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T mixing and decay widths of first two 1^- states in ^{10}B

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For the isospin-mixed 1^- states in ^{10}B, widths for deuteron and α decays provide an independent estimate of the amount of isospin mixing, which is consistent with results from γ decays. Analysis provides two solutions for the proton spectroscopic factors of the pure $T = 0$ and 1 states.

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I. INTRODUCTION

A great deal has been written about the isospin mixing of two 1^- states at 6.873(5) and 7.430(10) MeV in ^{10}B [1], beginning with the paper by Wilkinson and Clegg in 1956 [2]. The main features of the argument are summarized on p. 298 of the latest compilation [1]. Using the information available at the time, Wilkinson and Clegg estimated the $T = 1$ impurity in the 6.873-MeV state to be about 20%, and they predicted the existence of a predominantly $T = 1$ 1^- state within 1 MeV. They stated "This is distinctly the largest well-established isotopic spin impurity known in any light nucleus." Other estimates of the mixing were made by others. With the ^{9}Be(p,γ) reaction, Edge, and Gemmell [3] confirmed an isospin impurity of about 20%. Barker and Kondo [4] discussed the possibilities for the isospin-mixed partner of the first 1^- state.

II. ANALYSIS

The argument for T mixing is outlined on p. 298 of the compilation [1]. Briefly, $E1$ transitions in a self-conjugate nucleus must be isovector, and both of these 1^- states exhibit $E1$ decays to 1^+, $T = 0$ states, implying a $T = 1$ component in both 1^- states. These $E1$'s are summarized in Table I [1,5–8]. The ratios of the sums for decay from each state result in an estimate of the T mixing of $b^2/a^2 = 0.227(62)$, where I have assumed

$$|6.873\rangle = a|(T = 0)\rangle + b|(T = 1)\rangle;$$
$$|7.430\rangle = -b|(T = 0)\rangle + a|(T = 1)\rangle.$$

Transitions to the 0^+, $T = 1$ state at 1.74 MeV are also listed in Table I, and they imply a limit of $b^2/a^2 < 0.21(6)$, consistent with the other estimate.

Both of these 1^- states decay by proton, deuteron, and α emission. The latter two must involve decay only from the $T = 0$ components of the two states. It occurred to me that these decays could be used to estimate the $T = 0$ content in the upper state. The latest compilation [1] contains some inconsistent information about the p, d, and α widths of these two states [6,8–10]. I have attempted to arrive at best values for these quantities that are consistent with the latest total widths. These are listed in Table II. I used my best judgment and the procedures outlined in the footnotes to the table.

In order to use these widths to estimate T mixing in the second state, we need spectroscopic factors for decay of the first 1^- $T = 0$ state of ^{10}B to ^{6}Li + α and ^{8}Be + d. The 1^- wave function was obtained from a *psd* shell-model calculation using the Millener-Kurath interaction [11]. Wave functions for ^{8}Be and ^{6}Li ground states were taken from a shell-model calculation fully within the *1p* shell [12]. For ^{6}Li, the interaction was (6-16)2BME, for ^{8}Be, it was (8–16)POT. Cluster overlaps were computed from the shell-model wave functions using the procedure outlined by Kurath [13]. These are both $L = 1$ decays, and their shell-model S's [14] are listed in Table III. I have calculated the *sp* widths for these decays in a Woods-Saxon well, whose depth was adjusted to reproduce the decay energies. Geometric parameters were r_0, a, $r_{oc} = 1.37$, 0.60, 1.40 fm for the deuteron, and $R_0 = R_{0c} = 2.79$ fm, $a = 0.65$ fm for α. I calculated ^{6}Li + α and ^{8}Be + d elastic scattering and computed the *sp* widths from the $L = 1$ phase shifts. These *sp* widths are also listed in Table III. The spectroscopic factor is a measure of the single-particle nature of a state, and it is related to the decay width by the expression $S = \Gamma_{\mathrm{exp}}/\Gamma_{\mathrm{sp}}$. The calculated widths to be expected for the allowed decays are thus $\Gamma_{\mathrm{calc}} = S\,\Gamma_{\mathrm{sp}}$. Uncertainties are decreased if we sum the widths for d and α. The T mixing is then

$$b^2/a^2 = (\Gamma_d + \Gamma_\alpha)_{\mathrm{exp}}/(\Gamma_d + \Gamma_\alpha)_{\mathrm{calc}}.$$

The result is given in Table III. Perhaps surprisingly, this estimate of T mixing is very well consistent with the one derived from γ decays. I now compute the weighted average of these two and the limit from 1^- decays to 0^+, $T = 1$ to get $(b^2/a^2)_{\mathrm{ave}} = 0.243(40)$. These results are listed in Table IV and depicted graphically in Fig. 1.

As a check on the above procedure, I also apply it to d and α decays of the first 1^- state. Results are listed in Table V. Agreement is good.

The aim now is to use this amount of T mixing and experimental proton widths (Table VI) to estimate the proton spectroscopic factors for the pure $T = 0$ and 1 basis states. Defining $S = A^2$, and recalling that $C^2S = \Gamma_{\mathrm{exp}}/\Gamma_{\mathrm{sp}}$ (with $C^2 = 1/2$ here), we can use the proton widths to first determine experimental S's and then use those to determine the

TABLE I. Strengths of relevant γ transitions from first two 1^- states in ^{10}B [1,5–8].

Final		Initial	$B(E1)$ (w.u.)	Initial	$B(E1)$ (w.u.)	T mixing b^2/a^2
E_x (MeV)	J^π, T					
0.718	1^+,0	6.87 MeV	$4.2(11) \times 10^{-3}$	7.43 MeV	$2.3(5) \times 10^{-2}$	
2.154	1^+,0		$6.0(15) \times 10^{-3}$		$2.2(7) \times 10^{-2}$	
1.740	0^+,1		$1.9(5) \times 10^{-2}$		$<4.0 \times 10^{-3}$	<0.21(6)
Sum to 1^+,0			$1.02(19) \times 10^{-2}$		$4.5(9) \times 10^{-2}$	0.227(62)

TABLE II. Best decay widths (keV) for first two 1^- states in ^{10}B.

E_x (MeV)	Total	Proton	Alpha	Deuteron	Alpha + deuteron
6.873(5)	120(5)[a]	27.6(48)[b]	40(3)[d]	52(8)[f]	92(7)
7.430(10)	100(10)[a]	38(7)[c]	34(9)[e]	28(7)[e]	62(12)

[a] From master table [1].
[b] From $\Gamma_p/\Gamma = 0.23(4)$ [10].
[c] From $\Gamma_p/\Gamma = 0.38(6)$ [6].
[d] From $\Gamma_\alpha/\Gamma = 0.33(2)$ [10].
[e] Using BR and renormalizing Γ_{tot}
[f] From $\Gamma_d = \Gamma – \Gamma_p – \Gamma_\alpha$.

TABLE III. Results (energies in MeV, widths in keV) for second 1^- state.

Channel	E(decay)	Γ_{sp}	S	$\Gamma_{calc} = S\,\Gamma_{sp}$	Γ(exp)	T mixing b^2/a^2
^{8}Be+ d	1.403	755	0.1794	135	28(7)	
^{6}Li + α	2.969	2440	0.0304	75	34(9)	
Sum				210	62(12)	0.295(57)

TABLE IV. Estimates of T mixing in 1^- states.

Source	b^2/a^2
Gamma decay to 1^+, $T = 0$	0.227(62)
Gamma decay to 0^+, $T = 1$	<0.21(6)
Alpha + deuteron decay	0.295(57)
Weighted average	0.243(40)[a]

[a] Gives $a = 0.897$, $b = 0.442$.

TABLE V. Results (energies in MeV, widths in keV) for first 1^- state.

Channel	E(decay)	Γ_{sp}	S	$\Gamma_{calc} = a^2\,S\,\Gamma_{sp}$	Γ(exp)
^{8}Be + d	0.846	220	0.1794	32	52(8)
^{6}Li + α	2.412	2140	0.0304	52	40(3)
Sum				84	92(9)

TABLE VI. Proton widths and spectroscopic factors.

State	Γ_p(exp)	Γ_{sp}	$S = 2\,\Gamma_p(\mathrm{exp})/\Gamma_{sp}$	$A = S^{1/2}$
6.873	27.6(48)	54	1.02(18)	1.01(9)
7.430	38(7)	720	0.053(10)	±0.230(22)

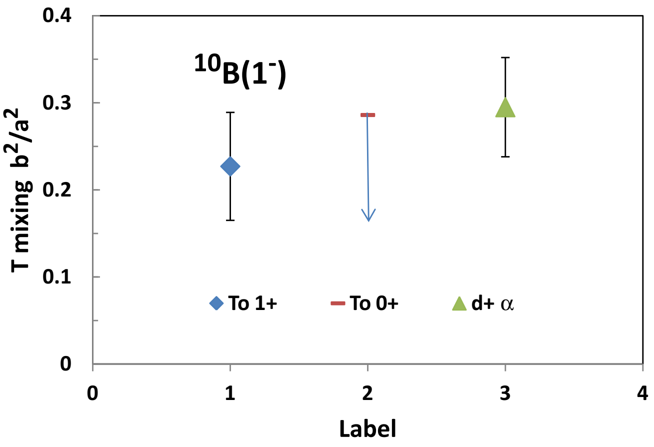

FIG. 1. Estimates of isospin mixing of first two 1^- states in ^{10}B: from γ decays to 1^+, $T = 0$ states, from γ decays to 0^+, $T = 1$ states, and from d and α decays.

basis-state S's. Relevant equations are

$$A(6.873) = aA_0 + bA_1;\ A(7.430) = -bA_0 + aA_1$$

(where the subscripts denote isospin), which can be manipulated to give

$$A_1 = bA(6.873) \pm aA(7.430) = 0.653(41) \text{ or } 0.240(41);$$

$$A_0 = aA(6.873) \pm bA(7.430) = 0.80(8) \text{ or } 1.01(8).$$

Basis-state spectroscopic factors are then $S_0 = {A_0}^2$ and $S_1 = {A_1}^2$, as listed in Table VII. Note that the sum of S is preserved. Of the two solutions, one has comparable strengths for the pure states, the other has virtually no strength in the $T = 1$ basis state. Most shell-model calculations would prefer the former (solution 2).

TABLE VII. Basis state spectroscopic factors.

Basis state	S (solution 1)	S (solution 2)
$1^-, T = 0$	1.02(17)	0.64(13)
$1^-, T = 1$	0.058(20)	0.426(54)
Sum	1.08	1.07

With an energy splitting of 0.557 MeV, the current T mixing results in a matrix element of $V(1^-) = ab\ \Delta E = 0.221(20)$ MeV. The T-mixed 2^- states are only 0.270(30) MeV apart, implying a matrix element of $V(2^-) < 0.135(15)$ MeV. The two would be consistent if V goes as $1/(2J + 1)$.

Barker and Kondo [4] have argued that these two 1^- states do not satisfy the conditions for two-state mixing. Additionally, they suggested that a second 1^-, $T = 0$ state should exist nearby. Such a state has not yet been identified. I see no evidence for large deviations from a two-state-mixing scenario.

III. SUMMARY

For the first two 1^- states in ^{10}B, I have used γ widths to 1^+, $T = 0$ states and to 0^+, $T = 1$ states to determine isospin mixing of $b^2/a^2 = 0.227(62)$ and $b^2/a^2 < 0.21(6)$, respectively, in agreement with earlier estimates. I have then used deuteron and α widths to obtain an independent estimate of $b^2/a^2 = 0.295(57)$, which is consistent with the results from γ decay. The weighted average of all the results is $b^2/a^2 = 0.243(40)$. With this value and the proton widths of the physical 1^- states, I have obtained the proton spectroscopic factors of the $T = 0$ and 1 1^- states. Of the two solutions, one has approximately equal S's for the two, the other has $S \approx 1$ for the $T = 0$ state and S very small for $T = 1$.

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