Half-lives of cluster radioactivity using the modified generalized liquid drop model with a new preformation factor

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Half-lives of clusters emitted from radioactive nuclei is determined using modified generalized liquid drop model (MGLDM), where a new preformation factor is added to the existing generalized liquid drop theoretical model with proximity 77 potential. Half-lives for radioactive nuclei whose mass numbers vary from 221 to 242, emitting C, O, F, Ne, Mg, and Si clusters, are calculated theoretically considering deformation and orientation effects and the values are in exact agreement with experimental data. Standard deviation of logarithm of half-lives using present model is found to be 0.755, which is a much better result in comparison with those of GLDM1 and GLDM2 of Bao *et al.* [J. Phys. G: Nucl. Part. Phys. **39**, 095103 (2012)], which are 1.84 and 1.114, respectively. Also, half-lives of various radioactive nuclei whose mass numbers vary from 220 to 250 are predicted. We hope that our present study on cluster half-lives may be helpful for future works in this field.

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I. INTRODUCTION

Before 1980, either α decay or spontaneous fission was the main disintegration process of radioactive nucleus. Then in 1980, Sandulescu *et al.* [1] proposed the idea of cluster radioactivity, which is the process by which particles heavier than α particle and lighter than fission fragments are emitted. And the experimental evidence of the prediction was observed [2] in 1984 for the first time by Rose and Jones and later it was confirmed by Aleksandrov *et al.* [3], where ^{14}C cluster was found to be emitted from ^{223}Ra leading to ^{209}Pb daughter nuclei. Later emission of other clusters, such as ^{20}O, ^{23}F, ^{22,24,26}Ne, ^{28,30}Mg, and ^{32,34}Si, were observed experimentally [4,5].

Many theoretical models were proposed to explain cluster emission. Earlier main arguments between various models were whether clusters should be treated like fission fragments obeying super asymmetric fission model [6–9] or considered as preformed within parent nucleus, as in Gamow's theory of the α-decay-preformed cluster model [10–12]. In unified fission theory [13–15], decay constant is equated as the product of assault frequency (ν_o) and barrier penetrability constant (P). But in the preformed cluster model [16–18], an additional factor, preformation factor is multiplied with above terms to get the decay constant.

Several approaches in the case of cluster decay included the contributions related to shell energies and variation of surface deformations of both cluster and daughter nucleus during cluster emission, because shell energies and surface deformations play an important role in cluster preformation model. Denisov [19] considered cluster emission using multidimensional cluster preformation model taking in account the shell correction to macroscopic potential energy and dynamical surface deformation of both daughter nuclei and cluster at barrier penetration path. Also, Mirea *et al.* [20,21] studied cluster decay within macroscopic-microscopic approximation, which includes microscopic shell effect. Cluster decay is computed in a fully microscopic model by Warda *et al.* [22,23], assuming it as a kind of hyper asymmetric spontaneous fission.

Generalized liquid drop model (GLDM) was developed by Royer [9,24–30] in 1984, which added nuclear proximity energy and quasimolecular shape to the conventional LDM. Cluster radioactivity half-lives were calculated using GLDM in 1998 and there was a deviation of about 3 orders of magnitude from observed data [8]. Thereafter, a preformation factor [31] ${}^{c}P_0 = {}^{\alpha}P_0^{(A_2-1)/3}$ is introduced after which deviation gets reduced by 2 orders of magnitude. In the above equation, A_2, ${}^{\alpha}P_o$ are the mass number of cluster and preformation factor of α decay, respectively. Then Bao *et al.* [32] included the effect of microscopic shell correction and shape-dependent pairing energy to LDM. Santhosh *et al.* [33] modified the GLDM of Royer [9,24–30], incorporating nuclear proximity potential proposed by Blocki *et al.* [30] and studied α-decay half-lives emitted from super heavy elements and could reproduce experimental half-lives exactly.

Blocki *et al.* [34] proposed the proximity potential in 1977, and for the first time Shi and Swiatecki [35] used it as a nuclear potential in decay process. After that, with lots of moderation, proximity potential has been used extensively [36,37]. Yao *et al.* [38] calculated α-decay half-lives using 14 types of proximity potential and found out proximity 77 as the perfect one with least deviation from experimental value. Ghodsi *et al.* [39] also made a study on α-decay half-lives using different potential and suggested proximity 77 as the right one. Again, work by Santhosh *et al.* on α decay of Po isotopes [40], Hg isotopes [41], and cluster decay of various nuclei [42]

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in trans lead region proves proximity 77 as a suitable potential with least standard deviation. The reliability of proximity 77 is very clear from above mentioned publications, and hence in our present model, potential included is also proximity 77 even though it was proposed long ago.

In our present model, we modify generalized liquid drop model with proximity 77 potential by a Q-value-dependent preformation factor [43], considering deformation and orientation effects so that we are able to generate half-life values that coincide well with the experimental data. Section II of this paper explains the theory of the modified generalized liquid drop model (MGLDM). Section III compares generated half-lives with the experimental data and also contains theoretical predictions. Our conclusion is presented in Sec. IV.

II. MODIFIED GENERALIZED LIQUID DROP MODEL

In GLDM, for a deformed nucleus, the macroscopic energy is defined as,

$$E = E_V + E_S + E_C + E_R + E_P. \tag{1}$$

Here the terms E_V, E_S, E_C, E_R, and E_P represent the volume, surface, Coulomb, rotational, and proximity energy terms, respectively.

For the prescission region, the volume, surface, and Coulomb energies in MeV are given by

$$E_V = -15.494(1 - 1.8I^2)A, \tag{2}$$

$$E_S = 17.9439(1 - 2.6I^2)A^{2/3}\left(S/4\pi R_0^2\right), \tag{3}$$

$$E_C = 0.6e^2(Z^2/R_0) \times 0.5 \int (V(\theta)/V_0)(R(\theta)/R_0)^3 \sin\theta d\theta. \tag{4}$$

Here I is the relative neutron excess and S the surface of the deformed nucleus, $V(\theta)$ is the electrostatic potential at the surface, and V_0 is the surface potential of the sphere.

For the post-scission region,

$$E_V = -15.494\left[\left(1 - 1.8I_1^2\right)A_1 + \left(1 - 1.8I_2^2\right)A_2\right], \tag{5}$$

$$E_S = 17.9439\left[\left(1 - 2.6I_1^2\right)A_1^{2/3} + \left(1 - 2.6I_2^2\right)A_2^{2/3}\right], \tag{6}$$

$$E_C = \frac{0.6e^2Z_1^2}{R_1} + \frac{0.6e^2Z_2^2}{R_2} + \frac{e^2Z_1Z_2}{r}. \tag{7}$$

Here A_i, Z_i, R_i, and I_i are the masses, charges, radii, and relative neutron excess of the fragments, and r is the distance between the centers of the fragments.

The nuclear proximity potential E_P is given by Blocki *et al.* [34] as

$$E_p(z) = 4\pi\gamma b\left[\frac{C_1C_2}{(C_1 + C_2)}\right]\Phi\left(\frac{z}{b}\right), \tag{8}$$

with the nuclear surface tension coefficient

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2/A^2]\ \text{MeV/fm}^2, \tag{9}$$

where N, Z, and A represent neutron, proton, and mass number of parent nucleus, respectively,Φ represents the universal proximity potential [44] given as

$$\Phi(\varepsilon) = -4.41e^{-\varepsilon/0.7176}, \quad \text{for} \quad \varepsilon > 1.9475, \tag{10}$$

$$\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.01696\varepsilon^2 - 0.05148\varepsilon^3, \quad \text{for} \quad 0 \leqslant \varepsilon \leqslant 1.9475, \tag{11}$$

with $\varepsilon = z/b$, where the width (diffuseness) of the nuclear surface $b \approx 1$ fm and Süsmann central radii C_i of fragments related to sharp radii R_i as

$$C_i = R_i - \left(\frac{b^2}{R_i}\right). \tag{12}$$

For R_i we use a semiempirical formula in terms of mass number A_i as [44]

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}. \tag{13}$$

The barrier penetrability P is calculated with the action integral

$$\mathrm{P} = \exp\left\{-\frac{2}{\hbar}\int_{R_{\text{in}}}^{R_{\text{out}}}\sqrt{2B(r)[E(r) - E(\text{sphere})]}dr\right\}, \tag{14}$$

where $R_{\text{in}} = R_1 + R_2$, $B(r) = \mu$, and $R_{\text{out}} = e^2Z_1Z_2/Q$. R_1 and R_2 are the radius of the daughter nuclei and emitted cluster, respectively, μ is the reduced mass, and Q is the released energy.

The partial half-life is related to the decay constant λ by

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P_C P}\right). \tag{15}$$

The assault frequency ν has been taken as $10^{20}\ \text{s}^{-1}$ and the preformation factor [43] is given as

$$P_C = 10^{aQ+bQ^2+C}, \tag{16}$$

with $a = -0.25736$, $b = 6.37291 \times 10^{-4}$, $c = 3.35106$, and Q is the Q value or the energy released in a radioactive nuclear reaction.

For the two deformed and oriented nuclei, the Coulomb interaction, which is taken from Ref. [45] and which includes higher multipole deformation [46,47], is given as

$$E_C = \frac{Z_1Z_2e^2}{r} + 3Z_1Z_2e^2\sum_{\lambda,i=1,2}\frac{1}{2\lambda+1}\frac{R_i^\lambda(\alpha_i)}{r^{\lambda+1}}Y_\lambda^{(0)}(\theta_i) \times\left[\beta_{\lambda i} + \frac{4}{7}\beta_{\lambda i}^2Y_\lambda^{(0)}(\theta_i)\delta_{\lambda,2}\right], \tag{17}$$

with

$$R_i(\alpha_i) = R_{0i}\left[1 + \sum_\lambda \beta_{\lambda i}Y_\lambda^{(0)}(\alpha_i)\right], \tag{18}$$

where $R_{0i} = [1.28A_i{}^{1/3} - 0.76 + 0.8A_i{}^{-1/3}]$. Here θ_i is the angle between the symmetry axis and axis of collision and α_i

TABLE I. The logarithmic half-lives predicted using MGLDM, CPPM, GLDM1, and GLDM2 and compared with experimental data.

Parent nuclei	Daughter nuclei	Cluster	Q (MeV)	$\log_{10}[T_{1/2}(s)]$				
				MGLDM	Expt.	CPPM	GLDM1	GLDM2
^{221}Fr	^{207}Tl	^{14}C	31.28	13.94	14.52	13.90	13.61	13.45
^{221}Ra	^{207}Pb	^{14}C	32.39	13.01	13.39	12.58	12.00	12.23
^{222}Ra	^{208}Pb	^{14}C	33.05	11.89	11.01	11.07	10.38	11.23
^{223}Ra	^{209}Pb	^{14}C	31.85	14.03	15.20	13.69	13.40	13.71
^{224}Ra	^{210}Pb	^{14}C	30.53	16.10	15.68	16.74	16.85	16.52
^{225}Ac	^{211}Bi	^{14}C	30.48	16.96	17.16	18.03	18.15	17.32
^{226}Ra	^{212}Pb	^{14}C	28.21	20.58	21.19	22.55	23.08	21.75
^{226}Th	^{212}Po	^{14}C	30.67	17.70	>15.30	18.74	19.15	18.30
^{226}Th	^{208}Po	^{18}O	45.88	20.13	>15.30	19.29	19.76	18.18
^{228}Th	^{208}Pb	^{20}O	44.72	21.79	20.72	21.66	22.36	21.64
^{231}Pa	^{208}Pb	^{23}F	51.84	25.50	26.02	24.26	25.28	24.26
^{230}U	^{208}Pb	^{22}Ne	61.59	20.59	>18.20	22.60	22.90	20.36
^{230}Th	^{206}Hg	^{24}Ne	57.78	25.55	24.61	26.00	26.92	25.18
^{232}Th	^{208}Hg	^{24}Ne	55.62	28.47	>29.20	30.36		
^{231}Pa	^{207}Tl	^{24}Ne	60.42	23.32	23.23	22.56	23.15	21.62
^{230}U	^{206}Pb	^{24}Ne	61.55	23.27	>18.20	22.37	23.18	21.97
^{232}U	^{208}Pb	^{24}Ne	62.31	22.18	21.08	20.72	21.04	20.20
^{233}U	^{209}Pb	^{24}Ne	60.5	24.39	24.83	24.15	24.80	23.15
^{234}U	^{210}Pb	^{24}Ne	58.84	26.54	25.92	27.39	28.26	25.94
^{235}U	^{211}Pb	^{24}Ne	57.36	28.54	27.42	30.37	31.34	28.51
^{232}Th	^{206}Hg	^{26}Ne	55.97	27.10	>29.20	29.54	31.38	29.75
^{234}U	^{208}Pb	^{26}Ne	59.47	24.71	25.92	25.88	27.32	25.97
^{236}U	^{210}Pb	^{26}Ne	56.75	28.51	>25.90	31.57		
^{234}U	^{206}Hg	^{28}Mg	74.13	26.34	27.54	27.55	27.82	25.46
^{232}U	^{204}Hg	^{28}Mg	74.32	25.36	>22.26	27.41	27.83	24.90
^{233}U	^{205}Hg	^{28}Mg	74.24	24.72	>27.59	27.45	27.90	25.18
^{235}U	^{207}Hg	^{28}Mg	72.2	28.31	>28.10	31.13	31.49	28.52
^{236}U	^{208}Hg	^{28}Mg	71.69	27.52	27.58	32.01		
^{236}Pu	^{208}Pb	^{28}Mg	79.67	21.35	21.67	21.73	21.41	20.46
^{238}Pu	^{210}Pb	^{28}Mg	75.93	25.25	25.7	28.31	28.46	25.73
^{237}Np	^{207}Tl	^{30}Mg	75.02	24.35	>26.93	27.34	28.74	27.20
^{236}U	^{206}Hg	^{30}Mg	72.51	26.99	27.58	30.03	31.91	29.18
^{238}Pu	^{208}Pb	^{30}Mg	77.03	25.88	25.7	25.70	26.90	25.15
^{240}Pu	^{206}Hg	^{34}Si	90.95	27.01	>25.52	28.11		
^{241}Am	^{207}Tl	^{34}Si	93.84	25.49	>24.41	25.40		
^{242}Cm	^{208}Pb	^{34}Si	96.53	24.24	23.24	23.20		

is the angle between the radius vector and symmetry axis of the ith nuclei (see Fig. 1 of Ref. [47]), and here the quadrupole interaction term is proportional to $\beta_{21}\beta_{22}$, which due to its short-range character is neglected.

In the case of proximity potential, the deformation comes only in the mean curvature radius, $E_P(z) = 4\pi\gamma b\overline{R}\Phi(\varepsilon)$. The mean curvature radius has been defined as $\overline{R} = \frac{C_1C_2}{C_1+C_2}$, for spherical nuclei. The mean curvature radius, $\overline{R}$, for two deformed nuclei lying in the same plane can be found by the relation [48]

$$\frac{1}{\overline{R}^2} = \frac{1}{R_{11}R_{12}} + \frac{1}{R_{21}R_{22}} + \frac{1}{R_{11}R_{22}} + \frac{1}{R_{21}R_{12}}, \tag{19}$$

where, the four principal radii of curvature R_{i1} and R_{i2}, with $i = 1, 2$, at the two points D and E (see Fig. 1 of Ref. [48]) of closest approach of the interacting nuclei are given by Baltz and Bayman [48] as

$$R_{i1} = \left| \frac{\left\{R_i^2(\alpha_i) + [R_i'(\alpha_i)]^2\right\}^{3/2}}{R_i''(\alpha_i)R_i(\alpha_i) - 2\left[R_i'(\alpha_i)\right]^2 - R_i^2(\alpha_i)} \right|, \tag{20}$$

$$R_{i2} = \left| \frac{R_i(\alpha_i)\sin\alpha_i\left[R_i^2(\alpha_i) + (R_i'(\alpha_i))^2\right]^{1/2}}{R_i'(\alpha_i)\cos\alpha_i - R_i(\alpha_i)\sin\alpha_i} \right|. \tag{21}$$

Here, $R'(\alpha)$ and $R''(\alpha)$ represent the first and second derivative of $R(\alpha)$ with respect to α, respectively.

The barrier penetrability of cluster in a deformed nucleus is different in different directions. The average penetrability over different directions is done using the equation

$$P = \frac{1}{2}\int_0^\pi P(\theta)\sin(\theta)d\theta, \tag{22}$$

TABLE II. The predicted logarithmic $T_{1/2}$ values of radioactive nuclei emitting clusters using MGLDM and compared with the values predicted by Bao *et al.* [28].

Parent nuclei	Cluster	Daughter nuclei	Q(MeV)	$\log_{10}[T_{1/2}(s)]$	
				MGLDM	Bao
^{220}Ra	^{12}C	^{208}Pb	32.13	12.14	11.75
^{221}Ra	^{15}N	^{206}Tl	35.24	20.74	21.86
^{222}Ra	^{18}O	^{204}Hg	39.94	27.20	28.08
^{223}Ra	^{18}O	^{205}Hg	40.45	24.17	26.83
^{226}Ra	^{20}O	^{206}Hg	40.96	25.72	26.57
^{225}Ac	^{18}O	^{207}Tl	43.60	20.13	21.11
^{224}Th	^{15}N	^{209}Bi	38.29	17.91	17.48
^{224}Th	^{24}Ne	^{200}Hg	55.63	27.84	29.49
^{226}Th	^{15}N	^{211}Bi	35.09	23.11	24.20
^{226}Th	^{24}Ne	^{202}Hg	56.68	26.52	26.90
^{228}Th	^{24}Ne	^{204}Hg	57.59	25.72	25.43
^{229}Th	^{21}O	^{208}Pb	43.43	23.22	25.34
^{229}Th	^{24}Ne	^{205}Hg	58.01	25.30	24.86
^{231}Pa	^{27}Na	^{204}Hg	63.84	27.04	29.34
^{232}Pa	^{25}Ne	^{207}Tl	59.22	23.38	24.46
^{232}Pa	^{28}Mg	^{204}Au	71.93	27.29	27.32
^{230}U	^{20}O	^{210}Po	43.93	25.37	26.18
^{230}U	^{24}Ne	^{206}Pb	61.54	23.28	21.97
^{230}U	^{32}Si	^{198}Pt	85.85	29.90	29.89
^{232}U	^{28}Mg	^{204}Hg	74.54	25.10	24.90
^{233}U	^{27}Na	^{206}Tl	64.90	28.30	29.32
^{234}U	^{27}Na	^{207}Tl	64.91	28.46	29.51
^{225}Np	^{12}C	^{213}Fr	35.26	11.55	10.38
^{225}Np	^{16}O	^{209}At	49.37	16.23	14.72
^{227}Np	^{16}O	^{211}At	49.11	16.01	15.28
^{227}Np	^{18}O	^{209}At	46.39	20.14	21.00
^{231}Np	^{20}O	^{211}At	43.64	27.03	28.23
^{233}Np	^{22}Ne	^{211}Bi	58.03	27.67	27.97
^{233}Np	^{25}Ne	^{208}Bi	59.08	26.55	27.81
^{234}Np	^{28}Mg	^{206}Tl	77.46	23.59	22.72
^{235}Np	^{29}Mg	^{206}Tl	74.13	27.77	28.04
^{236}Np	^{29}Mg	^{207}Tl	75.24	26.10	26.49
^{237}Np	^{32}Si	^{205}Au	88.12	28.55	28.26
^{234}Pu	^{27}Na	^{207}Bi	66.14	30.01	30.86
^{234}Pu	^{29}Al	^{205}Tl	82.63	27.45	27.15
^{236}Pu	^{24}Ne	^{212}Po	59.42	28.31	28.57
^{236}Pu	^{29}Al	^{207}Tl	82.40	27.82	27.71
^{237}Pu	^{29}Mg	^{208}Pb	77.68	24.33	23.83
^{237}Pu	^{32}Si	^{205}Hg	91.73	25.89	25.00
^{237}Am	^{28}Mg	^{209}Bi	80.09	22.76	21.89
^{237}Am	^{32}Si	^{205}Tl	94.74	24.56	23.00
^{238}Am	^{29}Mg	^{209}Bi	77.52	26.06	25.82
^{238}Am	^{33}Si	^{205}Tl	93.03	27.76	25.48
^{239}Am	^{32}Si	^{207}Tl	94.78	24.59	23.11
^{239}Am	^{34}Si	^{205}Tl	93.44	26.08	25.28
^{240}Am	^{34}Si	^{206}Tl	93.99	25.42	24.66
^{241}Am	^{34}Si	^{207}Tl	94.20	25.12	24.41
^{238}Cm	^{32}Si	^{206}Pb	97.58	23.34	21.43
^{240}Cm	^{30}Mg	^{210}Po	76.81	26.40	29.08
^{240}Cm	^{34}Si	^{206}Pb	95.74	25.21	24.28
^{242}Cm	^{32}Si	^{210}Pb	93.89	25.96	25.11
^{220}Ra	^{16}O	^{204}Hg	39.84	24.98	27.58
^{222}Ra	^{15}N	^{207}Tl	35.38	20.44	21.20
^{223}Ra	^{15}N	^{208}Tl	34.01	23.46	24.26
^{224}Ra	^{20}O	^{204}Hg	39.86	28.56	29.40
^{225}Ac	^{17}N	^{208}Pb	35.65	22.32	23.20
^{224}Th	^{14}C	^{210}Po	33.05	13.64	13.26
^{224}Th	^{16}O	^{208}Pb	46.63	19.38	15.81
^{226}Th	^{14}C	^{212}Po	30.66	17.72	18.30
^{226}Th	^{18}O	^{208}Pb	45.88	17.85	18.18
^{228}Th	^{14}C	^{214}Po	28.33	22.31	24.04
^{229}Th	^{14}C	^{215}Po	27.22	24.75	27.04
^{229}Th	^{23}F	^{206}Tl	48.70	28.23	29.52
^{231}Pa	^{22}O	^{209}Bi	42.56	27.13	29.40
^{231}Pa	^{28}Mg	^{203}Au	71.81	27.83	27.41
^{232}Pa	^{27}Na	^{205}Hg	63.95	28.35	29.26
^{230}U	^{14}C	^{216}Rn	28.46	24.04	26.04
^{230}U	^{21}F	^{209}Bi	50.10	26.99	27.76
^{230}U	^{28}Mg	^{202}Hg	74.20	26.85	25.15
^{232}U	^{23}F	^{209}Bi	49.73	28.65	29.97
^{232}U	^{32}Si	^{200}Pt	85.54	29.63	29.58
^{233}U	^{28}Mg	^{205}Hg	74.45	25.32	25.18
^{235}U	^{29}Mg	^{206}Hg	72.69	28.07	28.46
^{225}Np	^{14}C	^{211}Fr	32.83	16.52	16.49
^{227}Np	^{14}C	^{213}Fr	33.22	15.97	16.08
^{227}Np	^{17}O	^{210}At	45.50	22.18	22.11
^{229}Np	^{18}O	^{211}At	46.37	20.31	21.26
^{231}Np	^{22}Ne	^{209}Bi	62.10	22.31	21.11
^{233}Np	^{24}Ne	^{209}Bi	62.36	23.64	22.04
^{234}Np	^{25}Ne	^{209}Bi	60.90	23.99	24.26
^{235}Np	^{28}Mg	^{207}Tl	77.33	23.61	23.00
^{236}Np	^{28}Mg	^{208}Tl	75.37	25.77	24.90
^{236}Np	^{30}Mg	^{206}Tl	74.75	26.84	27.58
^{234}Pu	^{24}Ne	^{210}Po	62.45	24.45	23.46
^{234}Pu	^{28}Mg	^{206}Pb	79.39	22.60	21.73
^{234}Pu	^{32}Si	^{202}Hg	92.04	26.25	24.26
^{236}Pu	^{27}Na	^{209}Bi	66.89	28.94	28.91
^{236}Pu	^{32}Si	^{204}Hg	91.93	25.98	24.68
^{237}Pu	^{30}Al	^{207}Tl	82.25	28.20	28.23
^{238}Pu	^{31}Al	^{207}Tl	82.40	28.19	28.48
^{237}Am	^{29}Mg	^{208}Bi	76.28	27.80	27.52
^{238}Am	^{28}Mg	^{210}Bi	78.47	24.94	24.15
^{238}Am	^{32}Si	^{206}Tl	95.03	23.76	22.73
^{239}Am	^{30}Mg	^{209}Bi	76.78	27.31	27.32
^{239}Am	^{33}Si	^{206}Tl	92.43	27.11	26.40
^{240}Am	^{33}Si	^{207}Tl	93.33	27.27	25.28
^{241}Am	^{33}Si	^{208}Tl	90.47	28.87	29.15
^{238}Cm	^{28}Mg	^{210}Po	80.66	23.99	22.79
^{239}Cm	^{32}Si	^{207}Pb	97.95	22.55	21.15
^{240}Cm	^{32}Si	^{208}Pb	97.82	22.67	20.26
^{241}Cm	^{32}Si	^{209}Pb	95.67	24.45	22.86
^{242}Cm	^{34}Si	^{208}Pb	96.79	23.98	21.91
^{243}Cm	^{34}Si	^{209}Pb	95.03	26.01	24.08
^{242}Cf	^{32}Si	^{210}Po	99.70	24.32	21.53
^{242}Cf	^{34}Si	^{208}Po	97.10	26.74	25.15
^{244}Cf	^{34}Si	^{210}Po	97.67	25.98	24.46
^{246}Cf	^{38}S	^{208}Pb	113.02	23.32	23.20
^{249}Cf	^{46}Ar	^{203}Hg	125.08	27.23	26.83

TABLE II. (*Continued.*)

Parent nuclei	Cluster	Daughter nuclei	Q(MeV)	$\log_{10}[T_{1/2}(s)]$	
				MGLDM	Bao
^{249}Cf	^{50}Ca	^{199}Pt	137.09	29.33	27.74
^{244}Cm	^{34}Si	^{210}Pb	93.42	27.23	26.18
^{242}Cf	^{33}Si	^{209}Po	96.55	28.48	25.57
^{242}Cf	^{36}S	^{206}Pb	114.16	25.25	22.92
^{249}Cf	^{42}S	^{207}Pb	110.18	28.39	27.49
^{249}Cf	^{48}Ca	^{201}Pt	138.07	28.22	26.38
^{251}Cf	^{46}Ar	^{205}Hg	126.51	26.86	24.93

where $P(\theta)$ is the penetrability of cluster in the direction θ from the symmetry axis for axially symmetric deformed nuclei. The penetrability $P(\theta)$ can be calculated using Eq. (14) with the interaction potential taken as the sum of deformed Coulomb potential and deformed proximity potential.

III. RESULTS AND DISCUSSION

Half-lives of radioactive nuclei whose mass numbers vary from 221 to 242, emitting clusters like C, O, F, Ne, Mg, Si, using MGLDM, and considering deformation and orientation effects are illustrated in Table I. The deformation values β_2 and β_4 are taken from recent table of Moller *et al.* [49]. In MGLDM, the conventional liquid drop model developed by Royer is modified by proximity potential 77. To this equation, to calculate the decay constant, the preformation factor proposed by us [43] is multiplied. Preformation factor mentioned is Q value-dependent, and by introducing it, we could generate a more accurate result compared with the one without using it. Thus, in this model we assume cluster is preformed within parent nuclei before emission. When we take a close look at the tabulated value, it is clearly understood that using the present model, we are able to reproduce the observed value with great accuracy. Also, half-lives predicted using CPPM, GLDM1, and GLDM2 are also listed in the table. CPPM is the Coulomb and proximity potential model suggested by Santhosh *et al.* [50]. GLDM1 and GLDM2 are the models put forward by Bao *et al.* [32] with and without consideration of microscopic parameters like shell correction and pairing energy.

Standard deviation of logarithm of half-lives using MGLDM, CPPM, GLDM1, and GLDM2 are calculated and found to be 0.755, 1.58, 1.84, and 1.14, respectively. This is clear-cut evidence that the present theoretical approach using MGLDM is successful when compared with the other three models.

In Table II, half-lives of about 116 various isotopes of actinides like Ra, Ac, Th, Pa, U, Np, Pu, Am, Cm, and No, whose mass numbers vary from 220 to 251, are predicted using MGLDM. In the above reactions, clusters like C, N, O, F, Ne, Na, Mg, Al, Si, S, Ar, and Ca are emitted. Thus, even half-lives of heavy clusters like Ca with mass number 48 can be predicted using our present model. The prediction by Bao *et al.* [32] for the above-mentioned decay process is also listed in Table II. On close examination of the values predicted by MGLDM and by Bao *et al.*, it is evident that both values are comparable.

IV. SUMMARY AND CONCLUSION

In our approach, cluster radioactivity is described as a process in which preformed cluster within parent nuclei is emitted and tunnel through a potential barrier constructed by improving conventional liquid drop model by Royer with proximity 77 potential. Emission of clusters like C, F, O, Ne, Mg, and Si are studied using MGLDM including preformation factor considering deformation and orientation effects. Half-lives of such heavy clusters using our model almost replicate observed values with great precision. Least standard deviation in MGLDM exactly proves it as a good model compared to GLDM. And the results are also comparable with values predicted by Bao *et al.* Therefore, we extend our work in predicting half-lives of various radioactive nuclei emitting clusters. We hope that this would help possible candidates to calculate half-lives experimentally in the near future.

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