

Forbidden nuclear reactions

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Exothermal nuclear reactions that become forbidden due to Coulomb repulsion in the $\varepsilon \rightarrow 0$ limit [$\lim_{\varepsilon \rightarrow 0} \sigma(\varepsilon) = 0$] are investigated. [$\sigma(\varepsilon)$ is the cross section and ε is the center of mass energy.] It is found that any perturbation may mix states with small but finite amplitude to the initial state resulting in finite cross section (and rate) of the originally forbidden nuclear reaction in the $\varepsilon \rightarrow 0$ limit. The statement is illustrated by modification of nuclear reactions due to impurities in a gas mix of atomic state. The change of the wave function of reacting particles in nuclear range due to their Coulomb interaction with impurity is determined using standard time-independent perturbation calculation of quantum mechanics. As an example, cross section, rate and power densities of impurity-assisted nuclear pd reaction are numerically calculated. With the aid of astrophysical factors cross section and power densities of the impurity-assisted $d(d, n)^3\text{He}$, $d(d, p)t$, $d(t, n)^4\text{He}$, $^3\text{He}(d, p)^4\text{He}$, $^6\text{Li}(p, \alpha)^3\text{He}$, $^6\text{Li}(d, \alpha)^4\text{He}$, $^7\text{Li}(p, \alpha)^4\text{He}$, $^9\text{Be}(p, \alpha)^6\text{Li}$, $^9\text{Be}(p, d)^8\text{Be}$, $^9\text{Be}(\alpha, n)^{12}\text{C}$, $^{10}\text{B}(p, \alpha)^7\text{Be}$, and $^{11}\text{B}(p, \alpha)^8\text{Be}$ reactions are also given. The affect of gas mix-wall interaction on the process is considered too.

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I. INTRODUCTION

The cross section (σ) of nuclear reactions between charged particles j and k of charge numbers z_j and z_k reads as [1]

$$\sigma(\varepsilon) = S(\varepsilon) \exp[-2\pi\eta_{jk}(\varepsilon)]/\varepsilon, \quad (1)$$

where $S(\varepsilon)$ is the astrophysical S factor and ε is the kinetic energy taken in the center of mass (CM) coordinate system.

$$\eta_{jk} = z_j z_k \alpha_f \frac{a_{jk} m_0 c}{\hbar |\mathbf{k}|} = z_j z_k \alpha_f \sqrt{a_{jk} \frac{m_0 c^2}{2\varepsilon}} \quad (2)$$

is the Sommerfeld parameter, where \mathbf{k} is the wave number vector of particles j and k in their relative motion, \hbar is the reduced Planck constant, c is the velocity of light in vacuum and

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \quad (3)$$

is the reduced mass number of particles j and k of mass numbers A_j and A_k and rest masses $m_j = A_j m_0$, $m_k = A_k m_0$. $m_0 c^2 = 931.494$ MeV is the atomic energy unit, α_f is the fine structure constant. However, in the latest few decades anomalies to (1) were reported, which are anomalous screening effect and the less well documented area of phenomena of the so-called low-energy nuclear reactions (LENRs).

Extraordinary observations in cross-section measurements of dd reactions in deuterated metal targets made in low-energy accelerator physics, which can not be explained by electron screening are named anomalous screening. (A systematic survey of anomalous screening effect was made [2] a decade ago.) However, the full theoretical explanation of the effect is still missing.

In low-energy nuclear reactions (LENRs), a new and problematic field that emerged after the notorious cold fusion

publication by Fleischmann and Pons in 1989 [3], results are reported that are in conflict with (1). Despite the fact that even the possibility of the phenomenon of nuclear fusion at low energies is received with due scepticism in mainstream physics [4] low-energy nuclear reactions (LENRs) are dealt with in a great number of laboratories and publications (mostly experimental), conferences and periodicals have been devoted to various aspects of the problem. (For summary of the field see, e.g., Refs. [5–8].) The aim of this paper is to show the possible reason for anomalies of cross sections of nuclear reactions of particles of like charges at low energy in general.

II. ROLE OF COULOMB REPULSION

The solution $\varphi_{jk}(\mathbf{R}, \mathbf{r})$ of the stationary Schrödinger equation

$$H_{jk} \varphi_{jk}(\mathbf{R}, \mathbf{r}) = E_{jk} \varphi_{jk}(\mathbf{R}, \mathbf{r}) \quad (4)$$

of particles of charge numbers z_j and z_k with

$$H_{jk} = -\frac{\hbar^2}{2m_0(A_j + A_k)} \nabla_{\mathbf{R}}^2 - \frac{\hbar^2}{2m_0 a_{jk}} \nabla_{\mathbf{r}}^2 + \frac{z_j z_k e^2}{|\mathbf{r}|} \quad (5)$$

is

$$\varphi_{jk}(\mathbf{R}, \mathbf{r}) = V^{-1/2} e^{i\mathbf{K}\mathbf{R}} \varphi_{\text{Cb}}(\mathbf{r}), \quad (6)$$

where $\mathbf{R} = (m_j \mathbf{r}_j + m_k \mathbf{r}_k)/(m_j + m_k)$ and $\mathbf{r} = \mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k$ are CM and relative coordinate of particles j and k of coordinate \mathbf{r}_j and \mathbf{r}_k , respectively. V denotes the volume of normalization and $\varphi_{\text{Cb}}(\mathbf{r})$ is the Coulomb solution [9], which is the wave function of the relative motion in repulsive Coulomb potential. $\nabla_{\mathbf{R}}^2$ and $\nabla_{\mathbf{r}}^2$ are Laplace operators in the CM and relative coordinates, \mathbf{K} is the wave vector of the CM motion

and $E_{jk} = E_{\text{CM}} + \varepsilon$ with $E_{\text{CM}} = \hbar^2 \mathbf{K}^2 / [2m_0(A_j + A_k)]$ and $\varepsilon = \hbar^2 \mathbf{k}^2 / (2m_0 a_{jk})$. e is the elementary charge with $e^2 = \alpha_f \hbar c$.

The contact probability density in the nuclear volume is $|\varphi_{\text{Cb}}(\mathbf{0})|^2 = f_{jk}^2/V$, where

$$f_{jk} = |e^{-\pi \eta_{jk}/2} \Gamma(1 + i \eta_{jk})| = \sqrt{\frac{2\pi \eta_{jk}}{\exp(2\pi \eta_{jk}) - 1}}. \quad (7)$$

The result of a first-order calculation of the cross section in standard perturbation theory of quantum mechanics is proportional to f_{jk}^2/v where v is the relative velocity in the CM system. Investigating the energy dependence of f_{jk}^2/v it is found that $f_{jk}^2/v \sim \exp[-2\pi \eta_{jk}(\varepsilon)]/\varepsilon$ in the $\varepsilon \rightarrow 0$ limit. Accordingly, the magnitude of the factor f_{jk} is crucial from the point of view of magnitude of the cross section.

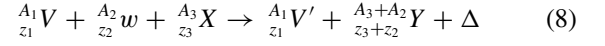
If the reaction energy $\Delta > 0$ (the difference between initial and final rest energies) of reaction between particles of likewise charge, the spontaneous process could be allowed by energy conservation. However, in the $\varepsilon \rightarrow 0$ limit $\lim_{\varepsilon \rightarrow 0} f_{jk}^2(\varepsilon) = 0$ with $\lim_{\varepsilon \rightarrow 0} |\varphi_{\text{Cb}}(\mathbf{0})|^2 = 0$ and the process becomes forbidden [$\lim_{\varepsilon \rightarrow 0} \sigma(\varepsilon) = 0$] due to Coulomb repulsion. (If one of the reacting particles is neutral, which is the case of neutron capture processes, the cross section has nonzero value in the $\varepsilon \rightarrow 0$ limit, see, e.g., thermal neutron absorption cross sections [10].)

III. STATEMENT AND EXAMPLES

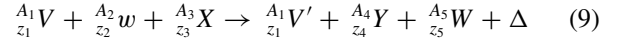
Experience in atomic physics indicates that in case of forbidden transitions the second-order process may play an important role. As, e.g., in the case of the hydrogen $2s_{1/2}$ - $1s_{1/2}$ transition, which is a forbidden electric dipole transition, the largest transition rate comes from a two photonic process [11] in which the sum of the energies of the simultaneously emitted photons equals the difference between the energies of states $2s_{1/2}$ and $1s_{1/2}$. The mean life time 1/7 s of the $2s_{1/2}$ state due to the two photonic process is much longer than the lifetime 1.6×10^{-9} s of state $2p_{1/2}$ for which electric dipole transition is allowed. Thus one can conclude that a second-order process from the point of view of perturbation calculation can result in small but finite transition rate. In the second-order process the state is changed in first order and states, which can produce allowed electric dipole transition rate, are mixed with small amplitude to the initial $2s_{1/2}$ state meanwhile two particles are emitted.

Similarly an essential change of the initial eigenstate of (4) of $\varepsilon = 0$ may happen due to any perturbation since it can mix states of $\varepsilon \neq 0$ with small but finite amplitude to the initial state resulting in much smaller (compared to neutron absorption) but finite rate of the nuclear reaction originally forbidden in the $\varepsilon \rightarrow 0$ limit. Consequently, cross section and rate of processes to be considered should be calculated by the rules of standard perturbation calculation of quantum mechanics. Our statement applies to every nuclear process for which $\sigma(\varepsilon)$ has the form of (1) and $\lim_{\varepsilon \rightarrow 0} \sigma(\varepsilon) = 0$ holds, and as such it concerns low-energy nuclear physics with charged participants in general.

Since the above statement is quite general it is only illustrated by modification of forbidden nuclear reactions due to Coulomb interaction with impurities (the initial state is defined in the next section). We demonstrate the mechanism on the



and



processes. Reaction (8) is an impurity (${}_{z_1}^{A_1}V$) assisted capture of particle ${}_{z_2}^{A_2}w$, e.g. capture of proton (p), deuteron (d), triton (t), ${}^3\text{He}$, ${}^4\text{He}$, etc. The impurity (${}_{z_1}^{A_1}V$)-assisted reaction (9) with two final fragments is possible with conditions $A_2 + A_3 = A_4 + A_5$ and $z_2 + z_3 = z_4 + z_5$. The reaction energy Δ is the difference between the sum of the initial and final mass excesses, i.e., $\Delta = \Delta_{A_2, z_2} + \Delta_{A_3, z_3} - \Delta_{A_3+A_2, z_3+z_2}$ in case of (8) and $\Delta = \Delta_{A_2, z_2} + \Delta_{A_3, z_3} - \Delta_{A_4, z_4} - \Delta_{A_5, z_5}$ in case of (9) where Δ_{A_j, z_j} and $\Delta_{A_3+A_2, z_3+z_2}$ are the corresponding mass excesses [12]. Since particle 1 merely assists the nuclear reaction its rest mass does not change.

Usually capture of particle ${}_{z_2}^{A_2}w$ may happen in the ${}_{z_3}^{A_3}X \rightarrow {}_{z_3+z_2}^{A_3+A_2}Y + \gamma$ (with $\Delta > 0$) reaction where γ emission is required by energy and momentum conservation. Accordingly (8) describes a new type of ${}_{z_2}^{A_2}w$ capture. In the usual ${}_{z_2}^{A_2}w$ -capture reaction particles ${}_{z_3+z_2}^{A_3+A_2}Y$ and γ take away the reaction energy and the reaction is governed by electromagnetic interaction. In reaction (8) the reaction energy is taken away by particles ${}_{z_1}^{A_1}V'$ and ${}_{z_3+z_2}^{A_3+A_2}Y$ while the reaction is governed by Coulomb as well as strong interactions.

IV. MECHANISM AND MODEL

It is assumed that initially all components of a three-body system are in atomic state. Atomic state can effectively be achieved, e.g., by dissociative chemisorption at metal (e.g., Pd, Ni, and Cu) surfaces from two atomic molecules [13] or simply by heating a molecular gas. So, as initial system three screened charged heavy particles of rest masses m_j and nuclear charges $z_j e$ ($j = 1, 2, 3$) are taken. The total Hamiltonian, which describes this three-body system is

$$H_{\text{tot}} = H_{\text{kin},1} + H_{23,\text{sc}} + V_{\text{Cb,sc}}(1, 2) + V_{\text{Cb,sc}}(1, 3), \quad (10)$$

where $H_{23,\text{sc}} = H_{\text{kin},2} + H_{\text{kin},3} + V_{\text{Cb,sc}}(2, 3)$ is the Hamiltonian of particles 2 and 3 whose nuclear reaction will be discussed. $H_{\text{kin},j}$ denotes the kinetic Hamiltonian of particle j and particle 1 is considered to be free.

$$V_{\text{Cb,sc}}(j, k) = \frac{z_j z_k e^2}{2\pi^2} \int \frac{\exp(i\mathbf{q}\mathbf{r}_{jk})}{q^2 + q_{\text{sc},jk}^2} d\mathbf{q}, \quad (11)$$

denotes the screened Coulomb interaction between particles j and k with screening parameter $q_{\text{sc},jk}$.

It is supposed that stationary solutions $|1\rangle$ and $|2, 3\rangle_{\text{sc}}$ of energy eigenvalues E_1 and E_{23} of the stationary Schrödinger equations $H_{\text{kin},1}|1\rangle = E_1|1\rangle$ with E_1 the kinetic energy of particle 1 and $H_{23,\text{sc}}|2, 3\rangle_{\text{sc}} = E_{23}|2, 3\rangle_{\text{sc}}$ with $E_{23} = E_{\text{CM}} + \varepsilon$ are known. Here ε and E_{CM} are the energies attached to the relative and CM motions (of wave numbers \mathbf{k} and \mathbf{K}) of

particles 2 and 3. Thus H_{tot} can be written as $H_{\text{tot}} = H_0 + H_{\text{Int}}$ with $H_0 = H_1 + H_{23,\text{sc}}$ as the unperturbed Hamiltonian and

$$H_{\text{Int}} = V_{\text{Cb,sc}}(1, 2) + V_{\text{Cb,sc}}(1, 3) \quad (12)$$

as the interaction Hamiltonian (time-independent perturbation). The stationary solution $|1, 2, 3\rangle_{0,\text{sc}}$ of $H_0|1, 2, 3\rangle_{0,\text{sc}} = E_0|1, 2, 3\rangle_{0,\text{sc}}$ with $E_0 = E_1 + E_{23}$ can be written as $|1, 2, 3\rangle_{0,\text{sc}} = |1\rangle|2, 3\rangle_{\text{sc}}$, which is the direct product of states $|1\rangle$ and $|2, 3\rangle_{\text{sc}}$. The states $|1, 2, 3\rangle_{0,\text{sc}}$ form complete system. The approximate solution of $H_{\text{tot}}|1, 2, 3\rangle_{\text{sc}} = E_0|1, 2, 3\rangle_{\text{sc}}$ in the screened case is obtained with the aid of standard time-independent perturbation calculation [14] and the first-order approximation is expanded in terms, which are called intermediate states, of the complete system $|1, 2, 3\rangle_{0,\text{sc}}$.

The solutions $|2, 3\rangle_{\text{sc}}$ in the screened case are unknown [their coordinate representation $\langle \mathbf{R}, \mathbf{r} | 2, 3 \rangle_{\text{sc}}$ is denoted by $\varphi_{23}(\mathbf{R}, \mathbf{r})_{\text{sc}}$] but the solution of $H_{23}|2, 3\rangle = E_{23}|2, 3\rangle$ in the unscreened case is known and the coordinate representation $\langle \mathbf{R}, \mathbf{r} | 2, 3 \rangle = \varphi_{23}(\mathbf{R}, \mathbf{r})$ of $|2, 3\rangle$, as it is said above, has the form $\varphi_{23}(\mathbf{R}, \mathbf{r}) = V^{-1/2} e^{i\mathbf{KR}} \varphi_{\text{Cb}}(\mathbf{r})$, where $\varphi_{\text{Cb}}(\mathbf{r})$ is the unscreened Coulomb solution [9] (now $\mathbf{r} = \mathbf{r}_{23}$).

The two important limits of $\varphi_{23}(\mathbf{R}, \mathbf{r})_{\text{sc}}$ are the solution $\varphi_{23}(\mathbf{R}, \mathbf{r}, \text{nucl})_{\text{sc}}$ in the nuclear volume and the solution $\varphi_{23}(\mathbf{R}, \mathbf{r}, \text{out})_{\text{sc}}$ in the screened regime. In the nuclear volume screening is negligible thus $\varphi_{23}(\mathbf{R}, \mathbf{r})_{\text{sc}} = \varphi_{23}(\mathbf{R}, \mathbf{r})$. Furthermore, in this case in $\varphi_{23}(\mathbf{R}, \mathbf{r})$ an approximate form $\varphi_{\text{Cb,a}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} f_{23}(|\mathbf{k}|)/\sqrt{V}$ of the (unscreened) Coulomb solution $\varphi_{\text{Cb}}(\mathbf{r})$ may be used. Here $f_{23}(|\mathbf{k}|)$ is the appropriate factor given by (7) corresponding to particles 2 and 3. Thus $\varphi_{23}(\mathbf{R}, \mathbf{r}, \text{nucl})_{\text{sc}} = f_{23}(|\mathbf{k}|) e^{i\mathbf{KR}} e^{i\mathbf{k}\mathbf{r}}/V$ is used in the range of the nucleus and in the calculation of the nuclear matrix element. In the screened (outer) range, where Coulomb potential is negligible, the solution becomes $\varphi_{23}(\mathbf{R}, \mathbf{r}, \text{out})_{\text{sc}} = e^{i\mathbf{KR}} e^{i\mathbf{k}\mathbf{r}}/V$ that is used in the calculation of the Coulomb matrix element.

In the screened range the initial wave function of zero energy is $\varphi_i = V^{-3/2}$. The intermediate states of particles 2 and 3 are determined by the wave number vectors \mathbf{K} and \mathbf{k} . In the case of the assisting particle 1 the intermediate and final state is a plane wave of wave number vector \mathbf{k}_1 .

The matrix elements $V_{\text{Cb,vi}}$ of the screened Coulomb potential between the initial and intermediate states are

$$V_{\text{Cb}}(1, s)_{\text{vi}} = \frac{z_1 z_s}{2\pi^2} e^2 \frac{(2\pi)^9}{V^3} \delta(\mathbf{k}_1 + \mathbf{K}) \frac{\delta(\mathbf{k} + a(s)\mathbf{k}_1)}{\mathbf{k}_1^2 + q_{\text{sc,1s}}^2}, \quad (13)$$

where $a(s) = (-A_3\delta_{s,2} + A_2\delta_{s,3})/(A_2 + A_3)$ and $s = 2, 3$.

V. CHANGE OF THREE-PARTICLE WAVE FUNCTION IN NUCLEAR RANGE

According to standard time-independent perturbation theory of quantum mechanics [14] the first-order change of the wave function in the range $r \lesssim R_0$ (R_0 is the nuclear radius of particle 3) due to screened Coulomb perturbation is determined as

$$\delta\varphi(\mathbf{r}) = \sum_{s=2,3} \delta\varphi(s, \mathbf{r}) \quad (14)$$

with

$$\delta\varphi(s, \mathbf{r}) = \iint \frac{V_{\text{Cb}}(1, s)_{\text{vi}}}{E_v - E_i} \frac{V}{(2\pi)^6} \times e^{i(\mathbf{KR} + \mathbf{k}_1\mathbf{r}_1)} \varphi_{\text{Cb,a}}(\mathbf{k}, \mathbf{r}) d\mathbf{K} d\mathbf{k}, \quad (15)$$

where E_i and E_v are the kinetic energies in the initial and intermediate states, respectively. The initial momenta and kinetic energies of particles 1, 2, and 3 are zero ($E_i = 0$) and $E_v = E_{23} + \hbar^2 \mathbf{k}_1^2 / (2m_0 A_1)$. Thus

$$\delta\varphi(s, \mathbf{r}) = z_1 z_s \alpha_f \frac{4\pi \hbar c}{V^{5/2}} \frac{e^{i(\mathbf{k}_1\mathbf{r}_1 - \mathbf{k}\mathbf{r})}}{\mathbf{k}_1^2 + q_{\text{sc,1s}}^2} \times \frac{2m_0 a_{1s}}{\hbar^2 \mathbf{k}_1^2} [f_{23}(k) e^{i\mathbf{k}\mathbf{r}}]_{\mathbf{k}=a(s)\mathbf{k}_1}. \quad (16)$$

It can be seen that the arguments of $f_{23}(|\mathbf{k}|)$ are $|\mathbf{k}| = \frac{A_3}{A_2 + A_3} k_1$ and $|\mathbf{k}| = \frac{A_2}{A_2 + A_3} k_1$, here $k_1 = |\mathbf{k}_1|$. Consequently, if particle 1 obtains large kinetic energy, as is the case in nuclear reactions [e.g., $\mathbf{k}_1^2 = k_0^2 = 2m_0 a_{14} \Delta \hbar^{-2}$ in the case of reaction (8)], then the factors $f_{23}(|\mathbf{k}|)$ and the rate of the process too will be considerable. [In this case one can neglect $q_{\text{sc,jk}}^2$ in the denominator of (16)]. Since $\lim_{\varepsilon \rightarrow 0} |\delta\varphi(\mathbf{0})|^2 \neq 0$, i.e., it remains finite in the $\varepsilon \rightarrow 0$ limit, and the expected reaction rate too remains finite. Furthermore, $\delta\varphi(\mathbf{r})$, which causes the effect, is temperature independent. (Temperature dependence is brought in by mechanisms responsible for producing atomic states.) Up to this point the calculation and the results are nuclear reaction and nuclear model independent.

VI. CROSS SECTION

When calculating the cross section of reaction ${}_{Z_1}^{A_1}V + p + d \rightarrow {}_{Z_1}^{A_1}V' + {}_2^3\text{He} + 5.493 \text{ MeV}$ the Hamiltonian $V_{\text{st}}(2, 3) = -V_0$ if $|\mathbf{r}_{23}| = |\mathbf{r}| \leq b$ and $V_{\text{st}}(2, 3) = 0$ if $|\mathbf{r}_{23}| = |\mathbf{r}| > b$ of strong interaction, which is responsible for nuclear reaction between particles 2 and 3, is used. For the final state of the captured proton the Weisskopf approximation is applied, i.e., $\Phi_f(\mathbf{r}) = \Phi_{\text{fW}}(\mathbf{r})$ with $\Phi_{\text{fW}}(\mathbf{r}) = \sqrt{3/(4\pi R_0^3)}$ if $r \leq R_0$, and $\Phi_{\text{fW}}(\mathbf{r}) = 0$ for $r > R_0$, where R_0 is the nuclear radius. We take $V_0 = 25 \text{ MeV}$ and $R_0 = b = 2 \times 10^{-13} \text{ cm}$ [10] in the case of pd reaction.

The matrix element of the potential of the strong interaction between intermediate ($e^{i\mathbf{KR}} \varphi_{\text{Cb,a}}(\mathbf{k}, \mathbf{r})/\sqrt{V}$) and final ($e^{i\mathbf{k}_4\mathbf{r}} \Phi_f(\mathbf{r})/\sqrt{V}$) states and in the Weisskopf approximation is

$$V_{\text{st,fv}}^{\text{W}} = -V_0 \frac{\sqrt{12\pi R_0}}{k} f_{23}(k) H(k) \frac{(2\pi)^3}{V^{3/2}} \delta(\mathbf{K} - \mathbf{k}_4) \quad (17)$$

where $H(k) = \int_0^1 \sin(kR_0 x) x dx$. According to standard time-independent perturbation theory of quantum mechanics [14] the transition probability per unit time ($W_{\text{fi}}^{(2)}$) of the process can be written as

$$W_{\text{fi}}^{(2)} = \frac{2\pi}{\hbar} \iint |T_{\text{fi}}^{(2)}|^2 \delta(E_f - \Delta) \frac{V^2}{(2\pi)^6} d\mathbf{k}_1 d\mathbf{k}_4 \quad (18)$$

with

$$T_{\text{fi}}^{(2)} = \iint \sum_{s=2,3} \frac{V_{\text{st,fi}} V_{\text{Cb}}(1, s)_{vi}}{E_v - E_i} \frac{V^2}{(2\pi)^6} d\mathbf{K} d\mathbf{k}. \quad (19)$$

Substituting everything obtained above into (19) and (18), where E_f is the sum of kinetic energies of the final particles (1 and 4), one can calculate $W_{\text{fi}}^{(2)}$. The cross section $\sigma_{23}^{(2)}$ of the process is defined as $\sigma_{23}^{(2)} = N_1 W_{\text{fi}}^{(2)} / (v_{23}/V)$ where N_1 is the number of particles 1 in the normalization volume V and v_{23}/V is the flux of particle 2 of relative velocity v_{23} .

$$v_{23}\sigma_{23}^{(2)} = n_1 S_{\text{pd}}, \quad (20)$$

where $n_1 = N_1/V$ is the number density of particles 1 and

$$S_{\text{pd}} = 24\pi^2 \sqrt{2cR_0} \frac{z_1^2 \alpha_f^2 V_0^2 (\hbar c)^4}{\Delta^{9/2} (m_0 c^2)^{3/2}} \times \frac{(A_2 + A_3)^2}{a_{14}^{7/2}} [F(2) + F(3)]^2 \quad (21)$$

with

$$F(s) = \frac{z_s a_{1s}}{A_3 \delta_{s,2} + A_2 \delta_{s,3}} f_{23}[a(s)k_0] H[a(s)k_0], \quad (22)$$

$s = 2, 3$ and $k_0 = \hbar^{-1} \sqrt{2m_0 a_{14} \Delta}$.

In the case of reactions with two final fragments [see (9)] the nuclear matrix element can be derived from $S(\varepsilon)$ [see (1)], i.e., in long wavelength approximation from $S(0)$, which is the astrophysical S factor at $\varepsilon = 0$, in the following manner. Calculating the transition probability per unit time $W_{\text{fi}}^{(1)}$ of the usual (first-order) process in standard manner

$$W_{\text{fi}}^{(1)} = \int \frac{2\pi}{\hbar} |V_{\text{st,fi}}|^2 \delta(E_f - \Delta) \frac{V}{(2\pi)^3} d\mathbf{k}_f, \quad (23)$$

where \mathbf{k}_f is the relative wave number of the two fragments of rest masses $m_4 = m_0 A_4$, $m_5 = m_0 A_5$ and atomic numbers A_4 , A_5 , and $E_f = \hbar^2 \mathbf{k}_f^2 / (2m_0 a_{45})$ is the sum of their kinetic energy. For the magnitude of nuclear matrix element $V_{\text{st,fi}}$ we take the form $|V_{\text{st,fi}}| = f_{23}(k_i) |h_{\text{fi}}| / V$, where $f_{23}(k_i)$ is the Coulomb factor of the initial particles 2 and 3 with k_i the magnitude of their relative wave number vector \mathbf{k}_i . [The Coulomb factor $f_{45}(k_f) \approx 1$ of the final particles 4 and 5 with k_f the magnitude of their relative wave number vector \mathbf{k}_f .] It is supposed that $|h_{\text{fi}}|$ does not depend on \mathbf{k}_i and \mathbf{k}_f namely the long wavelength approximation is used. In this case the product of the relative velocity v_{23} of the initial particles 2, 3 and the cross section $\sigma_{23}^{(1)}$ is

$$v_{23}\sigma_{23}^{(1)} = \frac{|h_{\text{fi}}|^2 f_{23}^2(k_i) (m_0 a_{45})^{3/2} \sqrt{2\Delta}}{\pi \hbar^4}. \quad (24)$$

On the other hand, $v_{23}\sigma_{23}^{(1)}$ is expressed with the aid of (1) and $v_{23} = \sqrt{2\varepsilon / (m_0 a_{23})}$. From the equality of the two kinds of $v_{23}\sigma_{23}^{(1)}$ one gets

$$|h_{\text{fi}}|^2 = \frac{(\hbar c)^4 S(0)}{z_2 z_3 \alpha_f (m_0 c^2)^{5/2} \sqrt{2\Delta} a_{45}^{3/2} a_{23}}. \quad (25)$$

In the case of the impurity-assisted, second-order process $|V_{\text{st,fi}}| = f_{23}(k) |h_{\text{fi}}| (2\pi)^3 \delta(\mathbf{K} - \mathbf{K}_f) / V^2$ where \mathbf{K}_f and \mathbf{k}_f are

TABLE I. S_{reaction} and power density of Xe assisted reactions with two final fragments in long wavelength approximation. $S(0)$ is the astrophysical S factor at $\varepsilon = 0$ in MeVb [1, 16, 17]. S_{reaction} (in $\text{cm}^6 \text{s}^{-1}$) is calculated using (27) with (28) taking $z_1 = 54$ (Xe), Δ is the energy of the reaction in MeV and $p_{\text{reaction}} = n_1 n_2 n_3 S_{\text{reaction}} \Delta$ is the power density in Wcm^{-3} that is calculated with $n_1 = n_2 = n_3 = 2.65 \times 10^{20} \text{ cm}^{-3}$. In the case of ${}^9\text{Be}(\alpha, n){}_{6}^{12}\text{C}$ and ${}^{10}\text{B}(p, \alpha){}_{4}^7\text{Be}$ reactions the astrophysical S factor $[S(\varepsilon)]$ has strong energy dependence therefore the calculation was carried out with two characteristic values of $S(\varepsilon)$.

Reaction	$S(0)$	S_{reaction}	Δ	p_{reaction}
$d(d, n){}_2^3\text{He}$	0.055	1.01×10^{-48}	3.269	9.82
$d(d, p)t$	0.0571	1.10×10^{-48}	4.033	13.2
$d(t, n){}_2^4\text{He}$	11.7	1.06×10^{-46}	17.59	5.57×10^3
${}_3^6\text{He}(d, p){}_2^4\text{He}$	5.9	1.51×10^{-48}	18.25	82.6
${}_3^6\text{Li}(p, \alpha){}_2^3\text{He}$	2.97	1.99×10^{-49}	4.019	2.38
${}_3^6\text{Li}(d, \alpha){}_2^4\text{He}$	16.9	1.33×10^{-49}	22.372	8.84
${}_3^7\text{Li}(p, \alpha){}_2^4\text{He}$	0.0594	3.85×10^{-51}	17.347	0.199
${}_4^9\text{Be}(p, \alpha){}_3^6\text{Li}$	17	1.79×10^{-49}	2.126	1.13
${}_4^9\text{Be}(p, d){}_4^8\text{Be}$	17	1.66×10^{-49}	0.56	0.277
${}_4^9\text{Be}(\alpha, n){}_6^{12}\text{C}$	2.5×10^3	6.22×10^{-51}	5.701	0.106
	6×10^5	1.49×10^{-48}	5.701	25.4
${}_{5}^{10}\text{B}(p, \alpha){}_4^7\text{Be}$	4	1.04×10^{-50}	1.145	0.0356
	2×10^3	5.21×10^{-48}	1.145	17.8
${}_{5}^{11}\text{B}(p, \alpha){}_4^8\text{Be}$	187	5.16×10^{-49}	8.59	13.2

the final wave number vectors attached to CM and relative motions of the two final fragments, particles 4 and 5. \mathbf{k}_f appears in E_f in the energy Dirac δ . Repeating the calculation of the transition probability per unit time of the impurity assisted, second-order process applying the above expression of $|V_{\text{st,fi}}|$ one gets

$$v_{23}\sigma_{23}^{(2)} = n_1 S_{\text{reaction}}, \quad (26)$$

where $\sigma_{23}^{(2)}$ is the cross section of the process and

$$S_{\text{reaction}} = \frac{8\alpha_f^2 z_1^2 S(0)c}{a_{23}^3 a_{123}^3 m_0 c^2} \left(\frac{\hbar c}{\Delta} \right)^3 I \quad (27)$$

with

$$I = \int_0^1 \left(\sum_{s=2,3} \frac{z_s a_{1s} \sqrt{A_s}}{\sqrt{e^{b_{23} A_s} \frac{1}{x} - 1}}} \right)^2 \frac{\sqrt{1-x^2}}{x^7} dx. \quad (28)$$

Here $b_{23} = 2\pi z_2 z_3 \alpha_f \sqrt{m_0 c^2 / (2a_{123} \Delta)}$ with $a_{123} = A_1(A_2 + A_3) / (A_1 + A_2 + A_3)$. In the index reaction the reaction resulting the two fragments will be marked (see Table I).

It is plausible to extend the investigation to the atomic gas-solid (e.g., wall) interaction. In this case the role of particle 1 is played by one atom of the solid (metal), which is supposed to be formed from atoms with nuclei of charge and mass numbers z_1 and A_1 . For initial state a Bloch function of the form

$$\varphi_{\mathbf{k}_1, i}(\mathbf{r}_1) = N_1^{-1/2} \sum_{\mathbf{L}} e^{i\mathbf{k}_1 \cdot \mathbf{L}} a(\mathbf{r}_1 - \mathbf{L}) \quad (29)$$

is taken, that is localized around all of the lattice points [15]. Here \mathbf{r}_1 is the coordinate, $\mathbf{k}_{1,i}$ is wave number vector of the first Brillouin zone (BZ) of the reciprocal lattice, $a(\mathbf{r}_1 - \mathbf{L})$ is the Wannier function, which is independent of $\mathbf{k}_{1,i}$ within the BZ and is well localized around lattice site \mathbf{L} . N_1 is the number of lattice points of the lattice of particles 1. Repeating the cross section calculation applying Bloch function it is obtained that cross-section results remain unchanged and $n_1 = N_{1c}/v_c$, where v_c is the volume of elementary cell of the solid and N_{1c} is the number of particles 1 in the elementary cell.

A. Numerical values of cross sections

The cross section $\sigma_{23}^{(2)}$ of the process ${}^{A_1}V + p + d \rightarrow {}^{A_1}V' + {}^3\text{He} + 5.493 \text{ MeV}$ is $\sigma_{23}^{(2)} = n_1 S_{pd}/v_{23}$, where $S_{pd} = 1.89 \times 10^{-53} z_1^2 \text{ cm}^6 \text{ s}^{-1}$ with z_1 the charge number of the assisting nucleus. $\sigma_{23}^{(2)}$, similarly to thermal neutron capture cross sections, has $1/v_{23}$ dependence. In case of 0.1 eV initial kinetic energy ($T = 1160 \text{ K}$ if $kT = 0.1 \text{ eV}$) and with $z_1 = 54$ (Xe) $\sigma_{23}^{(2)} = n_1 \times 2.5 \times 10^{-31} \text{ b}$ from which $\sigma_{23}^{(2)} = 0.0066 \text{ nb}$ at $n_1 = 2.65 \times 10^{19} \text{ cm}^{-3}$ (which equals the number density of an atomic gas in normal state). This value of $\sigma_{23}^{(2)}$ is 10–15 orders of magnitude less than the thermal neutron capture cross sections.

In anomalous electron screening investigations accelerator of low-energy beams, e.g., in case of [2] an accelerator line powered by a highly stabilized 60-kV supply is applied. The targets are deuterium implanted metals. Since our model is valid if the magnitude of initial kinetic energies of particles $j = 1-3$ are negligible compared to the reaction energy Δ , it can be applied. In this case in our model the role of particle 1 is played by one atom of the solid (metal). We focus on the $d(d,t)p$ reaction investigated in Ref. [2] and we compare the cross section of the assisted, second-order process $\sigma_{23}^{(2)}$ to the cross section $\sigma_{23}^{(1)}$ of the usual reaction. We take Pd as host metal. $v_c(\text{Pd}) = d^3/4$ since Pd has *fcc* crystal structure and $N_{1c} = 2$ resulting $n_1 = N_{1c}/v_c = 1.36 \times 10^{23} \text{ cm}^{-3}$ [$d(\text{Pd}) = 3.89 \times 10^{-8} \text{ cm}$]. We have calculated $S_{d(d,t)p}(\text{Pd})$ taking $z_1 = 46$ and $A_1 = 106$ producing $S_{d(d,t)p}(\text{Pd}) = 7.9 \times 10^{-49} \text{ cm}^6 \text{ s}^{-1}$ and $n_1 S_{d(d,t)p}(\text{Pd}) = 1.08 \times 10^{-25} \text{ cm}^3 \text{ s}^{-1}$. Taking $v_{23} = c\sqrt{2\varepsilon/(m_0c^2)}$, $S(0) = 0.0571 \text{ MeVb}$ (see Table I) and $2\pi\eta_{23} = 2\pi\alpha_f\sqrt{m_0c^2/(2\varepsilon)} = 0.990/\sqrt{\varepsilon(\text{in MeV})}$ one obtains $\sigma_{23}^{(2)} = n_1 S_{d(d,t)p}/v_{23} = 7.77 \times 10^{-11}/\sqrt{\varepsilon(\text{in MeV})} \text{ b}$ and $\sigma_{23}^{(1)} = 0.0571 \exp(-0.990/\sqrt{\varepsilon(\text{in MeV})})/\varepsilon(\text{in MeV}) \text{ b}$ from (1). If $\sigma_{23}^{(2)} > \sigma_{23}^{(1)}$ then the second-order process dominates, i.e., if $7.35 \times 10^8 \exp(-0.990/\sqrt{\varepsilon})/\sqrt{\varepsilon} < 1$, which is the case if $\varepsilon < 0.001762 \text{ MeV}$. Consequently the anomalous screening phenomenon may be connected to the processes discussed here. Moreover the experimental difficulties, which accompanied anomalous screening investigations indicate that the phenomenon discussed by us is difficult to observe and examine, and partially answers the question why it was not observed up till now.

B. Experimental proposal

The ground of the method, which seems to be capable to show and to investigate in detail the phenomenon may be the

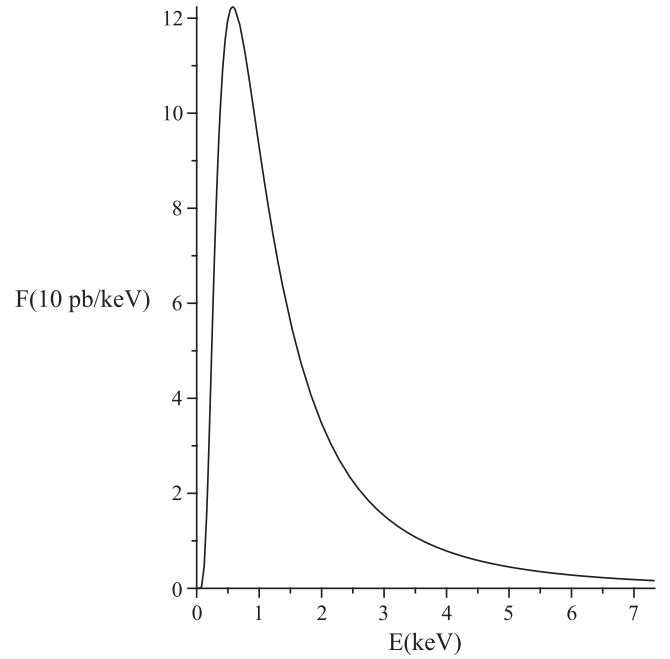


FIG. 1. The energy (E) dependence of the differential cross section $d\sigma_{23}^{(2)}/(dEd\Omega) = F(E)$ of $d(d,t)p$ reaction in the case of deuterized Pd. F is given in 10 pb/keV units. E is the energy of the assisting particle 1 (in this case Pd) in keV units. The accelerator potential $U = 1 \text{ keV}$. The possible maximum value of E is 146.65 keV.

measurement of the assisting particle and one from the two reaction products of, e.g., metal assisted $d(d,t)p$ reaction in coincidence.

For this it is useful to determine the differential cross section

$$\frac{d\sigma_{23}^{(2)}}{dEd\Omega} = F(E) = \frac{n_1}{v_{23}} \frac{A_1 \alpha_f^2 z_1^2}{\pi a_{23} a_{123}^4} \frac{S(0)c}{m_0 c^2 \Delta} \left(\frac{\hbar c}{\Delta} \right)^3 \chi[x(E)], \quad (30)$$

where $x(E) = k_1/k_m = \sqrt{A_1 E/(a_{123} \Delta)}$ with $k_m = \sqrt{2m_0 c^2 a_{123} \Delta/(\hbar c)}$, $k_1 = |\mathbf{k}_1|$, E and \mathbf{k}_1 are the energy and wave vector of particle 1, and

$$\chi(x) = \left(\sum_{s=2,3} \frac{z_s a_{1s} \sqrt{A_s}}{\sqrt{e^{b_{23} A_s \frac{1}{x}} - 1}} \right)^2 \frac{\sqrt{1-x^2}}{x^8}. \quad (31)$$

Figure 1 shows the E dependence of the differential cross section $d\sigma_{23}^{(2)}/(dEd\Omega) = F(E)$. If Φ_2 is the incident flux of particles 2 then $dN_1/dt = N_3 F(E) \Phi_2 \delta E \delta \Omega$ is the rate of particles 1 of energy in the energy interval $E \pm \delta E/2$ emitted in solid angle $\delta \Omega$ around the direction determined by \mathbf{k}_1 . N_3 is the total number of particles 3 irradiated by the beam of flux Φ_2 . It can be seen from Fig. 1 that particles 1 have kinetic energy mostly below 7 keV. Thus the wave vectors (\mathbf{k}_4 and \mathbf{k}_5) of the other two final particles 4 and 5 have approximately opposite direction. Their kinetic energies (E_4 and E_5) are peaked around $3\Delta/4$ and $\Delta/4$.

The accelerating electric potential U seems to be worth decreasing below 1 keV since $d\sigma_{23}^{(2)}/(dEd\Omega) \sim 1/v_{23} \sim 1/\sqrt{U}$. Furthermore decreasing U admits higher accelerator

current compared to the maximum of possible current of low-energy accelerators used in anomalous screening experiments [2]. However, decreasing U results decreasing penetration depth of the beam leading to decreasing interaction volume so that the optimal value of U needs further study.

VII. RATE AND POWER DENSITIES

The rate in volume V is

$$\frac{dN_{\text{reaction}}}{dt} = N_3 \Phi_{23} \sigma_{23}^{(2)}, \quad (32)$$

where $\Phi_{23} = n_2 v_{23}$ is the flux of particles 2 with $n_2 = N_2/V$ their number density. N_2 and N_3 are the numbers of particles 2 and 3 in the normalization volume. The rate and power densities are defined as

$$r_{\text{reaction}} = \frac{1}{V} \frac{dN_{\text{reaction}}}{dt} = n_3 n_2 n_1 S_{\text{reaction}} \quad (33)$$

and

$$p_{\text{reaction}} = r_{\text{reaction}} \Delta = n_1 n_2 n_3 S_{\text{reaction}} \Delta, \quad (34)$$

respectively, where $n_3 = N_3/V$ is the number density of particles 3. r_{reaction} and p_{reaction} are both temperature independent.

The rate (r_{pd}) and power (p_{pd}) densities of reaction ${}_{z_1}^{A_1}V + p + d \rightarrow {}_{z_1}^{A_1}V' + {}_2^3\text{He}$ are determined taking $z_1 = 54$ (Xe) and $n_1 n_2 n_3 = 1.86 \times 10^{61} \text{ cm}^{-9}$ (which is the case, e.g., at $n_1 = n_2 = n_3 = 2.65 \times 10^{20} \text{ cm}^{-3}$, n_1 , n_2 , and n_3 are the number densities of Xe, p , and d , i.e., particles 1, 2, and 3) for which considerable values are obtained: $r_{\text{pd}} = 1.02 \times 10^{12} \text{ cm}^{-3} \text{ s}^{-1}$ and $p_{\text{pd}} = 0.901 \text{ Wcm}^{-3}$. If the impurity is Hg or U then these numbers must be multiplied by 2.2 or 2.9, respectively. The results of S_{reaction} and power density calculations of a number of Xe-assisted reactions with two final fragments in long wavelength approximation and with $n_1 = n_2 = n_3 = 2.65 \times 10^{20} \text{ cm}^{-3}$ can be found in Table I.

To reach the order of magnitude 10^{61} cm^{-9} of $n_1 n_2 n_3$ is a great challenge. It may be done, e.g., with the aid of dissociative chemisorption at metal (e.g., Pd, Ni, and Cu) surfaces from two atomic molecules, e.g., H_2 , HD , or D_2 by heating molecular gas [13]. In this case $n_1 > 10^{22} \text{ cm}^{-3}$ is the number density of metal atoms in the solid and $n_1 n_2 n_3 = 10^{61} \text{ cm}^{-9}$ can be reached if $kT \sim 0.5\text{--}1 \text{ eV}$ producing $n_2 = n_3 > 5.3 \times 10^{19} \text{ cm}^{-3}$. It can be achieved in a two atomic gas in the 4–8 atm pressure, 600–1200 K temperature range, respectively, at the surface. In the case of powdered samples of small grain size or nanoparticles one may reach interaction volume large enough to be able to generate heat produced by power densities of some of nuclear reactions listed in Table I that is observable with the aid of precise calorimetric measurements.

Since in (8) and (9) the reaction energy is taken away by particles ${}_{z_1}^{A_1}V'$, ${}_{z_3+z_2}^{A_3+A_2}Y$ and ${}_{z_1}^{A_1}V'$, ${}_{z_4}^{A_4}Y$, ${}_{z_5}^{A_5}W$, respectively, as their kinetic energy that they lose in a very short range to their environment converting the reaction energy efficiently into heat if the state of matter of atomic state is dense, so their direct observation is difficult in this case. In the experimental conditions stated above the creation of new elements due to nuclear reactions, i.e., the presence of nuclear transmutation in the system may be a way to confirm our predictions experimentally.

VIII. CONCLUSION

It is found that any perturbation may lead to nonzero cross section and rate of nuclear reactions forbidden in the $\varepsilon \rightarrow 0$ limit. Since this statement applies to every nuclear process forbidden in the $\varepsilon \rightarrow 0$ limit it concerns low-energy nuclear physics with charged participants in general. Thus, it may be stated that a very great number of reactions, which are determined by different initial states, different perturbations and different processes of second and higher order and which may be attached to forbidden reactions, have not been investigated up till now.

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