Parity-violating neutron spin rotation in ^{4}He

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The parity-violating neutron spin rotation in ^{4}He is studied at vanishing incident neutron energy limit. Calculations have been realized by solving five-body Faddeev-Yakubovsky equations in configuration space employing modern strong-interaction Hamiltonian based on chiral perturbation theory including three-nucleon force. Parity-violating nucleon-nucleon interaction of the Desplanques, Donoghue, and Holstein model is employed. An implication of the recent theoretical large-N_c estimation of weak couplings [Phillips *et al.*, Phys. Rev. Lett. **114**, 062301 (2015)] is also discussed.

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I. INTRODUCTION

The presence of parity-non-conserving hadronic interactions was established almost 50 years ago [1]. However, an accurate description of parity-violating (PV) hadronic interactions is still missing. Indeed, PV have been observed mostly in heavy nuclei [2], where it can be strongly enhanced due to the complex structure of the compound states. Unfortunately, this complexity does not allow one to extract the parameters of the weak nucleon-nucleon (*NN*) interaction from the the experimental data without uncertainty. One is obliged to focus on few-nucleon systems, where strong interaction physics can be handled accurately, providing grounds to link experimental observables with underlying weak *NN* force. Success of this program obviously requires improvement of the experimental accuracy but also expansion in the outreach of few-body theory.

The natural scale of PV observables in few-nucleon systems is 10^{-7}, requiring exceptional effort to isolate these tiny weak effects and to provide quantitatively significant predictions. Until now only very few experiments have been able to meet this challenge in $A < 5$ systems [3–7], where accurate numerical calculations exist for the scattering process. The creation of a high-intensity cold neutron beamline at the Spallation Neutron Source as well as upgrades at National Institute of Standards and Technology (NIST) are bringing hopes to improve measurements in few-nucleon $A = 2$–5 systems significantly. Some measurements have already been accomplished [7]. In particular, $A = 5$ system looks quite promising with already-existing measurement of the longitudinal asymmetry in proton scattering on ^{4}He at 46 MeV [8]; a NIST study has also yielded the upper bound for neutron spin rotation of transversely polarized neutrons in passage through a ^{4}He target [9]. The last measurement is being repeated with improved apparatus, aiming to provide much better statistics and expecting to provide prediction with accuracy reaching the 10% level. On the theoretical side, one requires a reliable numerical method to solve the $A = 5$ problem employing accurate strong interactions. Recently, one of the authors has developed numerical code allowing us to solve the five-body Faddeev-Yakubovsky (FY) equations in configuration space and for the first time applied it to calculate n-^{4}He elastic phase shifts [10,11]. Thus we dispose a tool allowing us to study the neutron spin rotation in ^{4}He *ab initio* without approximations on the nuclear dynamics.

The accurate measurements of the parity-violating observables in few-body systems once combined with existing theoretical apparatus in $A = 2$–5 systems will finally allow us to carry out a systematic program to determine low-energy constants of PV *NN* interaction. Eventually, knowledge of these constants will promote theoretical parity-violation studies to the level of quantitative predictions. As a part of this program, we have already studied PV effects in $A = 3$ systems [12,13] based on Faddeev equations formalism. In this work, we compute the neutron spin rotation in ^{4}He by solving five-body FY equations with strong interactions derived from chiral effective field theory and weak interaction model of Desplanques, Donoghue, and Holstein (DDH) [14]. An implication of the recent theoretical large-N_c estimation of weak couplings [15,16] is also discussed.

II. THE FORMALISM

A. Faddeev-Yakubovsky equation scheme

We start by briefly highlighting the employed formalism. A general Hamiltonian of the system is defined as

$$\widehat{H} = \widehat{H}_0 + \sum_{i<j=1}^{A} V_{ij} + \sum_{i<j<k=1}^{A} W_{ijk}$$
$$= \widehat{H}_0 + \sum_{i<j=1}^{A} V_{ij}^{\rm PC} + \sum_{i<j=1}^{A} V_{ij}^{\rm PV} + \sum_{i<j<k=1}^{A} W_{ijk}^{\rm PC} + \sum_{i<j<k=1}^{A} W_{ijk}^{\rm PV}, \quad (1)$$

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where $\widehat{H}_0$ is a free Hamiltonian. Throughout this work we denote the total NN potential acting between the nucleons (ij) as V_{ij} and the three-nucleon interaction acting between nucleons (ijk) is denoted as W_{ijk}. These potentials represent the sums of parity-conserving V_{ij}^{PC} (W_{ijk}^{PC}) and parity-violating V_{ij}^{PV} (W_{ijk}^{PV}) terms. Furthermore, in this work we neglect the presence of parity-violating three-nucleon interactions ($W_{ijk}^{\mathrm{PV}} \equiv 0$); therefore, in the following we drop superscript PC in front of the three-nucleon interaction terms by supposing $W_{ijk} \equiv W_{ijk}^{\mathrm{PC}}$. Physical wave functions are solutions of the Hamiltonian, defined in Eq. (1):

$$\widehat{H}\Psi = E\Psi. \tag{2}$$

This wave function might be represented as a sum of positive- and negative-parity components $\Psi = \Psi^{(+)} + \Psi^{(-)}$, satisfying:

$$\begin{aligned}
&\left[E - \widehat{H}_0 - \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PC}} - \sum_{i<j<k=1}^{A} W_{ijk}^{\mathrm{PC}}\right]\Psi^{(+)} = \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PV}}\Psi^{(-)},\\
&\left[E - \widehat{H}_0 - \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PC}} - \sum_{i<j<k=1}^{A} W_{ijk}^{\mathrm{PC}}\right]\Psi^{(-)} = \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PV}}\Psi^{(+)}.
\end{aligned} \tag{3}$$

We are interested in the low-energy process of nucleon scattering on nucleus of a positive parity. At very low energies the process is dominated by a relative nucleon-nucleus S wave and thus the system wave function is predominantly of a positive parity. By neglecting very small term $V_{ij}^{\mathrm{PV}}\Psi^{(-)}$, the last set of equations might be approximated as:

$$\begin{aligned}
&\left[E - \widehat{H}_0 - \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PC}} - \sum_{i<j<k=1}^{A} W_{ijk}^{\mathrm{PC}}\right]\Psi^{(+)} \approx 0,\\
&\left[E - \widehat{H}_0 - \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PC}} - \sum_{i<j<k=1}^{A} W_{ijk}^{\mathrm{PC}}\right]\Psi^{(-)} = \sum_{i<j=1}^{A} V_{ij}^{\mathrm{PV}}\Psi^{(+)}.
\end{aligned} \tag{4}$$

The last set of equations corresponds to the result where only the first-order terms in a parity-violating interaction are considered. Furthermore, parity-violating potential couples two mutually orthogonal spaces; therefore, this approximation is fully equivalent with other perturbative schemes used in evaluating weak transition amplitudes like the Distorted Wave Born Approximation (DWBA) [12,17] approximation or the approximation proposed by Sloan for a three-body system [18,19].

In order to impose physical boundary conditions we employ a Faddeev-Yakubovsky equations approach to solve the Hamiltonian problem formulated above. Faddeev-Yakubovsky equations for a five-body system in their differential form have been formulated in Ref. [20] and were solved for the first time a few years ago by one of us [10,11]. In this work we further adjust the former equations in order to implement three-nucleon forces. For the convenience of its implementation, the three nucleon force W_{ijk} acting between the particles (ijk) is written in a symmetrized form as:

$$W_{ijk} = \mathcal{W}_{ij}^{k} + \mathcal{W}_{jk}^{i} + \mathcal{W}_{ki}^{j}, \tag{5}$$

and, accordingly we modify expressions of the four-body-like FY components derived in Refs. [11,21] for Hamiltonians involving NN interactions only:

$$\begin{aligned}
(E - \widehat{H}_0 - V_{ij})\psi_{ij}^{ijk} &= V_{ij}(\phi_{jk} + \phi_{ki}) + G_{ij}\mathcal{W}_{ij}^{k}\Psi\\
(E - \widehat{H}_0 - V_{ij})\psi_{ij}^{ij,kl} &= V_{ij}\phi_{kl},
\end{aligned} \tag{6}$$

where Ψ is the total wave function of five-nucleon system, which breaks into a sum of three-body-like ϕ_{ij} and then into four-body-like FY components ψ_{ij}^{ijk}, $\psi_{ij}^{ij,kl}$ by:

$$\Psi = \sum_{i<j=1}^{5} \phi_{ij} \tag{7}$$

and

$$\phi_{ij} = \psi_{ij}^{ijk} + \psi_{ij}^{ijl} + \psi_{ij}^{ijm} + \psi_{ij}^{ij,kl} + \psi_{ij}^{ij,km} + \psi_{ij}^{ij,lm}. \tag{8}$$

The final step in deriving five-body FY equations consists of decomposing four-body-like FY components into five-body ones as:

$$\begin{aligned}
\psi_{ij}^{ijk} &= \mathcal{K}_{ij,k}^{l} + \mathcal{K}_{ij,k}^{m} + \mathcal{T}_{ij,k}\\
\psi_{ij}^{ij,kl} &= \mathcal{H}_{ij,kl} + \mathcal{S}_{ij,kl} + \mathcal{F}_{ij,kl}.
\end{aligned} \tag{9}$$

These five-body FY components satisfy the following set of equations:

$$\begin{aligned}
&(E - \widehat{H}_0 - V_{12})\mathcal{K}_{12,3}^{4}\\
&\quad = V_{12}\big(\mathcal{K}_{13,2}^{4} + \mathcal{K}_{23,1}^{4} + \psi_{13}^{134} + \psi_{23}^{234} + \psi_{13}^{13,24} + \psi_{23}^{23,14}\big)\\
&\qquad + \mathcal{W}_{12}^{3}\big(\mathcal{K}_{12,3}^{4} + \mathcal{K}_{13,2}^{4} + \mathcal{K}_{23,1}^{4} + \psi_{13}^{134} + \psi_{23}^{234} + \psi_{13}^{13,24}\\
&\qquad + \psi_{23}^{23,14} + \psi_{12}^{12,34} + \psi_{12}^{124} + \phi_{14} + \phi_{24} + \phi_{34}\big)\\
&(E - \widehat{H}_0 - V_{12})\mathcal{H}_{12,34}\\
&\quad = V_{12}\big(\mathcal{H}_{34,12} + \psi_{34}^{134} + \psi_{34}^{234}\big)\\
&(E - \widehat{H}_0 - V_{12})\mathcal{T}_{12,3}\\
&\quad = V_{12}\big(\mathcal{T}_{13,2} + \mathcal{T}_{23,1} + \psi_{13}^{13,45} + \psi_{23}^{23,45}\big)\\
&\qquad + \mathcal{W}_{12}^{3}\big(\mathcal{T}_{12,3} + \mathcal{T}_{13,2} + \mathcal{T}_{23,1} + \psi_{13}^{13,45}\\
&\qquad + \psi_{23}^{23,45} + \psi_{12}^{12,45} + \phi_{45}\big)\\
&(E - \widehat{H}_0 - V_{12})\mathcal{S}_{12,34}\\
&\quad = V_{12}\big(\mathcal{F}_{34,12} + \psi_{34}^{34,15} + \psi_{34}^{34,25}\big)\\
&(E - \widehat{H}_0 - V_{12})\mathcal{F}_{12,34}\\
&\quad = V_{12}\big(\mathcal{S}_{34,12} + \psi_{34}^{345}\big).
\end{aligned} \tag{10}$$

In the last set of equations some of three-body ϕ_{ij} and four-body-like (ψ_{ij}^{ijk}, $\psi_{ij}^{ij,kl}$) components are not decomposed into five-body components to make expressions more compact. In order to simplify representation of these expressions, we present the last set of equations in a compact form:

$$(E-\widehat{H}_0-V_{12})F_{a_i}=V_{12}\sum_{b_j}\widehat{P}_V^{a_ib_j}F_{b_j}+\mathcal{W}_{12}^3\sum_{b}\widehat{P}_W^{a_ib_j}F_{b_j},\tag{11}$$

where indexes a, b denote the topology of the FY components $\mathcal{K}, \mathcal{H}, \mathcal{T}, \mathcal{S}$, or $\mathcal{F}$, whereas indexes (i, j) denote ordering of particle numbers within provided topology (12345). Thus F_{a_i} (or F_{b_j}) stands for one of the 180 FY components, defining the wave function of a five-body system. The operators $\widehat{P}_V^{a_ib_j}$ and $\widehat{P}_W^{a_ib_j}$, by taking one of two possible values 1 or 0, select the right combinations of FY components present in the right-hand side of Eq. (10) for two-body and three-body potentials, respectively. When considering the presence of the *PV* interactions, in analogy to total systems wave function Ψ, its FY components might be decomposed into positive-parity and negative-parity ones: $F_{a_i}=F_{a_i}^+ + F_{a_i}^-$. The set of FY equations following the Schrödinger Eq. (4), turns to be

$$\begin{aligned}\big(E-\widehat{H}_0-V_{12}^{\rm PC}\big)F_{a_i}^+ &\approx V_{12}^{\rm PC}\sum_{b_j}\widehat{P}_V^{a_ib_j}F_{b_j}^+ +\mathcal{W}_{12}^3\sum_{b}\widehat{P}_W^{a_ib_j}F_{b_j}^+,\\ (E-\widehat{H}_0-V_{12}^{\rm PC})F_{a_i}^- &= V_{12}^{\rm PC}\sum_{b_j}\widehat{P}_V^{a_ib_j}F_{b_j}^- +\mathcal{W}_{12}^3\sum_{b}\widehat{P}_W^{a_ib_j}F_{b_j}^-\\ &\quad + V_{12}^{\rm PV}\sum_{b_j}\widehat{P}_V^{a_ib_j}F_{b_j}^+.\end{aligned}\tag{12}$$

To solve the last set of equations we employ the same techniques, as explained in Ref. [11]. The FY components depend on four vector variables $\Psi(\vec{x},\vec{y},\vec{z},\vec{w})$, described by five-body Jacobi coordinates appropriately adapted to the chosen component, see Ref. [10]. Then a partial wave expansion on each vector variable is performed:

$$\begin{aligned}&F^{JM}(\overrightarrow{x},\overrightarrow{y},\overrightarrow{z},\overrightarrow{w})\\ &\quad=\sum\frac{f_\alpha(x,y,z,w)}{xyzw}|\{\{l_xl_y\}_{l_{xy}}\{l_zl_w\}_{l_{zw}}\}_L\{S\}\rangle_{JM}\{T\}_{TT_z},\end{aligned}\tag{13}$$

where $\alpha\equiv(l_x,l_y,l_z,l_w,l_{xy},l_{zw},L,\{S\},\{T\})$ is an index representing a set of intermediate quantum numbers, coupled to total angular momentum J and total isospin T with its projection T_z (for an n-^{4}He scattering considered in this work, total isospin and its projection are fixed to $T=1/2$ and $T_z=-1/2$). In the last expression $\{S\}$ and $\{T\}$ represent, respectively, partial-wave basis dependence on spin and isospin, which is provided by

$$\{S\}=|\{\{\{s_1s_2\}_{s_x}\{s_3s_4\}_{s_y}\}_{s_{xy}}s_5\rangle_{SS_Z},\tag{14}$$

where s_1 to s_5 are spins of individual nucleons, whereas s_x, s_y, s_{xy}, S represent quantum numbers of intermediate couplings. An equivalent expression is used to develop isospin dependence $\{T\}$ of FY components. The reduced components $f_\alpha(x,y,z,w)$ represent dependence on radial parts of the Jacobi coordinates. This dependence is expressed using Lagrange-Laguerre basis functions [22].

B. Boundary conditions

Solution of the differential equations is not complete, unless proper boundary conditions are formulated and imposed. The reduced components are regular functions both for bound states as well as for scattering problems,

$$\begin{aligned}f_\alpha(0,y,z,w)&=f_\alpha(x,0,z,w)=f_\alpha(x,y,0,w)\\ &=f_\alpha(x,y,z,0).\end{aligned}\tag{15}$$

It is the boundary condition for the asymptotic region (at large radial distances) which turns out to be more complicated when a scattering problem is considered. For a bound-state problem FY components are compact and thus square-integrable basis functions might be readily used to describe behavior of the reduced components. For the scattering problems, which does not involve systems decomposition into more than two clusters (a case considered in this work), reduced components still remain compact in the x, y, z directions. On the other hand, asymptotic parts of the elastic incoming (outgoing) wave of the scattered clusters are expressed in the w-radial dependence of the reduced FY components. In order to satisfy this criteria, but at the same time to be able to use square-integrable basis functions in solving the scattering problem, the reduced components are split in two terms,

$$f_{\alpha,a}(x,y,z,w)=\widetilde{f}^{\rm sh}_{\alpha,a}(x,y,z,w)+\widetilde{f}^{\rm asy}_{\alpha,a}(x,y,z,w).\tag{16}$$

In the last expression index a indicates an incoming channel number, for which we search for a solution. The term $\widetilde{f}^{\rm sh}_{\alpha,a}(x,y,z,w)$ is intended to describe only the interior part of the component $f_{\alpha,a}(x,y,z,w)$ based on expansion employing compact basis functions. The term $\widetilde{f}^{\rm asy}_{\alpha,a}(x,y,z,w)$ complements the expression in order to describe properly the asymptotic part of the reduced FY components. When solving the first equation of (12) this term takes the following form:

$$\begin{aligned}&\big[\widetilde{f}^{\rm asy}_{\alpha,a}(x,y,z,w)\big]^+\\ &\quad=\sum_b\sum_{\beta\subset b}\delta_{\beta,\alpha}\widetilde{\phi}_b^{(+)}(x,y,z)\bigg[w-\sqrt{\frac{8}{5}}\eta_0^{\rm reg}(w)a_0\bigg].\end{aligned}\tag{17}$$

In the last expression the first sum runs over all open channels b, whereas the second sum runs over all the partial-wave amplitudes $\beta\subset b$, contributing to expanding asymptotes of this channel. The term a_0 denotes neutron-^{4}He scattering length to be determined. The second equation of (12) is sought after solution for the first one is obtained, and the asymptotic term for this case is

$$\begin{aligned}&\big[\widetilde{f}^{\rm asy}_{\alpha,a}(x,y,z,w)\big]^-\\ &\quad=\sum_b\sum_{\beta\subset b}\delta_{\beta,\alpha}\widetilde{\phi}_b^{(-)}(x,y,z)\bigg[w-\sqrt{\frac{8}{5}}\eta_0^{\rm reg}(w)a_0\bigg]\\ &\qquad+\sum_b\sum_{\beta\subset b}\delta_{\beta,\alpha}\widetilde{\phi}_b^{(+)}(x,y,z)\sqrt{\frac{8}{5}}\bigg[\frac{K_{1,0}^{\rm PV}(p)}{p^2}\bigg]\\ &\qquad\times\bigg[p\widehat{n}_1\bigg(\sqrt{\frac{5}{8}}pw\bigg)\bigg]\eta_1^{\rm reg}(w),\end{aligned}\tag{18}$$

where relative neutron momentum $p=\sqrt{2\mu E_{\rm c.m.}/\hbar^2}=\sqrt{8E_{\rm c.m.}m/(5\hbar^2)}$ is introduced. In the last equations the terms $[\frac{K_{1,0}^{\rm PV}(p)}{p^2}]$ and $[p\widehat{n}_1(\sqrt{\frac{5}{8}}pw)]$ are separated in square brackets with appropriate power of momentum p in order to keep them finite when this momentum vanishes. Function $\widehat{n}_1(\sqrt{\frac{5}{8}}pw)$ denotes the Riccati-Bessel function; the term $K^{\rm PV}_{1,0}(p)$ is the K-matrix element to be determined describing the parity-violating transition between the S- and P-wave components. This K-matrix element is directly related with the parity-violating observables at low energies, in particular determining neutron-spin rotation in ^{4}He media.

In the former expressions function $\eta^{\rm reg}_l(w)$ is introduced in order to regularize $\widetilde{f}^{\rm asy}_{\alpha,a}(x,y,z,w)$ term at $w=0$. This regularization function is chosen in a form popularized in the numerical calculations of the Pisa group [23–25]:

$$\eta^{\rm reg}_l(w) = [1-\exp(w/w_0)]^{2l+k}. \tag{19}$$

In this parametrization, the power k parameter must be chosen to be $k \geqslant 1$, whereas values $k=1$ and $k=2$ turns to be optimal. The range parameter w_0 draws the matching region between dominance of $\widetilde{f}^{\rm sh}_{\alpha,a}$ and $\widetilde{f}^{\rm asy}_{\alpha,a}$ terms and was chosen in the range (1,2) fm. Calculated observables show very little sensitivity to two parameters encoded in $\eta^{\rm reg}_l(w)$.

Finally, functions $\widetilde{\phi}_\beta(x,y,z)$ represent bound-state-like solutions of the reduced five-body problem to a four-body case. For a case considered in this work it represents solution of bound state problem for ^{4}He nucleus. The FY components $\widetilde{\phi}^{(+)}_b(x,y,z)$ represent the standard (positive-parity) wave function $\Psi^{(+)}_{^4\rm He}(\vec{x},\vec{y},\vec{z})$ of the ^{4}He ground state, obtained from the Hamiltonian of the strong interaction ($H^{\rm PC}_{^4\rm He}$). Once this wave function and the corresponding eigenenergy $E_{^4\rm He}$ are obtained, the negative-parity component of the ^{4}He wave function $\Psi^{(-)}_{^4\rm He}(\vec{x},\vec{y},\vec{z})$ is calculated by solving the following equation:

$$\left(E_{^4\rm He} - H^{\rm PC}_{^4\rm He}\right)\Psi^{(-)}_{^4\rm He} = \sum_{i<j=1}^{4} V^{\rm PV}_{ij}\Psi^{(+)}_{^4\rm He}. \tag{20}$$

In practice these functions are obtained by reducing a five-body problem to a four-body one, which requires us simply to eliminate w dependence in Eq. (12) by equating the Laplacian operator in w as well as all permutation operators containing w dependence to zero.

C. Parity-violating interaction

For many years, the DDH model [14], which is based on a one-meson-exchange picture, has been the most popular choice of parity-violating NN potentials. The DDH PV potential includes contributions from pion, ρ, and ω meson exchange with one parity-conserving vertex and one PV vertex. Assuming h'^{1}_ρ is negligible, the DDH potential model has six weak-coupling constants, h^1_π, h^0_ρ, h^1_ρ, h^2_ρ, h^0_ω, and h^1_ω; the sizes of these coupling constants have been theoretically estimated [14], providing the DDH "best" values together with their "reasonable" range. The explicit form of the DDH potential can be written as

$$\begin{aligned}
V^{\rm PV}_{ij,\rm DDH} = {}& \frac{g_\pi}{2\sqrt{2}m_N}h^1_\pi(\tau_i\times\tau_j)^z(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot i[\boldsymbol{p}_{ij}, f_\pi(r_{ij})]_-,\\
&-\frac{g_\rho}{m_N}h^0_\rho(\tau_i\cdot\tau_j)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot[\boldsymbol{p}_{ij}, f_\rho(r_{ij})]_+,\\
&-\frac{g_\rho(1+\kappa_\rho)}{m_N}h^0_\rho(\tau_i\cdot\tau_j)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot i[\boldsymbol{p}_{ij}, f_\rho(r_{ij})]_-,\\
&-\frac{g_\omega}{m_N}h^0_\omega(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot[\boldsymbol{p}_{ij}, f_\omega(r_{ij})]_+,\\
&-\frac{g_\omega(1+\kappa_\omega)}{m_N}h^0_\omega(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot i[\boldsymbol{p}_{ij}, f_\omega(r_{ij})]_- + \ldots,
\end{aligned} \tag{21}$$

where $[\boldsymbol{p}_{ij}, f_\pi(r_{ij})]_\mp$ are the (anti-)commutation operator with $\boldsymbol{p}_{ij} \equiv \frac{(\boldsymbol{p}_i-\boldsymbol{p}_j)}{2}$, $f_x(r) = \frac{1}{4\pi r}e^{-m_x r}$ and ellipsis represent additional isovector meson exchange terms. Among these, the isovector pion exchange term, h^1_π, has been expected to be the dominant contribution. However, the isovector pion dominance picture is challenged by the measurement of $P_\gamma(^{18}\rm F)$, the circular polarization of photons from the decay of ^{18}F excited state [26–30] which constraints the value of h^1_π significantly below the DDH "best" value. There exist other theoretical estimations of these LECs such as Dubovik and Zenkin (DZ) [31] and Feldman *et al.* (FCDH) [32] which provide smaller pion weak-coupling values; however, they are not consistent with the observation of the near-zero h^1_π term.

To remove possible model dependence of the DDH potential, an alternative parametrization of the PV interaction based on effective field theory has been developed [33–35]. In the low-energy limit, the NN interaction is described by short range contact interactions. For parity violation at leading order, the hadronic PV interaction [2] have five independent low-energy constants which corresponds to Danilov's partial wave analysis [36],

$$\begin{aligned}
V^{\rm PV}_{\rm LO,EFT} = {}& \Lambda_0^{^1S_0-^3P_0}\left[\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) - \frac{1}{i}\frac{\overleftrightarrow{\nabla}_S}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\right]\\
&+\Lambda_0^{^3S_1-^1P_1}\left[\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + \frac{1}{i}\frac{\overleftrightarrow{\nabla}_S}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\right] + \Lambda_1^{^1S_0-^3P_0}\left[\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\left(\tau_1^z+\tau_2^z\right)\right]\\
&+\Lambda_1^{^3S_1-^3P_1}\left[\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\left(\tau_1^z-\tau_2^z\right)\right] + \Lambda_2^{^1S_0-^3P_0}\left[\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta^3(\boldsymbol{r})}{m_\rho^2}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)(\tau_1\otimes\tau_2)_{20}\right],
\end{aligned} \tag{22}$$

where $(\tau_1 \otimes \tau_2)_{20} = (3\tau_1^z\tau_2^z - \tau_1\tau_2)/\sqrt{6}$. These EFT low-energy constants can be approximately matched with the DDH coupling constants in the low energy limit [2],

$$\Lambda_0^{^1S_0-^3P_0} = -g_\rho(2+\kappa_\rho)h_\rho^0 - g_\omega(2+\kappa_\omega)h_\omega^0, \quad \Lambda_0^{^3S_1-^1P_1} = -3g_\rho\kappa_\rho h_\rho^0 + g_\omega\kappa_\omega h_\omega^0, \quad \Lambda_1^{^1S_0-^3P_0} = -g_\rho(2+\kappa_\rho)h_\rho^1 - g_\omega(2+\kappa_\omega)h_\omega^1,$$
$$\Lambda_1^{^3S_1-^3P_1} = \frac{1}{\sqrt{2}}g_{\pi NN}\left(\frac{m_\rho}{m_\pi}\right)^2 h_\pi^1 + g_\rho\left(h_\rho^1 - h_\rho^{\prime 1}\right) - g_\omega h_\omega^1, \quad \Lambda_2^{^1S_0-^3P_0} = -g_\rho(2+\kappa_\rho)h_\rho^2. \tag{23}$$

Recently, new large N_c limit estimation of weak couplings became available [15,16] and predicted presence of only two independent combinations of parameters at the leading order,

$$\Lambda_0^+ = \tfrac{3}{4}\Lambda_0^{^3S_1-^1P_1} + \tfrac{1}{4}\Lambda_0^{^1S_0-^3P_0} \sim N_c, \quad \Lambda_2^{^1S_0-^3P_0} \sim N_c, \tag{24}$$

whereas other couplings can be considered as higher order,

$$\Lambda_0^- = \frac{1}{4}\Lambda_0^{^3S_1-^1P_1} - \frac{3}{4}\Lambda_0^{^1S_0-^3P_0} \sim \frac{1}{N_c},$$
$$\Lambda_1^{^1S_0-^3P_0} \sim \sin^2\theta_W, \quad \Lambda_1^{^3S_1-^3P_1} \sim \sin^2\theta_W. \tag{25}$$

The suppression of isovector pion exchange contribution is prominent. A new analysis of experiments based on this order counting shows very promising consistency between experiments and theory [37]. Also, the new interpretation implies that the $P_\gamma(^{18}\mathrm{F})$ experiment and NPDGamma $A_\gamma(\vec{n}p \to d\gamma)$ experiment can be consistent in the sense that they are sensitive to different combination of LECs at higher order. For reference, different estimations of LECs: DDH, DZ, FCDH as well as values of the new analysis based on large N_c are summarized in the Table I. The implication of these estimation on the neutron spin rotation in ^{4}He will be discussed later.

D. Parity-violating observables

The n-^{4}He scattering amplitude is written in terms of the K-matrix elements, as

$$M_{S'M'T',SMT}(E,\theta) = \sqrt{4\pi}\sum_{JLL'}\sqrt{2L+1}C^{JM}_{L'0,S'M}C^{JM}_{L0,SM}Y_{L'0}(\theta) \times \frac{1}{p}\left[\frac{K}{1-iK}\right]^J_{L'S',LS}, \tag{26}$$

where $|(LS)_{JM}\rangle$ denotes the coupled channels of of n-^{4}He system with a total spin (S) and angular momentum (L), coupled to the total angular momentum JM. Since ground state of ^{4}He (α particle) has zero angular momentum, at very low energies only $J^\pi = \frac{1}{2}^\pm$ states should be considered. In this limit, the slow neutron spin rotation angle is written as

$$\frac{d\phi}{dz} = -\frac{2\pi\rho}{p}\mathrm{Re}\Big[M^{\mathrm{PV}}_{\frac12\frac12\frac12,\frac12\frac12\frac12}(E,0) - M^{\mathrm{PV}}_{\frac12-\frac12\frac12,\frac12-\frac12\frac12}(E,0)\Big] = \frac{8\pi\rho K_{1\frac12,0\frac12}}{p^2},$$

where $\rho = 0.188 \times 10^{23}\,\mathrm{atoms/cm^3}$ denotes the density of the liquid ^{4}He target traversed by a neutron.

Another closely related quantity is the longitudinal total asymmetry, $A(E)$, which can be written at very low-energy limit as

$$A(E) = \frac{\mathrm{Im}\Big[M^{\mathrm{PV}}_{\frac12\frac12\frac12,\frac12\frac12\frac12}(E,0) - M^{\mathrm{PV}}_{\frac12-\frac12\frac12,\frac12-\frac12\frac12}(E,0)\Big]}{\mathrm{Im}\Big[M^{\mathrm{PC}}_{\frac12\frac12\frac12,\frac12\frac12\frac12}(E,0) + M^{\mathrm{PC}}_{\frac12-\frac12\frac12,\frac12-\frac12\frac12}(E,0)\Big]} = \frac{2K_{1\frac12,0\frac12}}{K_{0\frac12,0\frac12}} = \frac{p}{4\pi\rho a_0}\frac{d\phi}{dz}, \tag{27}$$

where a_0 is a S-wave scattering length. Thus, the longitudinal asymmetry is rather trivial in the zero-energy limit. At higher energies this simple relation is broken by the contribution of higher partial waves. In particular, due to the presence of $J^\pi = 3/2^-$ neutron resonance situated at ~1 MeV above the ^{4}He threshold, this relation is expected to break already at $E_n \sim 0.1$ MeV.

TABLE I. Various estimations of PV low-energy constants in units of 10^{-7}. Large N_c analysis is a fit using existing PV experimental data [37].

	Λ_0^+	$\Lambda_2^{^1S_0-^3P_0}$	Λ_0^-	$\Lambda_1^{^1S_0-^3P_0}$	$\Lambda_1^{^3S_1-^3P_1}$
DDH best [14]	319	151	−70	21	1340
DZ [31]	246	108	−79	30	347
FCDH [32]	127	108	−74	42	819
Large N_c analysis [37]	717	324	0	0	0

III. RESULTS

A. Parity-conserving Hamiltonian

The parity-conserving, strong-interaction Hamiltonian in this work is based on χEFT approach, derived up to next-to-next-to-next-to-leading order in chiral perturbation theory [38], denoted by I-N3LO. This Hamiltonian has been also supplemented with a three-nucleon force (3NF), developed up to next-to-next-to-leading order in chiral perturbation theory [39], recently reparameterized by Marccuci *et al.* [40]. Regardless of some inconsistency between the chiral order of three and two nucleon interaction, as pointed out in series of previous works, this model describes very well properties of the light nuclei as well as the low-energy scattering in few-body systems [41–43]. Of particular importance in this study is description of the low energy n-^{4}He scattering. This feature has been already studied in [11].

The I-N3LO Hamiltonian based on two-nucleon interaction only underestimates binding energy of α particle and as consequence slightly overestimates n-^{4}He scattering length. Models based on two nucleon interactions only also fail to reproduce the splitting of the resonant n-^{4}He P-waves [11,43]. I-N3LO NN Hamiltonian provides too much attraction in

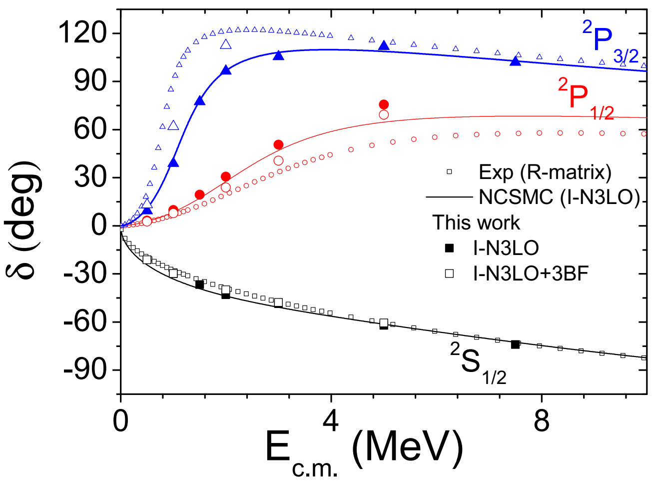

FIG. 1. Energy dependence of n-^{4}He S- and P-wave scattering phase shifts. Our calculated results obtained using I-N3LO NN potential without three-nucleon force (large hollow symbols) are compared with calculations performed within NCSMC framework (continuous line) [43]. Our results for Hamiltonian based on I-N3LO NN interaction and N2LO three-nucleon force parameterized by Marcucci *et al.* [40] are presented by full symbols. Finally, phase shifts extracted from the experimental data, employing R-matrix analysis are presented using small hollow symbols [44].

doublet P-wave but lacks attraction in quartet one, see Fig. 1. Presence of three-nucleon force, developed in Refs. [39,40], significantly improves description of the few-nucleon systems. In particular, binding energy and RMS radius of alpha particle are properly reproduced [39]. Consequently description of n-^{4}He scattering length is also improved [45]. However, note the presence of large discrepancy in experimentally determined n-^{4}He scattering length, which complicates quantitative comparison of the theoretical and the experimental results. Thus it is worth to note the ongoing experimental activity, aiming to settle this issue.

Finally, presence of three-nucleon interaction improves considerably description of the resonant n-^{4}He P-waves. Our calculated n-^{4}He $^2P_{1/2}$-wave phase shifts agrees perfectly with the ones determined from the experimental data using R-matrix analysis, see Fig. 1. Our calculated $^2P_{3/2}$-wave phase shifts still slightly underestimate R-matrix data, however demonstrating significant improvement relative to results obtained by neglecting 3NF. In this respect our results slightly differ from the ones obtained by employing No Core Shell Model with Continuum (NCSMC) technique [43], asserting perfect agreement between the calculations and R-matrix analysis. Estimation of the low-energy neutron spin rotation relies on the doublet n-^{4}He states, $^2S_{1/2}$ and $^2P_{1/2}$, well described by the PC Hamiltonian based I-N3LO NN interaction and 3NF from Ref. [40] and thus providing reliable PC wave function input.

TABLE II. Contribution of different operators from Eq. (21) to slow neutron spin rotation in 10^{-7} rad/m for the case of a DDH-best value parameters. Results for two different strong interaction models, namely I-N3LO+3BF and I-N3LO, are presented by considering liquid helium density $\rho = 0.188\ 10^{23}$ atoms/cm^3.

Ref. [12]	$c_n^{\rm cm}$	I-N3LO+3BF	I-N3LO
1	$\frac{g_\pi}{2\sqrt{2}m_N}h_\pi^1$	−0.654(3)	−0.711
2	$\frac{g_\rho}{m_N}h_\rho^0$	0.063(17)	−0.005
3	$\frac{g_\rho(1+\kappa_\rho)}{m_N}h_\rho^0$	4.52(1)	4.43
8	$\frac{g_\omega}{m_N}h_\omega^0$	−0.274(22)	−0.259
9	$\frac{g_\omega(1+\kappa_\omega)}{m_N}h_\omega^0$	0.161(8)	0.179
	Total	3.82(2)	3.64

B. Effects of parity nonconserving Interaction

The main goal of this work is to provide reliable calculations of the parity-violating matrix elements contributing to low-energy neutron spin rotation in ^{4}He. We have decided to concentrate on PV operators in DDH potential, which can be also linked with the models based on EFT as demonstrated in Refs. [12,48].

Due to high numerical costs we were not able to consider all the operators spanning DDH potential. Rather, in consideration of the large N_c estimation of LECs, we limited ourselves to isovector pion exchange, which would be dominant one in DDH-best values and isoscalar rho and omega meson exchanges, which would be dominant ones according large N_c estimation. These potential terms are summarized in Eq. (21) and denoted with indexes $n = 1, 2, 3, 8, 9$ in Table I of Ref. [12]. In Table II, we present contribution from each of these operators to n-^{4}He spin rotation based on DDH-best parameter set [14]. Calculations have been performed for nuclear Hamiltonian based on I-N3LO NN interaction alone and by including N2LO three-nucleon force parameterized by Marcucci *et al.* [40]. Two independent numerical computations for the case including 3NF force have been performed, employing slightly different grids, which allowed us to perform conservative estimate of the numerical accuracy—turning out to be of the order of a few percentages. Though the inclusion of 3BF improves much the binding energy of ^{4}He, the PV amplitudes are not sensitive to the 3BF.

By summing all the contributions, the total n-^{4}He spin rotation can be expressed as

$$\frac{d\phi}{dz} = \begin{cases} -\left(0.156h_\pi^1 - 0.0419h_\omega^0 + 0.388h_\rho^0 + \dots\right) \text{ rad/m for I-N3LO} \\ -\left(0.144(1)h_\pi^1 - 0.058(8)h_\omega^0 + 0.402(1)h_\rho^0 + \dots\right) \text{ rad/m for I-N3LO+3BF} \end{cases}. \tag{28}$$

This result may be compared with the only previous study of neutron spin rotation in ^{4}He, performed by Dmitriev *et al.* [49] employing simplistic nuclear wave functions,

$$\frac{d\phi}{dz} = -\left(0.97h_\pi^1 + 0.22h_\omega^0 + 0.32h_\rho^0 - 0.22h_\omega^1 - 0.11h_\rho^1 - 0.02h_\rho^{\prime 1}\right) \text{ rad/m}. \tag{29}$$

TABLE III. Convergence of calculated ^{4}He binding and n-^{4}He scattering lengths a_0 with a size of the partial wave basis used in calculation, being limited by partial angular-momentum values $\max(l_x, l_y, l_z, l_w)$. In the last line fully converged values of the ^{4}He binding energy are provided, obtained using considerably larger basis to discretize its wave function.

PW	I-N3LO		I-N3LO+3BF	
$\max(l_x, l_y, l_z, l_w)$	B(^{4}He) (MeV)	a_0 (n-^{4}He) (fm)	B(^{4}He) (MeV)	a_0(n-^{4}He) (fm)
2,1,1,1	21.86	2.79	22.41	2.78
2,2,1,1	24.32	2.73	26.36	2.69
2,2,2,2	24.66	2.74	26.99	2.71
3,3,3,3	25.19	2.69	27.72	2.65
4,4,4,3	25.21	2.71	28.10	2.67
4,4,4,4	25.22	2.71	28.11	2.67
Full.	25.38(1) [43,46,47]		28.24 (this work)	

Compared to the result of Dmitriev *et al.* [49], our results shows very different coefficients of h^1_π and h^0_ω. For DDH-best value, the total turns to be 3.64×10^{-7} rad/m for I-N3LO and $3.82(2) \times 10^{-7}$ rad/m for I-N3LO+3BF strong interaction Hamiltonians, significantly different from the result estimated by Dmitriev *et al.* [49] $\sim - 10^{-7}$ rad/m.

The suppression of h^1_π term may need closer inspection. Compared to the amplitudes related with n-^{2}H spin rotation form Ref. [12], or even with ones related with longitudinal asymmetry in $^3\mathrm{He}(\vec{n}, p)^3\mathrm{H}$ reaction [17], one may observe strong suppression of the currents related with a pion exchange. This feature may be due to the spherical symmetry of ^{4}He. PV one pion exchange operator is closely related with the proton density asymmetry along the projectile neutron axis, whereas this asymmetry is washed out due to perfect symmetry of ^{4}He. The Dmitriev *et al.* [49] employing simplistic nuclear wave functions, suggested much weaker suppression of one pion exchange currents. Interestingly, if we perform calculations using PC Hamiltonian based on phenomenological MT I-III NN interaction [50], being limited to S-wave, contribution of one pion exchange operator to ^{4}He spin rotation increases by factor four showing the sensitivity to NN interaction. This remarkable feature might be very useful in testing large N_c hypothesis [15,16] of weak couplings.

By adopting the conversion of Eq. (23), our result can be summarized in terms of EFT parameters as

$$\frac{d\phi}{dz} = \begin{cases} +0.0158\Lambda_0^+ + 0.010\Lambda_0^- - 0.0005\Lambda_1^{^3S_1-^3P_1} \text{ rad/m for I-N3LO} \\ +0.0166(1)\Lambda_0^+ + 0.0127(3)\Lambda_0^- - 0.000483(3)\Lambda_1^{^3S_1-^3P_1} \text{ rad/m for I-N3LO+3BF} \end{cases}. \tag{30}$$

The same manipulation for results of Dmitriev *et al.* yields [37],

$$\frac{d\phi}{dz} = 0.0095\left(\Lambda_0^+ - \left[1.61\Lambda_0^- + 0.92\Lambda_1^{^1S_0-^3P_0} + 0.35\Lambda_1^{^3S_1-^3P_1}\right]\right) \text{ rad/m}. \tag{31}$$

Though the coefficients of h^0_ρ is similar between two cases, because of the different combination of h^0_ρ and h^0_ω in the spin rotation, our result shows almost twice larger contribution of Λ_0^+. For different estimations of LECs, we get following predictions

$$\frac{d\phi}{dz} = \begin{cases} 3.8 \times 10^{-7} \text{ rad/m for DDH best} \\ 3.0 \times 10^{-7} \text{ rad/m for DZ} \\ 0.88 \times 10^{-7} \text{ rad/m for FCDH} \\ 1.2 \times 10^{-6} \text{ rad/m for large } N_c \text{ analysis} \end{cases}. \tag{32}$$

One finds that the large N_c estimation of LECs in our calculation favors larger neutron spin rotation values compared to that of Dmitriev *et al.*

As demonstrated in Eq. (27) the longitudinal asymmetry coefficient, $A(E_n)$, at very low energies can be expressed in terms of n-^{4}He spin rotation angle as $A(E_n) = C\sqrt{E_n}\frac{d\phi^n}{dz}$. Employing scattering lengths from Table III, one gets $C = 2.74 \times 10^{-4}$ and 2.79×10^{-4} eV$^{-1/2}$ m rad^{-1} for I-N3LO and I-N3LO+3BF models respectively. Thus for neutron energies of order 100 keV, where relation of Eq. (27) still holds, longitudinal asymmetry is not expected to exceed 10^{-7}.

Our n-^{4}He spin rotation results are inline with the current experimental bound $[1.7 \pm 9.1(\text{stat}) \pm 1.4(\text{syst})] \times 10^{-7}$ rad/m from Ref. [9] and new analysis of the same experiment, $[2.1 \pm 8.3(\text{stat}) \pm 1.2(\text{syst})] \times 10^{-7}$ rad/m from Ref. [51], nevertheless due to apparent drawbacks in determining mesonic coupling constants of the PV interaction this comparison should be considered only on very qualitative grounds. As discussed previously, for a better quantitative estimation consistent program based on few-body data is necessary in order to determine coupling constants defining PV interaction, which following EFT arguments may strongly depend on parity-conserving Hamiltonian. Determination of the PV amplitudes related with the higher order terms of EFT may worth as a future work.

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