Coexistence and mixing in ^{76}Se

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(Received 5 February 2019; revised manuscript received 7 April 2019; published 17 May 2019)

A simple two-state mixing model has been applied to the 0^+, 2^+, and 4^+ states in the first two 0^+ bands in ^{76}Se. The first two 0^+ states are found to be almost maximally mixed. The excited basis-state band has the energy and E2 strengths characteristic of a $K = 0$ rotational band, but the lower basis-state band does not. The e band is significantly more collective than the g band.

DOI: 10.1103/PhysRevC.99.054320

I. INTRODUCTION

The low-lying states of ^{76}Se have long been thought to be an example of shape coexistence. The 0^+ ground-state (g.s.) band and a presumed $K = 2$ band are well known [1]. With γ-γ coincidence measurements following β decay of ^{76}As, McMillan and Pate [2] proposed 14 excited states below 3 MeV. Wells *et al.* [3] used the ^{71}Ga(^{7}Li, $2n$) reaction to investigate ^{76}Se and reported evidence for a strongly excited quasirotational band. With the ^{74}Ge($\alpha, 2n\gamma$)^{76}Se reaction, Matsuzaki and Taketani [4] identified the first 0^+ and 2^+ bands. Barrette *et al.* [5] used Coulomb excitation to populate low-lying levels. In proton inelastic scattering, Delaroche *et al.* [6] found evidence for different deformations for the first and second 2^+ states. Kavka *et al.* [7] used Coulomb excitation with ^{16}O, ^{48}Ti, and ^{208}Pb ions and concluded that ^{76}Se exhibits features characteristic of anharmonic vibrational collective motion, but that "the simplest collective vibrational models do not reproduce the fine details." Xu *et al.* [8] used the ^{70}Zn(^{12}C, $\alpha 2n$)^{76}Se fusion evaporation reaction to populate the yrast band of ^{76}Se. They concluded that a shape transition from prolate to oblate occurs along the yrast line.

Much of the interest in the structure of ^{76}Se involves the need for nuclear matrix elements for the calculation of neutrino-less double beta decay [9]. Rodríguez [10] studied ^{76}Se with calculations involving configuration mixing methods based on the Gogny D1S interaction. He stated that "the ratios between different reduced electromagnetic transition probabilities are consistent with a rigid-γ deformed nucleus." He used the generator coordinate method (GCM), and calculated intrinsic states with the variation after particle number projection (PN-VAP) method. Much of his work was for axial symmetry, but he also considered triaxiality. He found that calculated $B(E2)$'s for $2_1 \rightarrow 0_1$, $2_2 \rightarrow 0_1$, $2_2 \rightarrow 2_1$, and $4_1 \rightarrow 2_1$ are in better agreement with experimental values for axial than for triaxial. The triaxial transition probabilities are systematically larger than the experimental ones. He pointed out that the closeness of the 0_2, 4_1, and 2_2 states in ^{76}Se could resemble a two-quadrupole phonon structure, but that the form of the 0_2 wave function (and those of other states belonging to the same band) does not support this interpretation. He stated that the 0_2 state also cannot be interpreted as a β vibration, which is to be associated with the 0_3 state at larger excitation energies. He found that shape mixing is small and negligible for J different from zero. The ground- state band he obtained with axial calculations had an oblate character with a peak at $\beta_2 \sim -0.25$ and the first-excited band was prolate with $\beta_2 \sim +0.40$. He concluded that ^{76}Se is "triaxial deformed and the axial approximation only captures part of such a triaxial structure by mixing prolate and oblate configurations."

Recently, Gupta and Hamilton [11] pointed out the "limitation in expressing the shape of a nucleus in terms of the hydro-dynamics, viz the spherical or anharmonic vibrator, or soft triaxial rotor, etc."

In other theoretical work, Bhat *et al.* [12] used the triaxial projected shell model for Ge and Se isotopes, with most of the attention to ^{76}Ge. They concluded that "the distinct γ -soft feature in these nuclei is shown to result from configuration mixing of the ground state with multi-quasiparticle states." Calculations with a collective Hamiltonian [13] were mostly for $^{72-82}$Ge. Barfield *et al.* [14] used the proton-neutron interacting-boson model (IBM-2) to extract effective neutron- and proton-boson charges from experimental E2 matrix elements for nine light Se, Kr, and Sr isotopes. They stated that the 0_2 level is outside of their model space. Deloncle *et al.* [15] used a Bohr quadrupole collective Hamiltonian to investigate the low-lying collective spectra of four transitional nuclei ^{74}Ge, ^{76}Se, ^{110}Cd, and ^{186}Pt, but they did not consider $E2$ strengths.

In a recent ^{76}Se(n,n'γ) experiment [16], 2^+ and 4^+ members of a band based on the second 0^+ state at 1122 keV were identified. Energies of members of the first three bands are listed in Table I and plotted versus $J(J + 1)$ in Fig. 1. These identifications leave the third 4^+ state at 2485 keV unassigned. It could presumably be the head of a $K = 4$ band.

Reference [16] measured several lifetimes and determined $E2$ transition strengths. These, and ones from nuclear data sheets [1], are listed in Table II, along with values of $M(E2)$, where $M(E2) = [(2J_i + 1)B(E2; i \rightarrow f)]^{1/2}$. Reference [16] compared experimental results with IBM and shell-model calculations and found some agreement. The aim here is to subject these transition matrix elements to a simple two-state mixing analysis [17,18] for the two 0^+ bands. The other band in Fig. 1 has $K = 2$ and is therefore unlikely to mix

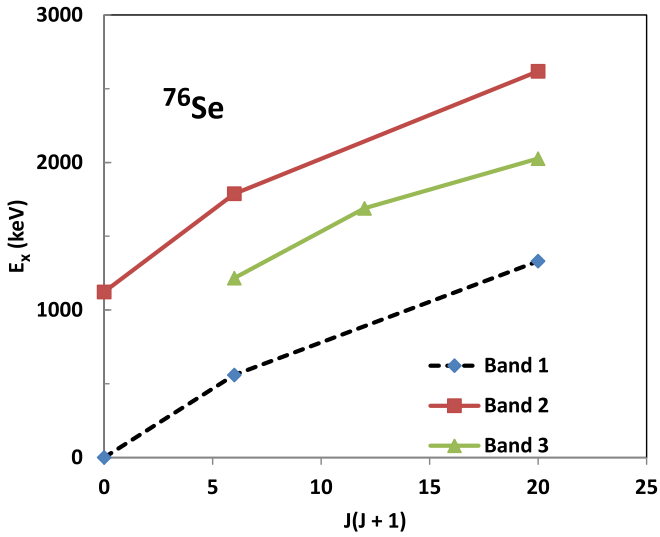

FIG. 1. Energies of first three bands in ^{76}Se [1,16].

significantly with the $K = 0$ bands. Evidence that such mixing is very small is the weakness of the transition from the second 2^+ to the g.s.—$B(E2) = 1.2(1)$ W.u., compared with 44(1) and 37(10) for 2_1 to 0_1 and 2_3 to 0_2, respectively [16].

II. ANALYSIS AND RESULTS

I denote 0^+ basis states as 0_g and 0_e, 2^+ basis states as 2_g and 2_e, and 4^+ basis states as 4_g and 4_e, where the subscripts g and e refer to ground and excited bands, respectively. I write

$$\text{g.s.} = a0_g + b0_e, \quad 0^+{}_2 = -b0_g + a0_e;$$

$$2^+{}_1 = A2_g + B2_e, \quad 2^+{}_3 = -B2_g + A2_e,$$

$$4^+{}_1 = C4_g + D4_e, \quad 4^+{}_4 = -D4_g + C4_e$$

I define $M_g = \langle 0_g \| E2 \| 2_g \rangle$, $M_e = \langle 0_e \| E2 \| 2_e \rangle$, $M'_g = \langle 4_g \| E2 \| 2_g \rangle$, $M'_e = \langle 4_e \| E2 \| 2_e \rangle$, and I assume there is no strength connecting g to e. No other assumptions are needed about the nature of the basis states. Such information emerges from the fit. Then, we have four unknowns, viz, two mixing amplitudes and two basis-state $E2$ matrix elements, and four experimental numbers to fit, with labels 0 to 3 in Table II.

In my phase convention, M_0 and M_3 must be positive, but M_1 and M_2 can have either sign, because they involve destructive interference. If only $B(E2)$'s and not the M's are known, a sign ambiguity will exist, and both possibilities of sign must be investigated. However, if M_e is significantly larger than M_g, all four matrix elements will be positive.

Whenever all four of the relevant matrix elements are available, the four parameters (0^+ mixing, 2^+ mixing, M_g and M_e) can be uniquely determined. Results of the procedure for $0 \leftrightarrow 2$ transitions in ^{76}Se are listed in Table III. The fact that the two 0^+ states are approximately maximally mixed

TABLE I. Energies (keV) of members of first three bands in ^{76}Se [1,16].

Band 1		Band 2		Band 3	
$J^\pi{}_n$	E_x	$J^\pi{}_n$	E_x	$J^\pi{}_n$	E_x
$0^+{}_1$	0	$0^+{}_2$	1122	$2^+{}_2$	1216
$2^+{}_1$	559	$2^+{}_3$	1788	$3^+{}_1$	1689
$4^+{}_1$	1331	$4^+{}_4$	2618	$4^+{}_2$	2026

TABLE II. E2 strengths of $0 \leftrightarrow 2$ and $2 \leftrightarrow 4$ transitions in ^{76}Se.

Label	Initial	Final	$B(E2)$ (W.u.)[a]	$M(E2)$ (W.u.)$^{1/2}$ [b]
M_0	2_1	0_1	44(1)	14.83(17)
M_1	2_1	0_2	47(22)	6.9(16)
M_2	2_3	0_1	0.2(1)	1.00(25)
M_3	2_2	0_2	37(10)	13.6(18)
M'_0	$4^+{}_1$	$2^+{}_1$	71(2)	25.28(36)
M'_1	$2^+{}_3$	$4^+{}_1$	23(6)	10.7(14)
M'_2	$4^+{}_4$	$2^+{}_1$	Unknown	–
M'_3	$4^+{}_4$	$2^+{}_3$	31(5)	16.7(14)

[a]Refs. [1,16].
[b]$M^2(E2) = (2J_i + 1)B(E2; i \to f)$.

TABLE III. Results of mixing for $0 \leftrightarrow 2$ transitions in ^{76}Se.

Quantity	Value
b	0.690(11)
B	0.822(16)
M_g	10.54(41) (W.u.)$^{1/2}$
M_e	18.49(30) (W.u.)$^{1/2}$

TABLE IV. Results of the central solution for $4 \leftrightarrow 2$ transitions in ^{76}Se.

Quantity	Value
B	0.822
D	0.875
M'_g	11.52 (77) (W.u.)$^{1/2}$
M_e	30.73(44) (W.u.)$^{1/2}$

TABLE V. Mixing potential matrix elements in ^{76}Se.

J	V (keV)
0	560
2	576
4	545

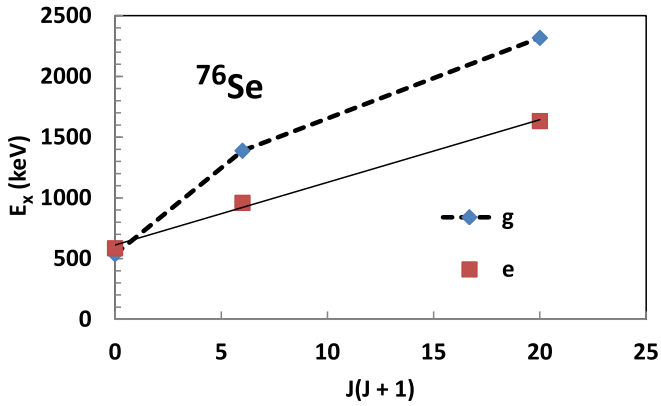

FIG. 2. Energies of the basis-state bands that result from the fit.

is apparent from the near equality of the relevant ΣM^2, i.e., ${M_0}^2 + {M_2}^2 = 221(7)$ W.u., ${M_1}^2 + {M_3}^2 = 232(55)$ W.u. The lower 2^+ physical state is found to contain the preponderance of the e basis-state 2^+. Results for M_g and M_e indicate that the e basis-state band is more collective than the g band.

For the $4 \leftrightarrow 2$ transitions, only three matrix elements are known. But, with the 2^+ mixing having been determined, only three M's are needed for the $4 \leftrightarrow 2$ analysis. Here, a range of solutions exists, with the center of the range characterized by $M'_g/M'_e = 0.375$. Results of this fit are listed in Table IV. Combining with the $0 \leftrightarrow 2$ fit, we note that $M'_e/M_e = 1.66(4)$, reasonably close to the value of 1.60 expected for a $K = 0$ rotational band. However, the g basis-state band has $M'_g/M_g = 1.09(8)$.

Now, with the mixing amplitudes determined above, the physical energy splitting can be used to evaluate the potential matrix elements responsible for the mixing. For example, $V_0 = ab\Delta E_0$. These are listed in Table V. The near equality of these V's is remarkable. With the V's and the physical energies, the energies of the basis states can be computed. These are plotted versus $J(J + 1)$ in Fig. 2. Note that the e energies exhibit a linear dependence, whereas the g ones do not.

III. SUMMARY

I have used a simple two-state mixing model to examine the 0^+, 2^+, and 4^+ states in the first two 0^+ bands in ^{76}Se. The first two 0^+ states are almost maximally mixed. The excited basis-state band is found to have the energy and $E2$ strengths characteristic of a $K = 0$ rotational band, but the lower basis-state band does not. The e band is significantly more collective than the g band.

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