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Exploring the production of new superheavy nuclei with proton and α-particle evaporation channels

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Excitation functions for fusion-xn evaporation reaction channels induced by ^{48}Ca as well as by heavier projectiles (usually leading to smaller cross sections) on actinide targets were calculated in the framework of the fusion-by-diffusion (FBD) model. For the first time, in this approach, channels in which a proton (pxn) or alpha particle (αxn) is evaporated have been included in the first step of the de-excitation cascade. To calculate the synthesis cross sections entry data such as fission barriers, ground-state masses, deformations, and shell effects of the superheavy nuclei calculated in a consistent way within the Warsaw macroscopic-microscopic model were used. The only adjustable parameter of the FBD model is the injection point distance s_{inj}. The systematics determined in our previous analysis of experimental cross sections for the synthesis of superheavy nuclei of $Z = 114$–118 has been used. Excitation functions for the synthesis of selected (cross section above a few fb) new superheavy nuclides in the range of atomic numbers 112–120 are presented. Observation of 21 new heaviest isotopes is also predicted. A realistic discussion of the FBD model uncertainties is presented for the first time.

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I. INTRODUCTION

The recently discovered elements have brought the existing periodic table to completion of the seventh row [1–11]. The question remains though: can we still synthesize new superheavy nuclides including elements heavier than oganesson. We know that superheavy elements are highly unstable systems with extremely low production cross sections. Existing experimental facilities limit the possibilities for discovery of new nuclides to those synthesized with cross sections above 100 fb, however the perspectives for future high current accelerators in which this limit could be lowered up to two orders of magnitude motivated us to show how new superheavy nuclei could be produced in hot fusion reactions. When successful, hot fusion creates a heavy compound nucleus in a highly excited state. Until now, in theoretical models, the de-excitation cascade was described assuming two competing processes, neutron evaporation and fission. However, it cannot be excluded that in the first stage of the de-excitation cascade proton or even alpha particle is evaporated. This possibility should be considered in addition as competing processes. The evaporation of the charge particle lowers the excitation energy of the daughter nucleus much more than the emission of a neutron. Therefore in the latter steps the emission of charged particles is not considered. Predictions of cross sections for (αxn) and (pxn) calculated within the fusion-by-diffusion (FBD) model are presented. At every stage of the de-excitation cascade the compound nucleus and the daughter nuclei must be resistant to nuclear fission. Therefore, fission barriers and ground and saddle point masses controlling the competition process are essential in the calculations. In Sec. II the basis of the FBD model and in Sec. III its application for the production of new superheavy nuclides are presented. As the uncertainties of the predicted cross sections are important when planning very difficult and expensive experiments, they are discussed in Sec. IV.

II. THE BASIS OF THE FBD MODEL

The FBD model was proposed by Świątecki *et al.* [12,13] as a simple tool to calculate cross sections and optimum bombarding energies for a class of reactions leading to the synthesis of superheavy nuclei. Here, we summarize the idea and main assumptions of this approach.

As in other theoretical models, in the FBD model the partial evaporation-residue cross section for the synthesis of superheavy nuclei, $\sigma_{ER}(l)$, is factorized as the product of the partial capture cross section $\sigma_{\text{cap}}(l) = \pi\lambdabar^2(2l+1)T(l)$, the fusion probability $P_{\text{fus}}(l)$, and the survival probability $P_{\text{surv}}(l)$:

$$\sigma_{ER} = \pi\lambdabar^2 \sum_{l=0}^{\infty}(2l+1)T(l)\cdot P_{\text{fus}}(l)\cdot P_{\text{surv}}(l). \tag{1}$$

Here, $\lambdabar$ is the wavelength, $\lambdabar^2 = \hbar^2/2\mu E_{\text{c.m.}}$, and μ is the reduced mass of the colliding system.

The key assumption which allows us to investigate the reaction mechanism in such a way is Bohr's hypothesis, which states that the whole reaction process is a Markov type stochastic process which means that there are no memory effects. This implies that the exit channel is completely independent of the intermediate stage leading to the compound nucleus as well as of the entrance channel. This hypothesis is justified by the different time scale of the particular reaction stages.

The capture transmission coefficients $T(l)$ in Eq. (1) are calculated in a simple sharp cut off approximation, where the upper limit $l_{\max}$ of full transmission, $T(l) = 1$, is determined by the capture cross sections. To calculate this, we assume that in very heavy systems overcoming the potential-energy barrier does not necessarily lead to fusion. In accordance with experimental results entrance channel barrier is not described by a single value. Neglecting structural effects, we assumed a Gaussian shape distribution characterized by two parameters, the mean barrier B_0 and the distribution width ω. Folding this distribution with the classical expression for fusion cross section leads to the formula for capture cross section:

$$\begin{aligned}\sigma_{\text{cap}} &= \pi R^2 \frac{\omega}{E_{\text{c.m.}}\sqrt{2\pi}}\{X\sqrt{\pi}[1+\text{erf}(X)] + \exp(-X^2)\}\\ &= \pi ƛ^2(2l_{\max}+1), \qquad (2)\end{aligned}$$

where

$$X = \frac{(E_{\text{c.m.}} - B_0)}{\omega\sqrt{2}}. \qquad (3)$$

The empirical systematics of the two parameters were obtained from analyzing precisely measured fusion excitation functions for about 50 heavy nuclear systems [14].

The second factor, the fusion probability $P_{\text{fus}}(l)$, is the probability that after reaching the capture configuration, the colliding system will eventually overcome the saddle point and fuse, avoiding reseparation. For very heavy and less asymmetric systems, $P_{\text{fus}}(l)$ is much smaller than 1 and thus is mainly responsible for the dramatically small cross sections for the production of superheavy nuclei. The fusion hindrance in these reactions is caused by the fact that for heaviest compound nuclei the saddle configuration is more compact than the configuration of the two initial nuclei at sticking. It is assumed in the FBD model that after sticking, a neck between the two nuclei grows rapidly at an approximately fixed mass asymmetry and constant length of the system [12,13] bringing the system to the "injection point" somewhere along the bottom of the asymmetric fission valley. To overcome the saddle point and fuse, the system must climb uphill from the injection point to the saddle in a process of thermal fluctuations in the shape degrees of freedom. It was shown in Ref. [12] by solving the Smoluchowski diffusion equation that the probability that a system injected on the outside of the saddle point at an energy H below the saddle will achieve fusion is

$$P_{\text{fus}} = \tfrac{1}{2}(1 - \text{erf}\sqrt{H/T}\,), \qquad (4)$$

where T is the temperature of the fusing system.

The energy threshold H opposing fusion in the diffusion process is calculated using simple algebraic expressions that approximate the potential energy surface [15]. The shape parametrization used to describe the interacting system was that of two spheres joined smoothly by a third quadratic surface. The corresponding values of the rotational energy at the injection point and at the saddle point were calculated assuming the rigid-body moments of inertia for the respective

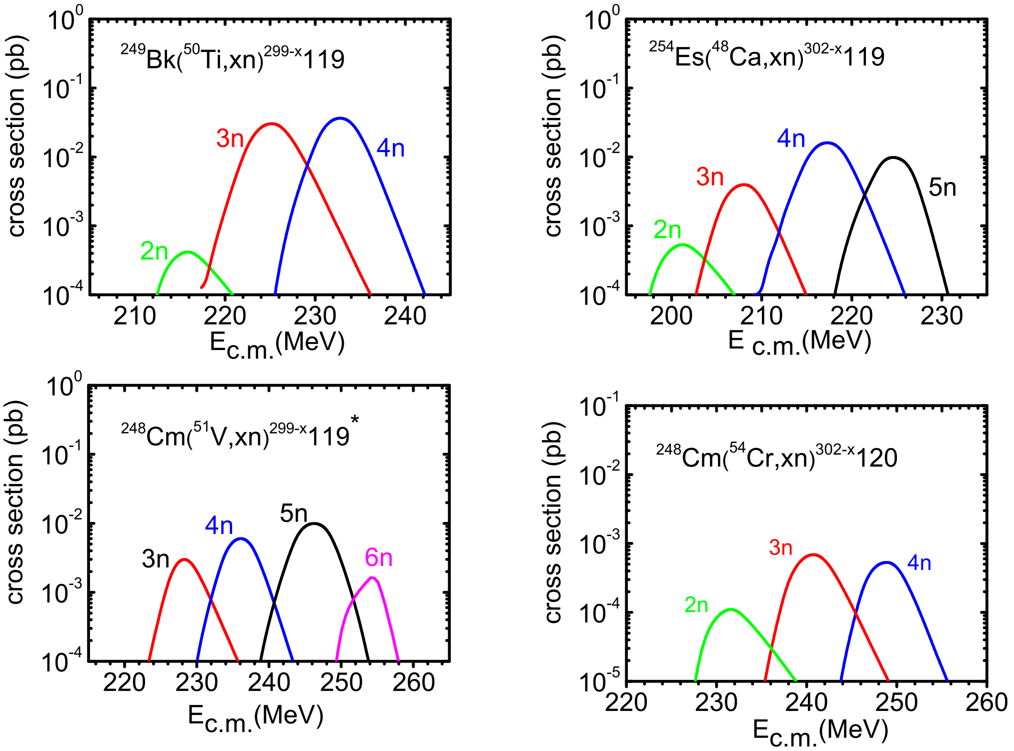

FIG. 1. Cross sections for the synthesis of superheavy nuclei of atomic number $Z = 119$ and 120 predicted in the fusion-by-diffusion (FBD) model with the fission barriers and ground-state masses of Kowal *et al.* [18,19] and the systematics of the injection-point distance given by Eq. 5 (see text).

shapes. The fact that H depends on the angular momentum causes that for higher partial waves, fusion is strongly reduced in comparison with central collisions (the rotational energy of the saddle point rises faster with angular momentum than the rotational energy at the injection point).

The last factor in Eq. (1), $P_{\text{surv}}(l)$, is the probability for the compound nucleus to decay to the ground state of the residual nucleus. On the first stage of the de-excitation the possibilities of light particles (neutrons, protons, or alpha particles) and fission are included. After the first particle evaporation the excitation energy of the daughter nucleus is so low that only neutron and fission channels will compete. The process ends by γ de-excitation and the final superheavy nucleus is synthesized. To calculate the survival probability P_{surv}, the standard statistical model was used by applying the Weisskopf formula for the particle emission width and the standard expression of the transition-state theory for the fission width. The level density parameters for the particle evaporation channels were calculated as proposed by Reisdorf [16] with shell effects accounted for by the Ignatyuk formula [17]. All details can be found in Ref. [15].

As follows from the above description, cross section calculations require knowledge of the individual characteristics of the synthesized compound nuclei and their decay products, all along the decay chain. The fission barriers, ground-state masses, deformations, and shell corrections of the superheavy nuclei predicted using the Warsaw macroscopic-microscopic model were used [18,19]. The assignment and recognition of magic numbers associated with increased stability is different in various theoretical models. What is important from the point of view of the fit protocol is to take consistent and coherent input data set from the one theoretical source.

The only adjustable parameter of the FBD model is the injection point distance, s_{inj}, defined as the excess of length of the deformed system at the injection point configuration over the sum of the target and projectile diameters. Its value was calculated from the systematics determined in our previous analysis of experimental cross sections for the synthesis of superheavy nuclei of $Z = 114$–118 [20]. As seen in Fig. 5 the systematics can be approximated by the straight line:

$$S_{inj} = 4.09\,\text{fm} - 0.192(E_{\text{c.m.}} - B_0)\,\text{fm/MeV}. \tag{5}$$

This parametrization is of course model dependent. Therefore, using this parametrization to predict synthesis cross sections requires the use of the same theoretical input data (fission barriers, ground-state masses, deformations, pairing, and shell corrections). This consistency is essential for the FBD calculations. It should be also mentioned that in this paper calculations are restricted only to hot fusion reactions (mostly ^{48}Ca induced reactions with $Z_1Z_2 < 2000$).

III. RESULTS

A. New elements

To synthesize new elements, $Z = 119$ and 120 in ^{48}Ca induced fusion-xn evaporation reactions, targets of Es or Fm are

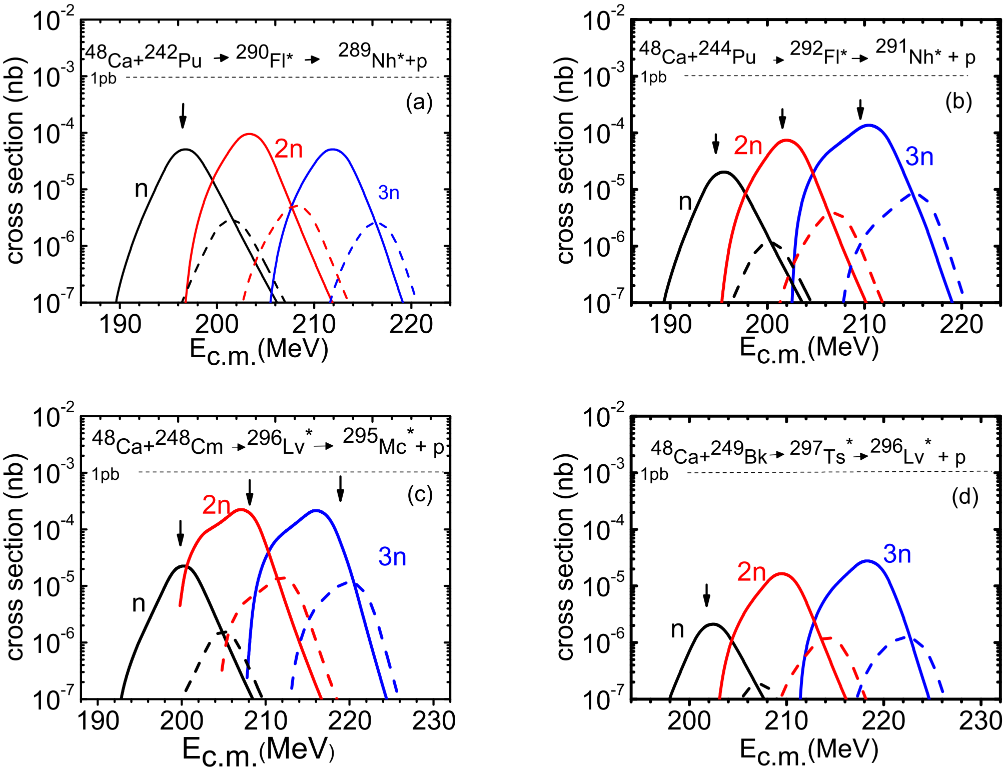

FIG. 2. Cross sections for the synthesis of superheavy nuclei in *pxn* fusion evaporation processes, predicted by the fusion-by-diffusion (FBD) model with the fission barriers and ground-state masses of Kowal *et al.* [18,19] and the systematics of the injection-point distance (see text). Solid lines - Coulomb barrier (Vp) parametrization for proton emission given by Eq. (6). Dashed lines - Vp increased by 4 MeV.

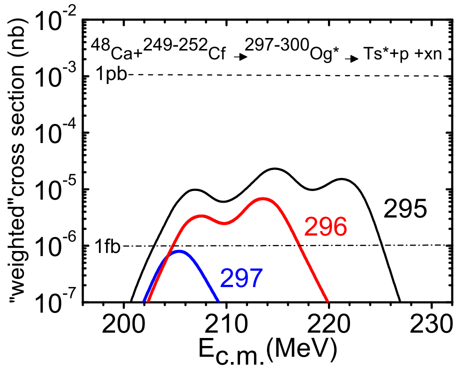

FIG. 3. The weighted cross section for the production of $^{295\text{-}297}$Ts on the mix californium target.

required, respectively. Since they are not currently available, reactions with heavier projectiles are also considered here. In Fig. 1 excitation functions for ^{50}Ti + ^{249}Bk, ^{48}Ca + ^{254}Es, ^{51}V + ^{248}Cm, and ^{54}Cr + ^{248}Cm are presented. Calculations for the above mentioned systems were also performed using other models, see, e.g., [21–27] and citations therein. These cross sections are at least one order of magnitude smaller than cross sections for the production of lighter superheavy elements. However, the perspectives of high current beams in planned new experimental facilities at RIKEN and DUBNA (SHE - FACTORY) give hope for success. An experiment with a ^{51}V beam is already under way at Riken.

B. New isotopes of known heaviest elements

With the perspectives of a higher beam current, one might expect that it will be feasible to synthesize heavier isotopes of already known superheavy elements. Most of these known elements were produced in the 3*n* or 4*n* fusion-evaporation channels. Although, the 2*n* evaporation channels have smaller cross sections they could lead to the synthesis of several new nuclei ^{290}Fl, ^{294}Lv, ^{295}Ts, ^{295}Og (see Ref. [20]).

As was mentioned in the introduction in addition to the (*xn*) fusion-evaporation processes, one could also consider the fusion process in which a proton or α particle is evaporated (in the first step of the compound nucleus de-excitation cascade). The excited nucleus of mass number A_{CN-1} and atomic number Z_{CN-1} or A_{CN-4}, Z_{CN-2}, respectively, could then decay by the *xn* cascade. Schematically, $P_{Zp,Ap} + T_{Zt,At} \rightarrow CN^*_{Z_{CN},A_{CN}} \rightarrow ER_{Z_{CN-1},A_{CN-1-x}} + p + xn$, $P_{Zp,Ap} + T_{Zt,At} \rightarrow CN^*_{Z_{CN},A_{CN}} \rightarrow ER_{Z_{CN-2},A_{CN-4-x}} + \alpha + xn$ where, P is the projectile, T is the target, CN^* is the excited compound nucleus, and ER is the evaporation residue.

To be able to predict cross sections for the above-mentioned processes, in addition to the entry data used in calculation of P_{surv} in the *xn* processes, one needs to know the value of the Coulomb barrier between the evaporated charged particle and the heavy nucleus with atomic number $Z = Z_{CN-1}$ or $Z = Z_{CN-2}$. In our calculations we have used the Coulomb barrier parametrization for protons and α particles proposed by Parker *et al.* [28]:

$$V_p = (0.106 Z_{CN-1} - 0.9)\,\mathrm{MeV} \qquad (6)$$

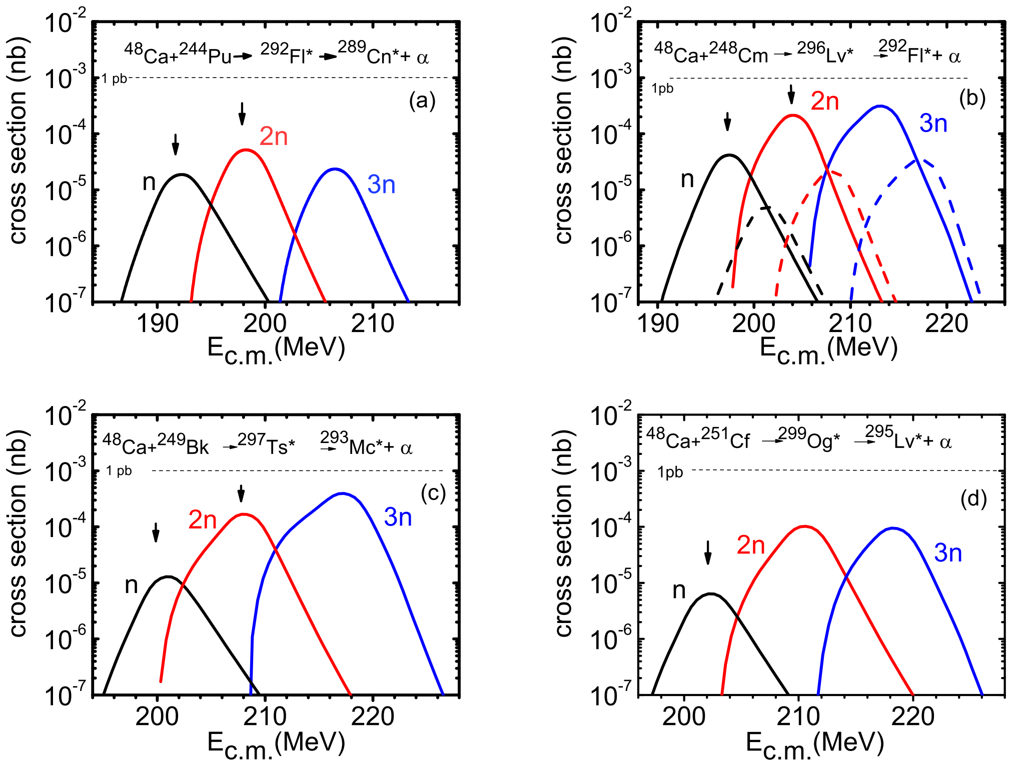

FIG. 4. Cross sections for the synthesis of superheavy nuclei in αxn fusion evaporation processes, predicted by the fusion-by-diffusion (FBD) model with the fission barriers and ground-state masses of Kowal *et al.* [18,19] and the systematics of the injection-point distance (see text). Solid lines - Coulomb barrier (Vα) parametrization for alpha particle emission given by Eq. (7). Dashed lines - Vα increased by 4 MeV.

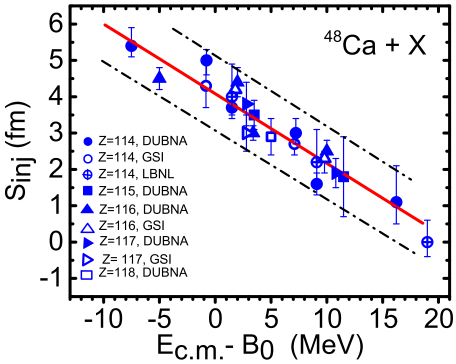

FIG. 5. Systematics of the s_{inj} parameter as a function of the kinetic energy excess $E_{\mathrm{c.m.}} - B_0$ above the mean barrier B_0. Solid line—approximation to experimental data, see Ref. [20]). Dashed dot lines—error corridor.

and

$$V_\alpha = \frac{2.88 Z_{CN-2}}{1.47\sqrt[3]{A_{CN-4}} + 4.642}\ \mathrm{MeV}. \quad (7)$$

Calculations were performed for all ^{48}Ca induced reactions used to produce superheavy nuclei with atomic numbers Z between 113 and 118. Excitation functions for reactions where new isotopes of known elements could be produced in pxn (^{242}Pu(^{48}Ca, pxn)$^{289-x}$Nh, ^{244}Pu(^{48}Ca, pxn)$^{291-x}$Nh, ^{248}Cm(^{48}Ca, pxn)$^{295-x}$Mc, ^{249}Bk(^{48}Ca, pxn)$^{296-x}$Lv) are presented in Fig. 2. The Fig. 3 corresponds to reactions on a mixed californium target $^{249-252}$Cf(^{48}Ca, pxn)$^{295-297}$Ts (for predictions for the synthesis of new isotopes of Og by the xn evaporation process see Ref. [29]). During the experiment, which is planned at Dubna with a new mixed californium target [30] in addition to synthesizing new Og isotopes it may also be feasible to look for new isotopes of tennesin. The cross section for synthesis of tennesin 295 in our predictions is about 25 fb and for 296 about 7 fb. Results for the αxn ($^{48}\mathrm{Ca} + {}^{244}\mathrm{Pu} \rightarrow {}^{288-x}\mathrm{Cn} + \alpha + xn$, $^{48}\mathrm{Ca} + {}^{249}\mathrm{Bk} \rightarrow {}^{293-x}\mathrm{Mc} + \alpha + xn$, $^{48}\mathrm{Ca} + {}^{248}\mathrm{Cm} \rightarrow {}^{292-x}\mathrm{Fl} + \alpha + xn$, and $^{48}\mathrm{Ca} + {}^{251}\mathrm{Cf} \rightarrow {}^{295-x}\mathrm{Lv} + \alpha + xn$) reactions are shown in Fig. 4. To illustrate the influence of the Coulomb barrier on the values of the cross sections, calculations were also made, for selected reactions with the Coulomb barriers increased by 4 MeV (shown as dashed lines in Figs. 2 and 4). This increase resulted in a shift of the maximum of the excitation functions to higher energies and a decrease of the cross section by at least one order of magnitude. The black arrows indicate those reaction channels which lead to the formation of undiscovered new isotopes. Although the value of the Coulomb barrier is not known exactly, the maximum of the synthesis cross sections is in most cases above 10 fb. Therefore, it should be possible to discover ten new isotopes in pxn fusion-evaporation reaction channels ($^{287\text{–}290}$Nh, $^{291\text{–}294}$Mc, and ^{295,296}Ts), and seven in αxn channels (^{286,287}Cn, ^{290,291}Fl, ^{291,292}Mc, and ^{294}Lv).

IV. UNCERTAINTIES

Different theoretical models give predictions of cross sections that may differ by a few orders of magnitude for the same fusion-evaporation reaction. Therefore, it is very important to estimate the uncertainties of the present calculations. As pointed out in the description of Eq. (1), the synthesis cross section consists of three factors: the partial capture cross section $\sigma_{\mathrm{cap}}(l) = \pi\lambdabar^2(2l+1)T(l)$, the fusion probability $P_{\mathrm{fus}}(l)$, and the survival probability $P_{\mathrm{surv}}(l)$. Each factor is calculated within some uncertainties. In our approach, the capture cross section should not change significantly from one system to another. The resulting uncertainties should not be large unless deeply sub-barrier reactions are studied. The fusion probability depends on the asymmetry of the colliding system and the entrance channel energy. Predictions may result in large uncertainties of even several orders of magnitude for the unexplored region of heavy systems. The survival probability is very sensitive to the value of the fission barrier (a 1 MeV difference in the fission barrier height may result in a one

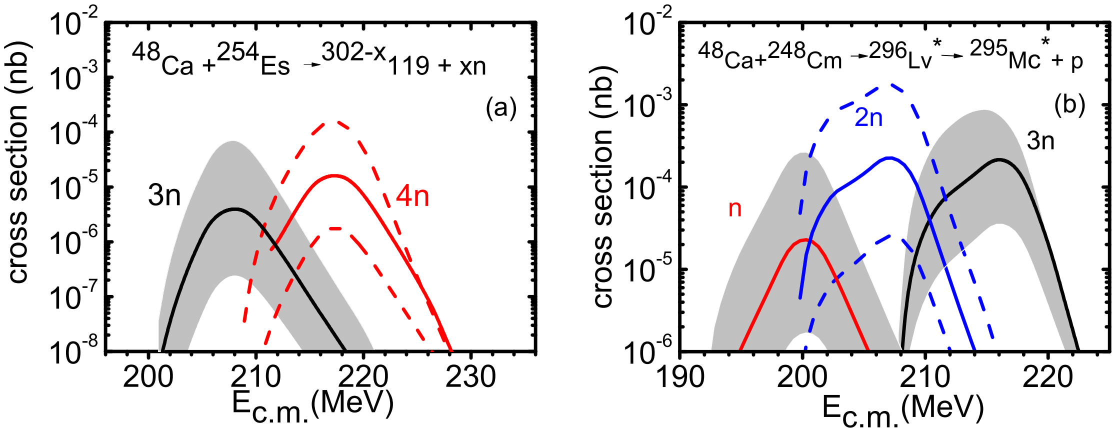

FIG. 6. Excitation functions for the synthesis of superheavy nuclei in the ^{254}Es(^{48}Ca, xn)$^{302-x}$119 and ^{248}Cm(^{48}Ca, pxn)$^{295-x}$Mc fusion evaporation processes. Solid lines correspond to calculations performed with the straight line approximation of the s_{inj}. Uncertainties are defined by the dashed lines or shaded areas.

order of magnitude difference in the value of the cross section at each step of the de-excitation cascade). Therefore, it is very important to do systematic calculations using the same entry data and compare to already measured excitation functions. In our approach there is one free parameter, s_{inj}. The systematics of s_{inj} as a function of the kinetic energy excess $E_{\mathrm{c.m.}} - B_0$ above the mean barrier B_0, was studied using all available experimental data for ^{48}Ca induced reactions. As shown in Fig. 5 this parameter can be approximated by a straight line [20]. Deviations from this line incorporate all uncertainties. The error corridor shown by the dashed lines (see Fig. 5) should allow the accuracy of our predictions to be estimated. As an example, two ^{48}Ca induced reactions are presented in Fig. 6. Solid lines correspond to calculations performed with the straight line approximation of the s_{inj}. Uncertainties are defined by the dashed lines or shaded areas. Calculations were made for all studied systems. The conclusion, based on this study, is that in our approach the uncertainties of the predicted cross sections for ^{48}Ca induced reactions on actinide targets are no better than one order of magnitude. Calculations of the *pxn* and *αxn* processes in ^{48}Ca induced reactions on actinide targets were also performed by Hong *et al.* in Ref. [31]. Our predictions agree with Hong's predictions within one order of magnitude in most cases, although the model and entry data used in the calculations are different.

V. CONCLUSIONS

The FBD model with fission barriers and ground state masses calculated within the Warsaw macroscopic-microscopic model was applied to predict synthesis cross sections of superheavy nuclei in fusion-evaporation *xn*, *pxn*, and *αxn* processes. Anticipating the use of high current accelerators and more effective experimental setups, calculations of the excitation functions for the synthesis of new superheavy nuclei in the atomic number range $Z = 112$–120 were presented. Calculations predict the possibility of observing 21 new heaviest nuclei with cross sections above 10 fb, among them two new elements ^{295,296}119 and ^{296,297}120. The accuracy of the predicted cross sections was discussed.

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