deuteron d state [21], in all our calculations of deuteron-channel nonlocal kernels.

We have generated the deuteron-channel NLDOM kernels for each nonlocality range and then summed them. To test our procedure, we calculated the kernels in the leading-order approximation $U_i(|\mathbf{R} + \mathbf{R}'|/2) \approx U_i(\mathbf{R})$. For one nonlocality range, this approximation gives a very similar result to the calculations performed with a local-equivalent potential U_{loc} obtained as a solution of a transcendental equation (see Ref. [15]). In the case of NLDOM, U_{loc} is the solution of the generalized transcendental equation [4]

$$U_{\text{loc}} = \sum_{i=1}^7 U_i \exp\left(\frac{\mu_d \beta_i^2}{2\hbar^2} (E_d - U_c - U_{\text{loc}})\right). \quad (34)$$

We have checked that just as in Ref. [15], the calculations with U_{loc} differ from the leading-order calculations at the cross section peak by 1% at 11.8 and 20 MeV, respectively, but differ by up to 5% for 56 MeV. Exact solutions of the nonlocal Eq. (29) reduce the leading-order (d, p) cross sections in the main peak by no more than 10% for all the deuteron energies, with the largest differences due to a small change in the location of the peak. This holds for all the optical models in the proton channel that were considered here. In the p channel, exact solutions of nonlocal Eq. (30) again reduce the (d, p) cross sections in the main peak by no more than 10% for all investigated energies.

It was also found that the difference between exact and transcendental methods are smaller when I3B terms are accounted for, with said differences in cross sections reduced in both the d and p channels to no more than 5% at the main peak for all investigated energies. This change is because the nonlocal wave functions obtained from Eqs. (29) and (30) are smaller in the nuclear interior than those obtained from the local Schrödinger equation used in conjunction with Eq. (34). This difference is given by the Perey factor [22]

$$f(r) = \exp\left(\frac{\mu_d \beta^2}{4\hbar^2} U_{\text{loc}}\right), \quad (35)$$

which is equal to unity outside the nucleus. With increased absorption, the contribution from the nuclear interior becomes less important, so applying the Perey factor does not have the same effect. This leads to results closer to those found using transcendental methods, reducing the difference between the nonlocal and local solutions.

V. THE $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ RESULTS

We have carried out numerical calculations of the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reaction using the NLDOM nucleon potential in the entrance deuteron channel evaluated at E_{eff} according to Eq. (18) and Ref. [3]. The shift of $\langle T_r \rangle / 2 = 57$ MeV was applied, consistent with the Hulthén model, which does not contain high relative n - p momenta. In the exit channel the NLDOM was used at the actual proton energy. The calculations have been carried out at $E_d = 11.8, 20,$ and 56 MeV. These

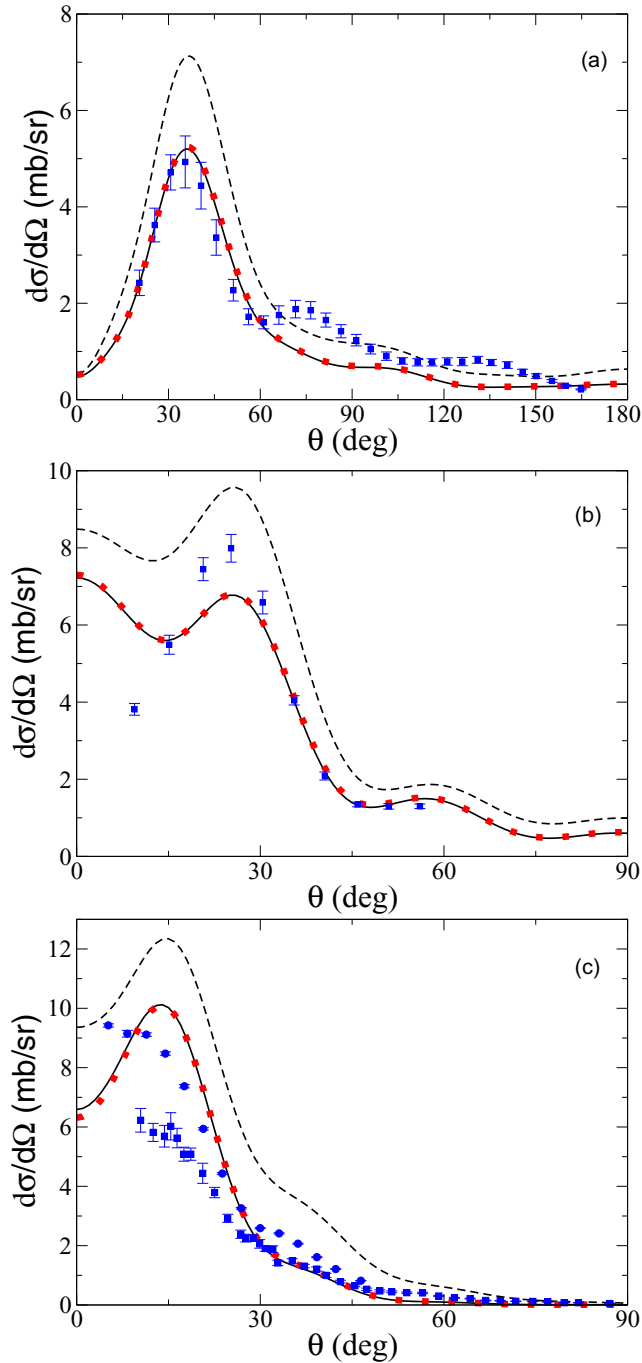


FIG. 1. Comparison of $^{40}\text{Ca}(d, p)^{40}\text{Ca}$ cross sections for the ground state at $E_d = 11.8$ MeV (top), 20 MeV (middle), and 56 MeV (bottom) using the DOM optical potentials. Cross sections are found with (solid lines) and without (dashed lines) I3B effects. We also present results for when $\text{Re}[\Delta V_{NA}^{\text{dyn}}]$ is left unmodified (dotted lines). Experimental data are from Refs. [23–26].

cross sections have been measured in Refs. [23–26] and we note that, at 56 MeV, the two measured data sets differs significantly. In all of the nonlocal ADWA calculations presented, the exact solutions of Eqs. (29) and (30) for deuteron and proton distorted waves in the entrance and exit channels are read into the transfer reactions code TWOFNR [27] and the transition

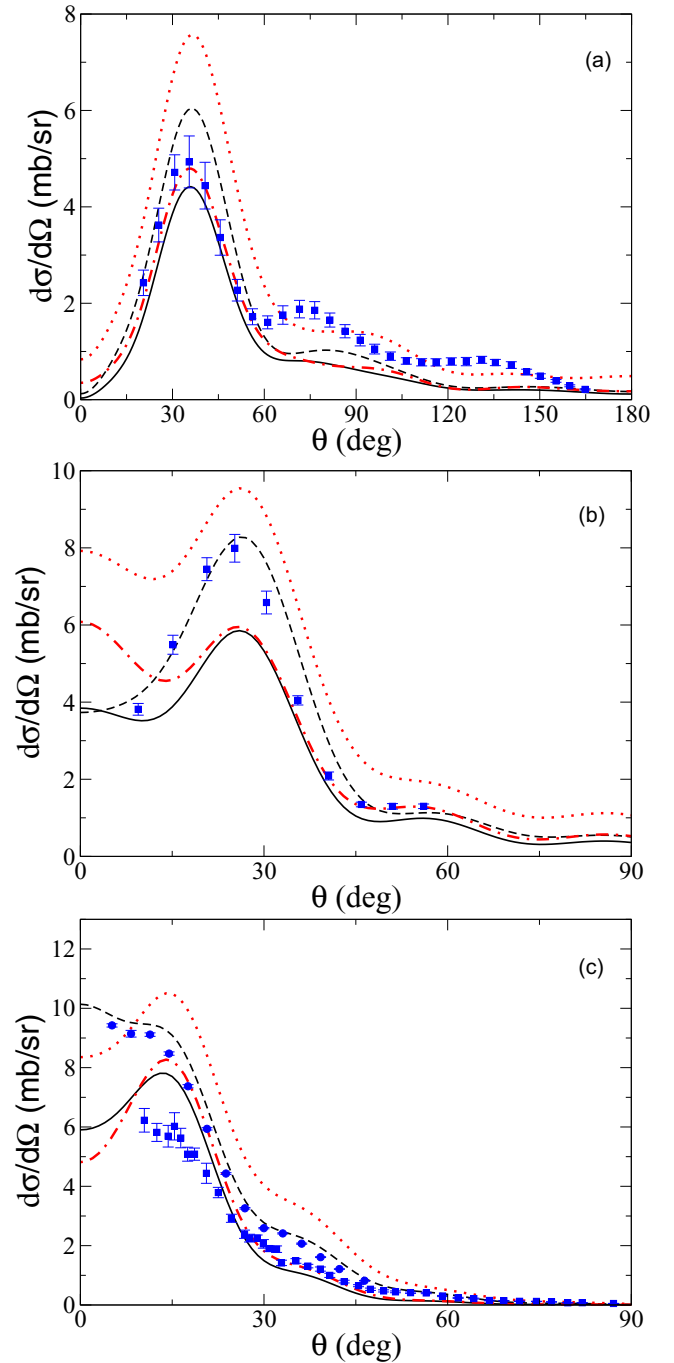


FIG. 2. Comparison of $^{40}\text{Ca}(d, p)^{40}\text{Ca}$ cross sections for the ground state at $E_d = 11.8$ MeV (top), 20 MeV (middle), and 56 MeV (bottom) using the GR potential with (solid lines) and without (dashed lines) I3B effects, along with cross sections found with the GRZ optical potential with (dot-dashed lines) and without (dotted lines) I3B effects. Experimental data are from Refs. [23–26].

amplitude is calculated in the zero-range approximation by using a standard value of $D_0 = -126.15$ MeV fm $^{3/2}$.

The overlap integral between the ^{41}Ca and ^{40}Ca ground-state wave functions was taken from Ref. [4], where the exact NLDOM overlap function was approximated, with good accuracy, by the single-particle wave function

TABLE I. The factor by which I3B terms change the size of the cross-section peaks. Presented as the ratio between the peak maximum when including first-order I3B, $\sigma_{I3B}^{\text{peak}}$, and without them, σ^{peak} . Peaks from Figs. 1 and 2 for each investigated optical potential at $E_d = 11.8, 20$, and 56 MeV.

E_d (MeV)	Ratio of $\sigma_{I3B}^{\text{peak}}/\sigma^{\text{peak}}$		
	NLDM	GR	GRZ
11.8	0.725	0.729	0.630
20	0.691	0.706	0.621
56	0.821	0.842	0.786

calculated in a Wood–Saxon potential well with the radius $r_0 = 1.252$ fm, diffuseness $a = 0.718$ fm, and spin-orbit depth $V_{s.o.} = 6.25$ MeV. This single-particle wave function has been multiplied by the square root of the NLDM spectroscopic factor, $S = 0.73$.

The results of the calculations are plotted in Fig. 1. It was already noticed in Ref. [4] that the NLDM overestimates significantly the $^{40}\text{Ca}(d, p)^{40}\text{Ca}$ cross section at 11.8 MeV. The cross sections at the two other energies we investigate are also overestimated. Moreover, the shape of neither of the two available 56 MeV data sets are reproduced. Including first-order I3B terms, doubling the dynamical real and imaginary NLDM parts decreases the cross sections due to increased absorption by the factors shown in Table I, bringing them to much better agreement with experimental data at 11.8 and 20 MeV, with 56 MeV data remaining difficult to reproduce.

To investigate the role of the dynamical real part, we present calculations where only the imaginary part of the NLDM is doubled. Using $\text{Re}[\Delta V_{NA}^{\text{dyn}}]$ instead of $2\text{Re}[\Delta V_{NA}^{\text{dyn}}]$ makes no notable difference to the calculated cross sections (see Fig. 1). We have found that this effect is only present when working with a doubled imaginary part, and the large absorption this implies. When the absorption is increased in such a manner, the effect of altering the dynamical real part of the potential becomes less notable, because it corresponds to a much smaller proportional change to the absolute optical potential than it would for an unmodified imaginary component. This could be a useful insight because several phenomenological systematics that do not include dynamical dispersive corrections are available [28–31], and our findings suggest that omitting a real dispersive term may not be significant in (d, p) calculations when I3B terms are present.

It is important to note that this statement is also true for cross sections produced from deuteron-target potentials found by using the transcendental method (27) and for its linear approximation, making it clear that this behavior is a result of the potential itself, rather than the method used to generate results.

Further calculations were carried out with phenomenological nonlocal optical potentials, the energy-independent Giannini–Ricco (GR) [28] and energy dependent Giannini–Ricco–Zucchiatti (GRZ) [29]. Energy-dependent n - A and p - A potentials based on the Tian, Pang, Ma [32] optical potentials have become available recently [30,31]. Unfortunately these do not cover the nucleon energy range based on Eq. (10) that is needed here. The GR and GRZ potentials include only the imaginary term W_{NA} from $\Delta V_{NA}^{\text{dyn}}$, so accounting for

first-order I3B terms would imply doubling the well depths of the imaginary part only, with no changes being made to the real part of each potential. These imaginary terms feature a surface term only. The calculations with these two potentials are presented in Fig. 2 and they show, qualitatively, the same result found when using NLDM: that including first-order I3B terms decreases the cross sections by factors similar to those obtained with the NLDM, shown in Table I for each energy. The 11.8 MeV data are better reproduced with the GR potential whereas the 20 MeV data favor the GRZ calculations. The case of 56 MeV remains inconclusive. There are large discrepancies between two sets of measurements in the literature [25,26] and resolving them experimentally is an important task.

VI. SUMMARY AND CONCLUSIONS

We have considered new terms in the optical potential in the incident channel in the ADWA approximation for $A(d, p)B$ reactions. These terms arise because target excitations in the $A + n + p$ breakup channels are coupled to different nucleons in the deuteron. These couplings are usually neglected when U is approximated as the sum of n - A and p - A optical model operators. Treating the new terms in the ADWA approximation leads to an effective operator U in which the first-order I3B terms double the dynamical excitation contributions to the n - A and p - A optical potentials.

Numerical calculations, performed for the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reaction with the NLDM, GR, and GRZ potentials, have shown that these I3B terms decrease the ADWA cross sections by 20%–40%, depending on the deuteron energy, bringing the cross sections closer to available experimental data. It was found that, with stronger imaginary parts, the impact of modifying the dynamical real part becomes insignificant. This suggests that other (nondispersive) nonlocal optical potentials could be used if I3B terms are taken into account by simply doubling their imaginary parts in a standard ADWA calculation. Given that the NLDM is available only for nucleon scattering from ^{40}Ca [14] and ^{48}Ca [33], this finding could be useful for applications to all other nuclei, because it would allow for the use of existing global nucleon optical potentials without dispersive terms.

Finally, our estimations suggest that I3B effects could play an important role in forming both the shape and absolute magnitude of (d, p) differential cross sections. This could have important consequences for extracting spectroscopic information from (d, p) experiments. It is important to extend the investigation of I3B force effects in the $d + A$ system beyond the approximate methods used here.

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