Consistent relativistic mean-field models constrained by GW170817

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We have obtained the Love number and corresponding tidal deformabilities (Λ) associated with the relativistic mean-field parametrizations shown to be consistent (CRMF) with the nuclear matter, pure neutron matter, symmetry energy, and its derivatives [Dutra *et al.*, Phys. Rev. C **90**, 055203 (2014)]. Our results show that CRMF models present very good agreement with the recent data from binary neutron star merger event GW170817. They also confirm the strong correlation between $\Lambda_{1.4}$ and the radius of canonical stars ($R_{1.4}$). When a recent GW170817 constraint on $\Lambda_{1.4}$ and the corresponding radius $R_{1.4}$ is used, the majority of the models tested are shown to satisfy it.

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I. INTRODUCTION

The discovery of the first binary pulsar PSR1913+16 by Hulse and Taylor in 1974 [1] with its very stable and precise pulse period, and the observation in 1978 that its orbit period was declining with time [2], opened a clear possibility for the detection of gravitational waves (GW). A probable explanation for the change in the period was the loss of energy by the binary system in the form of GW. The detection of these waves was expected since then until in 2015 the first signal was clearly seen (GW150914) and shown to be produced by two colliding black holes [3]. Finally, in 2017, LIGO and Virgo made the first detection of GW170817 produced by colliding neutron stars [4] and the event was observed also as light in the optical, UV, IR, x-ray, and γ-ray emissions [5], what was then called a multimessenger observation.

When one of the neutron stars in a binary system gets close to its companion just before merging, a mass quadrupole develops as a response to the tidal field induced by the companion. This is known as tidal deformability [6,7] and can be used to constrain neutron star macroscopic properties [8,9], which in turn are obtained from appropriate equations of state (EOS). A nice and simple review on the basic ingredients necessary to construct an EOS is given in Ref. [10].

If one searches the literature for an equation of state, hundreds of models are found. Not too long ago, 263 relativistic mean-field (RMF) parametrizations were analyzed in Ref. [11] and confronted with different sets of constraints, all of them related to symmetric nuclear matter, pure neutron matter, symmetry energy, and its derivatives. The sets differ from one another in the choice of validity ranges of certain quantities and in the level of restriction. Only a small number of parametrizations of these models (35) were shown to adequately satisfy the chosen constraints. This fact reinforces the idea that the proliferation of models and the production of new parameter sets with a limited range of application should not be encouraged.

In Ref. [11], the relativistic models were divided into seven families, namely, linear finite range models (Walecka-type models [12], type 1), nonlinear σ models (Boguta-Bodmer models [13], type 2), nonlinear σ and ω models with a self-quartic interaction in the ω field (type 3), nonlinear σ and ω terms and cross terms involving these fields (type 4), density-dependent models [14] with couplings adjusted to nuclear properties (type 5), nonlinear point coupling models [15] (type 6), and models with δ mesons (type 7). Thirty of the approved models are of type 4, two are of type 5, one of type 6, and two of type 7, both being density-dependent.

Later on, 34 RMF models that were shown to satisfy several nuclear matter constraints in Ref. [11], namely, the same previous 35 models excluding the point-coupling one as discussed in Ref. [16], were confronted with astrophysical constraints [16]. The more important of these constraints are the neutron stars with maximum mass in the range of $1.93 \leqslant M/M_\odot \leqslant 2.05$ [17,18], but the direct Urca process and the sound velocity also give hints on the star cooling mechanism and its internal matter distribution. From the 34 analyzed models with nucleonic matter included, only 15 can sustain massive stars and none if hyperons are included, a result that accounts for the famous hyperon puzzle. Once hyperons are included in the calculations, the situation becomes more complicated, because the EOS must be soft at subsaturation densities and hard at higher densities to predict massive stars, but hyperons soften the EOS. A possibility that reconciles the measurements of massive stars with canonical stars with small radii present in the same family (another recently imposed constraint) is either the inclusion of strange mesons or of a new degree of freedom (not necessarily known) in the calculations [19].

The measurements and analyses of data from this specific gravitational wave established limits both on the dimensionless tidal deformability of the binary system $\tilde{\Lambda}$ and on the tidal deformability of the canonical star $\Lambda_{1.4}$ as being $\leqslant$800 for the low-spin priors upper boundary [4] and contributed to the exclusion of very stiff EOS that would give rise to values larger than 800. A lower limit was estimated as $\tilde{\Lambda} > 400$ [20]. Recently, a LIGO and Virgo collaboration updated the $\Lambda_{1.4}$

values to be constrained to the range of $70 \leqslant \Lambda_{1.4} \leqslant 580$ [21]. Moreover, the chirp mass, which relates the masses of both NS in the binary system was observed to be $\mathcal{M} = 1.188 M_\odot$ [4]. The above-mentioned boundaries combined with the chirp mass can be used to calculate the bounds on the tidal deformability of the individual neutron stars in the binary system [22].

Since the detection of GW170817, several studies were dedicated to look for correlations and sensitivity of important nuclear bulk properties, i.e., the symmetry energy, its slope, compressibility, and values of the tidal deformability for the canonical $1.4 M_\odot$ and other slightly less and slightly more massive stars. In Ref. [23] the authors analyzed four Skyrme-type models and one obtained from a density functional theory; in Ref. [22] 18 relativistic and 24 nonrelativistic models were analyzed; in Ref. [24] many Skyrme-type models were investigated; and in Ref. [25] 67 RMF models were considered. In the last three works, different correlations were found between, for instance, Λ and R, $\Lambda_{1.4}$ and $R_{1.4}$, $\Lambda_{1.4}$ and $M_{\rm max}$, or $\Lambda_{1.4}$ and $\tilde{\Lambda}$.

In Ref. [21], a parametrized EOS was built at high-densities and one Skyrme EOS at low densities and was confronted with GW170817 tidal deformability information to obtain NS radii. The suggested values for the two neutron stars in the binary system lie in the range $R_1 = 10.8^{+2.0}_{-1.7}$ km and $R_2 = 10.7^{+2.1}_{-1.5}$ km. If a further restriction is imposed to account for EOS that support massive stars, then both radii are constrained to the range of 11.9 ± 1.4 km. All models analyzed in Ref. [16] with a maximum mass of $(1.97 \pm 0.04) M_\odot$ or larger bear radii within the proposed range. In Ref. [25], the authors established the upper limit on the canonical stars radii as $R_{1.4} \leqslant 12.9$ km. In Ref. [16], only seven models are excluded by this constraint.

Also, in a recent paper [26], the authors show that an infinite number of combinations of EOS with large slopes and small compressibilities or small slopes and large compressibilities can lead to the same $\Lambda_{1.4}$ and $R_{1.4}$, pointing to the need of more observables so that the density dependence of the symmetry energy can be completely determined.

In all of the above-mentioned papers, the models used to test constraints and to look for correlations were randomly chosen. However, in the present work we follow a more direct line of work by choosing models we have already tested previously based on exactly the same constraints, avoiding models either with flamboyant degrees of freedom or that have been forcefully corrected with extra mixed meson interactions and return to the more conventional 34 RMF parametrizations that were shown to satisfy the nuclear matter constraints in Ref. [11] to confront them with tidal deformabilities inferred from GW170817.

II. RESULTS AND DISCUSSION

In a binary neutron star system, the tidal deformability is the measurement of the perturbation generated by the quadrupole moment in one star as a response to the external field created by its companion. From the mathematical point of view, the dimensionless tidal deformability, in terms of the Love number k_2, is given by

$$\Lambda = \frac{2k_2}{3C^5}, \qquad (1)$$

where $C = m/R$ is the compactness of the neutron star of mass m. The Love number k_2 is calculated by the following expression:

$$\begin{aligned} k_2 = {}& \frac{8C^5}{5}(1-2C)^2[2 + C(y_R - 1) - y_R] \\ & \times \{2C[6 - 3y_R + 3C(5y_R - 8)] \\ & + 4C^3[13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] \\ & + 3(1 - 2C^2)[2 - y_R + 2C(y_R - 1)]\ln(1 - 2C)\}^{-1}, \end{aligned} \qquad (2)$$

where $y_R \equiv y(r)$ is found from the solution of

$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0, \qquad (3)$$

with

$$F(r) = \frac{r - 4\pi r^3[\epsilon(r) - p(r)]}{r - 2M(r)} \qquad (4)$$

and

$$\begin{aligned} Q(r) = {}& \frac{4\pi r\left[5\epsilon(r) + 9p(r) + \frac{\epsilon(r)+p(r)}{\partial p(r)/\partial\epsilon(r)} - \frac{6}{4\pi r^2}\right]}{r - 2M(r)} \\ & - 4\left[\frac{M(r) + 4\pi r^3 p(r)}{r^2(1 - 2M(r)/r)}\right]^2. \end{aligned} \qquad (5)$$

The set of Eqs. (1)–(5) can also be found in Refs. [27–31], for instance, in which earlier new developments were performed by using a large number of EOSs used to calculate k_2 and Λ.

Actually, Eq. (3) must be solved together with the well known TOV equations [32], in which ϵ and p are the energy density and pressure, respectively, given as input. In our case, these quantities are given by the CRMF parametrizations with protons, neutrons, electrons, and muons with the charge neutrality and β-equilibrium conditions together with the Baym-Pethick-Sutherland (BPS) equation of state [33] in the low-density regime, namely, $0.1581 \times 10^{-10}\ \text{fm}^{-3} \leqslant \rho \leqslant 0.008907\ \text{fm}^{-3}$. The initial condition for Eq. (3) is $y(0) = 2$ (related to the Love number order) and $M(r)$ is the neutron star mass enclosed within the radius r. At the surface of the star, in which $r = R$, one has $M(R) = m$. For detailed discussions on such calculations, we address the reader to Refs. [28–31,34], for instance.

The compactness of one recently measured isolated neutron star [35] is equal to 0.105 ± 0.002. Notice that the GW170817 constrains NS in a binary system and, hence, more measurements are necessary before this value is used as a constraint. Nevertheless, in Table I, we show the compactness for the cases of $m = m_{\rm max}$ ($C_{\rm max}$), $m = 1.4 M_\odot$ ($C_{1.4}$), and three more cases obtained from the limits of possible masses in the binary system. The mass-radius diagrams used to calculate these C values for the CRMF parametrizations are found in Ref. [16].

Models belonging to the same families present very similar compactness both for the maximum mass star and for the

TABLE I. Compactness, in units of $M_\odot$/km, related to the maximum neutron star mass ($C_{\max}$), canonical one ($C_{1.4}$), and for $m_1 = 1.37$, 1.48, and 1.60 solar masses, with their respective values of m_2 for the CRMF models analyzed.

Model	$C_{\max}$	$C_{1.4}$	$C^{m_1}_{1.37}$	$C^{m_2}_{1.36}$	$C^{m_1}_{1.48}$	$C^{m_2}_{1.26}$	$C^{m_1}_{1.60}$	$C^{m_2}_{1.17}$
BKA20	0.170	0.105	0.103	0.103	0.113	0.095	0.123	0.088
BKA22	0.170	0.105	0.103	0.102	0.112	0.094	0.122	0.087
BKA24	0.169	0.104	0.102	0.101	0.111	0.093	0.121	0.086
BSR8	0.171	0.108	0.105	0.105	0.115	0.097	0.125	0.090
BSR9	0.170	0.108	0.105	0.105	0.115	0.097	0.125	0.090
BSR10	0.170	0.107	0.104	0.104	0.113	0.095	0.123	0.088
BSR11	0.169	0.105	0.095	0.095	0.112	0.094	0.123	0.087
BSR12	0.170	0.106	0.103	0.102	0.112	0.094	0.122	0.087
BSR15	0.160	0.112	0.109	0.108	0.119	0.099	0.132	0.092
BSR16	0.159	0.111	0.109	0.108	0.119	0.099	0.132	0.092
BSR17	0.159	0.111	0.108	0.108	0.119	0.098	0.131	0.091
BSR18	0.158	0.110	0.107	0.107	0.118	0.097	0.130	0.090
BSR19	0.158	0.109	0.106	0.106	0.117	0.096	0.130	0.089
BSR20	0.157	0.107	0.104	0.104	0.115	0.095	0.128	0.087
FSU-III	0.158	0.111	0.108	0.108	0.119	0.098	0.132	0.090
FSU-IV	0.160	0.114	0.111	0.111	0.122	0.101	0.135	0.094
FSUGold	0.159	0.113	0.110	0.109	0.121	0.100	0.134	0.092
FSUGold4	0.160	0.114	0.111	0.111	0.122	0.101	0.135	0.094
FSUZG03	0.170	0.108	0.105	0.105	0.115	0.097	0.125	0.090
FSUGZ06	0.159	0.112	0.108	0.108	0.119	0.099	0.132	0.092
G2*	0.176	0.111	0.108	0.108	0.118	0.099	0.129	0.092
IU-FSU	0.152	0.112	0.109	0.109	0.118	0.100	0.129	0.093
Z271s2	0.153	0.111	0.108	0.108	0.120	0.098	0.136	0.090
Z271s3	0.153	0.114	0.110	0.110	0.122	0.100	0.139	0.092
Z271s4	0.153	0.115	0.112	0.111	0.124	0.101	0.141	0.093
Z271s5	0.153	0.116	0.113	0.113	0.125	0.103	0.142	0.095
Z271s6	0.154	0.117	0.114	0.114	0.126	0.104	0.143	0.096
Z271v4	0.149	0.115	0.111	0.111	0.124	0.100	0.147	0.092
Z271v5	0.149	0.115	0.112	0.111	0.125	0.101	0.149	0.092
Z271v6	0.150	0.116	0.113	0.112	0.126	0.102	0.150	0.093
DD-F	0.193	0.117	0.114	0.114	0.125	0.105	0.137	0.097
TW99	0.196	0.114	0.111	0.111	0.121	0.102	0.132	0.095
DDHδ	0.192	0.111	0.109	0.109	0.118	0.100	0.127	0.094
DD-MEδ	0.191	0.118	0.115	0.115	0.126	0.105	0.137	0.097

canonical one. In the low-limit mass case, both stars of the binary system present practically the same mass, $m_1 \approx m_2 \approx 1.37M_\odot$. This is the reason they present the same compactness. However, this is no longer true in the other cases, when one of the stars is always more compact than its companion.

In Fig. 1 we display the Love number k_2 for the CRMF parametrizations, obtained through the definition given in Eq. (2) with y_R calculated from the solution of Eq. (3) coupled to the TOV equations. The pattern exhibited by k_2 is similar to that found in calculations involving other relativistic hadronic model, as one can verify in Ref. [34], for instance. It is important to point out that k_2 is very sensitive to the description of the crust of the star. Once k_2 is calculated, it is possible to analyze the dimensionless tidal deformability by using the definition presented in Eq. (1).

In Fig. 2 we display a diagram of the dimensionless tidal deformabilities of each star in the binary system. Λ_1 is associated to the neutron star with mass m_1, which corresponds to the integration of every EOS in the range $1.37 \leqslant m/M_\odot \leqslant 1.60$ obtained from GW170817. However, the mass m_2 of the companion star is determined by solving the chirp mass $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$, whose value is $1.188M_\odot$, as determined in Ref. [4]. One can see that all the investigated models lie in between the confidence lines, which corroborates the fact that the previously constrained models to satisfy nuclear bulk properties are reliable to investigate neutron stars in binary systems, although many of them do not describe massive stars, as explained in the Introduction of this paper.

As already shown in Refs. [22,24,25], we have also found a strong correlation between the tidal deformability of the canonical star and its radius both in linear and log scale (not shown), namely, $\Lambda_{1.4} = 2.65 \times 10^{-5} R_{1.4}^{6.58}$, as can be seen in Fig. 3. Since the second Love number k_2 depends on R, for a given EOS, through a nontrivial differential equation coupled to the TOV one, $\Lambda_{1.4}$ as a function of $R_{1.4}$ is not simply given by $\Lambda_{1.4} \propto R_{1.4}^5$, as Eq. (1) suggests.

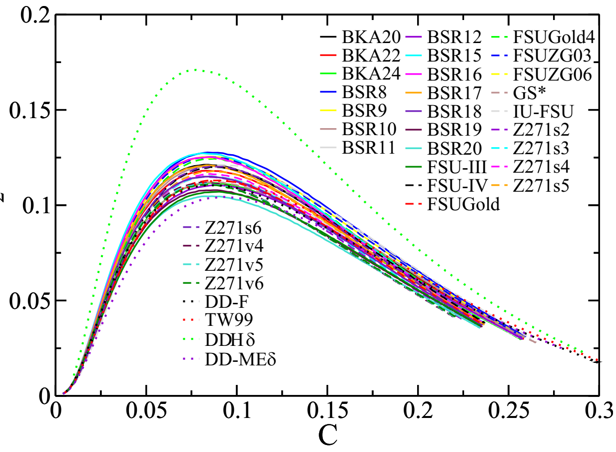

FIG. 1. Love number k_2 as a function of the compactness for the CRMF parametrizations.

When we analyze the constraint for $\Lambda_{1.4}$ in the range $70 \leqslant \Lambda_{1.4} \leqslant 580$, as proposed in Ref. [21], with corresponding $R_{1.4}$ values, depicted in Fig. 3 by the shaded square, we observe that 24 parametrizations (out of 34) are in accordance with this proposition. They are: BSR15, BSR16, BSR17, BSR18, BSR19, BSR20, FSU-III, FSU-IV, FSUGold, FSUGold4, FSUGZ06, G2*, IU-FSU, Z271s2, Z271s3, Z271s4, Z271s5, Z271s6, Z271v4, Z271v5, Z271v6, DD-F, TW99, and DD-MEδ. It is interesting to notice that not all models capable of describing massive stars in the range $1.93 \leqslant M/M_\odot \leqslant 2.05$ [17,18] discussed in Ref. [16] lie inside the box with values obtained from GW170817, and not all 24 models inside the gray area can describe massive stars. Only 5 models satisfy both constraints, namely, G2*, IU-FSU, DD-F, TW99, and DD-MEδ.

Although the current range for $\Lambda_{1.4}$ is not very restrictive, we remind the reader that its values were still more imprecise, as one can verify in Ref. [4] in which the range was computed as $\Lambda_{1.4} \leqslant 800$. Furthermore, there is a huge number of hadronic parameterizations coming from relativistic and nonrelativistic models, around 500 if we take into account only those from RMF and Skyrme models. Thus, it is important to find which particular set of parametrizations among these huge number is able to describe simultaneously different nuclear environments. In that sense, a constraint coming from the analysis of the recent GW170817, even being not so restrictive ($70 \leqslant \Lambda_{1.4} \leqslant 580$) can be useful for this purpose.

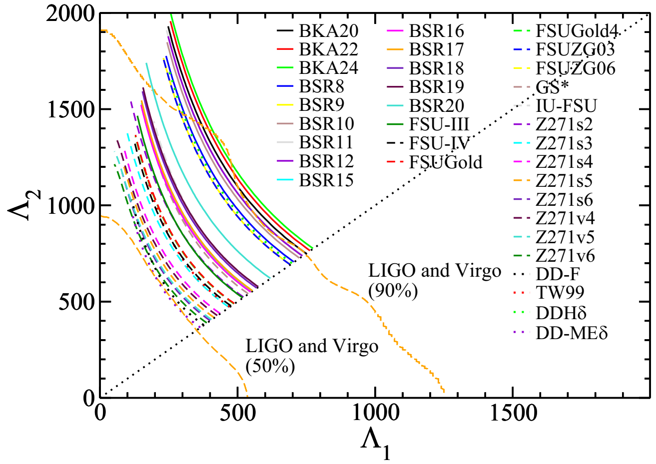

FIG. 2. Tidal deformability parameters for both components of the observed GW170817. The confidence lines (90% and 50%) are the recent results of LIGO and Virgo collaboration taken from Ref. [21].

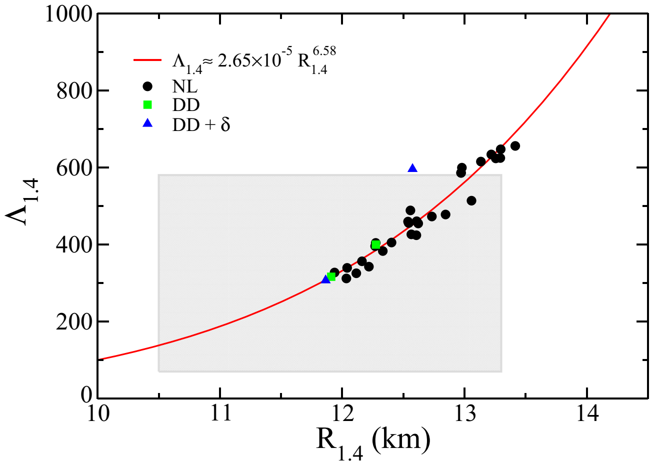

FIG. 3. $\Lambda_{1.4}$ as a function of $R_{1.4}$, obtained from the CRMF models. NL, DD, and DD + δ stand for nonlinear, density-dependent, and density-dependent with δ particle, respectively. Gray area: results given in Ref. [21].

III. FINAL REMARKS

In the present work, we have revisited 34 relativistic mean-field parametrizations shown to be consistent (CRMF) with the nuclear matter, pure neutron matter, symmetry energy, and its derivatives in Ref. [11] and used them to compute the Love number and corresponding tidal deformabilities. We have checked that all analyzed models lie in between the confidence lines in the plot Λ_2 versus Λ_1. They also confirm previously obtained correlation between the tidal deformability and the radius of canonical stars. Once we use the GW170817 constraints on the tidal deformabilities to identify the corresponding neutron star radii range, as proposed in Ref. [21], 24 parametrizations are shown to satisfy them. As far as the compactness, an important ingredient in the calculation of the Love numbers, is investigated, we have seen that, generally, one of the star is always more compact than its companion, except in the low limit mass case $m_1 = 1.37$, when both stars in the binary system present the same compactness.

It is also worth pointing out that only five parametrizations of the CRMF models, namely, G2*, IU-FSU, DD-F, TW99, and DD-MEδ, can simultaneously describe massive stars in the range $1.93 \leqslant M/M_\odot \leqslant 2.05$ [17,18], as shown in Ref. [16], and constraints from GW170817.

To further constrain the existing EOS or confirm the results obtained so far, we look forward to the next detections of gravitational waves.

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