

α decay in intense laser fields: Calculations using realistic nuclear potentialsJintao Qi,¹ Tao Li,² Ruihua Xu,¹ Libin Fu,¹ and Xu Wang^{1,*}¹Graduate School, China Academy of Engineering Physics, Beijing 100193, China²Beijing Computational Science Research Center, Beijing 100193, China

(Received 28 January 2019; published 22 April 2019)

We calculate the effect of intense laser fields on nuclear α decay processes, using realistic and quantitative nuclear potentials. We show that α decay rates can indeed be modified by strong laser fields to some finite extent. We also predict that α decays with lower decay energies are relatively easier to be modified than those with higher decay energies, due to longer tunneling paths for the laser field to act on. Furthermore, we predict that modifications to angle-resolved penetrability are easier to achieve than modifications to angle-integrated penetrability.

DOI: [10.1103/PhysRevC.99.044610](https://doi.org/10.1103/PhysRevC.99.044610)**I. INTRODUCTION**

The past few decades witness rapid advancements in intense laser technologies. The chirped pulse amplification technique [1] enables table-top Ti:sapphire lasers to have intensities exceeding one atomic unit (3.5×10^{16} W/cm²), opening the door to the rich area of strong-field atomic, molecular, and optical physics with novel nonperturbative phenomena such as multiphoton and above-threshold ionization [2,3], high-order harmonic generation [4,5], nonsequential double and multiple ionization [6,7], and attosecond physics [8–11].

Even higher intensities can be achieved by larger laser systems of different kinds, for example, x-ray free electron lasers (XFELs) and the under-construction extreme light infrastructure (ELI) of Europe. XFELs can be focused to reach peak intensities on the order of 10^{20} W/cm² [12]. ELI is designed to reach peak intensities above 10^{23} W/cm² [13,14]. The laser electric field corresponding to such an intensity is comparable to the Coulomb field from the bare nucleus at a distance of order 10 fm. Direct influence on the nucleus may be possible from such an intense laser field. In fact, one of the major scientific goals of the ELI facility is to study laser-driven nuclear physics.

A direct light-nucleus interaction with much weaker light intensities has been realized using synchrotron radiations on the Mössbauer ⁵⁷Fe system. Using a grazing-incidence x-ray diffraction technique and a planar ⁵⁷Fe cavity, collective quantum optical effects have been demonstrated with photon energy 14.4 keV, such as single-photon superradiance [15], electromagnetically induced transparency [16], spontaneously generated coherence [17], and Rabi oscillation [18]. On the other hand, the nuclei, as the media of x-ray pulse propagation, can be used to modify the properties of the x-ray pulse, such as the pulse shape [19] and the group velocity [20]. Theoretical proposals have also been made on single-photon entanglement [21], single-photon storage and phase

modulation [22], nuclear battery using isometric transition [23–25], etc.

Nonresonant effects of intense laser fields on nuclear systems have also been reported in the literature. Among them the possible influence of intense laser fields on nuclear α decay has received attention [26–29]. Widely accepted as a quantum tunneling process [30], α decay is expected to be modified in the presence of a strong laser field through modifying the potential barrier, on which quantum tunneling depends very sensitively. Indeed, existing works all predict such modifications.

To what degree can an intense laser field, currently available or to be available in the forthcoming years, influence α decay? This quantitative question, however, remains unanswered. Mişicu and Rizea numerically solve a time-dependent Schrödinger equation using a one-dimensional (1D) model nuclear potential [27,28]. They focus on obtaining qualitative understandings instead of quantitative evaluations. Delion and Ghinescu [29] adopt a Kramers-Henneberger (KH) approximation [31,32] to describe the laser-nucleus interaction. However, as will be explained in detail in the following section, the KH approximation is not valid to describe the laser-nucleus interaction. This explains why unreasonable predictions are made in Ref. [29] that the laser field greatly suppresses (by orders of magnitude) α decay along the polarization direction, where the laser electric field is the strongest, and greatly enhances (by orders of magnitude) α decay along the perpendicular direction, where no laser electric field is present.

The goal of the current article is to quantitatively study the effect of intense laser fields on nuclear α decay. To achieve this goal we need to start with a realistic and quantitative α -nucleus potential. In this work we use the potentials proposed by Igo [33], which have a simple analytical form and can be applied to a variety of nuclei. These potentials were obtained by fitting to α -nucleus scattering data. Our numerical results show that α decay can indeed be modified by strong external laser fields, to some small but finite extent. For example, with a laser intensity of 10^{24} W/cm², which is expected to be achievable in the forthcoming years with ELI, a modification

*xwang@giscaep.ac.cn

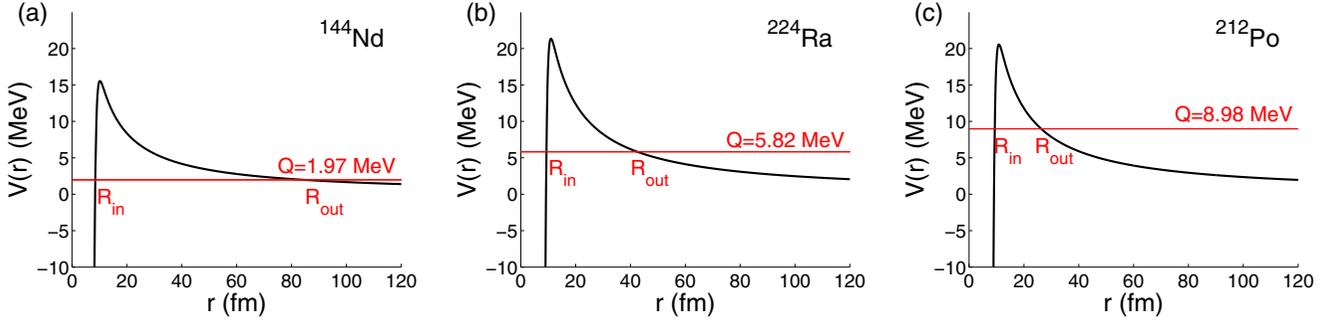


FIG. 1. Potentials between the α particle and the daughter nucleus, including the Igo α -nucleus potential and the Coulomb potential, for three nuclear elements (a) ^{144}Nd , (b) ^{224}Ra , and (c) ^{212}Po . The red horizontal line in each panel is the corresponding α decay energy Q , the intersections of which with the potential energy curve give the tunneling entrance point R_{in} and the tunneling exit point R_{out} .

of 0.1% to the α particle penetrability or the nuclear half-life is predicted. Besides, the α decay is modified along the laser polarization direction, as would be expected reasonably. A somewhat surprising result is that α decays with lower decay energies are relatively easier to be modified by external laser fields. This is due to longer tunneling paths under the potential barrier for the laser field to act on. We also explain that modifications to angle-resolved penetrability are easier to achieve than modifications to angle-integrated penetrability. The former modifications depend linearly on the laser electric field strength while the latter modifications depend quadratically on the laser electric field strength.

This article is organized as follows. In Sec. II we will introduce the methods that we use in our calculations. That includes the detailed form of the α -nucleus potentials, the form of the laser-nucleus interaction, and the method to calculate the α particle penetrability. Numerical results, analyses, and discussions are given in Sec. III. A conclusion is given in Sec. IV.

II. METHOD

A. α -nucleus potential

The potential energy felt by the α particle from the residue (daughter) nucleus can be written as

$$V(r) = V_N(r) + V_C(r), \quad (1)$$

where r is the distance between the α particle and the daughter nucleus, $V_N(r)$ is a short-range nuclear potential and $V_C(r) = 2Z/r$ is the Coulomb repulsive potential. Z is the charge of the daughter nucleus.

Igo proposed a quantitative yet simple α -nucleus potential [33] by fitting to α -nucleus scattering data

$$V_N(r) = -1100 \exp \left\{ - \left[\frac{r - 1.17A^{1/3}}{0.574} \right] \right\} \text{ MeV}, \quad (2)$$

where r is in units of fm ($1 \text{ fm} = 10^{-15} \text{ m}$) and A is the mass number of the daughter nucleus. The potential is given in MeV.

Figure 1 shows the potential $V(r)$ for three representative α -decay elements, namely, ^{144}Nd , ^{224}Ra , and ^{212}Po . The decay energy Q for the three elements are 1.97, 5.82, and

8.98 MeV, respectively. Typical α -decay energies range within 1 to 10 MeV, so the three elements chosen represent low-, medium-, and high-energy decays.

The decay energy Q is the sum of the kinetic energy of the α particle and the recoil energy of the daughter nucleus, and it can be obtained as

$$Q = \frac{A+4}{A} E_\alpha, \quad (3)$$

where E_α is the (detected) kinetic energy of the α particle, and A is the mass number of the daughter nucleus. Normally a very small electron-screening correction may be included due to energy loss of the α particle flying through the electron cloud of the atom. This small correction is not included in the current problem with the consideration that the electrons have been removed by the intense laser fields.

B. Laser-nucleus interaction

The interaction between the laser electric field and the nucleus is given in the length gauge as [27]

$$V_I(\vec{r}, t) = -Z_{\text{eff}} \vec{r} \cdot \vec{\varepsilon}(t) = -Z_{\text{eff}} r \varepsilon(t) \cos \theta, \quad (4)$$

where θ is the angle between \vec{r} and $\vec{\varepsilon}(t)$, and $Z_{\text{eff}} = (2A - 4Z)/(A + 4)$ is an effective charge. This effective charge indicates the tendency of the laser electric field separating the α particle and the daughter nucleus. One sees that if $Z/A = 1/2$, then $Z_{\text{eff}} = 0$. That is, if the daughter nucleus has the same charge-to-mass ratio as the α particle, then the daughter nucleus and the α particle will move in concert in the laser field and the laser electric field does not have an effect of separating the two. For the three nuclear elements shown in Fig. 1, $Z_{\text{eff}} = 0.33$ for ^{144}Nd , 0.43 for ^{224}Ra , and 0.42 for ^{212}Po .

The neglect of the magnetic part of the laser field is justified by the fact that the α particle moves much slower than the speed of light. Assuming a kinetic energy of 10 MeV, one gets an α particle speed of $2.2 \times 10^7 \text{ m/s}$, about 7% the speed of light. Therefore the effect of the magnetic field on the α particle is expected to be much smaller than that of the electric field.

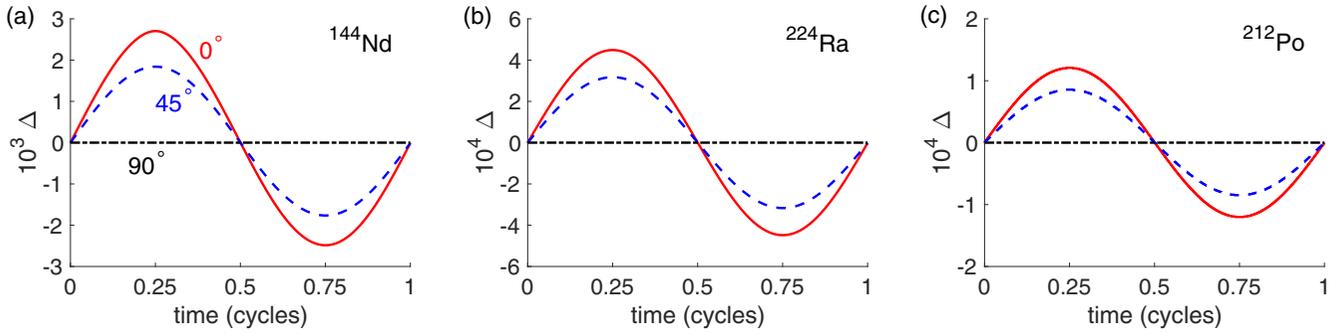


FIG. 2. Time-dependent modifications to the α particle penetrability, seen from three spatial angles, namely, $\theta = 0^\circ$ (red solid), 45° (blue dashed), and 90° (black dash dotted). The same three nuclear elements are used as in Fig. 1. The peak intensity used is 10^{24} W/cm² for all three elements.

C. Quasistatic approximation

The size of a typical nucleus is on the order of 1 fm. From a classical picture, the α particle oscillates back and forth within the nucleus. The frequency of this oscillation can be estimated to be $\sim 2 \times 10^7$ m s⁻¹ / 2 fm = 10^{22} Hz. Each time the α particle hits the potential wall, it has a small chance (which is called the penetrability) of tunneling out. If it does, we may estimate how much time the α particle needs to tunnel through the potential barrier. Referring to Fig. 1, the length of the potential barrier for the α particle to tunnel through is on the order of 10 fm. So the α particle needs about 10^{-21} s to travel through the potential barrier. This time is much smaller than an optical cycle of strong lasers. For the 800-nm near-infrared laser of ELI, one optical cycle is 2.6×10^{-15} s. For 10-keV x-ray lasers, one optical cycle is 4×10^{-19} s. Therefore during the time that the α particle penetrates through the potential barrier, the change of the laser field is negligible and the laser field can be viewed as static. This is the quasistatic approximation. In strong-field atomic physics, such approximation is routinely used in describing tunneling ionization of atoms [34–37].

It is obvious that the Kramers-Henneberger approximation [31,32] is not valid here. The KH approximation says that when the laser frequency is much higher than the particle oscillating frequency, the particle responds dominantly to the cycle-averaged laser field value (like our eyes' response to light). This high-frequency condition of validity for the KH approximation is well known in the literature [38,39]. Applying the KH approximation to the laser-assisted α decay process has led to unreasonable predictions by Delion and Ghinescu [29], as mentioned previously in the Introduction.

D. Penetrability of the α particle

Based on the quasistatic approximation, the penetrability of the α particle through the potential barrier can be calculated using the Wentzel-Kramers-Brillouin (WKB) method as

$$P(\theta, t) = \exp\left(-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu[V(r) - Q + V_I(r, \theta, t)]} dr\right), \quad (5)$$

where $V(r)$ and $V_I(r, \theta, t)$ are given in Eqs. (1) and (4), respectively. The laser polarization is assumed to be along the z axis and θ denotes the direction of α emission, with respect to the $+z$ axis. Understanding from the classical picture, the α particle oscillates back and forth inside the nucleus, and every time it hits the potential wall, it has a probability of $P(\theta, t)$ to tunnel out.

In this article we mainly look into the relative change of the penetrability induced by the laser field. The relative change of the penetrability is defined as

$$\Delta = \frac{P(\varepsilon) - P(\varepsilon = 0)}{P(\varepsilon = 0)}, \quad (6)$$

where ε is the laser field strength. Δ is also understood as a function of the emission angle θ and time t .

III. RESULTS AND DISCUSSIONS

A. Laser-induced modifications to the penetrability

First we show that the penetrability of the α particle can indeed be modified by strong external laser fields. Figure 2 shows time-dependent modifications to the penetrability seen from three spatial angles with respect to the $+z$ direction, namely, $\theta = 0^\circ$, 45° , and 90° . The same three nuclear elements are used as in Fig. 1. The modifications are strongest along $\theta = 0^\circ$, i.e., when the α emission direction is parallel to the laser polarization direction. No modifications are seen along $\theta = 90^\circ$, when the emission direction is perpendicular to the laser polarization direction.

A peak intensity of 10^{24} W/cm² is used for all the three nuclear elements. This intensity is expected to be achieved by ELI in the forthcoming years. One sees that modifications to the α penetrability are on the order of 0.1% for ¹⁴⁴Nd and 0.01% for ²²⁴Ra or ²¹²Po. The same amount of modifications are made to the nuclear half-lives.

It may seem unexpected at first that ¹⁴⁴Nd, with a lower decay energy than ²²⁴Ra and ²¹²Po, is relatively easier to be modified by external laser fields. This is a consequence of the tunneling mechanism. ¹⁴⁴Nd has a longer tunneling path for the laser field to act on, as shown in Fig. 1, and the potential from the laser electric field is proportional to this path length.

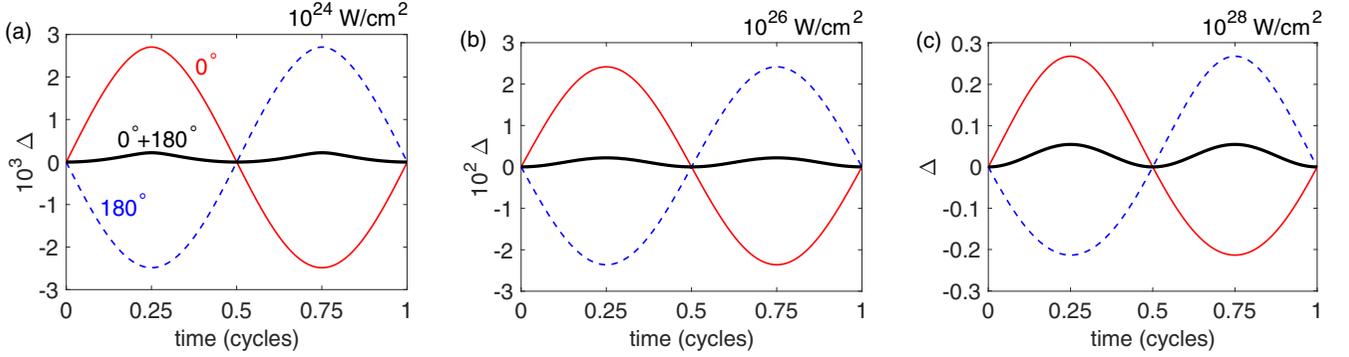


FIG. 3. Relative changes of the penetrability P seen from $\theta = 0^\circ$ and from $\theta = 180^\circ$, for three different laser intensities, namely, 10^{24} W/cm² (a), 10^{26} W/cm² (b), and 10^{28} W/cm² (c). The nuclear element used here is ^{144}Nd . Positive-negative asymmetry between 0° and 180° can be more clearly seen from the sum of the two, as shown by the thick black curve in each panel.

B. Laser potential as a perturbation to the α -nucleus potential

Compared to the potential energy between the α particle and the daughter nucleus, the laser potential has much smaller magnitudes, even with an intensity of 10^{24} W/cm². We can gain insights into the laser-modification process by treating the laser potential as a perturbation to the α -nucleus potential.

Let us start from the penetrability exponential given in Eq. (5) and write it in the following form:

$$P(\theta, t) = \exp \left\{ -\frac{2\sqrt{2\mu}}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{V_0 + V_I} dr \right\} \quad (7)$$

$$= \exp \left\{ -\frac{2\sqrt{2\mu}}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{V_0} \sqrt{1 + \frac{V_I}{V_0}} dr \right\}, \quad (8)$$

where for convenience $V_0(r) \equiv V(r) - Q$. By assuming $|V_I| \ll |V_0|$, we have the following Taylor expansion:

$$P(\theta, t) = \exp \left\{ -\frac{2\sqrt{2\mu}}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{V_0} \times \left(1 + \frac{V_I}{2V_0} - \frac{V_I^2}{8V_0^2} + \dots \right) dr \right\} \quad (9)$$

$$\approx \exp(\gamma^{(0)} + \gamma^{(1)} + \gamma^{(2)}) \quad (10)$$

$$= \exp(\gamma^{(0)}) \exp(\gamma^{(1)} + \gamma^{(2)}) \quad (11)$$

$$\approx P(\varepsilon = 0, \theta, t) (1 + \gamma^{(1)} + \gamma^{(2)}), \quad (12)$$

where $\gamma^{(0)}$, $\gamma^{(1)}$, and $\gamma^{(2)}$ are defined as

$$\gamma^{(0)} = -\frac{2\sqrt{2\mu}}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{V_0(r)} dr, \quad (13)$$

$$\gamma^{(1)} = \varepsilon(t) \frac{\sqrt{2\mu} Z_{\text{eff}} \cos \theta}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{r dr}{\sqrt{V_0(r)}}, \quad (14)$$

$$\gamma^{(2)} = \varepsilon^2(t) \frac{\sqrt{2\mu} Z_{\text{eff}}^2 \cos^2 \theta}{4\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{r^2 dr}{V_0^{3/2}(r)}. \quad (15)$$

Note that $\gamma^{(0)}$ is independent of the laser electric field, $\gamma^{(1)}$ is proportional to $\varepsilon(t)$, and $\gamma^{(2)}$ is proportional to $\varepsilon^2(t)$.

C. 0° versus 180°

When there is no laser field, α emission to the direction $\theta = 0^\circ$ is the same as that to $\theta = 180^\circ$. When the laser electric field is on and pointing to 0° , the penetrability to the same direction increases, but at the same time the penetrability to the opposite direction (180°) decreases.

For intensities with which $\gamma^{(2)}$ is negligible, the response of the penetrability to the laser electric field is linear. This means that the amount that P increases along 0° is equal to the amount that P decreases along 180° . The same argument can be made to other emission directions as well. Then there will be no net gain in the total α decay rate integrating over all emission directions. Only for higher intensities with which $\gamma^{(2)}$ is not negligible does the total α decay rate increase. This can be seen from Eq. (15) that $\gamma^{(2)}$ is always positive so both 0° and 180° contribute positively to the decay rate. Therefore modifying the angle-integrated total decay rate requires much higher laser intensities than modifying the angle-resolved decay rates. The former is a second-order process in laser field strength, whereas the latter is a first-order process in laser field strength.

Figure 3 shows the comparison between the modification to the α decay rate seen from 0° and from 180° , for three different intensities, namely, 10^{24} , 10^{26} , and 10^{28} W/cm². The nuclear element used is ^{144}Nd . One can see that for the lower two intensities, 0° and 180° look quite symmetric to the naked eye, although small asymmetries do exist, as can be seen from the sum of the two angles (the thick black curve in each panel). For the relatively high intensity shown in Fig. 3(c), obvious positive-negative asymmetry can be seen, due to appreciable $\gamma^{(2)}$ values with this intensity.

D. Angle-resolved versus angle-integrated modifications

Figure 4 shows the angle-resolved (red squares) and angle-integrated modifications (blue circles) to the α decay penetrability. The relative modification Δ and the laser intensity I are plotted in the logarithmic scale, therefore both curves are linear. The slope of the red curves is 0.5, due to a linear dependency on the laser electric field, while the slope of the blue curves is 1.0, due to a quadratic dependency on the laser electric field. Similar linear dependencies have also been

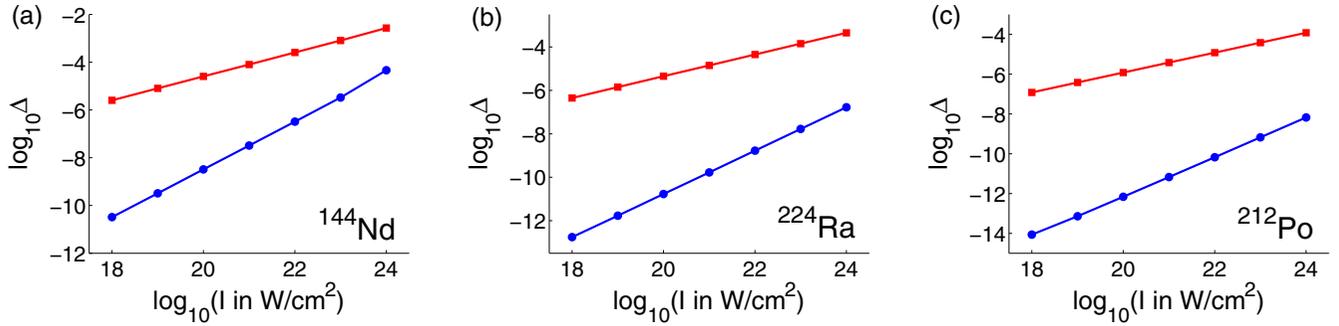


FIG. 4. Angle-resolved (red squares) and angle-integrated (blue circles) modifications to the α decay penetrability, as a function of laser intensity. Both Δ and the intensity I are plotted in the logarithmic scale, with which both curves are linear. The red (square) curves have slope 0.5, due to a linear dependency on the laser field strength. The blue (circle) curves have slope 1.0, due to a quadratic dependency on the laser field strength.

predicted in Ref. [26] with 1D nuclear models and various nuclear species.

As analyzed in the previous section, for the intensities used in the current article, as well as laser intensities available in the near future, the response of the α decay to the external laser field is dominantly linear. Observing from a particular spatial angle, the modification to the α penetrability depends (almost) linearly on the laser field strength. The linear response cancels if the angle-integrated modifications are considered, leaving the quadratic response dominating.

E. Possible experimental tests

In this section we propose an experimental scheme to test the predicted modifications to the α -decay penetrability by intense laser fields. We propose to use elliptically polarized laser fields to observe the laser-induced modifications. Elliptical polarization (EP) is able to “throw” α particles emitted at different times to different spatial angles, rendering the modifications free from time averaging. For if α particles emitted at two times arrive at the same position of the detector, and the increase to the α -decay penetrability at one time balances the decrease at the other time, then the time-integrated signal at the detector will not discriminate modifications of individual times. This is what would happen using linearly polarized laser fields. The problem can be cured using EP. This property of EP has been exploited in strong-field atomic physics to extract ionization information that is otherwise hidden with linear polarization [40–43].

An elliptically polarized laser field is illustrated in Fig. 5(a). The x - y plane is the polarization plane with the x direction the major direction, and the ellipticity value is 0.5. The electric field $E(t)$ rotates along the polarization ellipse. The effect of the elliptically polarized laser field on the α -emission process is twofold. First, the laser field adds an additional velocity (basically the vector potential) to the α particle, and the value of the additional velocity is determined by the time of emission. An example is illustrated in Fig. 5(b). Without the laser field, the α particle has equal probabilities emitting to all directions with the same velocity, forming a circle on the asymptotic velocity distribution. With the laser field, the circular velocity distribution will be shifted by an

amount determined by the time of emission. Second, the laser field modifies the probability emitting to different directions. The probability of emission to the right can be different from that to the left, for example, as illustrated in Fig. 5(b).

The above argument on a single time step applies to all time steps during the laser pulse. Different time steps lead to different velocity shifts and penetrability modifications. Integrating over a laser pulse (assuming an intensity of 10^{24} W/cm² and a 100-cycle, or 267-fs, trapezoidal pulse with a 2-cycle linear ramping on and a 2-cycle linear ramping off), the asymptotic velocity distribution is simulated as in Fig. 5(c). If there were no modifications to the α -decay penetrability, this velocity distribution would be slightly different,

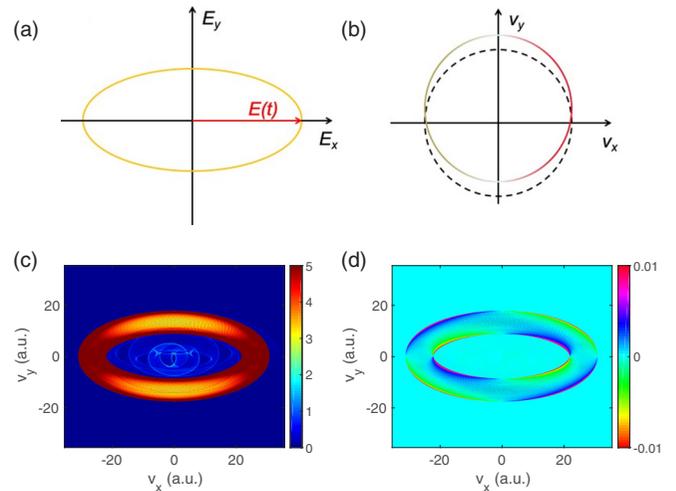


FIG. 5. (a) An elliptically polarized laser field $E(t)$ rotates along the polarization ellipse. (b) Asymptotic velocity distribution of the emitted α particle without (dashed circle) and with (solid grey circle) the laser field, the position of the circle will be shifted and the distribution over the circle will no longer be uniform. (c) Simulated α particle velocity distribution using a trapezoidal laser pulse. The velocities are in atomic units and the color scale is in arbitrary units. (d) Difference in the velocity distribution if there are no modifications to the penetrability. The color-scale units are the same as in (c).

and the difference is shown in Fig. 5(d). This difference, compared to the raw data, is roughly two orders of magnitude smaller in absolute values. The advantage of using EP can be seen: increase and decrease in the penetrabilities appear in different parts of the velocity distribution without overlapping. If a pattern similar to Fig. 5(d) could be identified by comparing an experimental distribution and a theoretical one assuming no modification to the penetrability, then the theory of the current article could be verified.

IV. CONCLUSION

We report in this article a combined theoretical and numerical study on the possible influences of strong laser fields on the nuclear α decay process. We use realistic and quantitative α -nucleus potentials and aim at obtaining quantitative evaluations of the laser influences.

We first show that the α penetrability (or equivalently the nuclear half-life) can indeed be modified by strong laser fields to some small but finite extent, with laser intensities expected to be achievable in the forthcoming years especially with the under-construction ELI facility. We also predict that α decays with lower decay energies are easier to be modified

than those with higher decay energies, due to longer tunneling paths for the laser electric field to act on. This is a somewhat counterintuitive result.

We point out that compared to the α -nucleus potential, the additional laser potential is weak, even with the highest laser intensities achievable in the near future. The response of α decay to the laser field is shown to be restricted to the lowest two orders (linear and quadratic). Angle-resolved α penetrability is shown to be a first-order process, depending linearly on the laser field strength. Whereas angle-integrated α penetrability is shown to be a second-order process, depending quadratically on the laser field strength, or linearly on the laser field intensity. Future experiments investigating laser-modified α decay processes should start with angle-resolved observables. An experimental scheme based on elliptically polarized laser fields is proposed.

ACKNOWLEDGMENTS

We acknowledge funding support from China Science Challenge Project No. TZ2018005, China NSF No. 11774323 and No. 11725417, National Key R&D Program No. 2017YFA0403200, and NSAF No. U1730449.

-
- [1] D. Strickland and G. Mourou, *Opt. Commun.* **56**, 219 (1985).
 - [2] P. Agostini, F. Fabre, G. Mainfray, G. Petite, and N. Rahman, *Phys. Rev. Lett.* **42**, 1127 (1979).
 - [3] G. G. Paulus, W. Nicklich, H. Xu, P. Lambropoulos, and H. Walther, *Phys. Rev. Lett.* **72**, 2851 (1994).
 - [4] A. McPherson, G. Gibson, H. Jara, U. Johann, T. S. Luk, I. A. McIntyre, K. Boyer, and C. K. Rhodes, *J. Opt. Soc. Am. B* **4**, 595 (1987).
 - [5] M. Ferray, A. L'Huillier, X. F. Li, L. A. Lompre, G. Mainfray, and C. Manus, *J. Phys. B* **21**, L31 (1988).
 - [6] B. Walker, B. Sheehy, L. F. DiMauro, P. Agostini, K. J. Schafer, and K. C. Kulander, *Phys. Rev. Lett.* **73**, 1227 (1994).
 - [7] S. Palaniyappan, A. DiChiara, E. Chowdhury, A. Falkowski, G. Ongadi, E. L. Huskins, and B. C. Walker, *Phys. Rev. Lett.* **94**, 243003 (2005).
 - [8] F. Krausz and M. Ivanov, *Rev. Mod. Phys.* **81**, 163 (2009).
 - [9] K. Zhao, Q. Zhang, M. Chini, Y. Wu, X. Wang, and Z. Chang, *Opt. Lett.* **37**, 3891 (2012).
 - [10] J. Li *et al.*, *Nat. Commun.* **8**, 186 (2017).
 - [11] T. Gaumnitz, A. Jain, Y. Pertot, M. Huppert, I. Jordan, F. Ardana-Lamas, and H. Jakob Wörner, *Opt. Exp.* **25**, 27506 (2017).
 - [12] P. H. Bucksbaum and N. Berrah, *Phys. Today* **68**, 26 (2015).
 - [13] D. Ursescu, O. Tesileanu, D. Balabanski, G. Cata-Danil, C. Ivan, I. Ursu, S. Gales, and N. V. Zamfir, *Proc. SPIE* **8780**, 878011 (2013).
 - [14] C. A. Ur *et al.*, *Nucl. Instr. Meth. B* **355**, 198 (2015).
 - [15] R. Röhlsberger, K. Schlage, B. Sahoo, S. Couet, and R. Ruffer, *Science* **328**, 1248 (2010).
 - [16] R. Röhlsberger, H. Wille, K. Schlage, and B. Sahoo, *Nature* **482**, 199 (2012).
 - [17] K. P. Heeg *et al.*, *Phys. Rev. Lett.* **111**, 073601 (2013).
 - [18] J. Haber, X. Kong, C. Strohm, S. Willing, J. Gollwitzer, L. Bocklage, R. Ruffer, A. Pálffy, and R. Röhlsberger, *Nat. Photon.* **11**, 720 (2017).
 - [19] F. Vagizov, V. Antonov, Y. V. Radeonychev, R. N. Shakhmuratov, and O. Kocharovskaya, *Nature* **508**, 80 (2014).
 - [20] K. P. Heeg *et al.*, *Phys. Rev. Lett.* **114**, 203601 (2015).
 - [21] A. Pálffy, C. H. Keitel, and J. Evers, *Phys. Rev. Lett.* **103**, 017401 (2009).
 - [22] W.-T. Liao, A. Pálffy, and C. H. Keitel, *Phys. Rev. Lett.* **109**, 197403 (2012).
 - [23] W.-T. Liao, A. Pálffy, and C. H. Keitel, *Phys. Lett. B* **705**, 134 (2011).
 - [24] W.-T. Liao, S. Das, C. H. Keitel, and A. Pálffy, *Phys. Rev. Lett.* **109**, 262502 (2012).
 - [25] W.-T. Liao, A. Pálffy, and C. H. Keitel, *Phys. Rev. C* **87**, 054609 (2013).
 - [26] H. M. Castañeda Cortés, C. Müller, C. H. Keitel, and A. Pálffy, *Phys. Lett. B* **723**, 401 (2013).
 - [27] Ş. Mişicu and M. Rizea, *J. Phys. G* **40**, 095101 (2013).
 - [28] Ş. Mişicu and M. Rizea, *Open Phys.* **14**, 81 (2016).
 - [29] D. S. Delion and S. A. Ghinescu, *Phys. Rev. Lett.* **119**, 202501 (2017).
 - [30] G. Gamow, *Z. Physik* **51**, 204 (1928).
 - [31] H. A. Kramers, *Collective Scientific Papers* (North-Holland, Amsterdam, 1956), p. 262.
 - [32] W. C. Henneberger, *Phys. Rev. Lett.* **21**, 838 (1968).
 - [33] G. Igo, *Phys. Rev. Lett.* **1**, 72 (1958).
 - [34] M. V. Ammosov, N. B. Delone, and V. P. Krainov, *Sov. Phys. JETP* **64**, 1191 (1986).
 - [35] T. Brabec, M. Y. Ivanov, and P. B. Corkum, *Phys. Rev. A* **54**, R2551 (1996).

- [36] J. Chen, J. Liu, L. B. Fu, and W. M. Zheng, *Phys. Rev. A* **63**, 011404 (2000).
- [37] G. L. Yudin and M. Y. Ivanov, *Phys. Rev. A* **63**, 033404 (2001).
- [38] M. Gavrilá and J. Z. Kaminski, *Phys. Rev. Lett.* **52**, 613 (1984).
- [39] M. Pont and M. Gavrilá, *Phys. Rev. Lett.* **65**, 2362 (1990).
- [40] P. Eckle, A. N. Pfeiffer, C. Cirelli, A. Staudte, R. Dörner, H. G. Müller, M. Büttiker, and U. Keller, *Science* **322**, 1525 (2008).
- [41] X. Wang and J. H. Eberly, *Phys. Rev. Lett.* **103**, 103007 (2009).
- [42] X. Wang and J. H. Eberly, *Phys. Rev. Lett.* **105**, 083001 (2010).
- [43] X. Wang, J. Tian, and J. H. Eberly, *Phys. Rev. Lett.* **110**, 073001 (2013).